Extraction of Resonance Parameter

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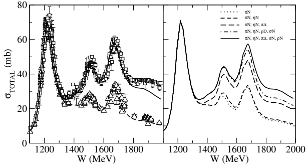
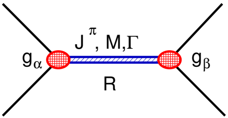
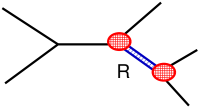
Introduction





formation process



production process

Characterize Resonances:

- Excitation spectrum Baryon : $J^{P,T}, M, \Gamma$
- Coupling constant : g_α, g_β, Branching ratio, electromagnetic form factor

Contents

Partial wave amplitude (PWA) $\rightarrow$ Resonance parameters

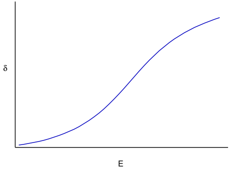
GW-VPI,Bonn-Gatchina,Jlab-Yerevan,MAID,CMB/Pitt-ANL,Zagreb,Giessen,KENT
Julich-Georgia, DMT,KVI,EBAC

- Breit-Wigner formula and pole of S-matrix
- Extraction of resonance parameters
- Simple exercise for extracting resonance parameter from ideal PWA
- Understanding resonance parameters

Breit-Wigner formula and pole of S-matrix

Breit-Wigner Formula

Resonance : $d\delta/dE$ has sharp maximum
(elastic scattering)



Breit-Wigner formula

$$T = \frac{e^{2i\delta_b}\Gamma_{BW}/2}{M_{BW} - E - i\Gamma_{BW}/2} + B$$

- resonance mass M_{BW}, width Γ_{BW}
- $\Gamma_{BW}, M_{BW}, B, \delta_b$ are E-independent constants

Breit-Wigner Formula

Extension for Multi-channel : Generalized BW formula

(Davies Baranger, McVoy)

$$T_{\beta,\alpha} = \frac{\gamma_{BW,\beta}\gamma_{BW,\alpha}}{M_{BW} - E - i\Gamma_{BW}/2} + B_{\alpha,\beta}$$

- Require unitarity assuming E-independent parameters

$$\gamma_{BW,\alpha} = e^{i\delta_\alpha}\sqrt{\Gamma_{BW,\alpha}/2}$$

$$\Gamma_{BW,\alpha} : \text{partial width}, \; \Gamma_{BW} = \sum_\alpha \Gamma_{BW,\alpha}$$

Pole

Resonance : pole of S-matrix on unphysical sheet

elastic scattering amplitude near pole position (Laurant expansion)

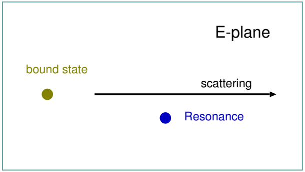
$$T = \frac{R}{M_P - E - i\Gamma_P/2} + B(E)$$

- Resonance mass M_P and width Γ_P.

  $M_P = M_{BW}, \Gamma_P = \Gamma_{BW}$
- If $B(E) = B$ is a good approximation

  $R \to e^{2i\delta_B}\Gamma_{BW}/2$
- multi-channel case:

  $R \to \gamma_\alpha \gamma_\beta$, multi-Riemann sheets structure



Pole

coupling constant, form factor from residue of amplitude at pole

$$\gamma_{em} =< \psi_{Res} | j_{em} | \psi_{Gr} >$$

- Resonance 'wave function' : 'Eigen state' of Hamiltonian with non-hermite outgoing boundary condition.(Siegert, Dalitz)

$$\partial \psi_{Res} / \partial r_\beta - i p_\beta \psi_{Res} |_\infty = 0$$

- γ_α need not be real

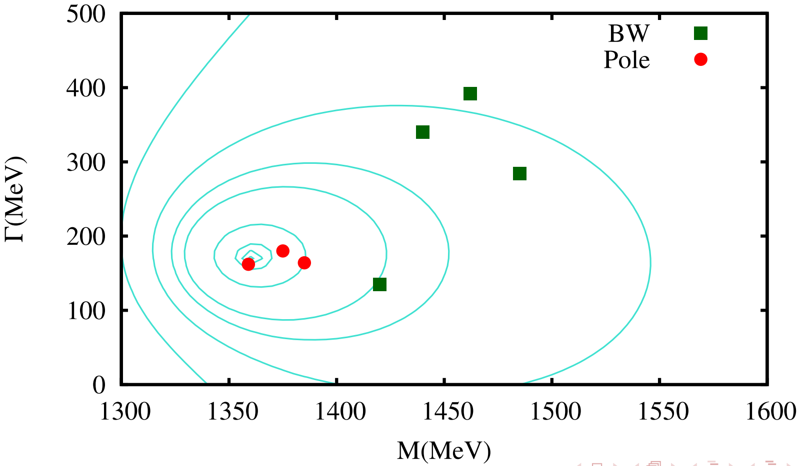
$$|\psi_{Res} >= |'bound' > + |'scattering' >$$

well defined resonance parameters can be a starting point to contact with hadron models.

Extraction of resonance parameters

P11

Mass and Width of P_{11} resonance from PDG



Breit-Wigner parametrization

BW parameters in practice

$$T = \frac{R(E)}{M_{BW} - E - i\Gamma(E)/2} + B(E)$$

- energy dependent $B(E), \Gamma(E)$

$$\Gamma(E) = (p/p_0)^{2l+1}\Gamma_{BW}$$

- K-matrix approach: invent recipe to match BW form
- $M_P < M_{BW}, \Gamma_P < \Gamma_{BW}$ (Lichtenberg, Manley)

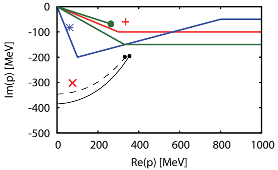
$$M_P \sim M_{BW} - \Gamma_{BW}/2(\alpha/(1+\alpha^2)), \alpha = \Gamma'/2$$

Pole parametrization

Pole and residue are automatically obtained from PWA of K-matrix(on-shell), dynamical(full off-shell dynamics) by analytic continuation of the amplitude on unphysical sheet

- K-matrix: use appropriate 'on-shell' momentum (Bonn-Gatchina, VPI,Giessen)

$$T = K \frac{1}{1 - i\rho K}$$



- Dynamical model:
  choose appropriate path of integration
  (EBAC, Juelich)

$$T(p', p; E) = V(p', p) + \int_C dqq^2 V(p', q) G_0(q; E) T(q, p; E)$$

for un-stable particle final state: need to take care of 3-body intermediate state.

Simple exercise

Simple Exercise

Stability of resonance parameters extracted from PWA

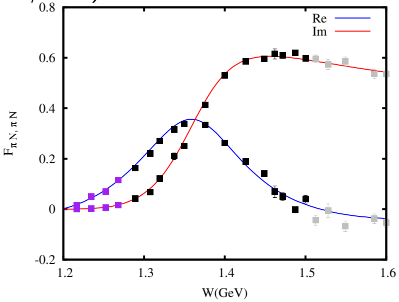
- Input: $T(E_i)$ from $\pi N - \pi N$ amplitude of VPI
- Output: $T(E)$ calculate pole and residue without physics input

Calculate $T(E)$ from known $T(E_i) i = 1, ...N$ (continued fraction)

$$T(E) = \frac{T(E_1)}{1+} \frac{a_1(E - E_1)}{1+} \frac{a_2(E - E_2)}{1+} .. \quad a_1 = \frac{T(E_1)/T(E_2) - 1}{E_2 - E_1}, ...$$

(Schlessinger)

πN input amplitude (GW/VPI)



Output:

Input	$M_P - i\Gamma_P/2$(MeV)	Residue(MeV)
E-dep [1210-1500] 59pt	1362 -89i	4.9 -44i
E-dep [1300-1500] 41pt	1362 -90i	4.6 -45i
E-ind [1217-1500] 17pt	1366 -128i	10 -68i
E-ind [1289-1500] 13pt	1347 -93i	-11 -52i

Simple Exercise

	$M_P - i\Gamma_P/2$(MeV)	Residue(MeV)
E-dep [1200-1500] 59pt	1362 -89i	4.9 -44i
Bonn-Gatchina	1377 -85i	5.7 -47i
EBAC	1357 -76i/ 1364 -83i	37 -110i/66 -99i
VPI/GW	1359 -82i/1388 -83i	38 -98i/ 86 -46i
Juelich	1387 -71i/1387 -74i	48 -64i/

- Obtained pole position is more stable than residue. In practice, PWA is obtained within certain accuracy. Resonance parameters may not be well determined from PWA such as E-ind PWA alone.
- Theoretical input (Tree diagram+ background, K-matrix, Dispersion relation, dynamical approach) is needed in extracting resonance parameters.

Branching ratio

Residue of the Pole gives coupling constant($D_{13}(1520)$)

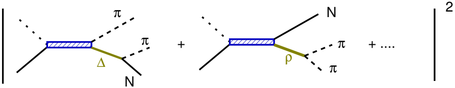
$$T_{res} = \frac{\gamma_\beta \gamma_\alpha}{M_P - E - i\Gamma_P/2}$$

B_α	πN	ηN	$\pi\Delta$	σN	ρN
EBAC	65%	0.02	33	4	1
Manley92	59	0	20	0	21
Vrana00	63	0	26	1	9

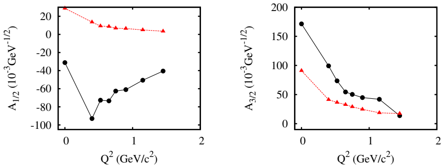
	$A_{3/2}$	$A_{1/2}$
EBAC	125 + 25i	-42 + 8i
Bonn-Gatchina	130 + 14i	-30+8i

($Gev^{-1/2} \times 10^3$)

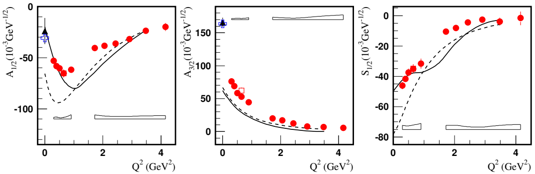
$B_\alpha = |\gamma_\alpha|^2/\Gamma_P$: (effective phase space factor for unstable particle channel $\pi\Delta$..)



Q^2 Dependence of A1/2,A3/2 (D13)



EBAC

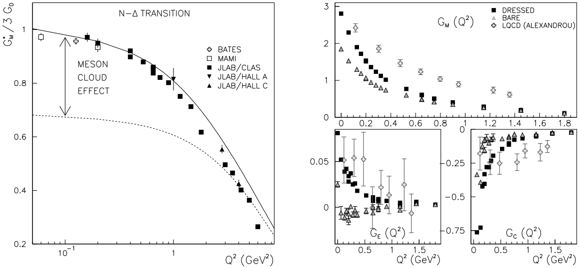


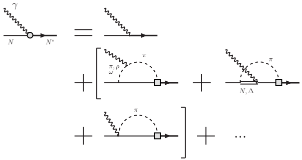
Jlab CLAS

Understanding resonance parameters

understanding resonance parameters

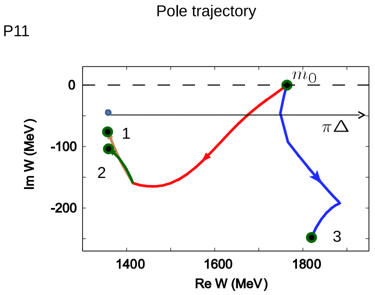
Example 1: $N\Delta$ electromagnetic form factor





- meson-nucleon continuum component is part of resonance property

Example 2: P11 resonance Roper



Analysis of EBAC

- reaction mechanism modifies resonance energy

  $m_{P11} = 1.76$GeV (Dyson-Schwinger H. L. L. Roberts et al.)

understanding resonance parameters

- Resonances are characterized by the pole and residue of the PWA
- reaction dynamics is part of the resonance properties (mass, couping constant, extracted resonance parameters). Resulting coupling constants are complex number.
- PWA analysis never gives us 100% accurate amplitudes for the whole energy region. Theoretical inputs on reaction dynamics are unavoidable/necessary both in extracting PWA, extraction of resonance parameters and to understand resonance.
- Combined analysis of 'reaction theory' + 'structure model of hadron' is one promising approach to understand resonance parameters.