

# Rare B decays with moving NRQCD and improved staggered quarks

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with

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#### Some experimental results

$$B(B^+ \to K^{*+}\gamma) = (40.3 \pm 2.6) \times 10^{-6}$$
$$B(B \to K l^+ l^-) = (0.39 \pm 0.06) \times 10^{-6}$$
$$B(B \to K^* l^+ l^-) = \left(0.97 \begin{array}{c} +0.17\\ -0.16 \end{array}\right) \times 10^{-6}$$

[Heavy Flavor Averaging Group, April 2008]

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### Rare B decays

More interesting than "tree-level decays"

b → s is FCNC process, very sensitive to new physics



(figure adapted from SLAC today 9/2006)

### More difficult than "tree-level decays"

- Iong-distance and spectator effects
- vector meson final states
- large recoil momenta



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General framework for weak B decays

Standard Model (or beyond) L OPE  $C_i(M_W) \mathcal{Q}_i^{\text{cont}}(M_W)$ ↓ RG running  $C_i(m_b) \mathcal{Q}_i^{\text{cont}}(m_b)$ ↓ matching  $C_i(m_b)Z_{ij}(am_b)\mathcal{Q}_{ij}^{\mathsf{latt}}(a^{-1})$ Non-perturbative lattice computation of  $\langle F | \mathcal{Q}_{ij}^{\mathsf{latt}} | B \rangle$ 

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Parametrization of matrix elements in terms of form factors

$$\begin{split} B &\to K l^+ l^- \\ &\langle K(p') | \bar{s} \gamma^\mu b | \bar{B}(p) \rangle &= f_+(q^2) \left[ p^\mu + p'^\mu - \frac{M_B^2 - M_P^2}{q^2} q^\mu \right] \\ &\quad + f_0(q^2) \frac{M_B^2 - M_P^2}{q^2} q^\mu, \\ &q_\nu \langle K(p') | \bar{s} \sigma^{\mu\nu} b | \bar{B}(p) \rangle &= \frac{i f_T(q^2)}{M_B + M_P} \left[ q^2 (p^\mu + p'^\mu) - (M_B^2 - m_P^2) q^\mu \right] \end{split}$$

 $B \to K^* \gamma$ 

$$q^{\nu} \langle K^{*}(p',\varepsilon) | \bar{s}\sigma_{\mu\nu}b | \bar{B}(p) \rangle = 4 T_{1}(q^{2})\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^{\rho}p'^{\sigma}$$

$$q^{\nu} \langle K^{*}(p',\varepsilon) | \bar{s}\sigma_{\mu\nu}\gamma_{5}b | \bar{B}(p) \rangle = 2iT_{2}(q^{2}) \left[\varepsilon_{\mu}^{*}(M_{B}^{2}-M_{K^{*}}^{2}) - (\varepsilon^{*} \cdot q)(p+p')_{\mu}\right]$$

$$2iT_{3}(q^{2})(\varepsilon^{*} \cdot q) \left[q_{\mu} - \frac{q^{2}}{M_{B}^{2}-M_{K^{*}}^{2}}(p+p')_{\mu}\right]$$

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#### Parametrization of matrix elements in terms of form factors

### $B \to K^* l^+ l^-$

$$\begin{array}{lcl} q^{\nu} \langle K^{*}(p',\varepsilon) | \bar{s}\sigma_{\mu\nu}b | \bar{B}(p) \rangle &=& 4 \, T_{1}(q^{2})\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^{\rho}p'^{\sigma} \\ q^{\nu} \langle K^{*}(p',\varepsilon) | \bar{q}\sigma_{\mu\nu}\gamma_{5}b | \bar{B}(p) \rangle &=& 2i T_{2}(q^{2}) \left[ \varepsilon^{*}_{\mu}(M_{B}^{2}-M_{K^{*}}^{2}) - (\varepsilon^{*} \cdot q)(p+p')_{\mu} \right] \\ && 2i T_{3}(q^{2})(\varepsilon^{*} \cdot q) \left[ q_{\mu} - \frac{q^{2}}{M_{B}^{2} - M_{K^{*}}^{2}}(p+p')_{\mu} \right] \\ \langle K^{*}(p',\varepsilon) | \bar{s}\gamma^{\mu}b | \bar{B}(p) \rangle &=& \frac{2i V(q^{2})}{M_{B} + M_{K^{*}}} \epsilon^{\mu\nu\rho\sigma}\varepsilon^{*}_{\nu}p'_{\rho}p_{\sigma}, \\ \langle K^{*}(p',\varepsilon) | \bar{s}\gamma^{\mu}\gamma_{5}b | \bar{B}(p) \rangle &=& 2M_{K^{*}}A_{0}(q^{2}) \, \frac{\varepsilon^{*} \cdot q}{q^{2}} q^{\mu} \\ && + (M_{B} + M_{K^{*}}) \, A_{1}(q^{2}) \left[ \varepsilon^{*\mu} - \frac{\varepsilon^{*} \cdot q}{q^{2}} q^{\mu} \right] \\ && - A_{2}(q^{2}) \, \frac{\varepsilon^{*} \cdot q}{M_{B} + M_{K^{*}}} \left[ p^{\mu} + p'^{\mu} - \frac{M_{B}^{2} - M_{K^{*}}^{2}}{q^{2}} q^{\mu} \right] \end{array}$$

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#### Non-perturbative lattice computation

Interpolating fields for  $\langle F(p') | \mathcal{Q}^{\mathsf{latt}} | \bar{B}(p) \rangle$ :

$$\begin{split} \Phi_B(x) &= \overline{\Psi}_q(x)\gamma_5\Psi_b(x), \\ \Phi_F(x) &= \overline{\Psi}_q(x)\gamma_F\Psi_s(x), \qquad \gamma_F = \gamma_j, \ \gamma_5 \\ \mathcal{Q}^{\mathsf{latt}}(x) &= \overline{\Psi}_s(x)\Gamma_\mathcal{Q}\Psi_b(x) \end{split}$$



Correlators we need:

$$C_{FQB}(\mathbf{p}', \mathbf{p}, x_0, y_0, z_0) = \sum_{\mathbf{y}} \sum_{\mathbf{z}} \left\langle \Phi_F(x) \ Q^{\mathsf{latt}}(y) \ \Phi_B^{\dagger}(z) \right\rangle e^{-i\mathbf{q} \cdot (\mathbf{y} - \mathbf{x})} e^{i\mathbf{p} \cdot (\mathbf{z} - \mathbf{x})}$$

$$C_{BB}(\mathbf{p}, x_0, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_B(x) \ \Phi_B^{\dagger}(y) \right\rangle e^{-i\mathbf{p} \cdot (\mathbf{x} - \mathbf{y})}$$

$$C_{FF}(\mathbf{p}', x_0, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_F(x) \ \Phi_F^{\dagger}(y) \right\rangle e^{-i\mathbf{p}' \cdot (\mathbf{x} - \mathbf{y})}$$

## Matrix elements (and energies) from correlators

For large 
$$|x_0 - y_0|$$
 and  $|y_0 - z_0|$   

$$C_{FQB} \longrightarrow e^{-E_F|x_0 - y_0|} e^{-E_B|y_0 - z_0|} A_{FQB},$$

$$C_{BB} \longrightarrow e^{-E_B|x_0 - y_0|} A_{BB},$$

$$C_{FF} \longrightarrow e^{-E_F|x_0 - y_0|} A_{FF},$$

$$A_{FQB} = \begin{cases} \frac{\sqrt{Z_{K^*}}}{2E_{K^*}} \frac{\sqrt{Z_B}}{2E_B} \sum_{s} \varepsilon_j(p', s) \langle K^*(p', \varepsilon(p', s)) | \mathcal{Q} | \bar{B}(p) \rangle, & F = K^*, \\ \frac{\sqrt{Z_K}}{2E_K} \frac{\sqrt{Z_B}}{2E_B} \langle K(p') | \mathcal{Q} | \bar{B}(p) \rangle, & F = K \end{cases}$$

$$A_{BB} = \frac{Z_B}{2E_B}$$

$$A_{FF} = \begin{cases} \sum_{s} \frac{Z_{K^*}}{2E_K^*} \epsilon_j^*(p', s) \epsilon_j(p', s), & F = K^*, \\ \frac{Z_K}{2E_K}, & F = K. \end{cases}$$

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### Fermion Actions

## Light quarks (u, d and s): AsqTad or HISQ

MILC configurations with sea quark masses in chiral regime

#### *b* quark: moving NRQCD

[Mandula & Ogilvie, Hashimoto & Matsufuru, Sloan, Davies-Dougall-Foley-Lepage-Wong]

- allows us to work directly at the physical b quark mass
- ▶ allows us to work at lower  $q^2$  compared to standard NRQCD, but  $M_B^2 q^2 \ll M_B^2$  is still required for convergence of heavy-quark expansion

- requires  $am_b > 1$
- applicable to both HL and HH mesons

Moving NRQCD: reducing discretization errors at large recoil

- ▶ give B meson large momentum to reduce final state momentum
- perform NRQCD expansion in B rest frame, boost to lattice frame and discretize



momentum of light degrees of freedom in B meson and residual momentum of b quark are of order

 $\gamma(v+1)a\Lambda_{QCD}$ 

(v – boost velocity)

## moving-NRQCD field redefinition (on Minkowski space)

$$\Psi_b(x) = S(\Lambda) \quad T_{\rm FWT} \quad e^{-im \, u \cdot x \, \gamma^0} \quad T_{\rm TD} \quad \frac{1}{\sqrt{\gamma}} \left( \begin{array}{c} \psi_v(x) \\ \xi_v(x) \end{array} \right)$$

with

$$T_{\rm FWT} = \exp\left(\frac{1}{2m} i\gamma^j \Lambda^{\mu}{}_j D_{\mu}\right) \dots$$

(FWT transformation in boosted frame),

$$T_{\rm TD} = \exp\left(\frac{i}{4\gamma m}\gamma^0 \left[(\gamma^2 - 1)D_0 + (\gamma^2 + 1)\mathbf{v}\cdot\mathbf{D}\right]\right)\dots$$

(removes unwanted time derivatives), and  $S(\Lambda)$  is Dirac spinor representation of Lorentz boost.

Euclidean mNRQCD Lagrangian correct through  $O(\Lambda_{QCD}^2/m^2)$  (HL) and  $O(v_{NR}^4)$  (HH)

$$\mathcal{L} = \psi_v^+ \left( D_4 + H_0 + \delta H \right) \psi_v + \xi_v^+ \left( D_4 - \overline{H_0} - \overline{\delta H} \right) \xi_v$$

with

$$\begin{split} H_{0} &= -i\mathbf{v}\cdot\mathbf{D} - \frac{\mathbf{D}^{2} - (\mathbf{v}\cdot\mathbf{D})^{2}}{2\gamma m} \\ \delta H &= -\frac{i}{4\gamma^{2}m^{2}}\left(\left\{\mathbf{D}^{2}, \ \mathbf{v}\cdot\mathbf{D}\right\} - 2(\mathbf{v}\cdot\mathbf{D})^{3}\right) \\ &+ \frac{1}{8\gamma^{3}m^{3}}\left(-\mathbf{D}^{4} + 3\left\{\mathbf{D}^{2}, \ (\mathbf{v}\cdot\mathbf{D})^{2}\right\} - 5(\mathbf{v}\cdot\mathbf{D})^{4}\right) \\ &- \frac{g}{2\gamma m}\boldsymbol{\sigma}\cdot\mathbf{B}' - \frac{g}{8\gamma m^{2}} \ \boldsymbol{\sigma}\cdot\left(\mathbf{D}\times\mathbf{E}'-\mathbf{E}'\times\mathbf{D}\right) \\ &- \frac{ig}{4\gamma^{2}m^{2}}\left\{\mathbf{v}\cdot\mathbf{D}, \ \boldsymbol{\sigma}\cdot\mathbf{B}'\right\}. \\ &+ \frac{g}{8(\gamma+1)m^{2}}\left\{\mathbf{v}\cdot\mathbf{D}, \ \boldsymbol{\sigma}\cdot(\mathbf{v}\times\mathbf{E}')\right\} \\ &+ \frac{g}{8m^{2}}\left(i\mathbf{D}^{\mathrm{ad}}\cdot\mathbf{E} + \mathbf{v}\cdot(\mathbf{D}^{\mathrm{ad}}\times\mathbf{B})\right) \\ &- \frac{(2-\mathbf{v}^{2})g}{16m^{2}}\left(D_{4}^{\mathrm{ad}} + i\mathbf{v}\cdot\mathbf{D}^{\mathrm{ad}}\right)(\mathbf{v}\cdot\mathbf{E}). \end{split}$$

#### mNRQCD external momentum renormalisation and energy shift

The residual B momentum is discretized on the lattice,

$$k_j = \frac{2\pi n_j}{L}$$

The physical momentum is given by

$$\mathbf{p} = \mathbf{k} + Z_p \mathbf{P}_0$$

with

$$\mathbf{P}_0 = \gamma m \mathbf{v}, \quad Z_p \approx 1.$$

B meson correlators decay with ground state energies

$$E_{\mathbf{v}}(\mathbf{k}) = \underbrace{\sqrt{(Z_p \mathbf{P}_0 + \mathbf{k})^2 + M_B^2}}_{\text{physical energy}} - \Delta_{\mathbf{v}}$$

with an energy shift  $\Delta_{\mathbf{v}}$ . In perturbation theory,

$$\Delta_{\mathbf{v}} = Z_m Z_\gamma \gamma m - E_0$$

#### Perturbative computation of renormalization parameters

▶  $E_0$ ,  $Z_{\psi}$ ,  $Z_m$ ,  $Z_v$  and  $Z_p$  can be extracted from heavy-quark self energy



$$E_{0} = 1 + \alpha_{s} \operatorname{Re}\{\Sigma\}|_{p=0}$$

$$Z_{\psi} = 1 + \alpha_{s} \left(\operatorname{Re}\{\Sigma\} + \operatorname{Im}\{\frac{\partial\Sigma}{\partial p_{0}}\}\right)\Big|_{p=0}$$

$$\vdots$$

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▶ computed to 1-loop for full  $\mathcal{O}(\Lambda^2_{QCD}/m^2)$  action [Lew Khomskii 2008]

Perturbative computation of renormalization parameters



Full improved  $O(\Lambda_{QCD}^2/m^2)$  mNRQCD action with Lüscher-Weisz gluon action, m = 2.8, n = 4. All errors are  $O(10^{-3})$ 

Preliminary data from [Lew Khomskii 2008]

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### Perturbative matching of vector and axial vector currents



 Computation of matching coefficients for all lattice operators up to O(Λ<sub>QCD</sub>/m) is underway [Lew Khomskii 2008]

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Lattice operators for temporal axial current [Lew Khomskii 2008]

$$\begin{split} I_{1} &= f_{0}(1+\gamma_{(0)})\,\bar{q}_{0}(x)\hat{\gamma}_{5}Q_{0}(x)\,,\\ I_{2} &= if_{0}\gamma_{(0)}\,\bar{q}_{0}(x)\hat{\gamma}_{5}\hat{\gamma}\cdot\boldsymbol{v}_{0}Q_{0}(x)\,,\\ I_{3} &= -if_{0}(1-\gamma_{(0)}^{2})\,\frac{1}{2m_{0}}\,\bar{q}_{0}(x)\hat{\gamma}_{5}\boldsymbol{v}_{0}\cdot\boldsymbol{D}Q_{0}(x)\,,\\ I_{4} &= -f_{0}\gamma_{(0)}(1+\gamma_{(0)})\,\frac{1}{2m_{0}}\,\bar{q}_{0}(x)\hat{\gamma}_{5}(\hat{\gamma}\cdot\boldsymbol{v}_{0})(\boldsymbol{v}_{0}\cdot\boldsymbol{D})Q_{0}(x)\,,\\ I_{5} &= \frac{f_{0}(1+\gamma_{(0)})}{2m_{0}}\,\bar{q}_{0}(x)\hat{\gamma}\cdot\boldsymbol{D}\hat{\gamma}_{5}Q_{0}(x)\,,\\ I_{6} &= \frac{if_{0}\gamma_{(0)}}{2m_{0}}\,\bar{q}_{0}(x)(\hat{\gamma}\cdot\boldsymbol{D})\hat{\gamma}_{5}(\hat{\gamma}\cdot\boldsymbol{v}_{0})Q_{0}(x)\,,\\ I_{7} &= -\frac{f_{0}(1+\gamma_{(0)})}{m_{0}}\,\bar{q}_{0}(x)(\hat{\gamma}\cdot\boldsymbol{D})\hat{\gamma}_{5}(\hat{\gamma}\cdot\boldsymbol{v}_{0})Q_{0}(x)\,,\\ I_{8} &= -\frac{if_{0}\gamma_{(0)}}{m_{0}}\,\bar{q}_{0}(x)(\hat{\gamma}\cdot\boldsymbol{D})\hat{\gamma}_{5}(\hat{\gamma}\cdot\boldsymbol{v}_{0})Q_{0}(x)\,,\\ I_{9} &= if_{0}(1+\gamma_{(0)})\,\frac{1}{2m_{0}}\,\bar{q}_{0}(x)(\boldsymbol{v}_{0}\cdot\boldsymbol{D})\hat{\gamma}_{5}(\hat{\gamma}\cdot\boldsymbol{v}_{0})Q_{0}(x)\,,\\ I_{10} &= -f_{0}\gamma_{(0)}\,\frac{1}{2m_{0}}\,\bar{q}_{0}(x)(\boldsymbol{v}_{0}\cdot\boldsymbol{D})\hat{\gamma}_{5}(\hat{\gamma}\cdot\boldsymbol{v}_{0})Q_{0}(x)\,.\\ \end{split}$$

Perturbative matching of tensor current

 matching coefficients computed to 1 loop for leading order operators [Eike Müller 2008]

$$\mathcal{Q}_{7,1}^{0\ell} = \sqrt{\frac{1+\gamma}{2\gamma}} m\left(\bar{q}\sigma_{0\ell}\tilde{\Psi}_{v}^{(+)}\right)$$
$$\mathcal{Q}_{7,2}^{0\ell} = v\sqrt{\frac{\gamma}{2(1+\gamma)}} m\left(\bar{q}\sigma_{0\ell}\hat{\boldsymbol{v}}\cdot\boldsymbol{\gamma}\gamma_{0}\tilde{\Psi}_{v}^{(+)}\right)$$

$$\begin{aligned} \mathcal{Q}_{7}^{(lat)0\ell} &= (1 + \alpha_{s}c_{1}^{0\ell})\mathcal{Q}_{7,1}^{0\ell} + (1 + \alpha_{s}c_{2}^{0\ell})\mathcal{Q}_{7,2}^{0\ell} \\ &= (1 + \alpha_{s}c_{+}^{0\ell})\mathcal{Q}_{7,+}^{0\ell} + \alpha_{s}c_{-}^{0\ell}\mathcal{Q}_{7,-}^{0\ell} \end{aligned}$$

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with  $\mathcal{Q}_{7,\pm}^{0\ell}=\mathcal{Q}_{7,1}^{0\ell}\pm\mathcal{Q}_{7,2}^{0\ell}$ 

Perturbative matching of tensor current (preliminary)

$$\mathcal{Q}_{7}^{(lat)0\ell} = (1 + \alpha_{s}c_{+}^{0\ell})\mathcal{Q}_{7,+}^{0\ell} + \alpha_{s}c_{-}^{0\ell}\mathcal{Q}_{7,-}^{0\ell}$$



Improved  $O(\Lambda_{QCD}/m)$  mNRQCD action with Symanzik improved gluon action, m = 2.8, n = 2 [Eike Müller 2008]

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- ▶ tadpole-improved  $O(v_{NR}^4, \Lambda_{QCD}^2/m_b^2)$  moving NRQCD action
- ► MILC  $20^3 \times 64$ , lattice spacing  $a^{-1} \approx 1.6$  GeV, sea quark masses  $am_u = am_d = 0.007$ ,  $am_s = 0.05$  ( $m_\pi \sim 300$  MeV)
- ▶ valence quark masses  $am_u = am_d = 0.007$ ,  $am_s = 0.04$ ,  $a m_b = 2.8$
- $\blacktriangleright \mathbf{v} = (v, 0, 0)$
- compute heavy-heavy and heavy-light meson energies and decay constants at different boost velocities
- heavy-heavy mesons: smearing with hydrogen wave functions in Coulomb gauge

- heavy-light mesons: use AsqTad valence quarks, Gaussian smearing in Coulomb gauge
- Bayesian multi-exponential fitting

Recall that

$$E_{\mathbf{v}}(\mathbf{k}) = \underbrace{\sqrt{(Z_p \mathbf{P}_0 + \mathbf{k})^2 + M^2}}_{\text{physical energy}} - \Delta_{\mathbf{v}}.$$

 $\Rightarrow$  compute

$$\begin{split} \Delta_{\mathbf{v}} &= \frac{\mathbf{k}_{\perp}^2 - \left(E_{\mathbf{v}}^2(\mathbf{k}_{\perp}) - E_{\mathbf{v}}^2(0)\right)}{2(E_{\mathbf{v}}(\mathbf{k}_{\perp}) - E_{\mathbf{v}}(0))}, \\ Z_p &= \frac{E_{\mathbf{v}}^2(\mathbf{k}_{\parallel}) - E_{\mathbf{v}}^2(-\mathbf{k}_{\parallel}) + 2\Delta_{\mathbf{v}}(E_{\mathbf{v}}(\mathbf{k}_{\parallel}) - E_{\mathbf{v}}(-\mathbf{k}_{\parallel}))}{4\mathbf{k}_{\parallel} \cdot \mathbf{P}_{\mathbf{0}}}, \\ M_{\mathrm{kin}} &= \sqrt{(E_{\mathbf{v}}(\mathbf{k}) + \Delta_{\mathbf{v}})^2 - (Z_p \mathbf{P}_{\mathbf{0}} + \mathbf{k})^2}. \end{split}$$

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#### Results for $\eta_b$ :

v	$Z_p$	$M_{\rm kin}$	$\Delta_v$
0		$6.240\pm0.033$	$5.815 \pm 0.033$
0.2	$1.029\pm0.015$	$6.40 \pm 0.10$	$6.09 \pm 0.10$
0.4	$1.014\pm0.069$	$6.28 \pm 0.42$	$6.37 \hspace{0.2cm} \pm \hspace{0.2cm} 0.45 \hspace{0.2cm}$
0.6	$0.929 \pm 0.062$	$6.47 \hspace{0.2cm} \pm \hspace{0.2cm} 0.41 \hspace{0.2cm}$	$7.24 \hspace{0.2cm} \pm \hspace{0.2cm} 0.49 \hspace{0.2cm}$

(lattice units)

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Energy splittings vs boost velocity (points are offset horizontally for legibility)



B and  $B_s$  decay constants  $\langle 0|\mathsf{A}^\mu(0)|\bar{B}_{(s)},\mathbf{p}\rangle=if_{B_{(s)}}\;p^\mu$  at different boost velocities

 $f_{B_s}$  vs. total momentum  $\mathbf{p}=\mathbf{k}+Z_p\gamma m\mathbf{v}$  (lattice units,  $Z_ppprox 1$ )



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- ▶ O(Λ<sub>QCD</sub>/m<sub>b</sub>) moving NRQCD action, leading current operators only
- ▶ MILC  $20^3 \times 64$ , lattice spacing  $a^{-1} \approx 1.6$  GeV, sea quark masses  $am_u = am_d = 0.007$ ,  $am_s = 0.05$  ( $m_\pi \sim 300$  MeV)
- ▶ valence quark masses  $am_u = am_d = 0.007$ ,  $am_s = 0.04$ ,  $a m_b = 2.8$
- 3-point functions must be fitted to

$$C_{FJB}(\mathbf{p}', \mathbf{p}, t, T) \to \sum_{\substack{k=0..K, \\ l=0..L}} A_{kl}^{(FJB)} \cdot (-1)^{kt} (-1)^{l(T-t)} e^{-F_k t} e^{-E_l(T-t)}$$

(oscillating contributions due to use of naive (AsqTad) quarks)

- Bayesian fitting
- $\blacktriangleright$  2-variable fits, varying both t and T







(not all data shown for clarity)



The points at lowest  $q^2$  have  ${\bf v}=0.4$ ,  ${\bf k}_{({\bf p})}=\frac{2\pi}{L}(1,0,0),~{\bf k}_{({\bf q})}=\frac{2\pi}{L}(2,0,0)$ 

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The points at lowest  $q^2$  have  $\mathbf{v} = 0.2$ ,  $\mathbf{k}_{(\mathbf{p})} = \frac{2\pi}{L}(1,0,0)$ ,  $\mathbf{k}_{(\mathbf{q})} = \frac{2\pi}{L}(2,0,0)$ 

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## Outlook

- 3-point fits shown here are just first attempts and can probably be improved
- ▶ for more data points and lower q<sup>2</sup>, need to work with off-axis lattice momenta and boost velocities
- ► random wall sources [Kit Wong, Lattice 2007] will improve statistics → larger K, K\* momentum and higher v

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smearing will reduce excited state contaminations