Lattice QCD equation of state
(an update on QCD thermodynamics from HotQCD)

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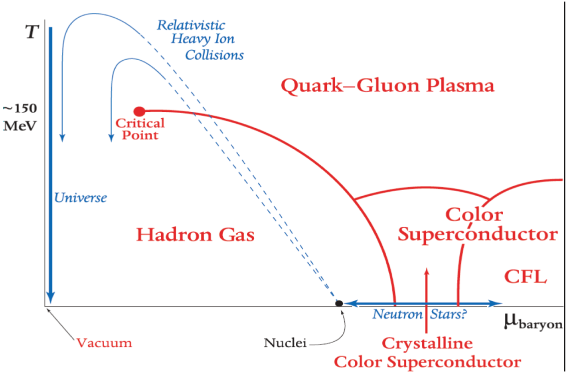
(Conjectured) QCD phase diagram



- Direct lattice simulations are possible only along the T axis.
- At non-zero chemical potential the action is complex and importance sampling fails.
- Focus on $\mu = 0$ in this talk.

Introduction: HotQCD thermodynamics program

Previous results:

- $N_\tau = 6, 8$, $m_l/m_s = 1/10$, asqtad, p4[1]
- $N_\tau = 8$, $m_l/m_s = 1/20$, p4[2]

New results:

- $N_\tau = 8, 12$, $m_l/m_s = 1/20$, asqtad and
  $N_\tau = 6, 8$, $m_l/m_s = 1/20$, HISQ[3]

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[1] Bazavov et al. [HotQCD], Phys. Rev. D 80, 014504 (2009)

[2] Cheng et al. [RBC-Bielefeld, Phys. Rev. D 81, 054504 (2010)

[3] Bazavov and Petreczky [HotQCD], J. Phys. Conf. Ser. 230, 012014 (2010); Söldner [HotQCD], PoS LAT2010, 215 (2010); HotQCD, work in progress.

Introduction

Physics:

- Deconfinement
- Chiral symmetry restoration
- QGP equation of state

Questions to address:

- Why to use yet another action?
- Do we have the cut-off effects under control?
- How lattice compares with the Hadron Resonance Gas model?

Lattice QCD

- An observable $\mathcal{O}$ in the path integral representation of QCD in the imaginary time (Euclidian) formalism:

$$\begin{aligned}\langle\mathcal{O}\rangle &= \frac{1}{Z}\int D\bar{\psi}D\psi DA\ \mathcal{O}\exp(-S),\\ Z &= \int D\bar{\psi}D\psi DA\exp(-S), \qquad S=\int d^4x\mathcal{L}_E,\end{aligned}$$

  where S is the action of the theory.
- Integrals may not be expanded (no small parameter), but may still be evaluated by other means.
- Lattice[4] – discrete Euclidian space-time, serves as a regulator (momenta are bound) and allows for stochastic evaluation of path integrals,
  - quarks live on sites and gluons on links as SU(3) matrices

$$U_{x,\mu}=\mathcal{P}\exp\left\{ig\int_x^{x+a\hat{\mu}}dy_\nu\, A_\nu(y)\right\}.$$

[4]Wilson, Phys. Rev. D 10, 2445 (1974)

Lattice QCD

- Lattice action
$$S = S_G + S_F, \qquad S_F = \sum_{x,y} \bar{\psi}_x M_{x,y} \psi_y$$
($M_{x,y}$ is the fermion matrix) preserves the gauge symmetry, but there is the infamous fermion doubling problem – 16 species of fermions in 4D.
- Staggered fermions[5] remove the 4-fold degeneracy, reduce 16 to 4 (call them tastes to distinguish from physical flavors), preserve a part of the chiral symmetry at non-zero lattice spacing.
- Rooting procedure[6] is used to further reduce the number of species.
- Irrelevant operators (that vanish in the continuum limit) can be added to the lattice action to remove leading discretization effects – the idea of improved actions[7].
- The p4[8], asqtad[9] and HISQ[10] actions have similar improvement at high temperatures and differ by the degree of improvement at low temperatures.

[5] Kogut and Susskind, Phys. Rev. D 11, 395 (1975)
[6] Sharpe, PoS LAT2006, 022 (2006), Creutz, PoS LAT2007, 007 (2007)
[7] Symanzik, Nucl. Phys. B 226, 187 (1983)
[8] Heller, Karsch and Sturm, Phys. Rev. D 60, 114502 (1999)
[9] Orginos and Toussaint, Phys. Rev. D 59, 014501 (1999)
[10] E. Follana et al., Phys. Rev. D 75, 054502 (2007)

Lattice QCD

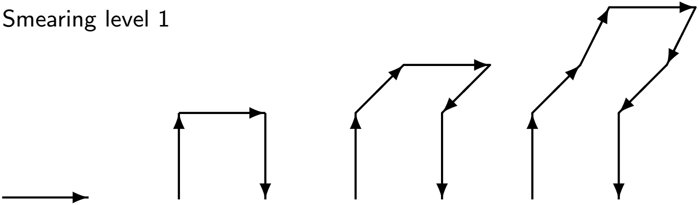
- Integrate fermionic degrees of freedom explicitly, then introduce bosonic fields to exponentiate the fermionic determinant:

$$
\begin{aligned}
Z &= \int \prod_{x,\mu} dU_{x,\mu} \left(\det M(U)\right)^{1/4} \exp\{-S_G\} \\
&= \int \prod_{x,\mu} dU_{x,\mu} \prod_{x} [d\Phi_x^\dagger d\Phi_x] \exp\{-S_G - \Phi^\dagger (M^\dagger M)^{-1/4} \Phi\}.
\end{aligned}
$$

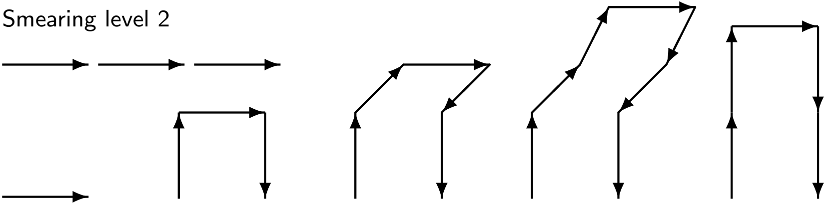
- If the weight is real this resembles canonical ensemble and we can use importance sampling techniques to estimate the integrals stochastically.
- Develop a Markov Chain Monte Carlo procedure to sample the phase space.
- Temperature is set by compactifying the temporal dimension, $T = 1/(N_\tau a)$, hold N_τ fixed and vary a.
- Lower temperatures – coarser lattices.
- Establish lines of constant physics (LCP), i.e. change bare quark masses with lattice spacing such that m_π, m_K are fixed.

HISQ action

Smearing level 1



Smearing level 2



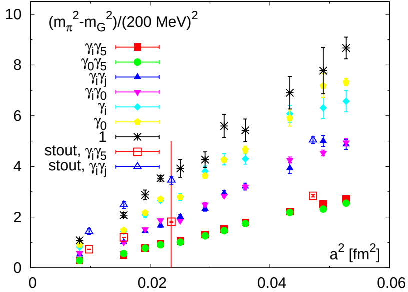
Taste symmetry

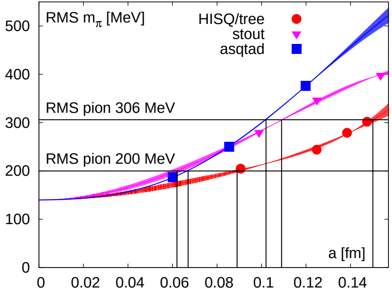
- Staggered fermion discretization describes a theory with four tastes. The rooting procedure (reducing four flavors to one by taking the fourth root of the fermion determinant) amounts to averaging between staggered tastes.
- Four tastes are not equivalent at non-zero lattice spacing because the taste symmetry is broken.
- As a result, only one of the pseudo-scalar mesons is massless in the chiral limit and the other 15 pseudo-scalar mesons have masses of order a^2.
- Violations of the taste symmetry have been identified as the dominant source of the cutoff effects at $O(a^2)$ in the asqtad and p4 actions. They lead to distortion of the hadron spectrum at non-zero lattice spacing.
- In thermodynamics calculations deviations from the physical hadron spectrum show up at low temperatures, where agreement with the Hadron Resonance Gas (HRG) model is expected.
- The cutoff effects can be reduced either by going to finer lattices (e.g., asqtad $N_t = 8$ to $N_t = 12$) or by using an action with higher degree of improvement (e.g. HISQ).

Taste symmetry

- Taste violations affect the pseudo-scalar meson sector most.
- The quadratic mass splitting of non-Goldstone mesons and the Goldstone meson is of order $\alpha^2 a^2$ (left panel).
- These splittings are to a good approximation mass independent.
- The root-mean-squared (RMS) pion mass for asqtad, stout and HISQ (right panel)[11]:

$$m_\pi^{RMS} = \sqrt{\frac{1}{16}\left(m^2_{\gamma_5} + m^2_{\gamma_0\gamma_5} + 3m^2_{\gamma_i\gamma_5} + 3m^2_{\gamma_i\gamma_j} + 3m^2_{\gamma_i\gamma_0} + 3m^2_{\gamma_i} + m^2_{\gamma_0} + m^2_1\right)}.$$

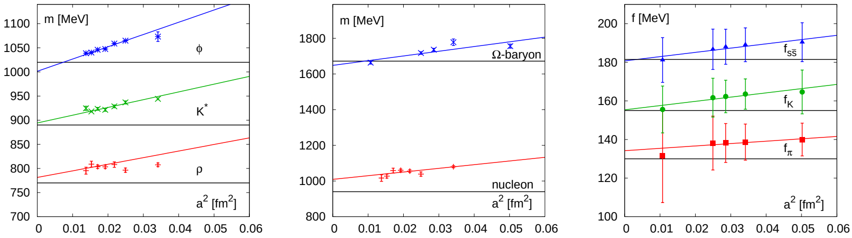




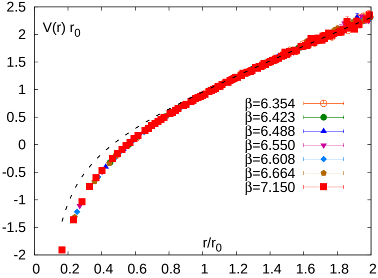
[11] Bazavov and Petreczky, PoS LAT2010, 169 (2010)

Taste symmetry and spectrum

- Effects of taste symmetry breaking are also seen in other channels, increasing masses of hadrons at non-zero lattice spacing comparing to their continuum values.
- Preliminary results for the masses of ρ, K^*, ϕ, N, Ω^- (left and central panels) and the decay constants f_π, f_K and $f_{s\bar{s}}$ (right panel).



Setting the lattice spacing



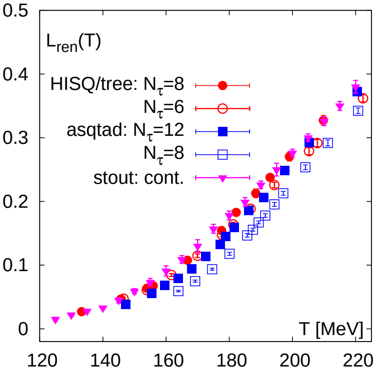
- The lattice spacing is determined from the static quark anti-quark potential, which does not show any noticeable cutoff dependence.
- Sommer scale[12]

$$\left(r^2 \frac{dV_{q\bar{q}}(r)}{dr} \right)_{r=r_n} = \begin{cases} 1.65\,, & n=0 \\ 1.0\,, & n=1 \end{cases}$$

$r_0 = 0.469$ fm or $r_1 = 0.318$ fm is used to convert to physical units.

[12] Sommer, Nucl. Phys. B 411, 839 (1994)

Deconfinement: renormalized Polyakov loop



- Related to the free energy of a static quark anti-quark pair at infinite separation:

$$L_{ren}(T) = \exp(-F_\infty(T)/(2T)).$$

- The renormalization constant

$$z(\beta) = \exp(-c(\beta)/2),$$

where $c(\beta)$ is the additive renormalization of the static potential.

$$L_{ren}(T) = z(\beta)^{N_\tau} L_{bare}(\beta), \qquad L_{bare}(\beta) = \left\langle \frac{1}{3} \mathrm{Tr} \prod_{x_0=0}^{N_\tau - 1} U_0(x_0, \vec{x}) \right\rangle.$$

- The increase of $L_{ren}(T)$ (and decrease of $F_\infty(T)$) is related to the onset of screening at higher temperatures.

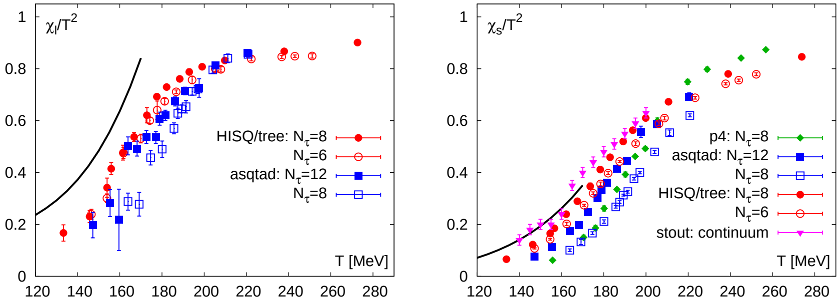
Fluctuations of conserved charges

- Fluctuations and correlations of conserved charges:

$$\begin{aligned}\frac{\chi_i(T)}{T^2} &= \frac{1}{T^3V}\frac{\partial^2 \ln Z(T,\mu_i)}{\partial(\mu_i/T)^2}\bigg|_{\mu_i=0},\\ \frac{\chi_{11}^{ij}(T)}{T^2} &= \frac{1}{T^3V}\frac{\partial^2 \ln Z(T,\mu_i,\mu_j)}{\partial(\mu_i/T)\partial(\mu_j/T)}\bigg|_{\mu_i=\mu_j=0}.\end{aligned}$$

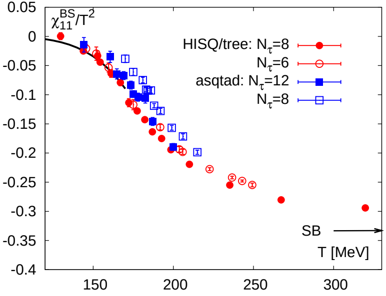
- Consider light and strange quark number susceptibility.
- At low temperatures they are carried by massive hadrons and their fluctuations are suppressed.
- At high temperatures they are carried by quarks and therefore can signal deconfiment.

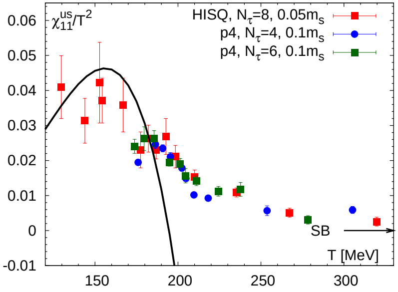
Fluctuations of conserved charges



- The light (left) and strange (right) quark number susceptibility, comparison with the hadron resonance gas (HRG) model (solid line).
- Quark number susceptibilities rapidly rise in the transition region and approach the ideal gas limit (up to the cut-off effects).
- The light quark number susceptibility is carried by the lightest states (pions) and therefore is more sensitive to the taste symmetry breaking effects resulting in poorer agreement with HRG.

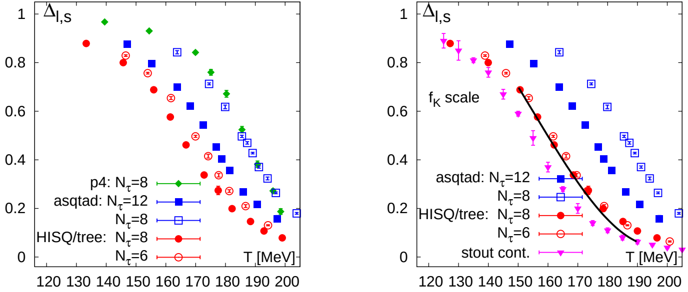
Correlations of conserved charges





- The strangeness-baryon number (left) and *u*- and *s*-quark number correlations (right).
- The *u*- and *s*-quark number correlations are compatible with HRG (but still too noisy to draw firm conclusions).

Chiral condensate

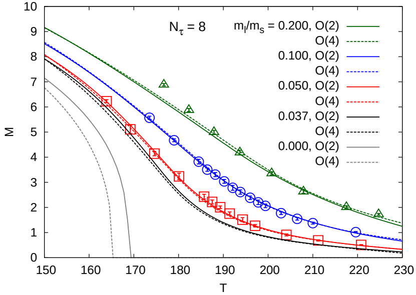


- The renormalized chiral condensate (left – r_0, right – f_K scale):

$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}, \qquad \langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m}.$$

- Improving the action reduces the lattice artifacts and shifts the transition region to lower temperatures (compare p4, asqtad and HISQ at fixed $N_\tau = 8$ in the left panel). Increasing N_τ gives the same effect.
- The solid line on the right panel represents the HISQ continuum limit taken from the data with r_0 scale.

Chiral condensate (scaling)



- The chiral condensate is the order parameter in the chiral limit.
- The multiplicatively renormalized chiral condensate for the asqtad action:

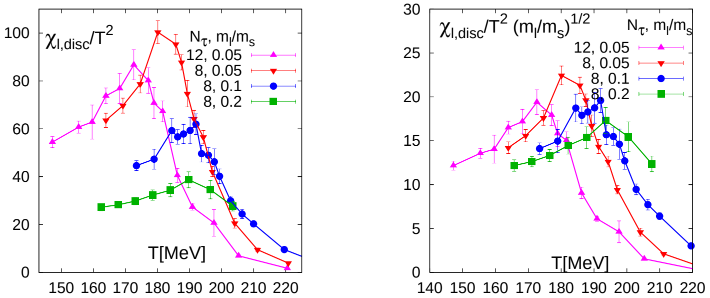
$$M_b = \frac{m_s}{T^4}\langle\bar{\psi}\psi\rangle_l$$

- At sufficiently low mass the chiral condensate is described by a universal scaling function f_G plus additional scaling violating terms:

$$M_b(T, m_l, m_s) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + a_t \Delta T H + b_1 H, \quad H = \frac{m_l}{m_s}, \quad \Delta T = \frac{T - T_c}{T_c},$$

$$h = H/h_0,\ t = \Delta T/t_0.$$

Chiral susceptibility



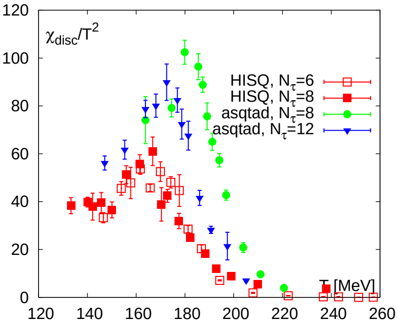
- The chiral susceptibility ($M_l = D + 2m_l$ is the staggered fermion matrix for light quarks):

$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle_l}{\partial m_l} = \frac{T}{V}\left(\langle(\mathrm{Tr}M_l^{-1})^2\rangle - \langle\mathrm{Tr}M_l^{-1}\rangle^2 - 2\langle\mathrm{Tr}M_l^{-2}\rangle\right).$$

- Disconnected chiral susceptibility for the Asqtad action at different quark masses[13] (left).
- Expected behavior $1/\sqrt{m_l}$ in the low-temperature phase (right).

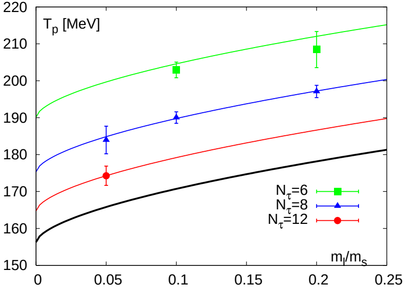
[13] Söldner, PoS LAT2010, 215 (2010)

Chiral susceptibility



- Disconnected chiral susceptibility, comparison of the asqtad and HISQ data.
- Peak locations agree between Asqtad $N_\tau = 12$ and HISQ $N_\tau = 8$.
- (Similar conclusion as from studying the pion splittings: HISQ at lattice spacing a is comparable to asqtad at $2/3a$.)

T_c in the physical mass limit



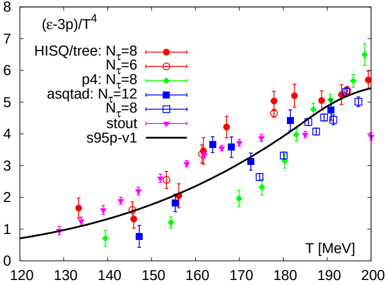
- Determine the peak location in the disconnected chiral susceptibility for Asqtad at different m_l and N_τ.
- Fit with the O(N) scaling inspired ansatz:

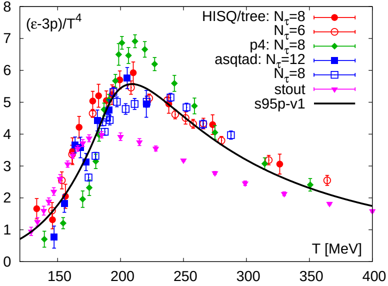
$$T_p(m_l, N_\tau) = T_c + b\left(\frac{m_l}{m_s}\right)^d + c\,\frac{1}{N_\tau^2}, \qquad d = \frac{1}{\beta\delta} \approx 0.54.$$

- At the physical mass $m_l/m_s \simeq 1/27$ the preliminary continuum estimate for the pseudocritical temperature is

$$T_p = (164 \pm 6)\,\text{MeV}.$$

Trace anomaly





- The trace anomaly at $m_l/m_s = 0.05$ for p4, asqtad and HISQ.
- Pressure and other thermodynamic quantities can be derived from the trace anomaly.
- At low temperatures HISQ results agree with stout[14] (left).
- At high temperatures p4, Asqtad and HISQ agree (as expected), but substantial disagreement with stout is observed (right).
- The solid curve is a parametrization based on the HRG model and lattice data[15].

[14] Borsanyi et al., JHEP11 (2010) 077

[15] Huovinen and Petreczky, Nucl. Phys. A 837, 26 (2010)

Conclusions

- Taste symmetry breaking effects are identified as the largest source of the cut-off effects in the low-temperature region for p4 and asqtad staggered actions.
- Thus, higher degree of improvement (e.g. the HISQ action) substantially reduces cut-off effects in many thermodynamic quantities at lattice spacings comparable to previously used.
- To control the systematic errors we compare different staggered actions.
- HISQ results are comparable to asqtad results at finer lattices.
- Using our asqtad data at $m_l/m_s = 1/5, 1/10, 1/20$ and $N_\tau = 6$, 8, 12 we perform continuum and chiral extrapolations and determine the pseudo-transition temperature in the limit of the physical light quark mass. HotQCD preliminary result for T_p, defined as the location of the peak in the disconnected chiral susceptibility is $T_p = 164 \pm 6$ MeV[16].
- For fluctuations and correlations of conserved charges improved agreement with the Hadron Resonance Gas model is observed for the HISQ action.
- HISQ $N_\tau = 8$ and asqtad $N_\tau = 12$ results for the trace anomaly agree with the stout data in the low-temperature region.
- In 250-350 MeV range p4, asqtad and HISQ data agree, but disagree with stout.

[16] HotQCD, work in progress