Recent results on QCD thermodynamics:

Lattice QCD

versus

Hadron Resonance Gas model

Claudia Ratti

Torino University and INFN, Italy

S.Borsanyi, Z.Fodor, C.Hölbling, S.Katz, S.Krieg, C.R. and K.Szabó, JHEP1009 (2010)

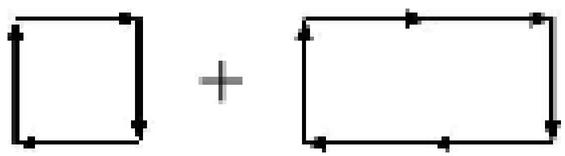
S.Borsanyi,G.Endrodi,Z.Fodor,A.Jakovac, S.Katz, S.Krieg, C.R. and K.Szabó JHEP1011 (2010)

Choice of the action

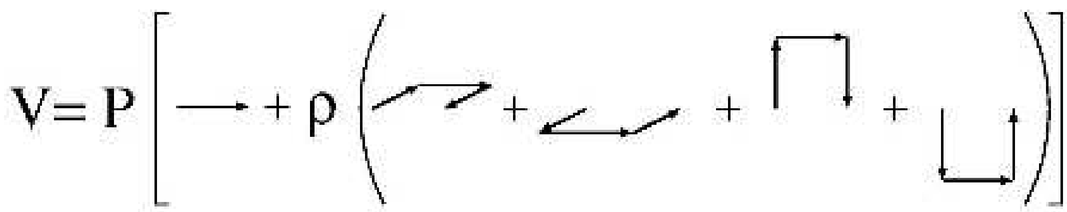
- no consensus: which action offers the most cost effective approach

  Aoki, Fodor, Katz, Szabo, JHEP 0601, 089 (2006)

- our choice tree-level $O(a^2)$-improved Symanzik gauge action



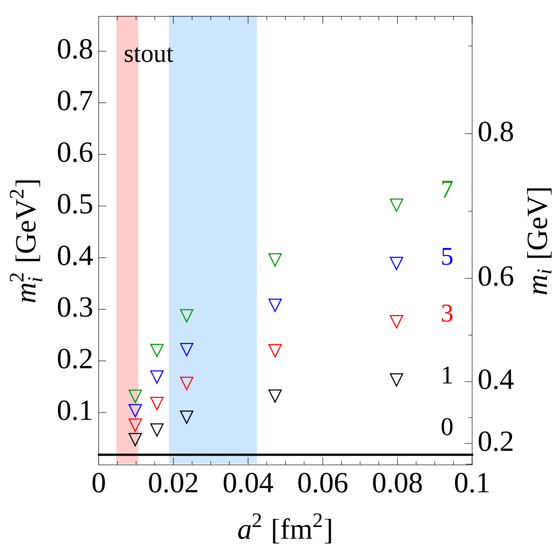
2-level (stout) smeared improved staggered fermions

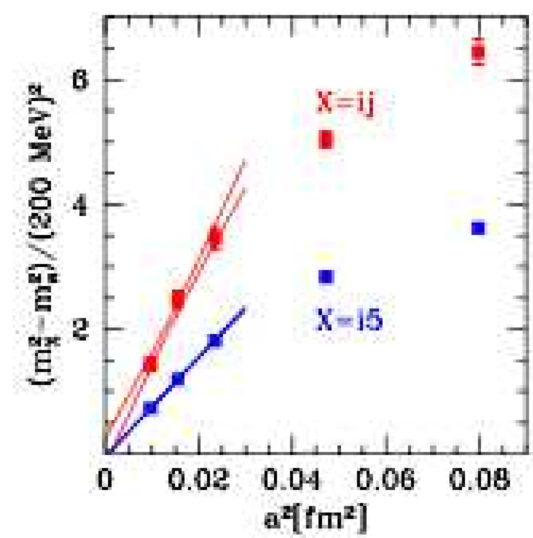


best known way to improve on taste symmetry violation

Pseudo-scalar mesons in staggered formulation

- Staggered formulation: four degenerate quark flavors (‘tastes’) in the continuum limit
- Rooting procedure: replace fermion determinant in the partition function by its fourth root
- At finite lattice spacing the four tastes are not degenerate
  - each pion is split into 16
  - the sixteen pseudo-scalar mesons have unequal masses
  - only one of them has vanishing mass in the chiral limit





- Scaling starts for $N_t \geq 8$.

Partition function of HRG model

- The pressure can be written as

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a}) \ ,$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{Vd_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2+m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left((\sum_a X^a_i \mu_{X^a})/T \right) \ .$$

X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

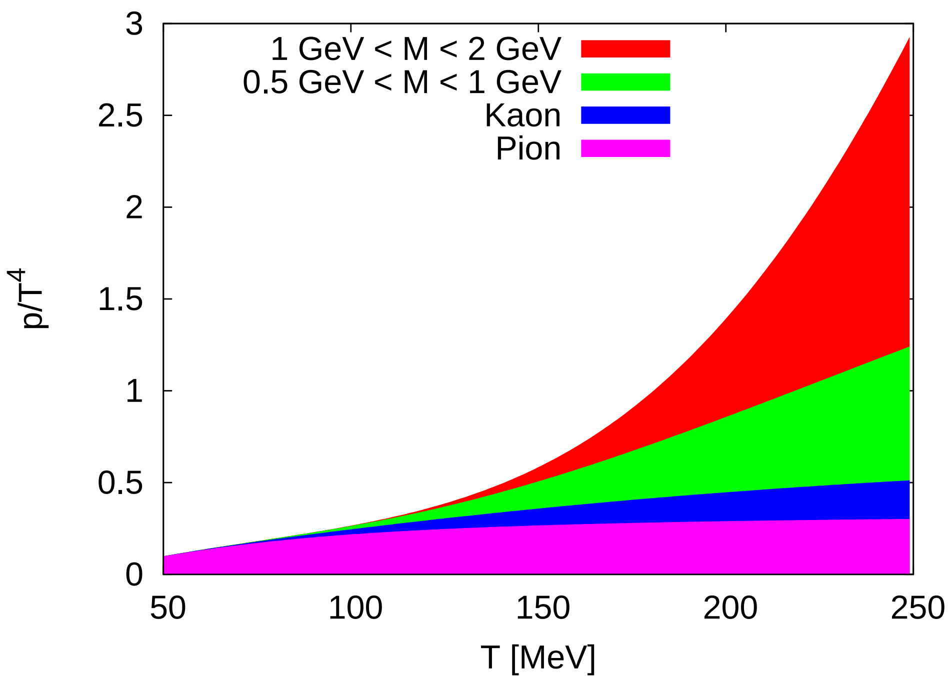
F. Karsch, A. Tawfik, K. Redlich; S. Ejiri, F. Karsch, K. Redlich

Hadronic species from the Particle Data Book

hadron	m_i (GeV)	d_i	B_i	S_i	I_i	hadron	m_i (GeV)	d_i	B_i	S_i	I_i
π	0.140	3	0	0	1	N (1535)	1.530	4	1	0	1/2
K	0.496	2	0	1	1/2	π_1 (1600)	1.596	9	0	0	1
$\overline{K}$	0.496	2	0	-1	1/2	Δ (1600)	1.600	16	1	0	3/2
η	0.543	1	0	0	0	Λ (1600)	1.600	2	1	-1	0
ρ	0.776	9	0	0	1	Δ (1620)	1.630	8	1	0	3/2
ω	0.782	3	0	0	0	η_2 (1645)	1.617	5	0	0	0
K^*	0.892	6	0	1	1/2	N (1650)	1.655	4	1	0	1/2
$\overline{K}^*$	0.892	6	0	-1	1/2	ω (1650)	1.670	3	0	0	0
N	0.939	4	1	0	1/2	Σ (1660)	1.660	6	1	-1	1
η'	0.958	1	0	0	0	Λ (1670)	1.670	2	1	-1	0
f_0	0.980	1	0	0	0	Σ (1670)	1.670	2	1	-1	1
a_0	0.980	3	0	0	1	ω_3 (1670)	1.667	7	0	0	0
ϕ	1.020	3	0	0	0	π_2 (1670)	1.672	15	0	0	1
Λ	1.116	2	1	-1	0	Ω^-	1.672	4	1	-3	0
h_1	1.170	3	0	0	1	N (1675)	1.675	12	1	0	1/2
Σ	1.189	6	1	-1	1	ϕ (1680)	1.680	3	0	0	0
a_1	1.230	9	0	0	1	K^* (1680)	1.717	6	0	1	1/2
b_1	1.230	9	0	0	1	$\overline{K}^*$ (1680)	1.717	6	0	-1	1/2
Δ	1.232	16	1	0	3/2	N (1680)	1.685	12	1	0	1/2
f_2	1.270	5	0	0	0	ρ_3 (1690)	1.688	21	0	0	1
K_1	1.273	6	0	1	1/2	Λ (1690)	1.690	4	1	-1	0
$\overline{K}_1$	1.273	6	0	-1	1/2	Ξ (1690)	1.690	8	1	-2	1/2
f_1	1.285	3	0	0	1	ρ (1700)	1.720	9	0	0	1
η (1295)	1.295	1	0	0	0	N (1700)	1.700	8	1	0	1/2
π (1300)	1.300	3	0	0	1	$\Delta(1700)$	1.700	16	1	0	3/2

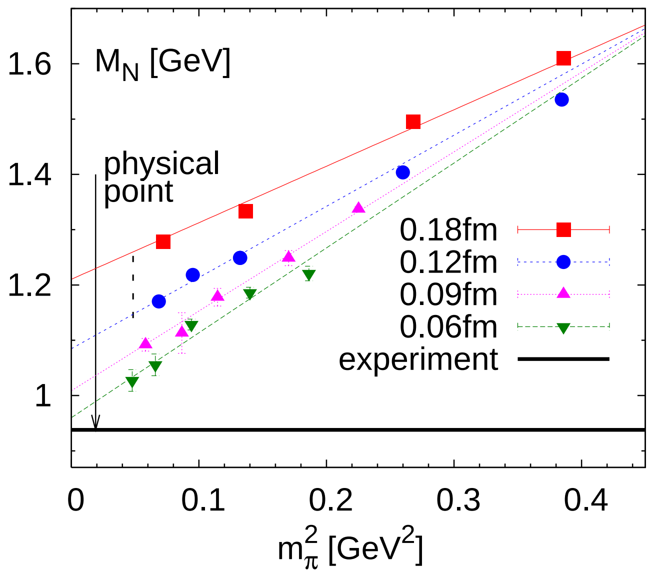
How many resonances do we include?

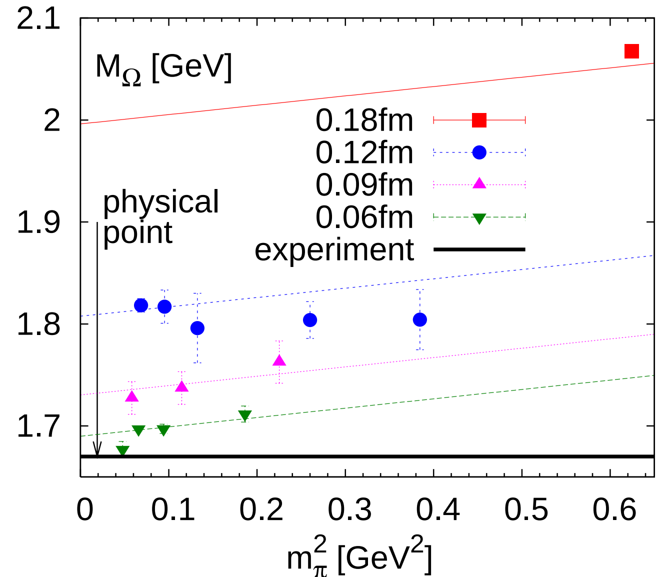
- With different mass cutoffs we can separate the contributions of different particles

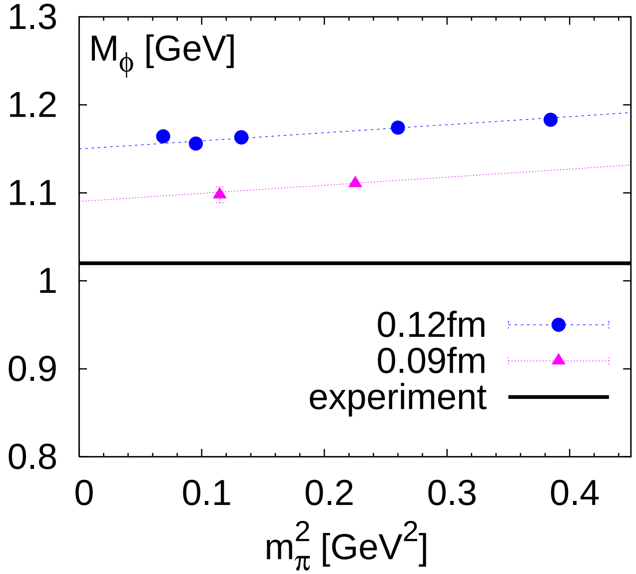


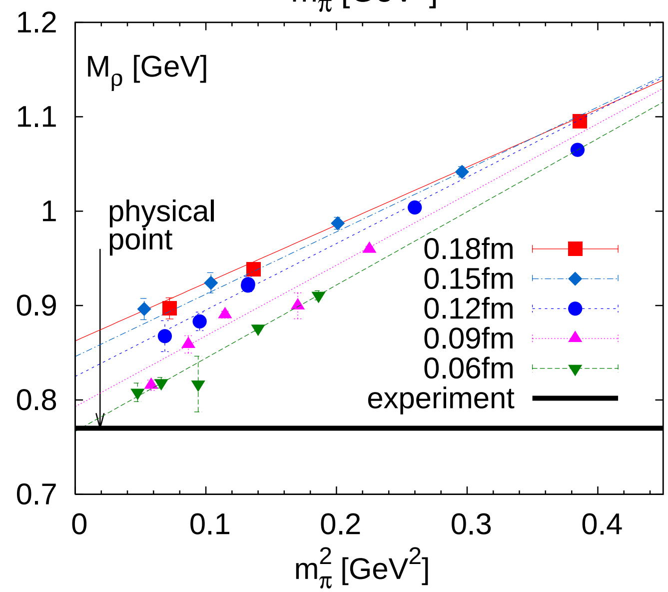
- No visible difference between cuts at 2 GeV and 2.5 GeV in our temperature regime
- We include all resonances with $M \leq$ 2.5 GeV
  - ➡ $\simeq 170$ different masses $\leftrightarrow$ 1500 resonances

Discretization effects









C. W. Bernard *et al.*, PRD (2001), C. Aubin *et al.*, PRD (2004), A. Bazavov *et al.*, 0903.3598.

Hadron masses

- Non-strange baryons and mesons:

$$r_1 m = r_1 m_0 + \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x}, \qquad x = (\frac{a}{r_1})^2$$

- Strange baryons and mesons:

$$r_1 \cdot m_\Lambda(a, m_\pi) = r_1 m_\Lambda^{phys} + \frac{2}{3} \frac{a_1 (r_1 m_\pi)^2}{1+a_2 x} + \frac{b_1 x}{1+b_2 x} + \frac{r_1 \cdot (m_\Lambda^{phys} - m_p^{phys})}{1+a_2 x} \left(\frac{m_s}{m_s^{phys}} \right),$$

$$r_1 \cdot m_\Sigma(a, m_\pi) = r_1 m_\Sigma^{phys} + \frac{1}{3} \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Sigma^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right),$$

$$r_1 \cdot m_\Xi(a, m_\pi) = m_\Xi^{phys} + \frac{1}{3} \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Xi^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right)$$

$$r_1 m_\Omega(a, m_\pi) = r_1 m_\Omega^{phys} + a_1 (r_1 m_\pi)^2 - a_1 (r_1 m_\pi^{phys})^2 + b_1 x + (m_\Omega^{phys} - m_\Delta^{phys}) \cdot 1.02 x$$

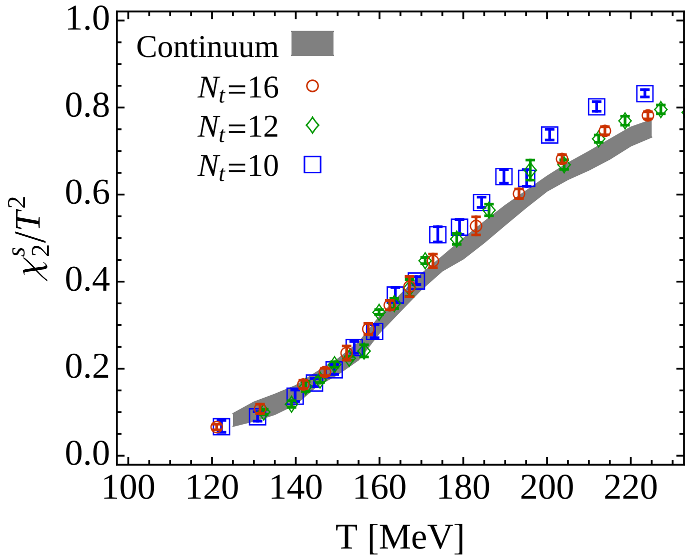
- Distorted spectrum implemented in the HRG model

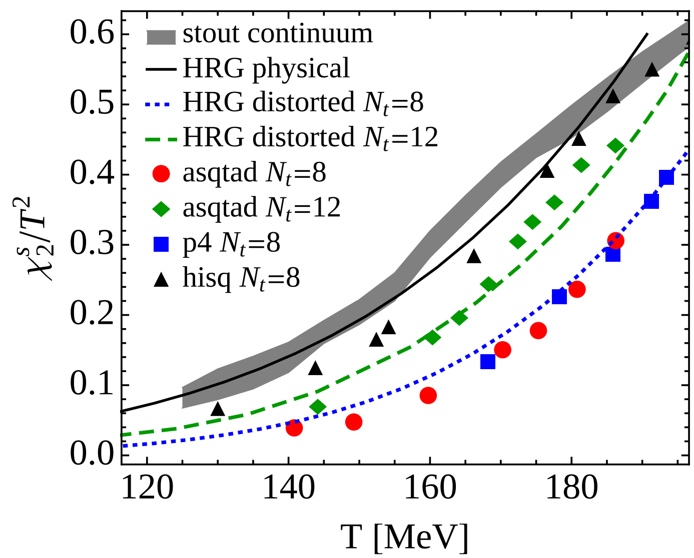
- Assumption: all resonances behave as their fundamental states

P. Huovinen and P. Petreczky (2009).

Results: strangeness susceptibilities

$$\chi_n^S = T^n \frac{\partial^n p(T,\mu_B,\mu_S,\mu_I)}{\partial \mu_S^n}\Big|_{\mu_X=0}$$



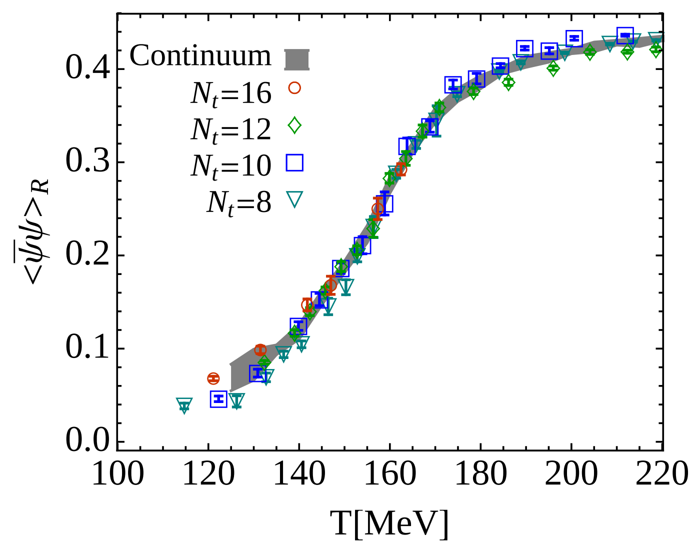


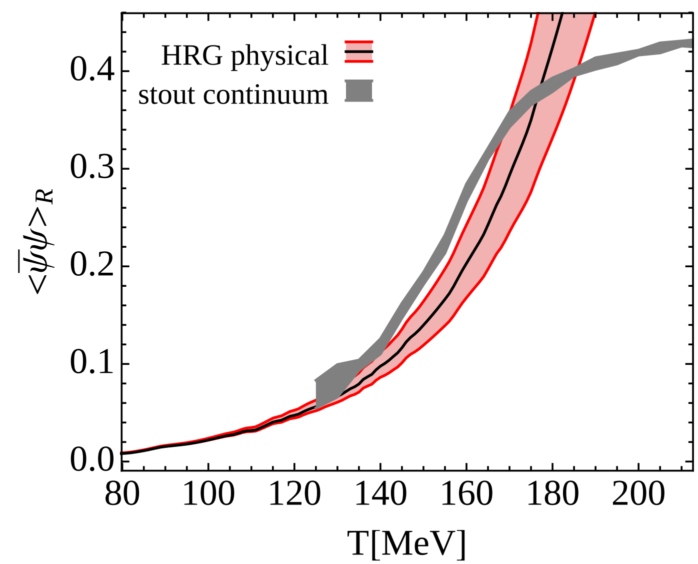
- HRG results in good agreement with stout action
- asqtad and p4 results show similar shape but shift in temperature
  - ➡ HRG results with corresponding distorted spectrum reproduce asqtad and p4 results

S. Borsanyi *et al.*, JHEP1009 (2010)

Results: chiral condensate

$$\langle\bar{\psi}\psi\rangle_R = -\left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right]\frac{m_l}{X^4} \quad \text{with} \quad \langle\bar{\psi}\psi\rangle_l = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$





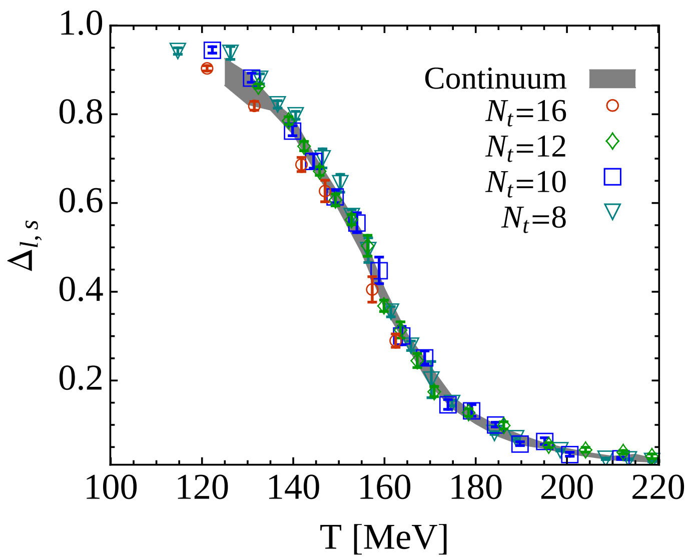
$$\langle\bar{\psi}\psi\rangle_l = \langle\bar{\psi}\psi\rangle_{l,0} + \langle\bar{\psi}\psi\rangle_\pi + \sum_{i\in mesons}\frac{\partial \ln Z^M_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial m_l} + \sum_{i\in baryons}\frac{\partial \ln Z^B_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial m_l}.$$

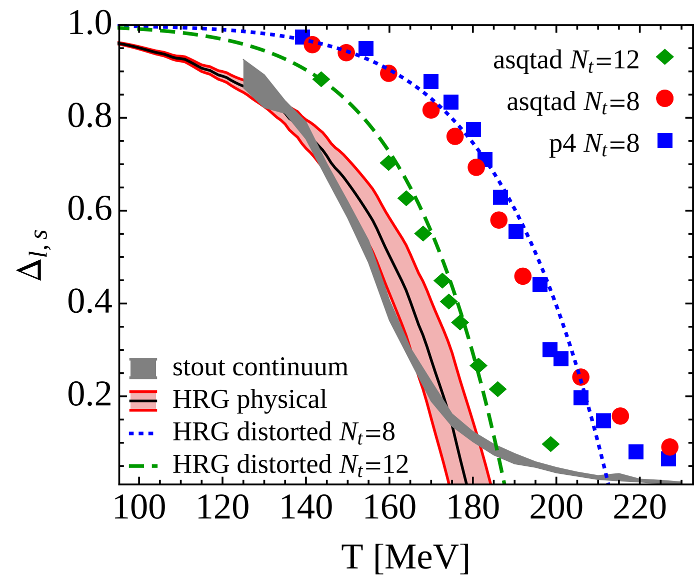
- Contribution of pions from Chiral Perturbation Theory Gerber and Leutwyler (1989)
- $\frac{\partial m_i}{\partial m_\pi^2}$ from fit to lattice data Camalich, Geng and Vacas (2010)

S. Borsanyi *et al.*, JHEP1009 (2010)

Results: subtracted chiral condensate

$$\Delta_{l,s} = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$





$$\langle\bar{\psi}\psi\rangle_s = \langle\bar{\psi}\psi\rangle_{s,0} + \langle\bar{\psi}\psi\rangle_K + \sum_{i\in mesons}\frac{\partial \ln Z^M_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_s} + \sum_{i\in baryons}\frac{\partial \ln Z^B_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_s}.$$

- $\frac{\partial m_i}{\partial m_s}$ from fit to lattice data Camalich, Geng and Vacas (2010)

S. Borsanyi *et al.*, JHEP1009 (2010)

Equation of state: integral method

- On the lattice, the dimensionless pressure is given by:

$$p^{lat}(\beta, m_q) = (N_t N_s^3)^{-1} \log Z(\beta, m_q)$$

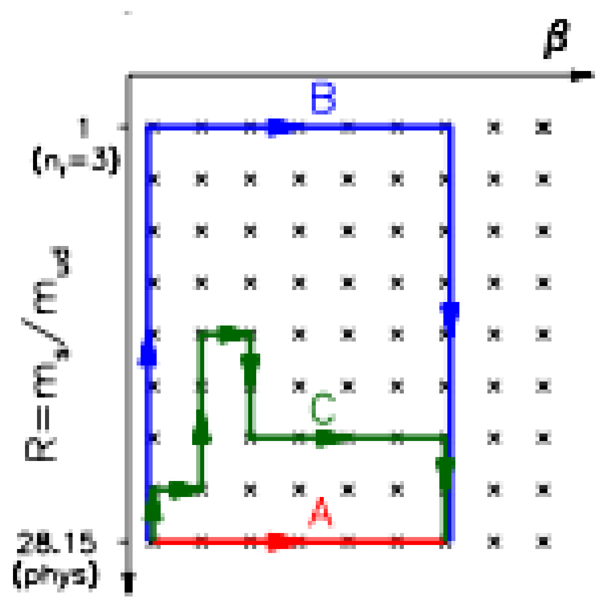
- only its derivatives are accessible using conventional algorithms:

$$p^{lat}(\beta, m_q) - p^{lat}(\beta^0, m_q^0) = (N_t N_s^3)^{-1} \int_{(\beta^0, m_q^0)}^{(\beta, m_q)} \left[d\beta \frac{\partial \log Z}{\partial \beta} + dm_q \frac{\partial \log Z}{\partial m_q} \right]$$

- the pressure has to be renormalized: subtraction at $T = 0$ (or $T > 0$)
- $T \neq 0$ simulations cannot go below $T \simeq 100$ MeV (lattice spacing is large)
- physical HRG gives here 5% contribution of SB
  - path of $M_\pi = 720$ MeV
  - distorted HRG no contribution at $T = 100$ MeV

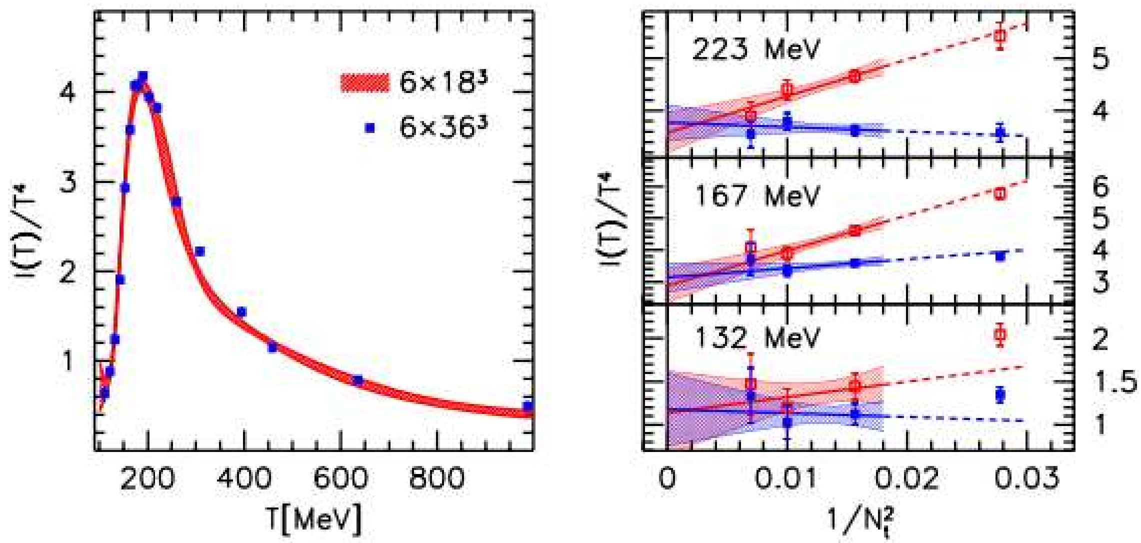
All path approach

- Our goal:
  - determine the equation of state for several pion masses
  - reduce the uncertainty related to the choice of β^0



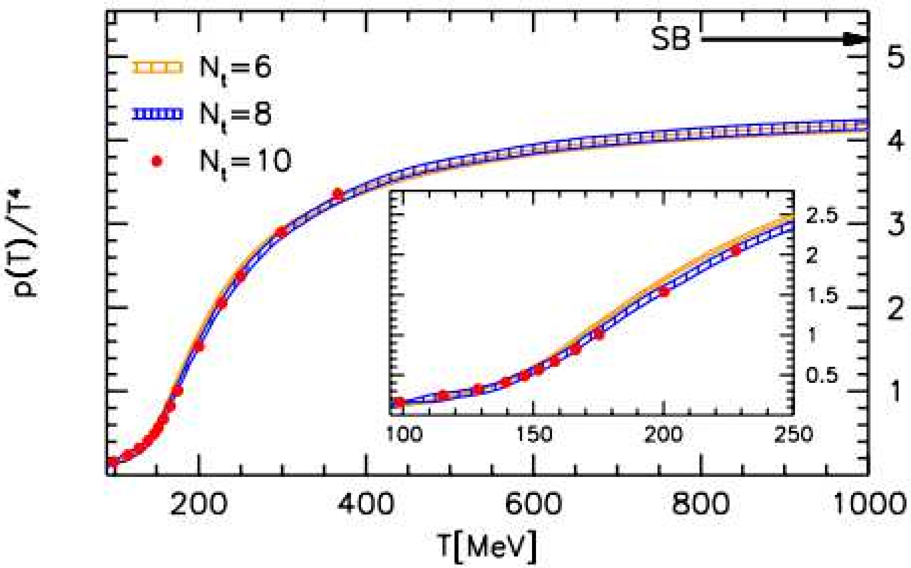
- conventional path: A, though B, C or any other paths are possible
- generalize: take all paths into account

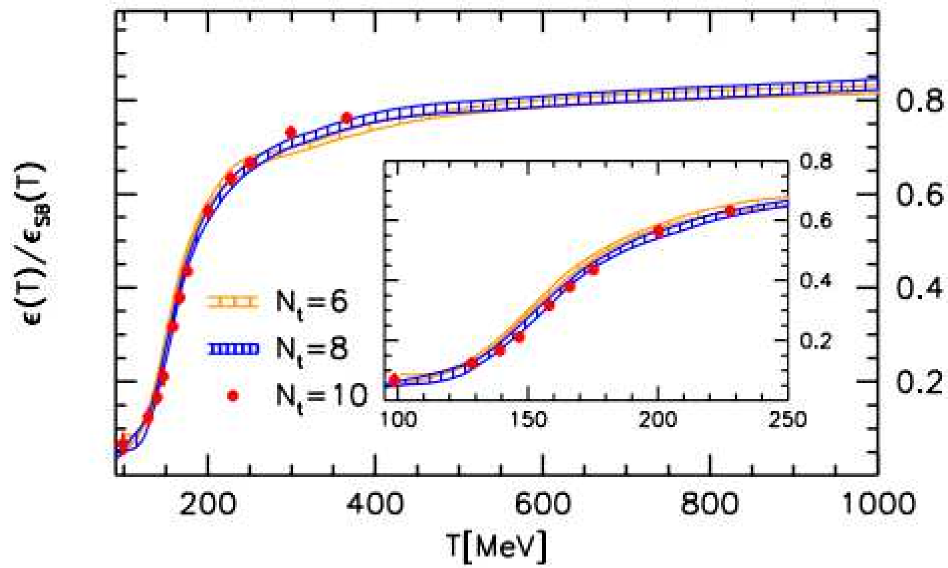
Finite volume and discretization effects



- finite V : $N_s/N_t = 3$ and 6 (8 times larger volume): no sizable difference
- finite a: improvement program of lattice QCD (action observables)
  - tree-level improvement for p (thermodynamic relations fix the others)
  - trace anomaly for three T-s: high T, transition T, low T
  - continuum limit $N_t = 6, 8, 10, 12$: same with or without improvement
- improvement strongly reduces cutoff effects: slope$\simeq 0$ ($1 - 2\sigma$ level)

Results: pressure and energy density

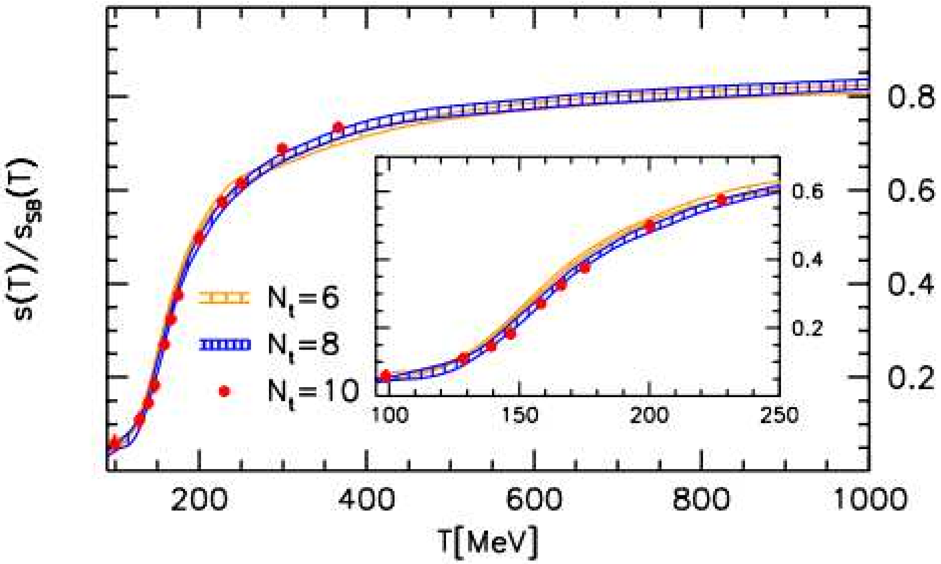


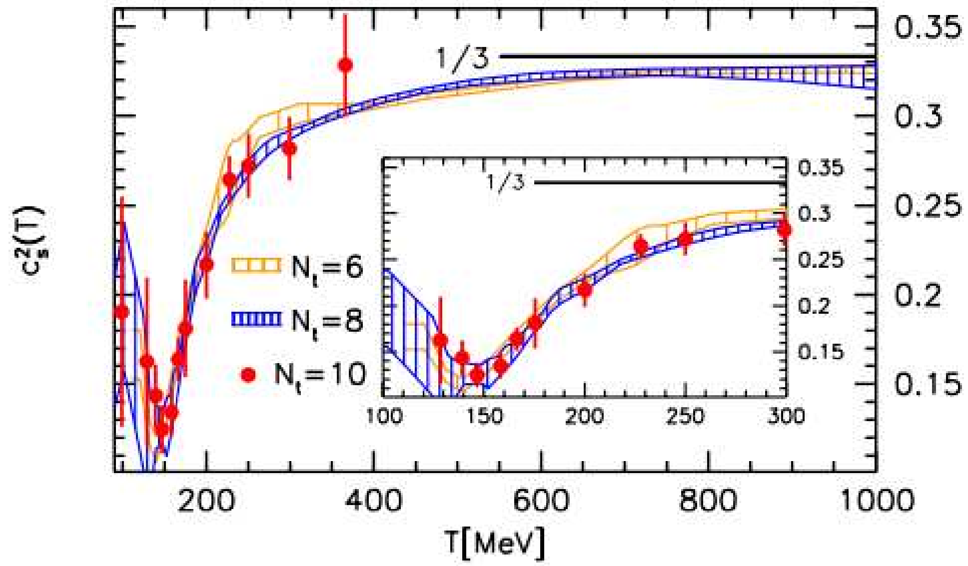


- The different N_t data are on top of each other
- The energy density is rescaled by the SB limit $\epsilon_{SB}/T^4 = 15.7$
- At T$\simeq 1000$ MeV these quantities reach $\sim$ 80% of the SB limit

Results: entropy and speed of sound

$$c_s^2 = \frac{dp}{d\epsilon}$$



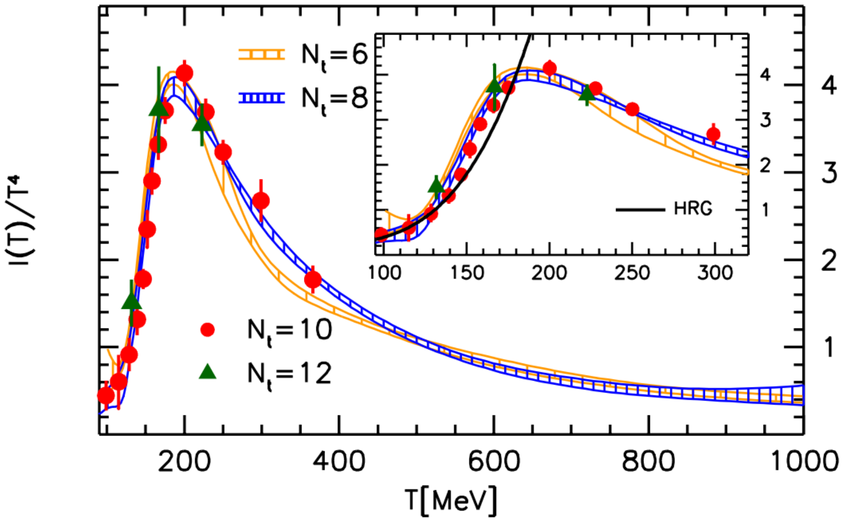


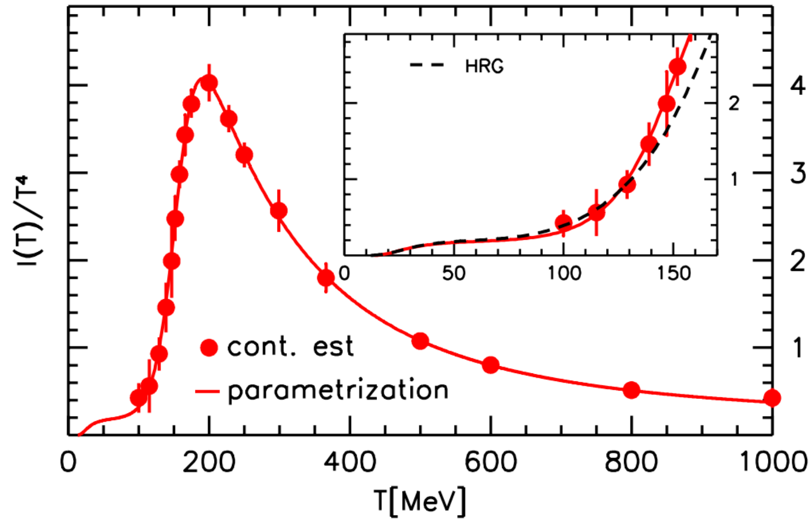
- The different N_t data are on top of each other
- The entropy is rescaled by the SB limit
- c_s^2 minimum value is about 0.13 at $T \simeq 145$ MeV

S. Borsanyi *et al.*, JHEP1011 (2010)

Results: trace anomaly and parametrization

$$\frac{\theta(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$



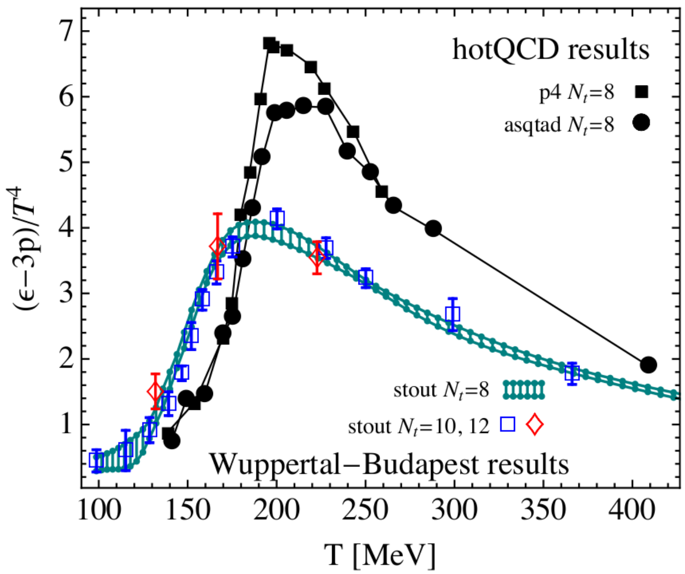


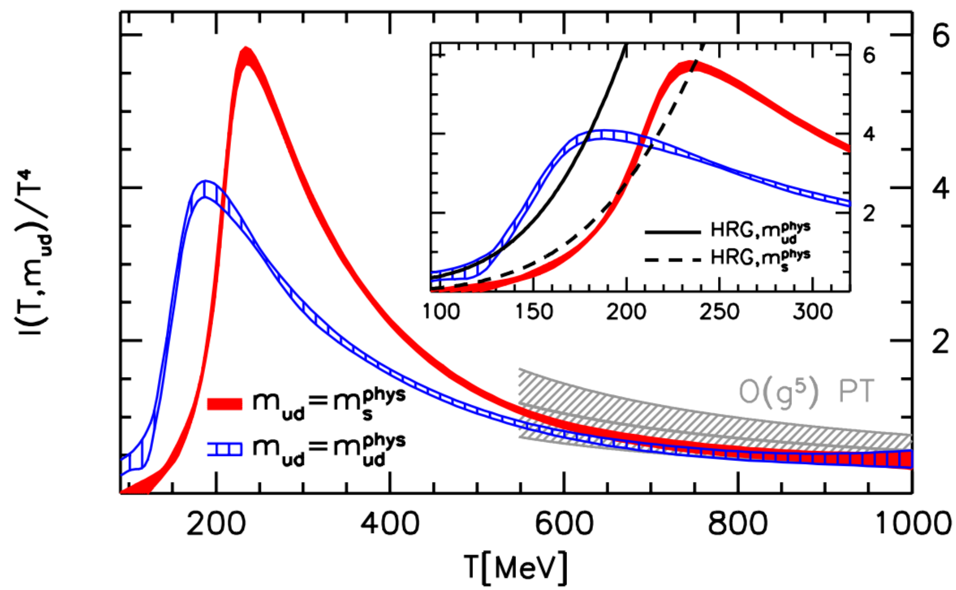
- The different N_t data are on top of each other (a few test points at $N_t = 12$)
- parametrization $T = 100...1000$ MeV ($t = T/200$ MeV)

$$\frac{I(T)}{T^4} = \exp(-h_1/t - h_2/t^2) \cdot \left(h_0 + \frac{f_0 \cdot [\tanh(f_1 \cdot t + f_2) + 1]}{1 + g_1 \cdot t + g_2 \cdot t^2} \right)$$

h_0	h_1	h_2	f_0	f_1	f_2	g_1	g_2
0.1396	-0.1800	0.0350	2.76	6.79	-5.29	-0.47	1.04

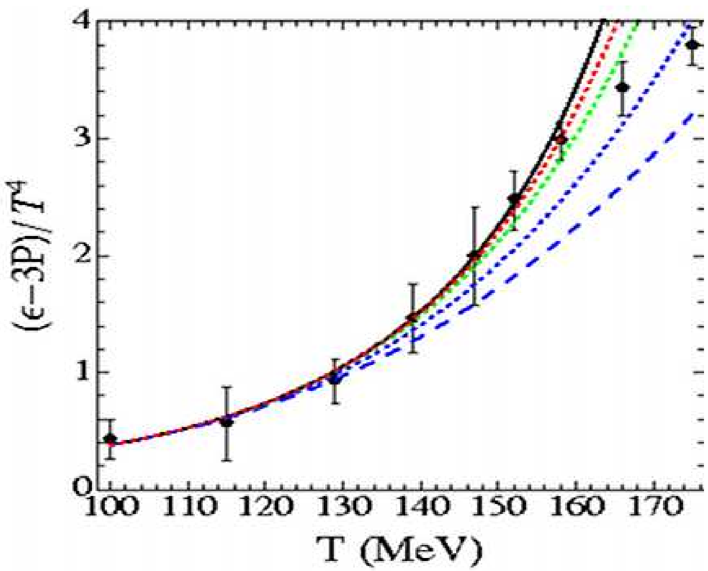
Trace anomaly





- comparison with the published results of the hotQCD collaboration
  - discrepancy: peak at $\simeq 20$ MeV larger T and $\simeq 50\%$ higher
- two different pion masses: $M_\pi = 135$ MeV and $M_\pi \simeq 720$ MeV
- good agreement with the HRG model up to the transition region
- quark mass dependence disappears for high T
- good agreement with perturbation theory

Inclusion of exponential spectrum in HRG model



- For large masses few states are known experimentally
- Inclusion of exponentially growing hadron mass spectrum

  J. Noronha-Hostler, C. Greiner, I. Shovkovy (2008); J. Noronha-Hostler, M. Beitel, C. Greiner, I. Shovkovy (2010)

- Agreement between lattice and HRG improved up to $T \sim 155$ MeV

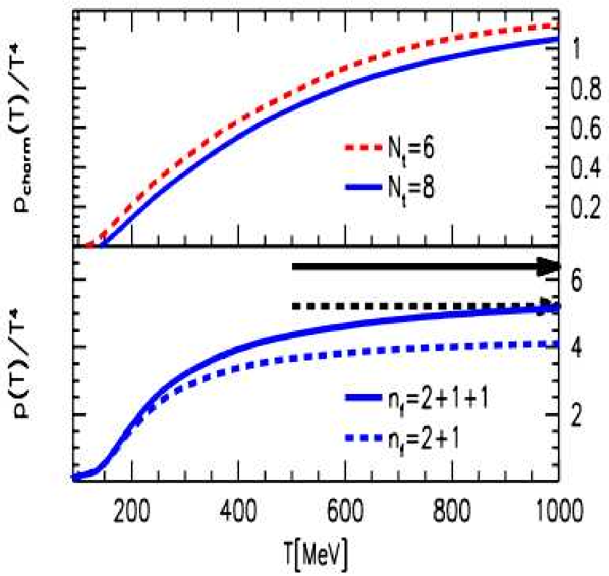
  (A. Majumder, B. Müller: 1008.1747)

Charm contribution

- perturbative indications: important already at $2T_c$

  M. Laine and Y. Schroder, Phys. Rev. D73 (2006)

- determine it within the partially quenched framework: $m_c/m_s = 11.85$



- charm contribution is indeed non-negligible from 200 MeV

- one has to extend this observation to the dynamical case

S. Borsanyi *et al.*, JHEP1011 (2010)

Conclusions

- Most recent results on QCD thermodynamics by the WB collaboration
- QCD transition temperature: results from 2006 and 2009 are improved:
  - physical quark masses used in simulations also at $T = 0$
  - finer lattices with $N_t = 16$
- The new results are in perfect agreement with those from 2006 and 2009
- QCD Equation of State:
  - results for physical quark masses
  - $N_t = 6, 8, 10, 12$
  - partially quenched charm contribution
- Good agreement between HRG model predictions and WB continuum results

Backup slides

What happens below T_c?

- At low T and $\mu = 0$, QCD thermodynamics is dominated by pions
- The interaction between pions is suppressed
  - ➟ chiral perturbation theory: pion contribution to the thermodynamic potential
  - ➟ the energy density of pions from 3-loop ChPT differs only less than 15% from the ideal gas value
    P. Gerber and H. Leutwyler (1989)
- as T increases, heavier hadrons start to contribute
- for $T \geq 120$ MeV heavy states dominate the energy density
- their mutual interactions are proportional to $n_i n_k \sim \exp[-(M_i + M_k)/T]$: they are suppressed
  - ➟ the virial expansion can be used to calculate the effect of the interaction

Why HRG?

- In the virial expansion, the partition function can be split into a non-interacting piece and a piece which includes all interactions Dashen, Ma and Bernstein (1969)
- virial expansion and experimental information on scattering phase shift Prakash and Venugopalan (1992)
  - ➡ interplay between attractive and repulsive interaction

Interacting hadronic matter

can be well approximated by

a non-interacting gas of resonances