

Light-Meson Two-Photon Decays in Full QCD

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Outline

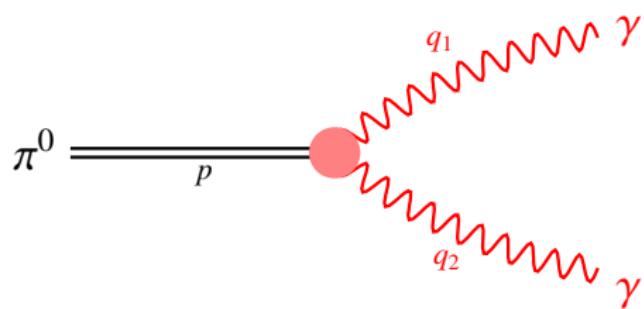
1 Introduction

2 Lehmann-Symanzik-Zimmermann Reduction

3 Lattice Calculation

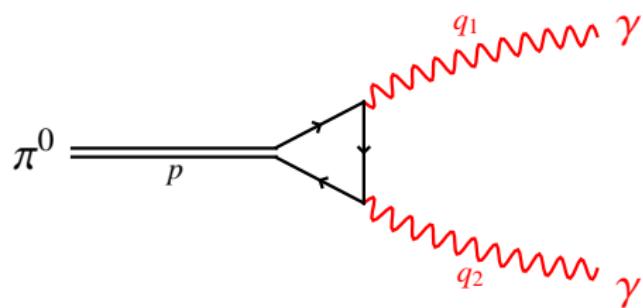
$$\pi \rightarrow \gamma\gamma$$

- Extends the reach of lattice techniques to QED
- Often difficult to measure experimentally
- Error in determination of the light quark mass ratio is dominated by uncertainty in $\Gamma(\pi \rightarrow \gamma\gamma)$



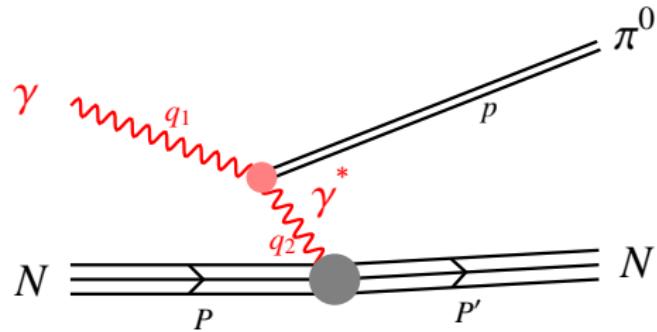
Anomalous Symmetry Breaking

- Directly tests one of the most striking predictions of the Standard Model:
anomalous symmetry breaking
- Pion is probably well described by the anomaly + XPT (2% uncertainties),
but heavier states might not be



Primakoff Effect

- Useful to photoproduction experiments (e.g. GlueX)
- Photon fusion often used to produce flavor-neutral mesons



QED Eigenstates without QED

- Since the photon is not an eigenstate of QCD, standard lattice techniques will fail; a 1^{--} interpolating operator yields ρ (or $\pi\pi$) rather than γ .
- An elegant solution is provided by Ji & Jung PRL86, 208. We want this matrix element:

$$\langle \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) | \Phi(p) \rangle$$

- Perform a Lehmann-Symanzik-Zimmermann (LSZ) reduction:

$$-\lim_{q' \rightarrow q} \epsilon_\mu^{(1)*} \epsilon_\nu^{(2)*} {q'_1}^2 {q'_2}^2 \int d^4x d^4y e^{iq'_1 \cdot y + iq'_2 \cdot x} \langle 0 | T\{A^\mu(y) A^\nu(x)\} | \Phi(p) \rangle$$

Perturbative QED

- Although we cannot treat the A fields in QCD, we can use perturbative QED to integrate them out:

$$\int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_{\text{QED}}} A^\mu(y) A^\nu(x) \approx \\ \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_0} (\dots + [\bar{\psi} \gamma^\rho \psi A_\rho](z) [\bar{\psi} \gamma^\sigma \psi A_\sigma](w) + \dots) A^\mu(y) A^\nu(x)$$

- Then we Wick contract the photon fields into propagators

$$-e^2 \lim_{q' \rightarrow q} \epsilon_\mu^{(1)*} \epsilon_\mu^{(2)*} q'_1{}^2 q'_2{}^2 \times$$

$$\int d^4x d^4y d^4w d^4z e^{iq'_1 \cdot x} D^{\mu\rho}(0, z) D^{\nu\sigma}(x, w) \langle 0 | T\{j_\rho(z) j_\sigma(w)\} | \Phi(p) \rangle$$

Into Euclidean Space

- Using the explicit form of the photon propagator, most of these integrals go to delta functions:

$$e^2 \epsilon_\mu^{(1)*} \epsilon_\mu^{(2)*} \int d^4x e^{iq_1 \cdot y} \langle 0 | T\{j^\mu(0)j^\nu(y)\} | \Phi(p) \rangle$$

- This, we can rotate into Euclidean space unless we hit a pole; we must keep $q^2 < M_\rho^2$ (or $E_{\pi\pi}^2$).

$$\frac{e^2 \epsilon_\mu^{(1)} \epsilon_\mu^{(2)}}{\frac{Z_\Phi(p)}{2E_\Phi(p)} e^{-E_\Phi(p)(t_f-t)}} \int dt_i e^{-\omega_1(t_i-t)} \times \\ \left\langle T \left\{ \int d^3\vec{x} e^{-i\vec{p}\cdot\vec{x}} \varphi_\Phi(\vec{x}, t_f) \int d^3\vec{y} e^{i\vec{q}_2\cdot\vec{y}} j^\nu(\vec{y}, t) j^\mu(\vec{0}, t_i) \right\} \right\rangle$$

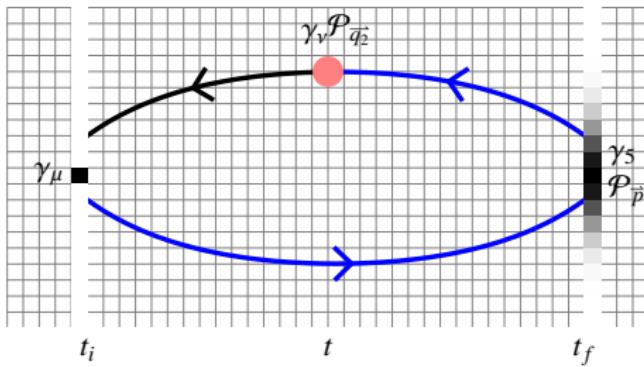
Lattice Three-Point Correlator

- This expression we can evaluate on the lattice.
The term between the angled brackets is just the three-point function with an arbitrary meson on one end and vector currents at the other end and inserted.

$$\frac{e^2 \epsilon_\mu^{(1)} \epsilon_\mu^{(2)}}{\frac{Z_\Phi(p)}{2E_\Phi(p)} e^{-E_\Phi(p)(t_f-t)}} \int dt_i e^{-\omega_1(t_i-t)} \times \\ \left\langle T \left\{ \int d^3 \vec{x} e^{-i\vec{p} \cdot \vec{x}} \varphi_\Phi(\vec{x}, t_f) \int d^3 \vec{y} e^{i\vec{q}_2 \cdot \vec{y}} j^\nu(\vec{y}, t) j^\mu(\vec{0}, t_i) \right\} \right\rangle$$

- The remaining parts describe how to combine QCD states into a photon of the appropriate energy.
- The most straightforward way to evaluate this is to compute the three-point function on all t_i and perform the integral explicitly.

Lattice Setup

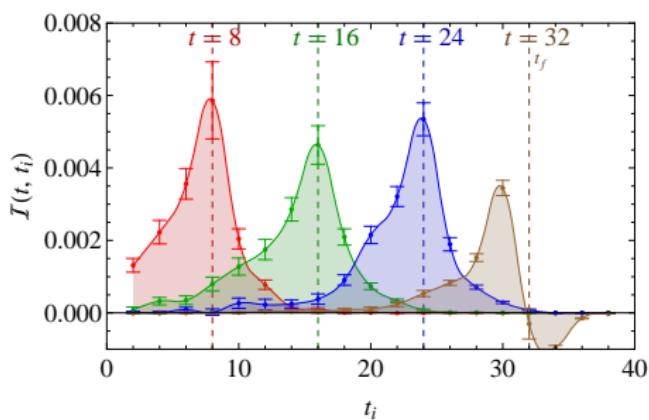


- CP-PACS $20^3 \times 40$ $N_f = 2$ lattices made available on the ILDG.
- $a^{-1} \approx 2.25$ GeV, $M_\pi = 725$ MeV
- Meson location fixed: $t_f = 32$, $\vec{p} = \{0, 0, 0\}$, Gaussian-smeared
- Sequential source starting from point-source at t_i
- Momentum projection $0 \leq |\vec{q}_2|^2 \leq 5$ at t

Integrand Evaluation

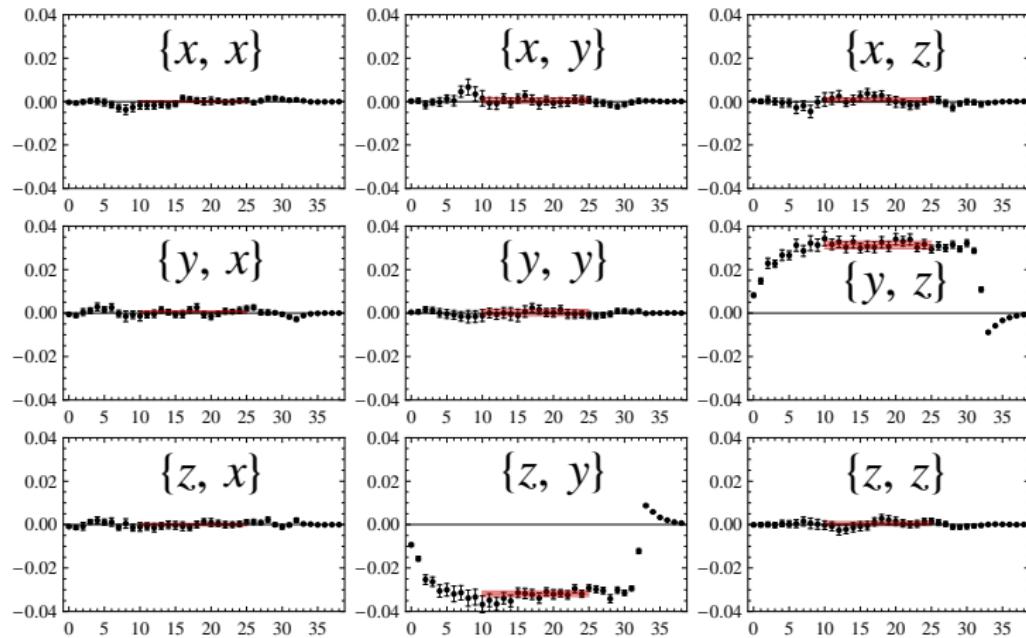
$$\frac{Z_\Phi(p)}{2E_\Phi(p)} \frac{e^{-\omega_1(t_i-t)}}{e^{-E_\Phi(p)(t_f-t)}} \mathcal{C}_{PVV}(t_f, t, t_i)$$

- Check width: If too narrow, cannot integrate accurately. If too wide, cannot find a plateau.
- Check distortion: Due to Dirichlet BCs and sink location.



The Integral

$$\vec{q} = \hat{x}$$

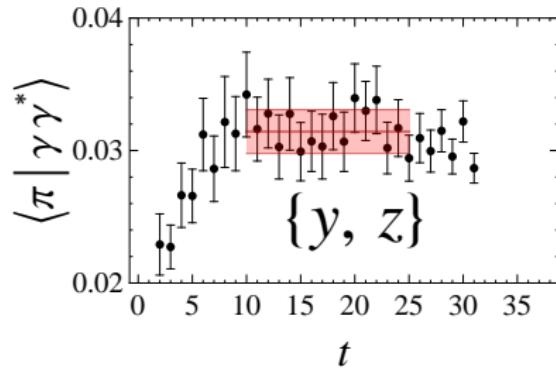


The Integral

A Non-Zero Element

- Integral only nonzero when $\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu}q_1^\rho q_2^\sigma \neq 0$
- We see a clear plateau in the expected region, away from $t = 0$ and $t = t_f$
- There may be exponential contamination due to excited states. Not seen here since the gap is large?

$$\vec{q} = \hat{x}$$

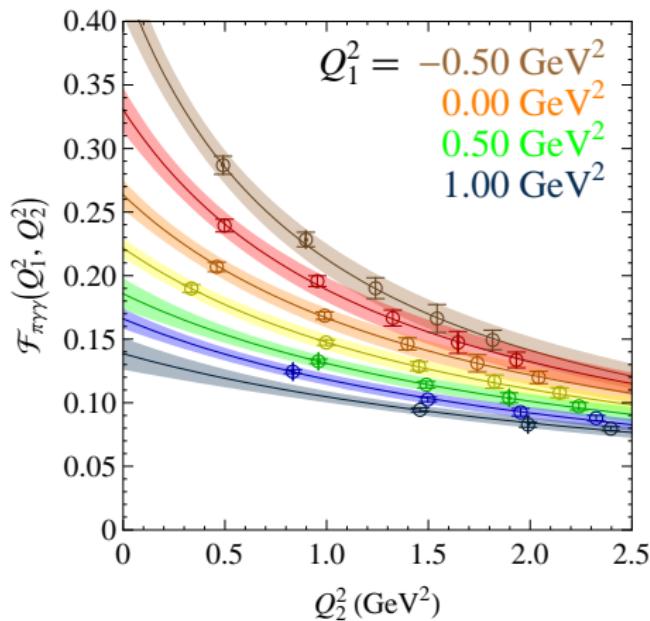


$$\mathcal{F}_{\pi\gamma\gamma}(Q_1^2, Q_2^2)$$

Monopole Fit

- We can set Q_1^2 arbitrarily.
Useful for photon fusion?
- The data are well described by a monopole fit:

$$\mathcal{F}(Q_1^2, Q_2^2) = \frac{F(Q_1^2)}{1 + Q_2^2/M_{\text{pole}}^2(Q_1^2)}$$

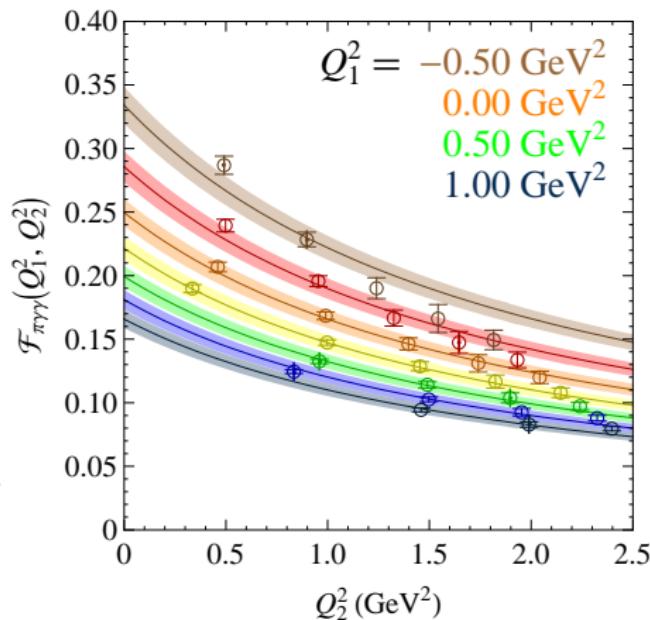


$\mathcal{F}_{\pi\gamma\gamma}(Q_1^2, Q_2^2)$

Double-Pole Fit

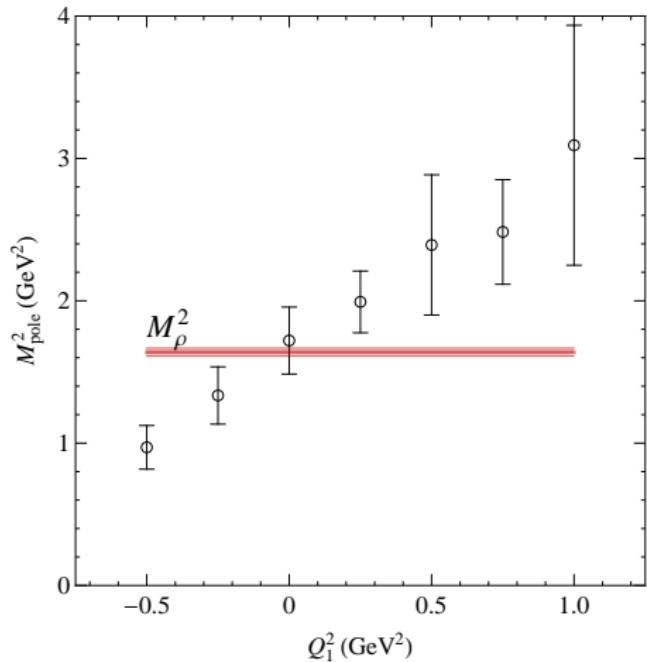
- Since there should be $Q_1 \leftrightarrow Q_2$ symmetry, we can try to fit all the data simultaneously to a double-pole form:

$$\mathcal{F}(Q_1^2, Q_2^2) = \frac{F}{(1 + Q_1^2/M_{\text{pole}}^2)(1 + Q_2^2/M_{\text{pole}}^2)}$$



Pole Mass

- We expect the pole mass to be approximately the rho mass due to vector-meson dominance.

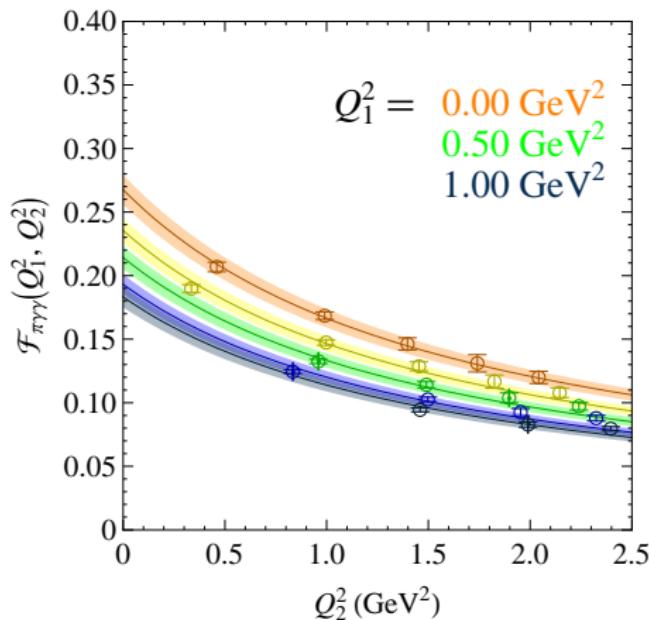


$$\mathcal{F}_{\pi\gamma\gamma}(Q_1^2, Q_2^2)$$

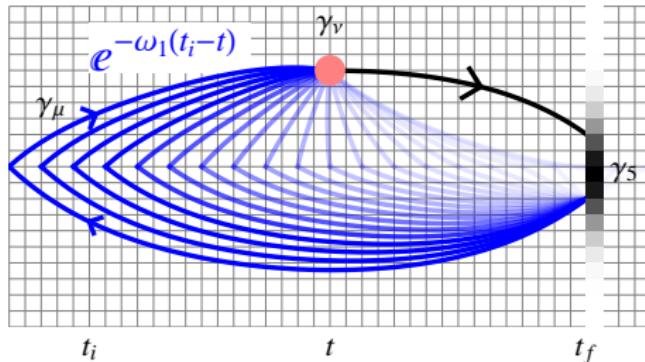
Rho-Pole Fit

- We fix the pole mass to the rho mass

$$\mathcal{F}(Q_1^2, Q_2^2) = \frac{F(Q_1^2)}{1 + Q_2^2/M_\rho^2}$$



Fast Method



- Fold the exponential and integral over t_i into a sequential source
- Disadvantages: Cannot directly examine integrand; cannot vary Q_1^2 (without recalculating sequential propagator)
- Advantages: T times faster than the slow method

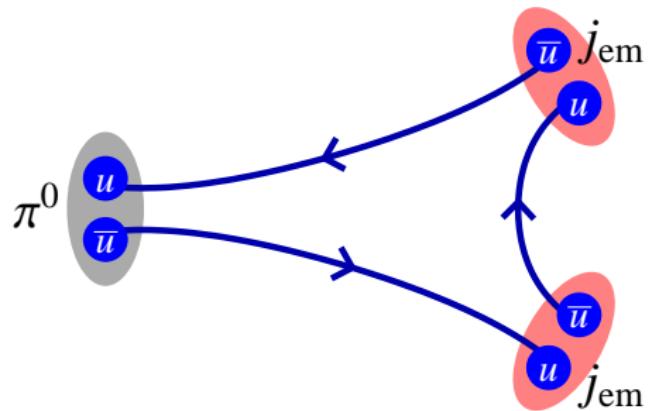
Summary

- Conclusions
 - The method of Jung & Ji allows access to the two-photon decays of neutral mesons.
 - We confirm the predictions of vector-meson dominance
- Previous Publications
 - Two-Photon Decays of Charmonia from Lattice QCD
Dudek & Edwards, PRL97:172001, 2006
- Future Work
 - Fast method with conserved currents
 - Calculation of two-photon decays of scalar and axial mesons

Quark Contractions

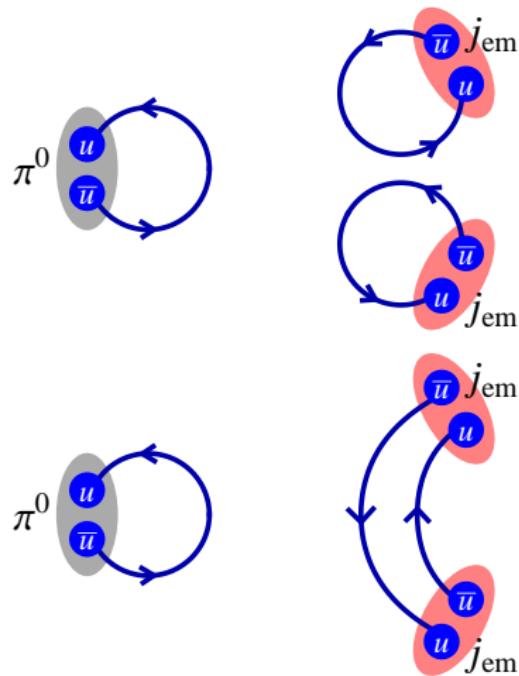
$$\langle \pi j^\mu j^\nu \rangle$$

$$\begin{aligned} & \left\langle \left(\frac{1}{\sqrt{2}} \bar{u} \gamma_5 u - \frac{1}{\sqrt{2}} \bar{d} \gamma_5 d \right) \right. \\ & \times \left(\frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d \right) \\ & \left. \times \left(\frac{2}{3} \bar{u} \gamma^\nu u - \frac{1}{3} \bar{d} \gamma^\nu d \right) \right\rangle \end{aligned}$$



Disconnected Diagrams That Cancel

- These terms cancel exactly under isospin symmetry



Remaining Diagrams

- These do not cancel
- We suspect they must be small, since $\frac{\Gamma(\omega \rightarrow \pi\gamma)}{\Gamma(\rho \rightarrow \pi\gamma)} \approx 9$, which is close to the value expected for the connected diagrams alone

