

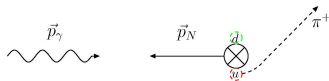
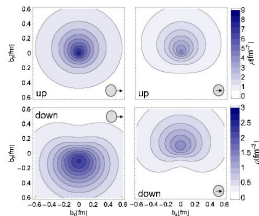
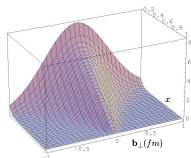
GPDs

Matthias Burkardt

New Mexico State University

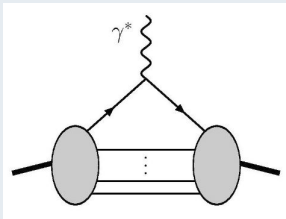
December 8, 2014

- GPDs:
 - impact parameter dependent PDFs
 - angular momentum sum rule
 - physics of form factors
- GPDs \rightsquigarrow SSA
- g_2 : \perp force on quarks in DIS
- Jaffe *vs.* Ji OAM
- DVCS $Q^2 \text{evol.}$ \rightsquigarrow GPDs
- GPDs for $x = \xi$
- Summary



form factor

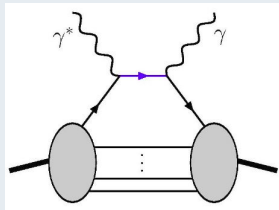
- electron hits nucleon & nucleon remains intact



- study amplitude that nucleon remains intact as function of momentum transfer $\rightarrow F(q^2)$
 - $F(q^2) = \int dx GPD(x, \xi, q^2)$
- \rightarrow GPDs provide momentum dissected form factors

Compton scattering

- electron hits nucleon, nucleon remains intact & photon gets emitted



- study both energy & q^2 dependence
- \rightarrow additional information about momentum fraction x of active quark
- \rightarrow generalized parton distributions $GPD(x, \xi, q^2)$

Quark (Orbital) Angular Momentum

X.Ji, 1996:

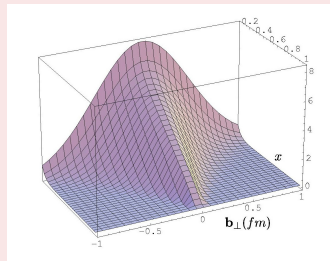
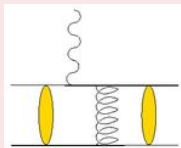
$$J_q = \frac{1}{2} \int dx x [H_q(x, \xi, 0) + E_q(x, \xi, 0)]$$

Transverse Imaging

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics of large Q^2 Form Factors

- Feynman: form factor at large Q^2 dominated by high- x quarks
- pQCD mechanism: dominated by typical valence quarks accompanied by hard gluon exchange

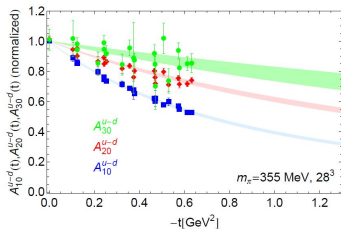
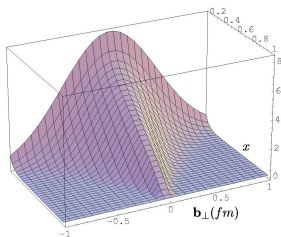
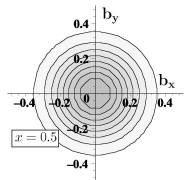
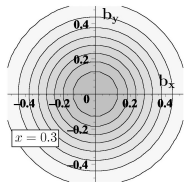
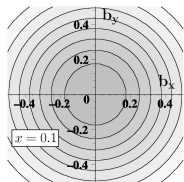


GPDs

$$F_1(Q^2) = \int dx H(x, \xi, Q^2)$$

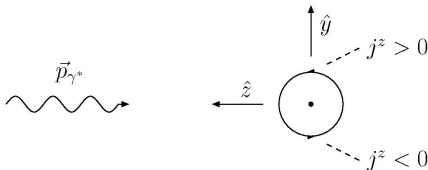
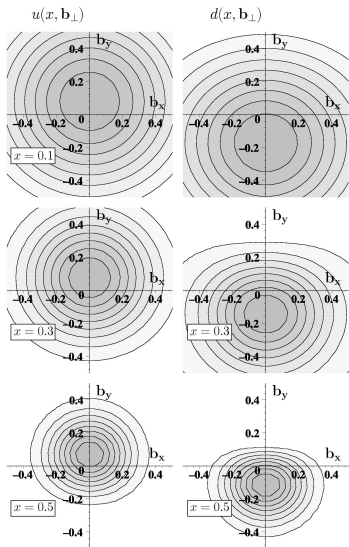
$$F_2(Q^2) = \int dx E(x, \xi, Q^2)$$

$q(x, \mathbf{b}_\perp)$ for unpol. p



unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$
 - x = momentum fraction of the quark
 - \vec{b}_\perp = \perp distance of quark from \perp center of momentum
 - small x : large 'meson cloud'
 - larger x : compact 'valence core'
 - $x \rightarrow 1$: active quark becomes center of momentum
- $\hookrightarrow \vec{b}_\perp \rightarrow 0$ (narrow distribution) for $x \rightarrow 1$

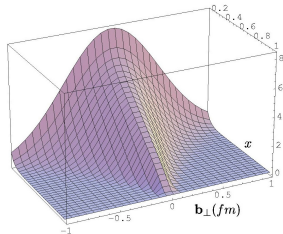


proton 'polarized in $+\hat{x}$ direction'

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\ - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3



nucleon size at large x

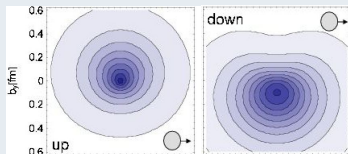
MB & G.A.Miller (2006): 'mathematical model'

$$H(x, 0, Q^2) = (1-x)^3 \exp(-\mu^2(1-x)^2 Q^2)$$

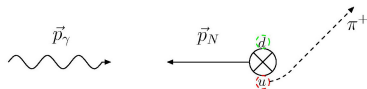
$$\hookrightarrow F_1(Q^2) = \int dx H(x, 0, Q^2) \sim \frac{1}{Q^4}$$

large Q^2 behaviour dominated by large x regime \rightarrow Feynman mechanism

Chromodynamic Lensing

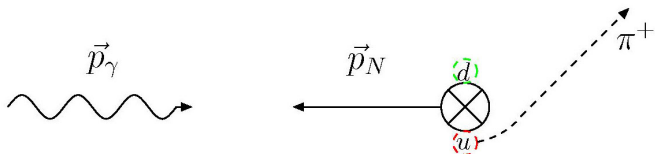


- qualitative connection between **deformation** and **SSA**
- one gluon exchange: SSA \leftrightarrow \perp density-density correlations



LHC physics

Frankfurt, Strikman, Weiss:
 \perp spatial distributions \rightarrow hard parton-parton processes at LHC

example: $\gamma p \rightarrow \pi X$ 

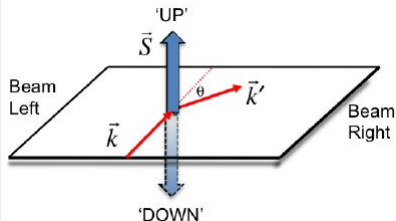
- u, d distributions in \perp polarized proton have left-right asymmetry in \perp position space (T-even!); sign “determined” by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum
- \hookrightarrow FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$ -direction into momentum asymmetry that favors $-\hat{y}$ direction \rightarrow **chromodynamic lensing**

 \Rightarrow $\kappa_p, \kappa_n \longleftrightarrow$ sign of SSA!!!!!!! (MB,2004)

- confirmed by HERMES (& COMPASS) data

inclusive SSA

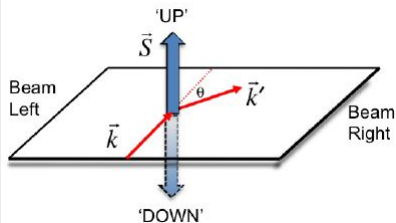
Hall A: inclusively scattered (summed over all final states) electrons off \perp polarized target show left-right asymmetry relative to target spin



for n target (i.e. ${}^3\text{He}$), the e^- is preferentially deflected to beam-right

inclusive SSA

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Metz et al. arXiv:1209.3138

- beam e^- approx. at same \perp position as struck quark

$$\hookrightarrow \langle \mathbf{k}_{\perp e} \rangle \sim \sum_q (-e) F_{FT}^q \text{ with}$$

$$F_{FT}^q \equiv \int dr^- \langle P, S | \bar{q}(0) \gamma^+ e F_{QED}^{+\perp}(0, r^-) q(0) | P, S \rangle$$

Metz et al. arXiv:1209.3138

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model (A. Metz *et al*, *Phys. Rev. D* **86**, 094039 (2012))

- if FSI in QCD is caused by one-gluon exchange between valence quarks then

$$F_{FT}^q = 2 \frac{e'_q \alpha}{\frac{4}{3} \alpha_s} F_{QCD}^q$$

- e'_s charge of spectators, e.g. $e'_q = \frac{1}{3}$ for u quarks in proton

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Flaw: (M. Burkardt, *Phys. Rev. D* **69**, 057501 (2004))

- on average, ISI/FSI of active quark with same-flavor spectator yields **zero** contribution to $\langle k_{\perp} \rangle$

Metz et al.

- multiply by charge of spectator flavor, e.g. $e'_q = \frac{1}{3}$ for u in p
- divide by $\frac{4}{3}\alpha_s$

$$F_{FT}^{u/p}(x, x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} g T_F^{u/p}(x, x)$$

$$F_{FT}^{d/p}(x, x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{d/p}(x, x)$$

$$F_{FT}^{u/n}(x, x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{d/p}(x, x)$$

$$F_{FT}^{d/n}(x, x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} g T_F^{u,p}(x, x).$$

- violates $\frac{2}{3}F_{FT}^{u/N} - \frac{1}{3}F_{FT}^{d/N} \stackrel{!}{=} 0$
- cancellation between u and d in p

assume $T_F^{q/p}$ saturate sum rule $\sum_{q=u,d} \int dx T_F^{q/p}(x, x) = 0$.

MB (evolution 2014)

- multiply by charge of spectator flavor, e.g. $e'_q = -\frac{1}{3}$ for u in p
- divide by $\frac{2}{3}\alpha_s$ for u in p or d in n

$$F_{FT}^{u/p}(x, x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{u/p}(x, x)$$

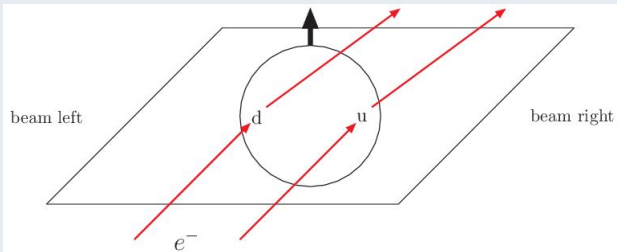
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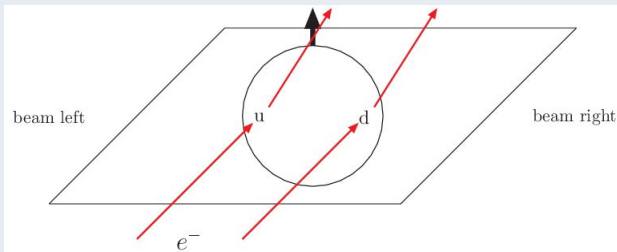
$$F_{FT}^{d/n}(x, x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{u,p}(x, x).$$

- consistent with $\frac{2}{3}F_{FT}^{u/N} - \frac{1}{3}F_{FT}^{d/N} \stackrel{!}{=} 0$
- same sign from u & d

neutron target



proton target



signs opposite to Afanasev *et al.*, *Phys.Rev. D* **77**, 014028 (2008)

$$F_{FT}^{u/p}(x, x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} g T_F^{u/p}(x, x)$$

$$F_{FT}^{u/p}(x, x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{u/p}(x, x)$$

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$$F_{FT}^{d/n}(x, x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{u,p}(x, x).$$

- cancellation between u and d in p
- violates $\frac{2}{3} F_{FT}^{u/N} - \frac{1}{3} F_{FT}^{d/N} \stackrel{!}{=} 0$

- consistent with $\frac{2}{3} F_{FT}^{u/N} - \frac{1}{3} F_{FT}^{d/N} \stackrel{!}{=} 0$
- same sign from u & d

- $\sigma_{UT}^p \propto \frac{2\alpha_{em}g}{3\pi C_F \alpha_s} \left(-T_F^{u/p} - T_F^{d/p} \right)$

- $\sigma_{UT}^p \propto \frac{2\alpha_{em}g}{3\pi C_F \alpha_s} \left(2T_F^{u/p} - T_F^{d/p} \right)$

- $\sigma_{UT}^n \propto \frac{2\alpha_{em}g}{3\pi C_F \alpha_s} \left(2T_F^{d/p} - \frac{1}{4}T_F^{u/p} \right)$

- $\sigma_{UT}^n \propto \frac{2\alpha_{em}g}{3\pi C_F \alpha_s} \left(2T_F^{d/p} - T_F^{u/p} \right)$

- $\sigma_{UT}^p \ll \sigma_{UT}^n$

- $\sigma_{UT}^p \approx -\sigma_{UT}^n$

- assume $T_F^d \approx -T_F^u$ (vanishing isoscalar Sivers)

$d_2 \leftrightarrow$ average \perp force on quark in DIS from \perp pol target

polarized DIS:

$$\bullet \sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2 \qquad \bullet \sigma_{LT} \propto g_T \equiv g_1 + g_2$$

\leftrightarrow 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

$$\bullet g_2 = g_2^{WW} + \bar{g}_2 \text{ with } g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2} S^x} \langle P, S | \bar{q}(0) \gamma^+ g F^{+y}(0) q(0) | P, S \rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2} F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

matrix element defining d_2

\leftrightarrow

1st integration point in QS-integral
 \hookrightarrow Siverts

$\int x^2 e(x)$ (scalar twist-3 PDF)

\leftrightarrow

'Boer-Mulders force':
 \perp pol. quarks; unpol. target

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S | \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) | P, S \rangle$$

color Lorentz force

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

$\hookrightarrow d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F_y^q \rangle = -2M^2 d_2^q$$

sign of $d_2 \leftrightarrow \perp$ imaging

- $\kappa_q/p \rightarrow$ sign of deformation

$$\hookrightarrow d_2^u > 0, d_2^d < 0$$

- cf. $f_{1T}^{\perp u} < 0, f_{1T}^{\perp d} < 0$

$$\bullet d_2 = \frac{4}{9}d_2^u + \frac{1}{9}d_2^d$$

\hookrightarrow partial cancellation!

lattice (Göckeler et al., 2005)

$$d_2^u \approx 0.010, d_2^d \approx -0.0056$$

magnitude of d_2

$$\bullet \langle F^y \rangle = -2M^2 d_2 = -10 \frac{\text{GeV}}{\text{fm}} d_2$$

- expect partial cancellation of forces in SSA

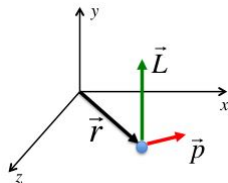
$$\hookrightarrow |\langle F^y \rangle| \ll \sigma \approx 1 \frac{\text{GeV}}{\text{fm}}$$

$$\hookrightarrow d_2 = \mathcal{O}(0.01)$$

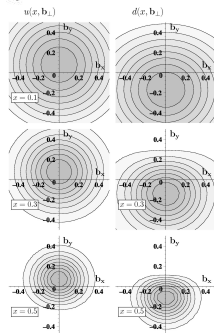
- consistent with chiral quark soliton model (\rightarrow Weiss)

angular momentum and energy momentum tensor

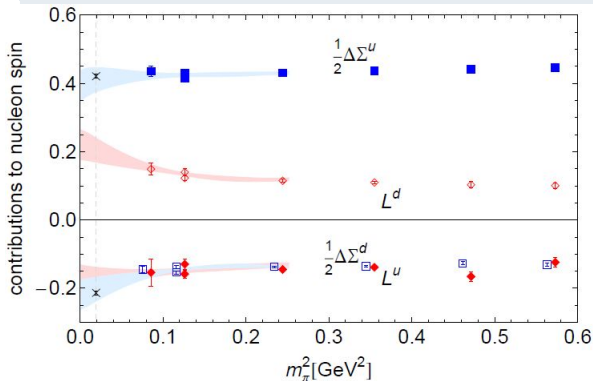
- $L_x = yp_z - zp_y$
- ↪ $J_x = \int d^3r [yT^{0z} - zT^{0y}]$
- if state invariant under rotations about \hat{x} axis then $\langle yp_z \rangle = -\langle zp_y \rangle$
- ↪ $\langle L_x \rangle = 2\langle yp_z \rangle \rightarrow J_z = 2 \int d^3r yT^{0z}$
- ↪ J_{\perp} sensitive to \perp shift of momentum distribution

GPDs \rightarrow \perp shift of momentum distribution

- GPDs provide simultaneous information about p_z & \mathbf{b}_{\perp}
- ↪ use quark GPDs to determine angular momentum carried by quarks
- ↪ $J_q^x = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$ (X.Ji, 1996)
- partonic interpretation in terms of 3D distribution (MB,2001,2005)



lattice: LHPC



- no disconnected quark loops
- chiral extrapolation

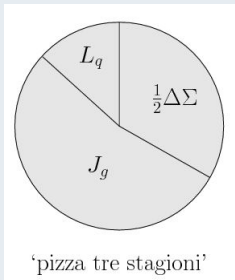
$$J^q = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$$

$$L^q = J^q - \frac{1}{2} \Delta\Sigma^q$$

$$L^u + L^d \approx 0$$

signs of L^q counter-intuitive

Ji decomposition



$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + L_q + J_g$$

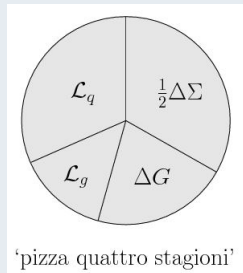
$$\frac{1}{2} \Delta q = \frac{1}{2} \int d^3x \langle P, S | q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) | P, S \rangle$$

$$L_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i \vec{D})^3 q(\vec{x}) | P, S \rangle$$

$$J_g = \int d^3x \langle P, S | \left[\vec{x} \times (\vec{E} \times \vec{B}) \right]^3 | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}$

Jaffe decomposition



light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i \vec{\partial})^z q(\vec{r}) | P, S \rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle$$

$$\mathcal{L}_g = 2 \int d^3r \langle P, S | \text{Tr} F^{+j} (\vec{x} \times i \vec{\partial})^z A^j | P, S \rangle$$

Ji decomposition

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- GPDs $\longrightarrow L^q$
- Δq evolution and/or $\vec{p} \vec{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i \in q, g} \mathcal{L}^i$
- $L^q \neq \mathcal{L}^q$ [MB & H. BC, *Phys. Rev. D* **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$
 - can we calculate/predict the difference?
 - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ q(\xi) | P S \rangle.$$

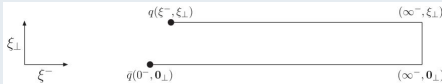
- TMDs: $f(x, \mathbf{k}_\perp) = \int d^2 \mathbf{b}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
- GPDs: $q(x, \mathbf{b}_\perp) = \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
- $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{l\xi}$ to connect 0 and ξ (Ji, Yuan; Hatta; Lorcé;...)

straight line (Ji et al.)

straight Wilson line from 0 to ξ yields Ji-OAM:

$$L^q = \int d^3 x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$ (Hatta)



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

straight line ($\rightarrow J_i$)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + L_q + J_g$$

$$L_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (\rightarrow Jaffe-Manohar)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

$$i\vec{D}^j = i\partial^j - gA^j(x^- = \infty, \mathbf{x}_\perp)$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -(\vec{E} + \vec{v} \times \vec{B})^y \text{ for } \vec{v} = (0, 0, -1)$$

Torque along the trajectory of q

$$T^z = [\vec{x} \times (\vec{E} - \hat{z} \times \vec{B})]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^{\infty} dr^- [\vec{x} \times (\vec{E} - \hat{z} \times \vec{B})]^z$$

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$$iD^j = i\partial^j - gA^j(x^-, \mathbf{x}_\perp) - g \int_{x^-}^{\infty} dr^- F^{+j}$$

difference $\mathcal{L}^q - L^q$

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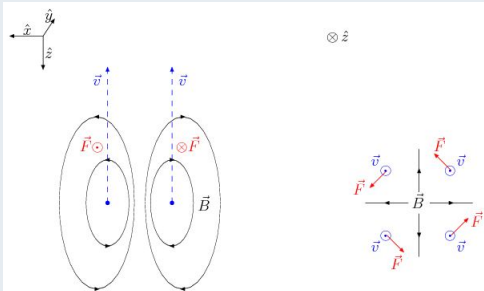
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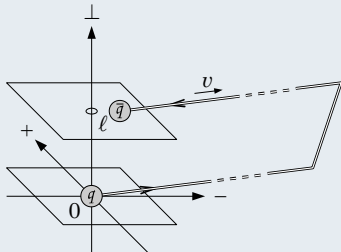
difference $\mathcal{L}^q - L^q$ (MB, PRD 88 (2013) 014014)

$\mathcal{L}^q - L^q = \Delta L_{FSI}^q =$ change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



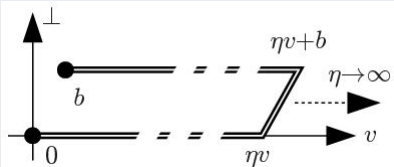
challenge



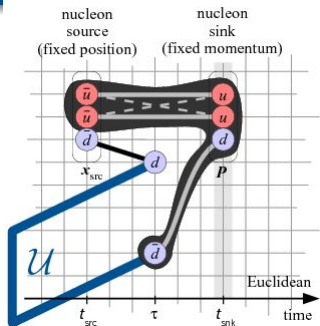
- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

TMDs in lattice QCD

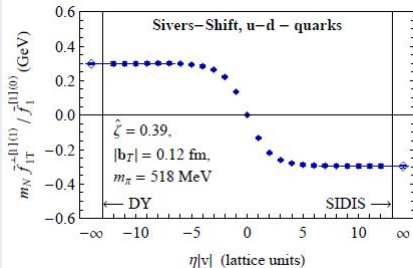
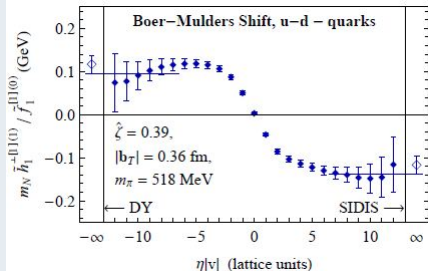
B. Musch, P. Hägler, M. Engelhardt



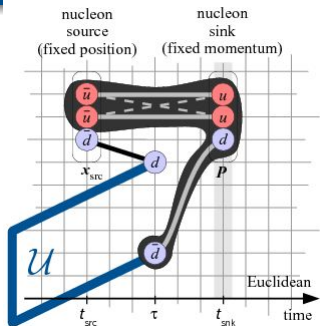
- calculate space-like staple-shaped Wilson line pointing in \hat{z} direction; length $L \rightarrow \infty$
 - momentum projected nucleon sources/sinks
 - remove IR divergences by considering appropriate ratios
- ↪ extrapolate/evolve to $P_z \rightarrow \infty$



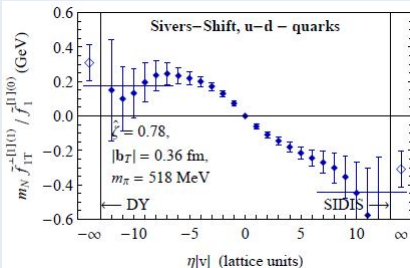
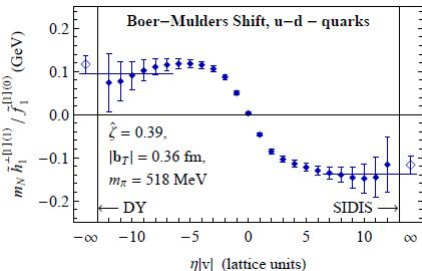
$$f_{1T,SIDIS}^{\perp} = -f_{1T,DY}^{\perp} \text{ (Collins)}$$



$f_{1T}^{\perp}(x, \mathbf{k}_{\perp})$ is \mathbf{k}_{\perp} -odd term in quark-spin averaged momentum distribution in \perp polarized target



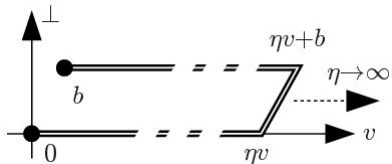
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TMDs in lattice QCD

B. Musch, P. Hägler, M. Engelhardt



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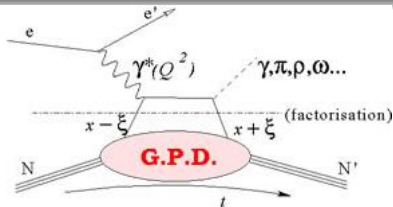
next: Orbital Angular Momentum

- same operator as for TMDs, only nonforward matrix elements:
 - momentum transfer provides position space information ($\rightarrow \mathbf{r}_\perp \times \mathbf{k}_\perp$)
 - staple with long side in \hat{z} direction
 - (large) nucleon momentum in \hat{z} direction
 - small momentum transfer in \hat{y} direction
- ↪ generalized TMD F_{14} (Metz et al.)
- quark OAM
 - renormalization same as f_{1T}^\perp
- ↪ study ratios...

$\mathcal{A}_{DVCS} \rightsquigarrow GPDs$

interesting GPD physics:

- $J_q = \int_0^1 dx x [H(x, \xi, 0) + E(x, \xi, 0)]$ requires $GPDs(x, \xi, 0)$ for (common) **fixed ξ for all x**
- \perp imaging requires $GPDs(x, \xi = 0, t)$



- ξ longitudinal momentum transfer on the target $\xi = \frac{p'^+ - p^+}{p'^+ + p^+}$
- x (average) momentum fraction of the active quark $x = \frac{k'^+ + p^+}{p'^+ + p^+}$

$\Im \mathcal{A}_{DVCS}(\xi, t) \rightarrow GPD(\xi, \xi, t)$

- only sensitive to 'diagonal' $x = \xi$
- limited ξ range

$\Re \mathcal{A}_{DVCS}(\xi, t) \rightarrow \int_{-1}^1 dx \frac{GPD(x, \xi, t)}{x - \xi}$

- limited ξ range
- most sensitive to $x \approx \xi$
- some sensitivity to $x \neq \xi$, but

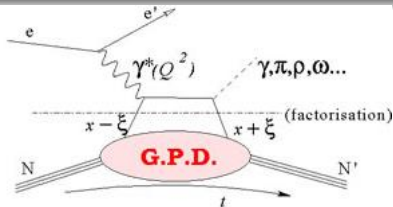
Polynomiality/Dispersion Relations (GPV/AT DI)

$$\Re \mathcal{A}(\xi, t, Q^2) = \int_{-1}^1 dx \frac{H(x, \xi, t, Q^2)}{x - \xi} = \int_{-1}^1 dx \frac{H(x, x, t, Q^2)}{x - \xi} + \Delta(t, Q^2)$$

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$$\Im \mathcal{A}_{DVCS}(\xi, t) \longrightarrow GPD(\xi, \xi, t)$$

$$\Re \mathcal{A}_{DVCS}(\xi, t) \longrightarrow \int_{-1}^1 dx \frac{GPD(x, \xi, t)}{x - \xi}$$

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- Can 'condense' all information contained in contained in \mathcal{A}_{DVCS} (fixed Q^2) into $GPD(x, x, t, Q^2)$ & $\Delta(t, Q^2)$
- if two models both satisfy **polynomiality** and are **equal for $x = \xi$** (but not for $x \neq \xi$) and have **same $\Delta(t, Q^2)$** then **DVCS** at fixed Q^2 **cannot distinguish** between the two models

Polynomiality/Dispersion Relations (GPV/AT DI)

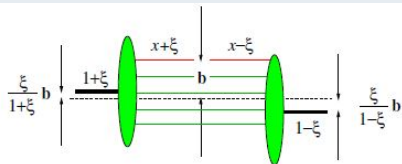
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need Evolution!

$$\mu^2 \frac{d}{d\mu^2} H^{q(-)}(x, \xi, t) = \int_{-1}^1 dx' \frac{1}{|\xi|} V_{NS}\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) H^{q(-)}(x', \xi, t)$$

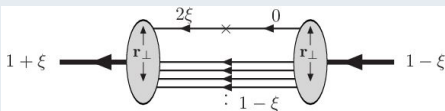
- Q^2 evolution changes x distribution in a known way for fixed ξ
 \hookrightarrow measure Q^2 dependence to disentangle x vs. ξ dependence

\perp imaging: $\zeta \neq 0$ 

- center of momentum of hadron **not** 'conserved' when $\xi \neq 0$,
- \hookrightarrow distance of active quark to COM **not** conserved
- \perp position of each parton **is** conserved, and so is (any ξ)
- \hookrightarrow distance \mathbf{r}_\perp of active quark to spectators (any $\xi/zeta$) diagonal
- variable conjugate to Δ_\perp is $\frac{1-x}{1-\zeta} \mathbf{r}_\perp$

 \perp imaging: $\xi = 0$

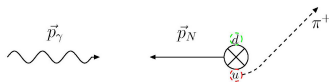
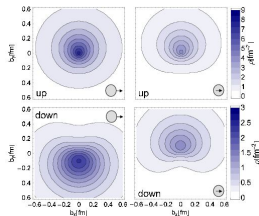
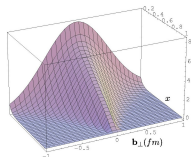
- probabilistic interpretation
- variable conjugate to Δ_\perp is $\mathbf{b}_\perp \equiv (1-x)\mathbf{r}_\perp$ distance to COM of hadron

 \perp imaging ($x = \xi$)

- no probabilistic interpretation
- variable conjugate to Δ_\perp is \mathbf{r}_\perp** distance to COM of spectators
- \hookrightarrow expect no dramatic ζ -dependence of Δ_\perp^2 -slope (size of system)
- \hookrightarrow t -slope $\sim (1-\zeta)$

- GPDs \rightarrow impact parameter dependent PDFs
- \hookrightarrow chromodynamic lensing
 - semi-inclusive SSA
 - inclusive SSA
 - d_2 (\perp force on quarks in DIS)
- Ji vs. Jaffe-Manohar quark OAM: interpretation of $\mathcal{L}_q - L_q$ as change in OAM of ejected quark
- parton interpretation for Ji-relation
- Jaffe-Manohar OAM from lattice
- (in principle) $GPD(x, \xi, Q^2)$ from QCD evolution of $GPD(\xi, \xi, Q^2)$

\hookrightarrow EIC (need wide Q^2 range)



- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q = \bar{q} \gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q$$

- \mathcal{L}_q^z matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q} \gamma^+ \left[\vec{r} \times i\vec{\partial} \right]^z q \Big|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q} \gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q$ vanishes (parity!)
- ↪ L_q identical to matrix element of $\bar{q} \gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q$ (nucleon at rest)
- ↪ even in light-cone gauge, L_q^z and \mathcal{L}_q^z still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A} \right)^z q \Big|_{A^+=0} = q^\dagger (r^x g A^y - r^y g A^x) q \Big|_{A^+=0}$
- how significant is that difference?

first: QED without electrons

- apply $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ to $\vec{E} \times (\vec{\nabla} \times \vec{A})$

$$\begin{aligned}\vec{J} &= \int d^3r \vec{x} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{x} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})] \\ &= \int d^3r \left[E^j (\vec{x} \times \vec{\nabla}) A^j - \vec{x} \times (\vec{E} \cdot \vec{\nabla}) \vec{A} \right]\end{aligned}$$

- integrate by parts (drop surface term)

$$\vec{J} = \int d^3r \left[E^j (\vec{x} \times \vec{\nabla}) A^j + (\vec{x} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]$$

- drop 2nd term (eq. of motion $\vec{\nabla} \cdot \vec{E} = 0$), yielding $\vec{J} = \vec{L} + \vec{S}$ with

$$\vec{L} = \int d^3r E^j (\vec{x} \times \vec{\nabla}) A^j \quad \vec{S} = \int d^3r \vec{E} \times \vec{A}$$

- note: \vec{L} and \vec{S} not separately gauge invariant as written, but can be made so (\rightarrow nonlocal)

QED with electrons

$$\begin{aligned}
 \vec{J}_\gamma &= \int d^3r \vec{r} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{r} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})] \\
 &= \int d^3r \left[E^j (\vec{r} \times \vec{\nabla}) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla}) \vec{A} \right] \\
 &= \int d^3r \left[E^j (\vec{r} \times \vec{\nabla}) A^j + (\vec{r} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]
 \end{aligned}$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j (\vec{x} \times \vec{\nabla}) A^j + \vec{E} \times \vec{A} \right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$
- ↪ decomposing \vec{J}_γ into spin and orbital also shuffles angular momentum from photons to electrons!
- can also be done for only part of $\vec{A} \rightarrow$ Chen/Goldman, Wakamatsu

Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r [yT_q^{0z}(\vec{r}) - zT_q^{0y}(\vec{r})]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor ($T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r})$)
- $T_q^{0i}(\vec{r})$ momentum density [$P_q^i = \int d^3r T_q^{0i}(\vec{r})$]
- think: $(\vec{r} \times \vec{p})^x = yp^z - zp^y$

relate to impact parameter dependent quark distributions $q_\psi(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x -axis

\hookrightarrow both terms in J_q^x equal:

$$J_q^x = 2 \int d^3r y T_q^{0z}(\vec{r}) = \int d^3r y [T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r})]$$

- $\int d^3r y T_q^{00}(\vec{r}) = 0 = \int d^3r y T_q^{zz}(\vec{r})$

$$\Rightarrow J_q^x = \int d^3r y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

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$$\Rightarrow J_q^x = \int d^3r y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

- $\int dx x q(x, \mathbf{r}_\perp) = \frac{1}{2m_N} \int dr^z T^{++}(\vec{r})$
(note: here x is momentum fraction and not r^x)

↪ $\langle \psi | J_q^x | \psi \rangle = m_N \int dx \int d^2b_\perp x b^y q_\psi(x, \mathbf{b}_\perp)$

distribution in delocalized wave packet (pol. in $+\hat{x}$ direction)

$$q_\psi(x, \mathbf{b}_\perp) = \int d^2 r_\perp q(x, \mathbf{b}_\perp - \mathbf{r}_\perp) \left(|\psi(\mathbf{r}_\perp)|^2 - \frac{1}{2M} \frac{\partial}{\partial r_y} |\psi(\mathbf{r}_\perp)|^2 \right) \text{ with}$$

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

two contributions to \perp shift

- intrinsic shift relative to center of momentum \mathbf{R}_\perp
- overall shift of \mathbf{R}_\perp for \perp polarized nucleon

insert into $\langle \psi | J_q^x | \psi \rangle = \int dx \int d^2 b_\perp q_\psi(x, \mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$$\langle \psi | J_q^x | \psi \rangle = \frac{1}{2} \int dx x [H_q(x, 0, 0) + E_q(x, 0, 0)] \quad (\text{here: derived for } \vec{p} = \vec{0} \text{ only!})$$

- X.Ji (1996): rotational invariance \Rightarrow apply to all components of \vec{J}
- result for J_q^z also applies to $p_z \neq 0$
- partonic interpretation (\perp shift) exists only for \perp components of $\vec{J}_q!$

insert into $\langle \psi | J_q^x | \psi \rangle = \int dx \int d^2 b_\perp q_\psi(x, \mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$$\langle \psi | J_q^x | \psi \rangle = \frac{1}{2} \int dx x [H_q(x, 0, 0) + E_q(x, 0, 0)] \quad (\text{here: derived for } \vec{p} = \vec{0} \text{ only!})$$

- X.Ji (1996): rotational invariance \Rightarrow apply to all components of \vec{J}
- result for J_q^z also applies to $p_z \neq 0$
- partonic interpretation (\perp shift) exists only for \perp components of $\vec{J}_q!$

gauge invariance

- matrix element of $T_q^{++} = \bar{q}\gamma^+i\partial^+q$ in $A^+ = 0$ gauge same as that of $\bar{q}\gamma^+(i\partial^+ - gA^+)q$ in any gauge
- \hookrightarrow identify $\frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$ with J_q in decomposition where
- $$\vec{L}_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D}) q(\vec{x}) | P, S \rangle$$

insert into $\langle \psi | J_q^x | \psi \rangle = \int dx \int d^2 b_\perp q_\psi(x, \mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

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caution!

- made heavily use of rotational invariance

\hookrightarrow identification $\langle \psi | J_q^x | \psi \rangle = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$ does not apply to unintegrated quantities

- $\int d^2 \Delta_\perp e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \frac{x}{2} [H(x, 0, -\Delta_\perp^2) + E(x, 0, -\Delta_\perp^2)]$ **not** equal to $J^z(\mathbf{b}_\perp)$
- $J_q(x) \equiv \frac{x}{2} [H_q(x, 0, 0) + E_q(x, 0, -\Delta_\perp^2)]$ **not** x -distribution of angular momentum $J_q^z(x)$ in long. pol. target (only x -distribution of 'half of $J_{q\perp}$ ')
regardless whether one takes gauge covariant definition or not