Transverse Momentum Dependent Distribution Functions of Definite Rank

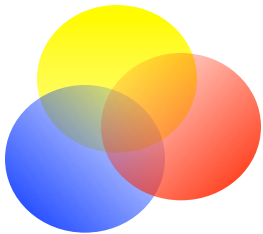Piet Mulders

mulders@few.vu.nl
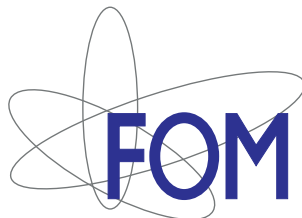
ABSTRACT

## TMDs of definite rank

Piet Mulders (Nikhef/VU University Amsterdam)

Transverse momentum dependent (TMD) parton distribution functions include path-dependent Wilson lines. Using moments in x and pT, it is possible to study the operator structure which is relevant for evolution as well as for modelling of TMDs or lattice calculations. Using the moments it is possible to categorize the TMDs according to their rank which labels the relevant azimuthal behavior in pT. For quark TMDs of rank 2, such as the Pretzelocity TMD, and gluon TMDs of rank 1 and higher, such as the TMD describing linear polarization of gluons, one finds multiple TMDs depending on the color structure of the operators. The explicit appearance of these TMDs in scattering processes, including diffractive scattering, involves factors depends on the color flow in the process in two ways, namely a factor depending on the gluonic rank of the TMD as well as an additional process-dependent factor if multiple TMDs are involved in a process, such as double Sivers asymmetries in the Drell-Yan process.

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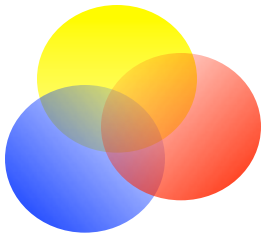
Starting point

- **TMD's**: color gauge invariant correlators, describing distribution and fragmentation functions including partonic transverse momentum.

- Nonlocality in field operators including transverse directions

- Observable in azimuthal dependence, i.e. noncollinearity in hard processes (convolution in kT)

- Transverse separation complicates gauge link structure

- TMDs encode novel aspects of hadronic structure, e.g. spin-orbit correlations, such as T-odd transversely polarized quarks or T-even longitudinally polarized gluons in an unpolarized hadron, thus possible applications for precision probing at the LHC, but for sure at a polarized EIC.
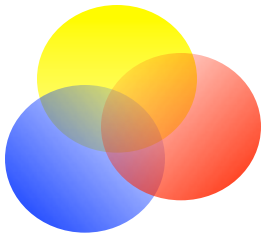
(Un)integrated correlators

$$\Phi(x, p_T, p.P) = \int \frac{d^4\xi}{(2\pi)^4} e^{i p.\xi} \left\langle P \middle| \bar{\psi}(0)\, \psi(\xi) \middle| P \right\rangle$$

■ unintegrated

$$\Phi(x, p_T; n) = \int \frac{d(\xi.P) d^2\xi_T}{(2\pi)^3} e^{i p.\xi} \left\langle P \middle| \bar{\psi}(0)\, \psi(\xi) \middle| P \right\rangle_{\xi.n = \xi^+ = 0}$$
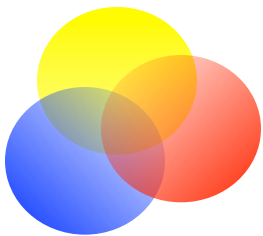
■ TMD (light-front)

   ■ σ = p⁻ integration makes time-ordering automatic.
     The soft part is simply sliced at the light-front

$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i p.\xi} \left\langle P \middle| \bar{\psi}(0)\, \psi(\xi) \middle| P \right\rangle_{\xi.n = \xi_T = 0 \;\; or \;\; \xi^2 = 0}$$

   ■ Is already equivalent to a point-like interaction

■ collinear (light-cone)

$$\Phi = \left\langle P \middle| \bar{\psi}(0)\, \psi(\xi) \middle| P \right\rangle_{\xi = 0}$$

■ local

   ■ Local operators with calculable anomalous dimension

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Simplest gauge links for quark TMDs

$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \langle P | \overline{\psi}_j(0) U_{[0,\xi]}^{[C]} \psi_i(\xi) | P \rangle_{\xi.n=0}$$
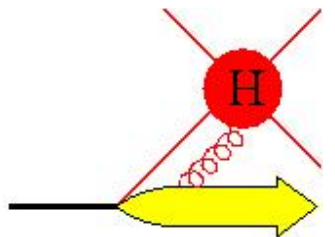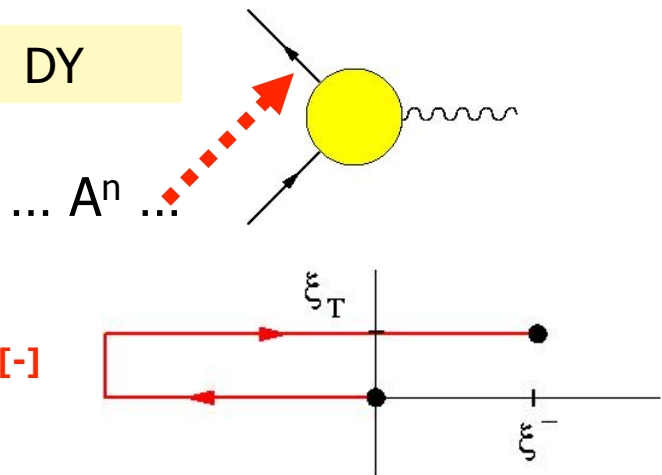
TMD

$$\Phi_{ij}^q(x; n) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \langle P | \overline{\psi}_j(0) U_{[0,\xi]}^{[n]} \psi_i(\xi) | P \rangle_{\xi.n=\xi_T=0}$$
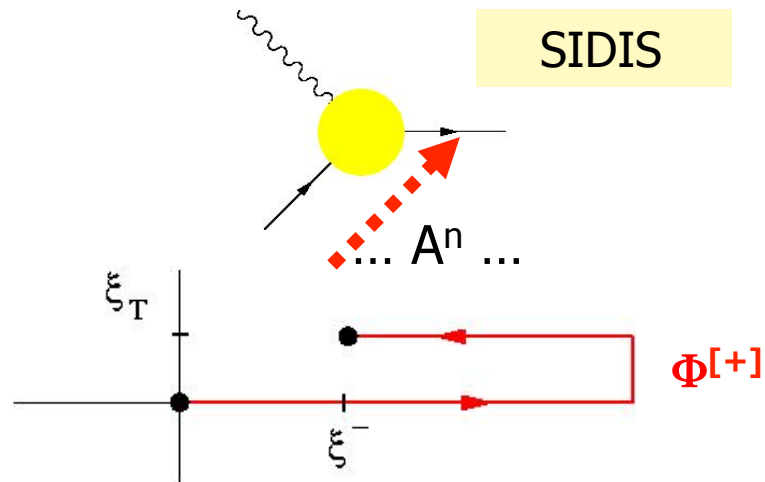
collinear

◆ Gauge links come from dimension zero (not suppressed!) collinear A.n gluons, but leads for TMD correlators to process-dependence:

DY
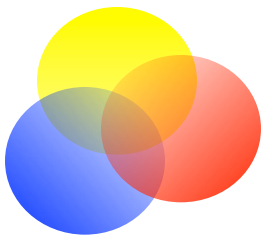
SIDIS

... A^n ...
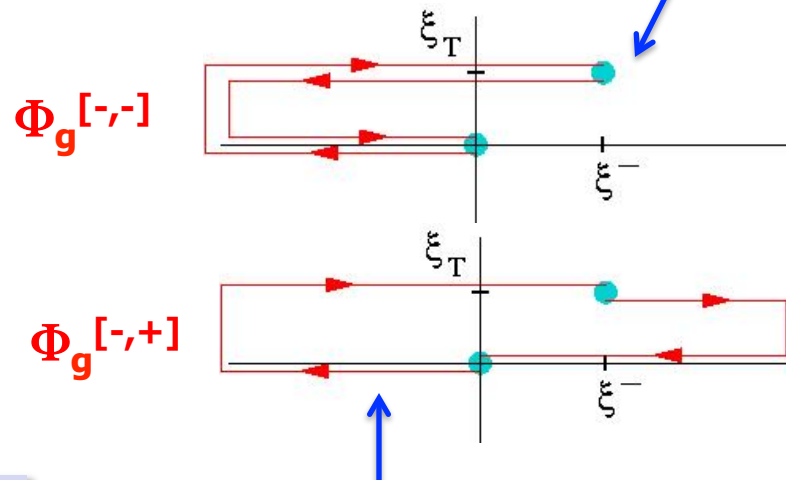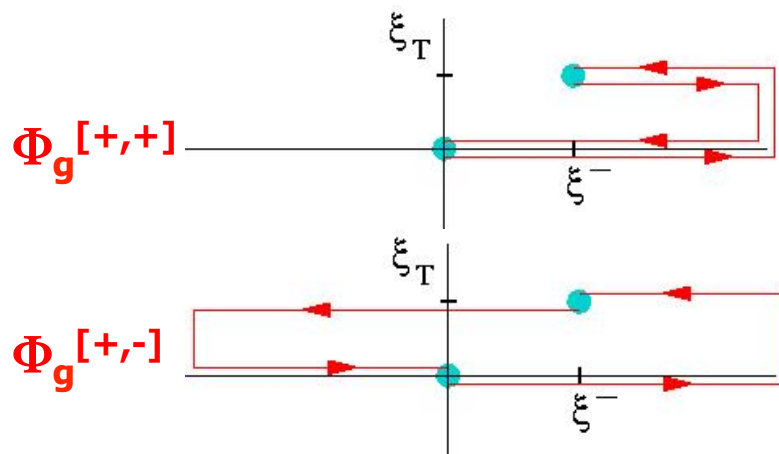
$\Phi^{[-]}$

ξ_T

ξ^-

Time reversal

ξ_T

ξ^-

$\Phi^{[+]}$

... A^n ...

AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJM and F Pijlman, NP B 667 (2003) 201

Simplest gauge links for gluon TMDs

$$\Phi_g^{\alpha\beta[C,C']}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \left| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \right| P \right\rangle_{\xi.n=0}$$

◆ The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'
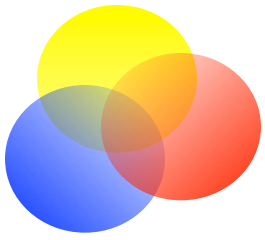
$$F^{\alpha\beta}(\xi) \to U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

◆ Basic (simplest) gauge links for gluon TMD correlators:

gg ➜ H

$\Phi_g^{[+,+]}$

$\Phi_g^{[-,-]}$

$\Phi_g^{[+,-]}$

$\Phi_g^{[-,+]}$

in gg ➜ Q$\overline{\text{Q}}$

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Color gauge invariant correlators

- Including gauge links we have well-defined matrix elements for TMDs but this implies multiple possiblities for gauge links depending on the process and the color flow in the diagram

- Leading quark TMDs

$$\Phi^{[U]}(x, p_T; n) = \left\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp[U]}(x, p_T^2) \frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T)\gamma_5 \right.$$

$$\left. + h_{1T}^{[U]}(x, p_T^2) \, \gamma_5 \, \slashed{S}_T + h_{1s}^{\perp[U]}(x, p_T) \frac{\gamma_5 \, \slashed{p}_T}{M} + i h_1^{\perp[U]}(x, p_T^2) \frac{\slashed{p}_T}{M} \right\} \frac{\slashed{P}}{2},$$

- Leading gluon TMDs:

$$2x \, \Gamma^{\mu\nu[U]}(x, p_T) = -g_T^{\mu\nu} \, f_1^{g[U]}(x, p_T^2) + g_T^{\mu\nu} \frac{\epsilon_T^{p_T S_T}}{M} \, f_{1T}^{\perp g[U]}(x, p_T^2)$$

$$+ i\epsilon_T^{\mu\nu} \, g_{1s}^{g[U]}(x, p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu} \frac{p_T^2}{2M^2} \right) h_1^{\perp g[U]}(x, p_T^2)$$

$$- \frac{\epsilon_T^{p_T\{\mu} p_T^{\nu\}}}{2M^2} \, h_{1s}^{\perp g[U]}(x, p_T) - \frac{\epsilon_T^{p_T\{\mu} S_T^{\nu\}} + \epsilon_T^{S_T\{\mu} p_T^{\nu\}}}{4M} \, h_{1T}^{g[U]}(x, p_T^2).$$
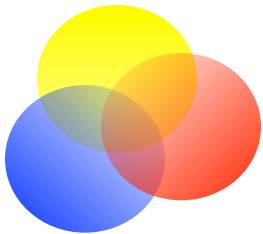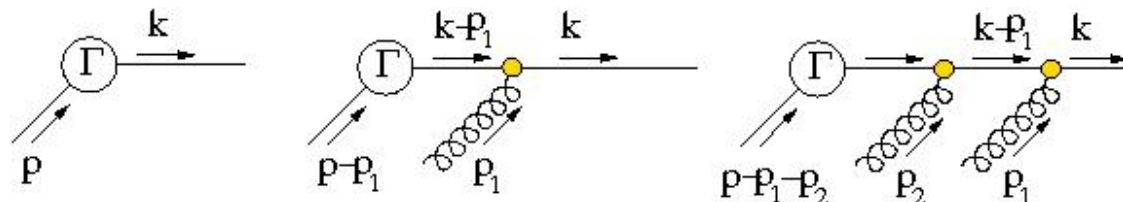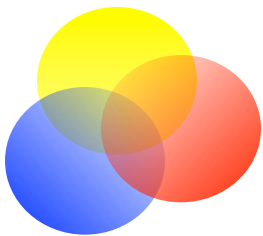
Some details on the gauge links (1)

■ Proper gluon fields (F rather than A, Wilson lines and boundary terms)

$$A^{\mu}(p_1) = n.A(p_1)\frac{P^{\mu}}{n.P} + i A_T^{\mu}(p_1) + \ldots = \frac{1}{p_1.n}\left[n.A(p_1)\,p_1^{\mu} + iG_T^{n\mu}(p_1) + \ldots\right]$$

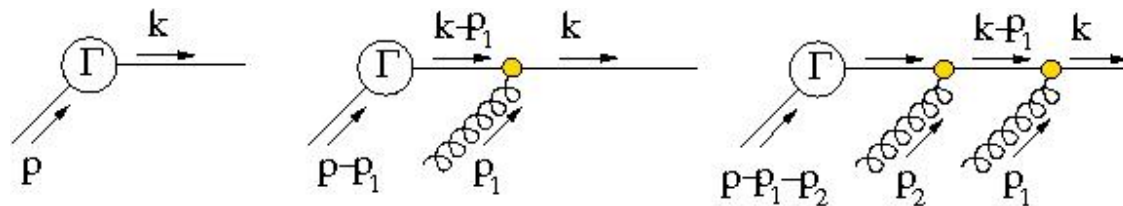■ Resummation of soft n.A gluons (coupling to outgoing color-line) for one correlator produces a gauge-line (along n)

■ Boundary terms give transverse pieces

■ Resummation of soft n.A gluons (coupling to outgoing color-line) for one correlator produces a gauge-line (along n)

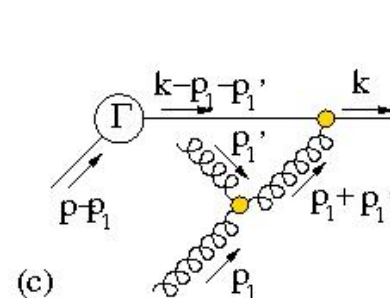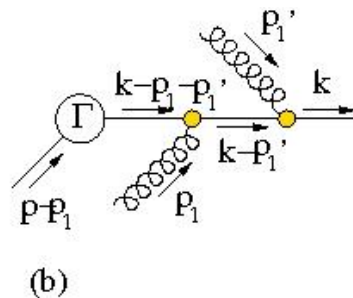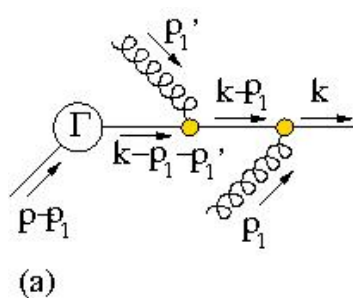■ The lowest order contributions for soft gluons from two different correlators coupling to outgoing color-line resums into gauge-knots: shuffle product of all relevant gauge-lines from that (external initial/final state) line.
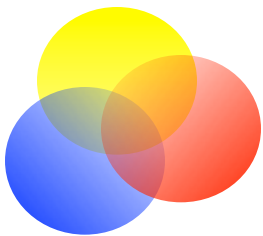
Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp

- Gauge knot $U_+[p_1, p_2, ...]$
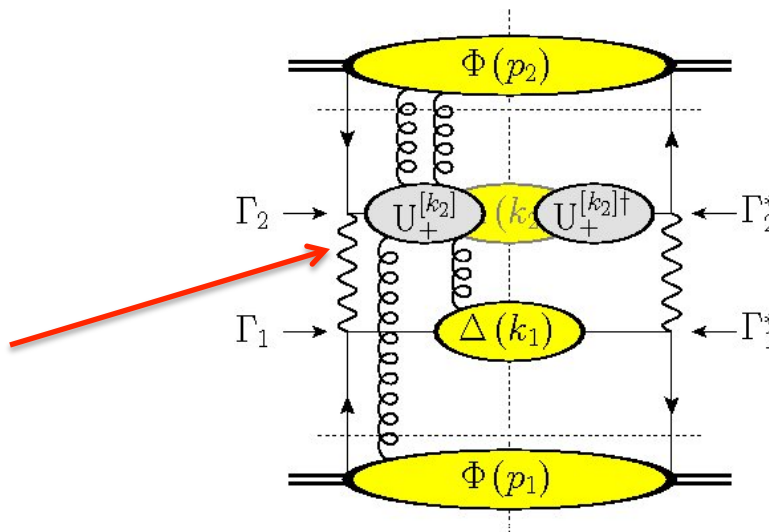
- Outgoing color contributes to a future pointing gauge link in $\Phi(p_2)$ and future pointing part of a gauge loop in the gauge link for $\Phi(p_1)$

- This causes trouble with factorization
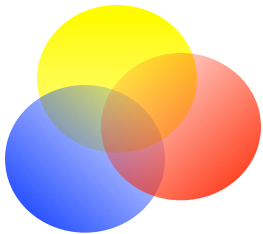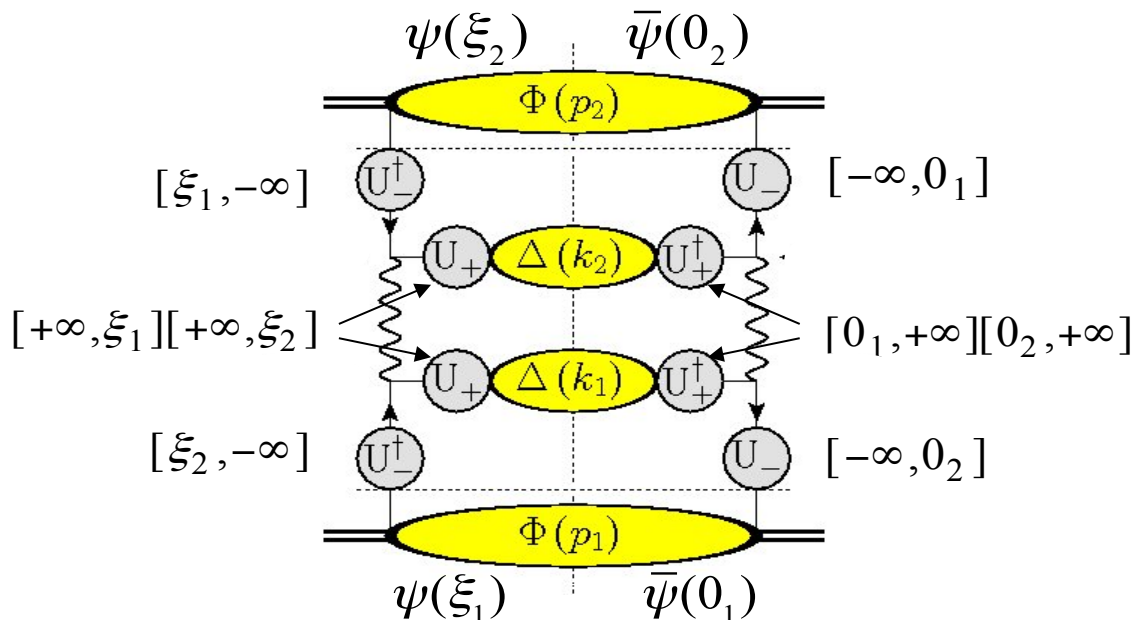
- Can be color-detangled if only p_T of one correlator is relevant (using polarization, ...) but must include Wilson loops in final U
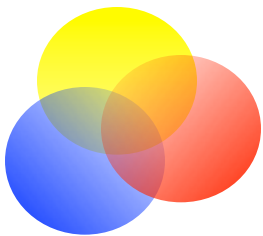
Trouble appears already in DY
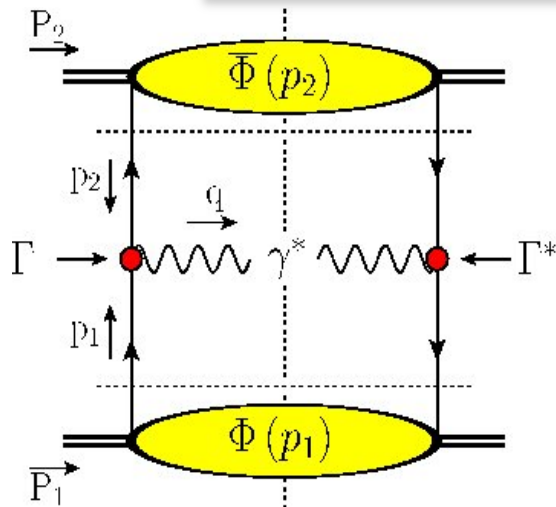
$$d\sigma_{\mathrm{DY}} \sim \mathrm{Tr}_c\left[\Phi(x_1, p_{1T})\Gamma^*\overline{\Phi}(x_2, p_{2T})\Gamma\right]$$
$$= \frac{1}{N_c}\Phi(x_1, p_{1T})\Gamma^*\overline{\Phi}(x_2, p_{2T})\Gamma,$$

- Complications for DY at measured Q_T if the transverse momentum of two initial state hadrons is involved

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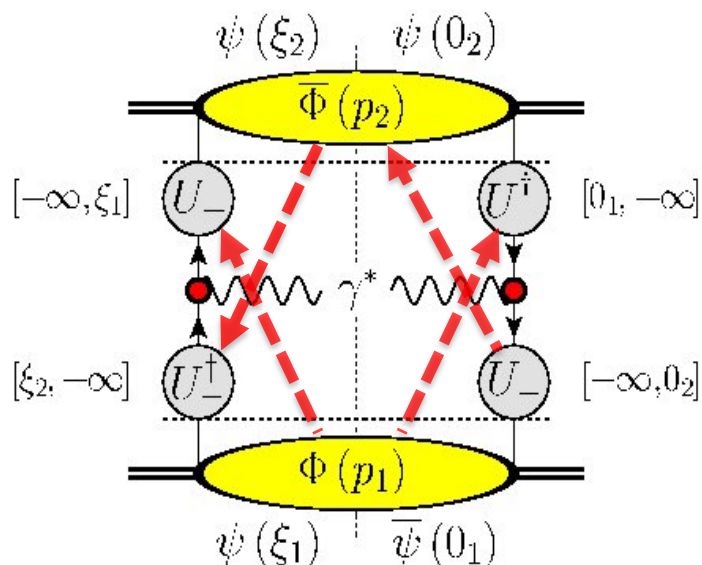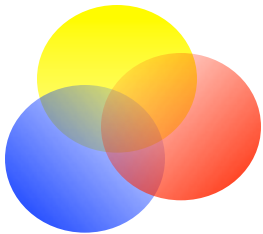
Basic strategy: operator product expansion

- Taylor expansion for functions around zero

$$f(z) = \sum_n \frac{f^n}{n!} z^n \qquad\qquad f^n = \left.\frac{\partial^n f}{\partial z^n}\right|_{z=0}$$

- Mellin transform for functions on [-1,1] interval

$$f(x) = -\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn\, x^{-n} M_n \qquad\qquad M_n = \int_0^1 dx\, x^{n-1} f(x)$$

- functions in (transverse) plane

$$f(p_T) = \sum_n \sum_{\alpha_1 \ldots \alpha_n} p_T^{\alpha_1} \ldots p_T^{\alpha_n} f_{\alpha_1 \ldots \alpha_n} \qquad\qquad f_{\alpha_1 \ldots \alpha_n} = \left. \partial_{\alpha_1} \ldots \partial_{\alpha_n} f(p_T)\right|_{p_T=0}$$
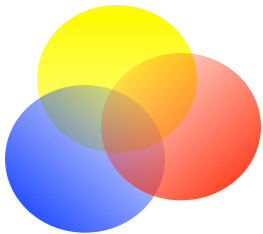
Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \left| \overline{\psi}(0) U_{[0,\xi]}^{[n]} \psi(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1} \Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \left| \overline{\psi}(0) (\partial_\xi^n)^{N-1} U_{[0,\xi]}^{[n]} \psi(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

$$\boxed{x = p.n}$$

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \left| \overline{\psi}(0) U_{[0,\xi]}^{[n]} (D_\xi^n)^{N-1} \psi(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

$$\Phi^{(N)} = \left\langle P \left| \overline{\psi}(0) (D^n)^{N-1} \psi(0) \right| P \right\rangle$$

- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.
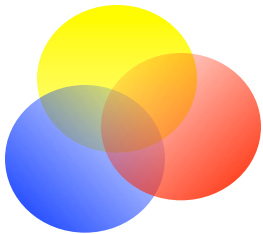
Operator structure in TMD case

- For TMD functions one can consider transverse moments

$$\Phi(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i p.\xi} \left\langle P \left| \overline{\psi}(0) \, U^{[\pm]}_{[0,\xi]} \, \psi(\xi) \right| P \right\rangle_{\xi.n=0}$$

$$p_T^\alpha \Phi^{[\pm]}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i p.\xi} \left\langle P \left| \overline{\psi}(0) U_{[0,\pm\infty]} D_T^\alpha U_{[\pm\infty,\xi]} \psi(\xi) \right| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha_1} p_T^{\alpha_2} \Phi^{[\pm]}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i p.\xi} \left\langle P \left| \overline{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha_1} D_T^{\alpha_2} U_{[\pm\infty,\xi]} \psi(\xi) \right| P \right\rangle_{\xi.n=0}$$

- Upon integration, these do involve collinear twist-3 multi-parton correlators
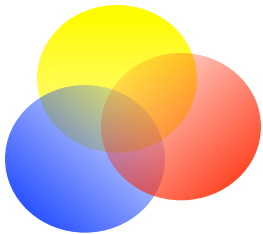
Operator structure in TMD case

- For first transverse moment one needs quark-gluon correlators

$$\Phi_D^\alpha(x - x_1, x_1 \mid x) = \int \frac{d\xi.P \, d\eta.P}{(2\pi)^2} e^{i(p-p_1).\xi + ip_1.\eta} \left\langle P \left| \bar{\psi}(0) D_T^\alpha(\eta) \, \psi(\xi) \right| P \right\rangle_{\xi.n = \xi_T = 0}$$

$$\Phi_F^\alpha(x - x_1, x_1 \mid x) = \int \frac{d\xi.P \, d\eta.P}{(2\pi)^2} e^{i(p-p_1).\xi + ip_1.\eta} \left\langle P \left| \bar{\psi}(0) F^{n\alpha}(\eta) \, \psi(\xi) \right| P \right\rangle_{\xi.n = \xi_T = 0}$$

- In principle multi-parton, but we need

$$\Phi_D^\alpha(x) = \int dx_1 \, \Phi_D^\alpha(x - x_1, x_1 \mid x)$$

$$\Phi_A^\alpha(x) = PV \int dx_1 \frac{1}{x_1} \Phi_F^{n\alpha}(x - x_1, x_1 \mid x)$$

$\Phi_F(p-p_1, p)$
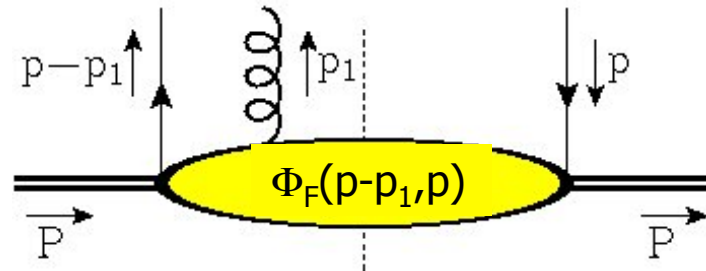
$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$ T-even (gauge-invariant derivative)

$$\boxed{\Phi_G^\alpha(x) = \pi \, \Phi_F^{n\alpha}(x, 0 \mid x)}$$ T-odd (soft-gluon or gluonic pole)

Efremov, Teryaev; Qiu, Sterman; Brodsky, Hwang, Schmidt; Boer, Teryaev, M; Bomhof, Pijlman, M

■ Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators (which are not suppressed!)

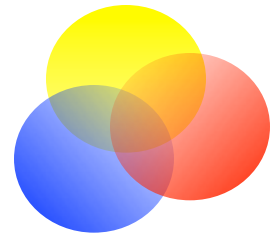$$\Phi_\partial^{\alpha[U]}(x) = \int d^2 p_T \; p_T^\alpha \Phi^{[U]}(x, p_T; n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]} \Phi_G^\alpha(x)$$

T-even

T-odd

T-even

T-even

T-odd

$$\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG,c}^{[U]} \Phi_{GG,c}^{\alpha\beta}(x) + C_G^{[U]}\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

$\mathrm{Tr}_c(GG\,\psi\bar{\psi})$ $\mathrm{Tr}_c(GG)\,\mathrm{Tr}_c(\psi\bar{\psi})$

■ $C_G^{[U]}$ calculable gluonic pole factors

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

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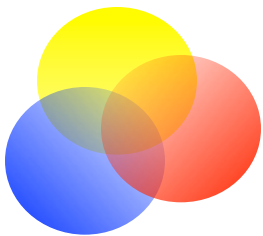
Distribution versus fragmentation functions

Operators:

$$\Phi^{[U]}(p \mid p) \sim \left\langle P \mid \bar{\psi}(0) U_{[0,\xi]} \psi(\xi) \mid P \right\rangle$$
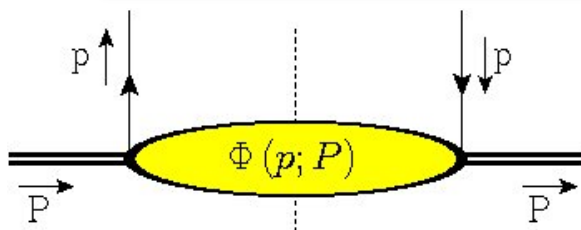
$$\Phi_{\partial}^{\alpha[U]}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]} \Phi_G^{\alpha}(x)$$

T-even   T-odd (gluonic pole)

$$\boxed{\Phi_G^{\alpha}(x) = \pi \Phi_F^{n\alpha}(x, 0 \mid x) \neq 0}$$

Operators:

out state

$$\Delta(k \mid k)$$
$$\sim \sum_X \left\langle 0 \mid \psi(\xi) \mid K_h X \right\rangle \left\langle K_h X \mid \bar{\psi}(0) \mid 0 \right\rangle$$
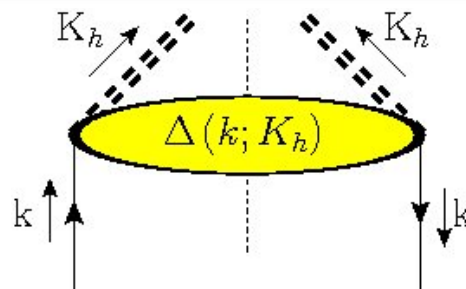
$$\boxed{\Delta_G^{\alpha}(x) = \pi \Delta_F^{n\alpha}(\tfrac{1}{z}, 0 \mid \tfrac{1}{z}) = 0}$$

$$\Delta_{\partial}^{\alpha[U]}(x) = \tilde{\Delta}_{\partial}^{\alpha}(x)$$

T-even operator combination, but still T-odd functions!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503
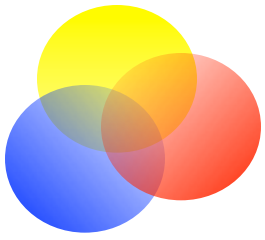
- Collecting right moments gives expansion into full TMD PDFs of definite rank

$$\Phi^{[U]}(x, p_T) \quad = \quad \Phi(x, p_T^2) + p_{Ti} \tilde{\Phi}^i_{\partial}(x, p_T^2) + p_{Tij} \tilde{\Phi}^{ij}_{\partial\partial}(x, p_T^2) + ...$$

$$+ \sum_c C^{[U]}_{G,c} \left[\qquad p_{Ti} \Phi^i_{G,c}(x, p_T^2) + p_{Tij} \tilde{\Phi}^{ij}_{\{\partial G\},c}(x, p_T^2) + ... \right]$$

$$+ \sum_c C^{[U]}_{GG,c} \left[p_T^2 \Phi_{G.G,c}(x, p_T^2) + \quad ... \quad + p_{Tij} \Phi^{ij}_{GG,c}(x, p_T^2) + ... \right]$$

- but for TMD PFFs

$$\Delta^{[U]}(z^{-1}, k_T) = \Delta(z^{-1}, k_T^2) + k_{Ti} \tilde{\Delta}^i_{\partial}(z^{-1}, k_T^2) + k_{Tij} \tilde{\Delta}^{ij}_{\partial\partial}(z^{-1}, k_T^2) + ...$$
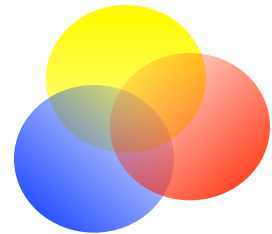
Classifying Quark TMDs

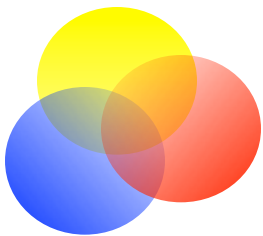| factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x, p_T^2)$	$\tilde{\Phi}_{\partial}(x, p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x, p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x, p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x, p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x, p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x, p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x, p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x, p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x, p_T^2)$

- Only a finite number needed: rank up to $2(S_{hadron}+s_{parton})$
- Rank m shows up as $\cos(m\phi)$ and $\sin(m\phi)$ azimuthal asymmetries
- No gluonic poles for PFFs

factor	TMD PFF RANK			
	0	1	2	3
1	$\Delta(z^{-1}, k_T^2)$	$\tilde{\Delta}_{\partial}(z^{-1}, k_T^2)$	$\tilde{\Delta}_{\partial\partial}(z^{-1}, k_T^2)$	$\tilde{\Delta}_{\partial\partial\partial}(z^{-1}, k_T^2)$

Classifying Quark TMDs

factor	QUARK TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Only a finite number needed: rank up to $2(S_{hadron} + s_{parton})$
- Rank m shows up as $\cos(m\phi)$ and $\sin(m\phi)$ azimuthal asymmetries
- Example: quarks in an unpolarized target are described by just 2 functions
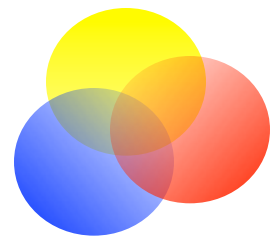
$$\Phi(x, p_T^2) = \left(f_1(x, p_T^2) \right) \frac{\not{P}}{2}$$

$$\Phi_G^\alpha(x, p_T^2) = \left(i h_1^\perp(x, p_T^2) \frac{\gamma_T^\alpha}{M} \right) \frac{\not{P}}{2}$$

T-even

T-odd

[B-M function]

Classifying Quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	f_1, g_1, h_1	g_{1T}, $h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp}$, $f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$	$h_1^{(B1)}$, $h_1^{(B2)}$		$h_{1T}^{\perp(B1)}$, $h_{1T}^{\perp(B2)}$	

Three pretzelocities:

$$A: \ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$B1: Tr_c\left[GG\psi\bar{\psi}\right]$$

$$B2: Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Process dependence in h_1
(broadening)

Classifying Quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	f_1, g_1, h_1	$g_{1T}, h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp}, f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$	$h_1^{(B1)}, h_1^{(B2)}$		$h_{1T}^{\perp(B1)}, h_{1T}^{\perp(B2)}$	

factor	QUARK TMD PFFs RANK SPIN ½ HADRON			
	0	1	2	3
1	D_1, G_1, H_1	$D_{1T}^{\perp}, G_{1T}, H_1^{\perp}, H_{1L}^{\perp}$	$H_{1T}^{\perp}$	

Just a single 'pretzelocity' PFF
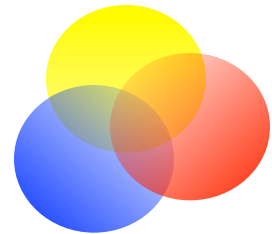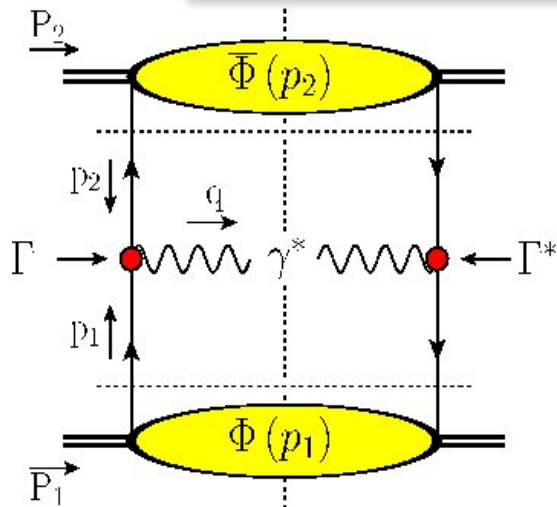
Classifying Gluon TMDs

factor	GLUON TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1		$h_1^{\perp(A)}$	
$C_{GG,c}^{[U]}$	$f_1^{(Bc)}$		$h_1^{\perp(Bc)}$	

factor	GLUON TMD PDF RANK SPIN ½ HADRON			
	0	1	2	3
1	f_1, g_1	g_{1T}	$h_1^{\perp(A)}$	
$C_{G,c}^{[U]}$	$g_1^{(A,c)}$	$f_{1T}^{\perp(Ac)}, h_{1T}^{(Ac)}$	$h_{1L}^{\perp(A,c)}$	$h_{1T}^{\perp(Ac)}$
$C_{GG,c}^{[U]}$	$f_1^{(Bc)}$		$h_1^{\perp(Bc)}$	
$C_{GGG,c}^{[U]}$		$h_{1T}^{(Bc)}$		$h_{1T}^{\perp(Bc)}$

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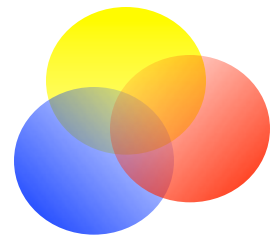
Multiple TMDs in cross sections

$$d\sigma_{DY} \sim \mathrm{Tr}_c\left[\Phi(x_1, p_{1T})\Gamma^*\overline{\Phi}(x_2, p_{2T})\Gamma\right]$$

$$= \frac{1}{N_c}\Phi(x_1, p_{1T})\Gamma^*\overline{\Phi}(x_2, p_{2T})\Gamma,$$

■ Complications if the transverse momentum of two initial state hadrons is involved, resulting for DY at measured Q_T in
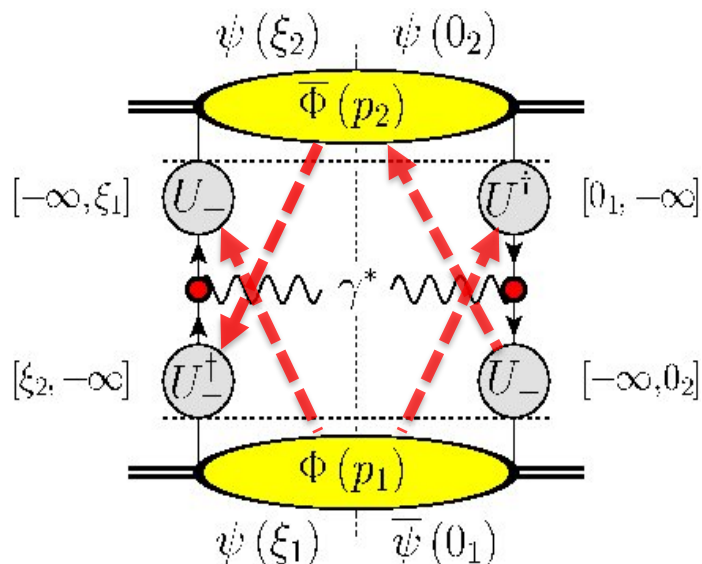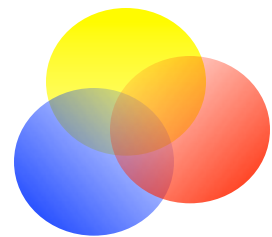
$$d\sigma_{DY} = \mathrm{Tr}_c\left[U_-^\dagger[p_2]\Phi(x_1, p_{1T})U_-[p_2]\Gamma^*\right.$$

$$\left. \times U_-^\dagger[p_1]\overline{\Phi}(x_2, p_{2T})U_-[p_1]\Gamma\right]$$

$$\neq \frac{1}{N_c}\Phi^{[-]}(x_1, p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2, p_{2T})\Gamma,$$

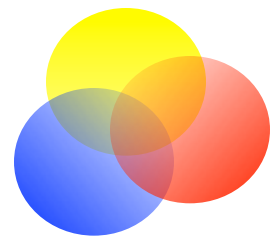Just as for twist-3 squared in collinear DY

Classifying Quark TMDs

factor	TMD RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_\partial(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

$$\sigma(x_1,x_2,q_T) \sim \frac{1}{N_c} f_{R_{G1}R_{G2}}^{[U_1,U_2]} \Phi^{[U_1]}(x_1,p_{1T})$$

$$\otimes \overline{\Phi}^{[U_2]}(x_2,p_{2T})\,\hat{\sigma}(x_1,x_2),$$

R_G for $\overline{\Phi}^{[-^\dagger]}$	R_G for $\Phi^{[-]}$		
	0	1	2
0	1	1	1
1	1	$-\frac{1}{N_c^2-1}$	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$
2	1	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$	$\frac{3N_c^4-8N_c^2-4}{(N_c^2-2)^2(N_c^2-1)}$

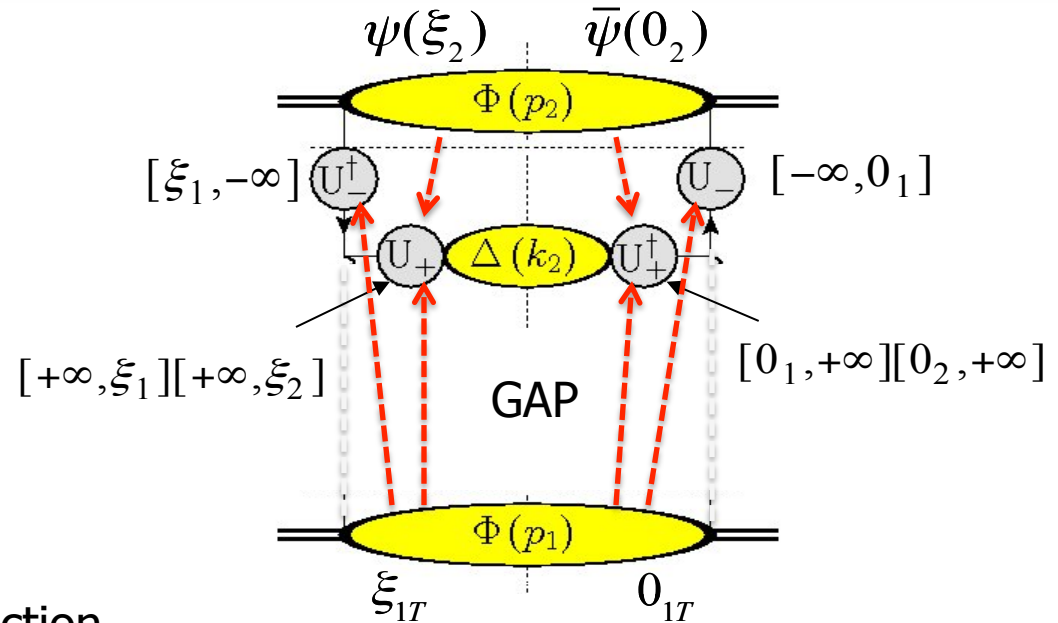$$\frac{\mathrm{Tr}_c[T^aT^bT^aT^b]}{\mathrm{Tr}_c[T^aT^a]\,\mathrm{Tr}_c[T^bT^b]} = -\frac{1}{N_c^2-1}\frac{1}{N_c}.$$
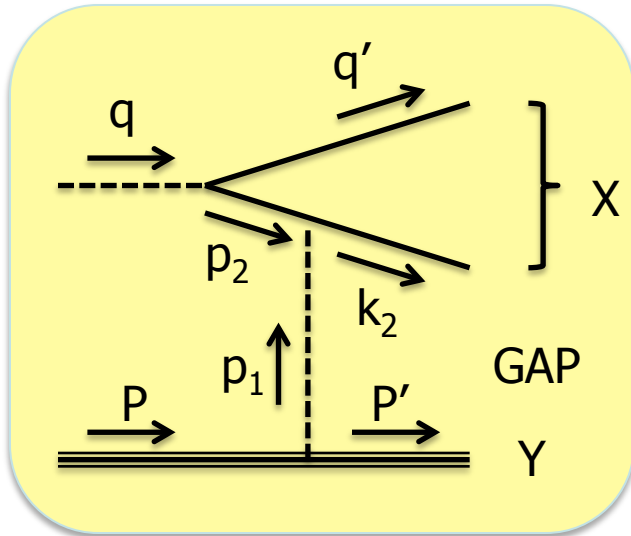
Remember classification of Quark TMDs

factor	QUARK TMD RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Example: quarks in an unpolarized target needs only 2 functions
- Resulting in cross section for unpolarized DY at measured Q_T

$$\sigma_{DY}(x_1, x_2, q_T) = \frac{1}{N_c} \Phi(x_1, p_{1T}) \otimes \bar{\Phi}(x_2, p_{2T})$$ contains f_1

$$-\frac{1}{N_c} \frac{1}{N_c^2 - 1} q_T^{\alpha\beta} \Phi_G^\alpha(x_1, p_{1T}) \otimes \bar{\Phi}_G^\beta(x_2, p_{2T})$$ contains $h_1^{perp.}$

D. Boer, PRD 60 (1999) 014012; MGA Buffing, PRL (2014) PJM, Arxiv: 1309.4681 [hep-ph]

A TMD picture for diffractive scattering

- Momentum flow in case of diffraction
  $x_1 \rightarrow M_X^2/W^2 \rightarrow 0$ and $t \rightarrow p_{1T}^2$
- Picture in terms of TMD and inclusion of gauge links
  (including gauge links/collinear gluons in $M \sim S - 1$)

- (Work in progress: Hoyer, Kasemets, Pisano, M)
- (Another way of looking at diffraction, cf Dominguez, Xiao, Yuan 2011 or older work of Gieseke, Qiao, Bartels 2000)
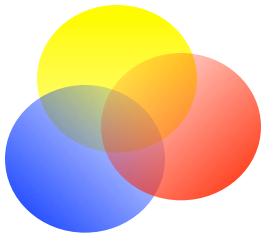
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A TMD picture for diffractive scattering
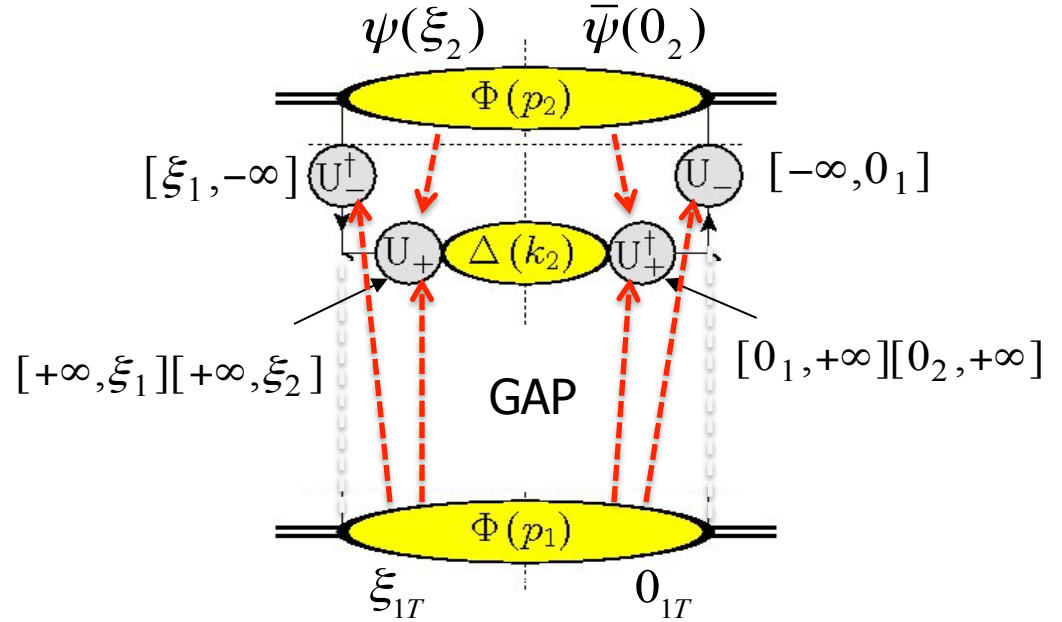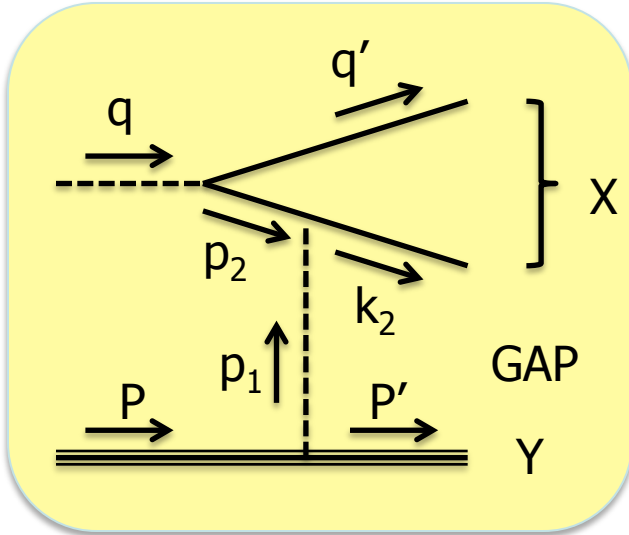
- **Cross section**

$$d\sigma = \Phi(p_{1T}; P) \operatorname{Tr}_c \left[U_-^\dagger[p_1] U_+[p_1, p_2] U_+^\dagger[p_1, p_2] U_-[p_1] \Phi(x_2, p_{2T}; q) \right]$$

- involving correlators for proton and photon

$$\Phi^{q/\gamma[+]}(x_2, p_{2T}; q) = \int \frac{d(\xi.q) d^2\xi_T}{(2\pi)^3} e^{i p_2 . \xi_2} \left\langle \gamma*(q) \left| \overline{\psi}(0) U_{[0,\xi]}^{[+]} \psi(\xi) \right| \gamma*(q) \right\rangle_{\xi.n=0}$$

$$\Phi_{DIF}^{[loop]}(x_1, p_{1T}; P) = \delta(x_1) \int \frac{d^2\xi_T}{(2\pi)^2} e^{i p_{1T}.\xi} \left\langle P \left| U^{[loop]} - 1 \right| P \right\rangle_{\xi.n=0}$$
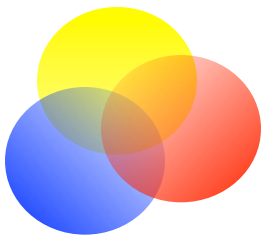
Ingredients

■ Photon PDF

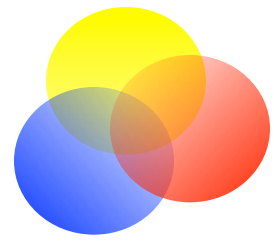$$\Phi^{q/\gamma[+]}(x, p_T; q) \sim \frac{3\alpha}{2\pi^2} \frac{x^2 + (1-x)^2}{p_T^2 + \frac{1-x}{x}Q^2}$$

■ Diffractive correlator

$$\Phi_{DIF}^{[loop]}(x, p_T; P) = \delta\left(\frac{M_X^2}{W^2 + M_X^2}\right) t\, f_{GG}(t)$$

Conclusion with (potential) rewards

- (Generalized) universality studied via operator product expansion, extending the well-known collinear distributions (including polarization 3 for quarks and 2 for gluons) to novel TMD PDF and PFF functions, ordered into functions of definite rank.

- Knowledge of operator structure is important for lattice calculations.

- ! Multiple operator possibilities for pretzelocity/transversity

- The rank m is linked to specific $\cos(m\phi)$ and $\sin(m\phi)$ azimuthal asymmetries.

- ! The TMD PDFs appear in cross sections with specific calculable factors that deviate from (or extend on) the naïve parton universality for hadron-hadron scattering.

- ! Applications in polarized high energy processes, but also in unpolarized situations (linearly polarized gluons) and possibly diffractive processes.

Thank you