Predictions for transverse-momentum dependence in e^+e^- annihilation

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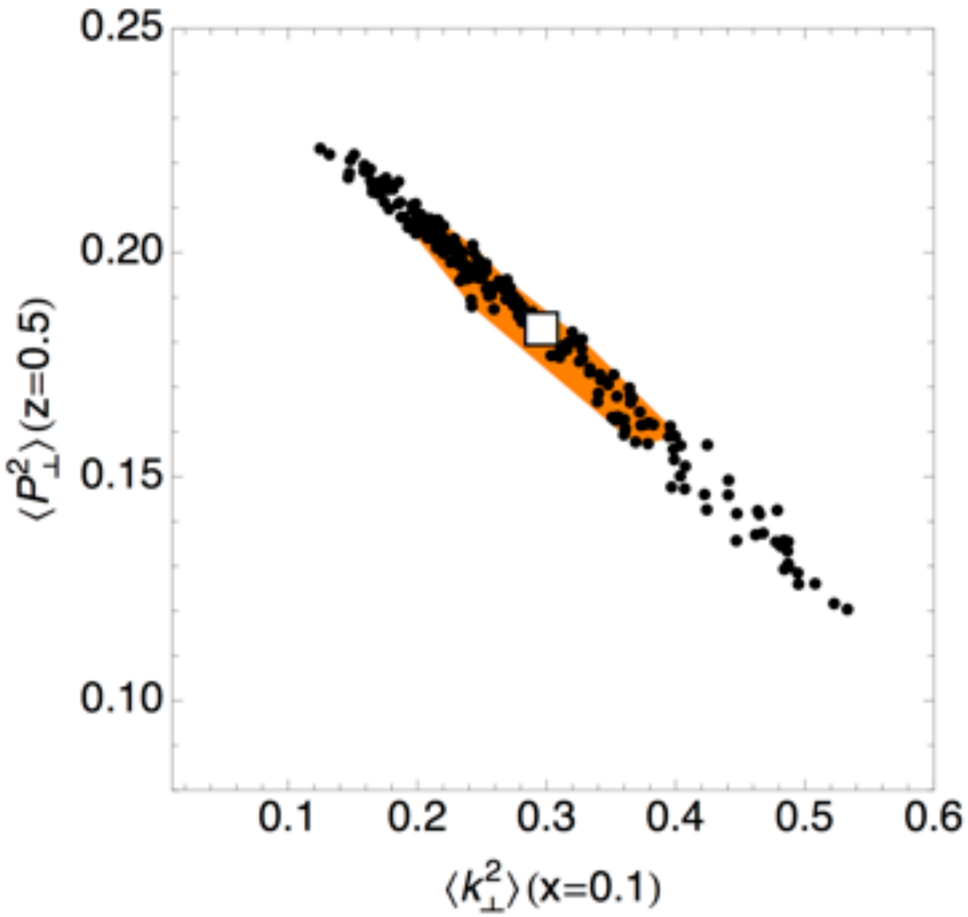
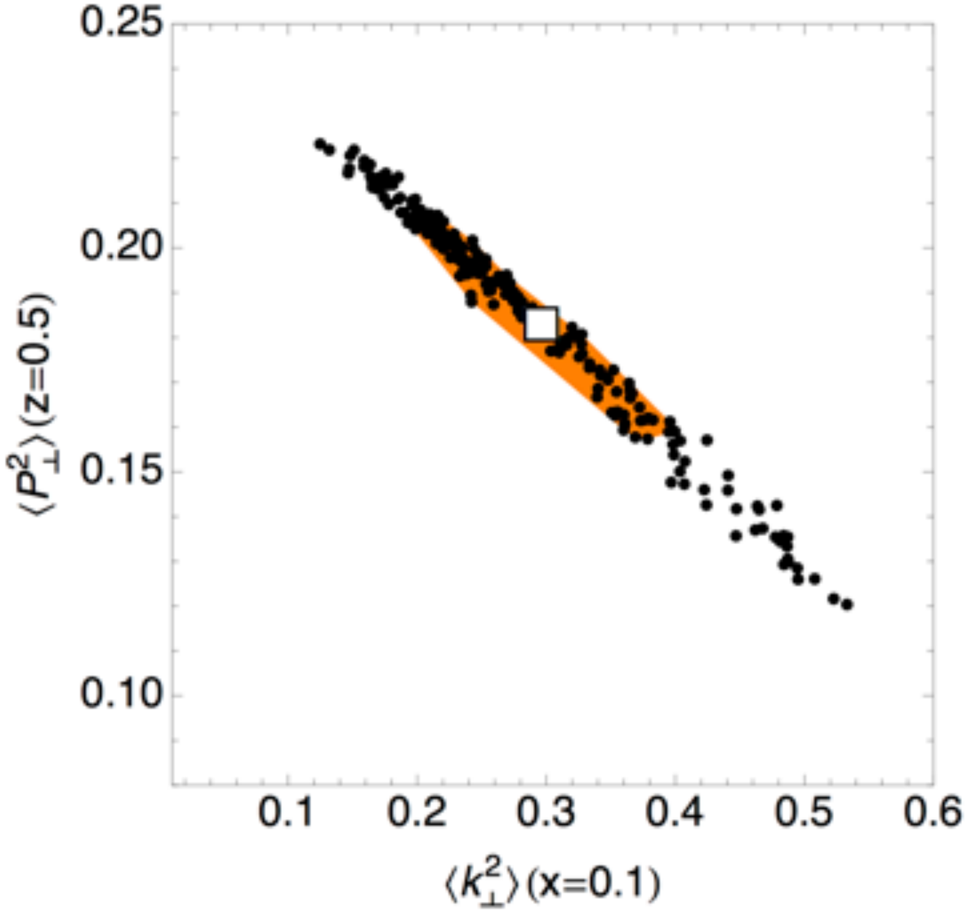
Lessons from SIDIS

Transverse momentum
in FFs



Transverse momentum
in PDFs

Lessons from SIDIS

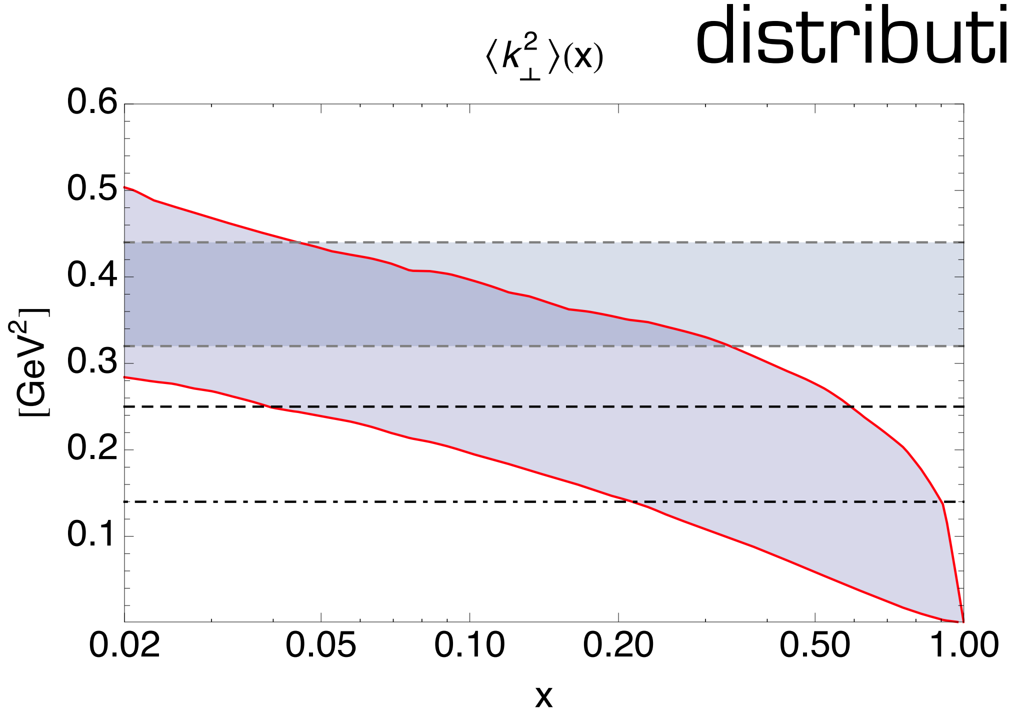


Transverse momentum in FFs

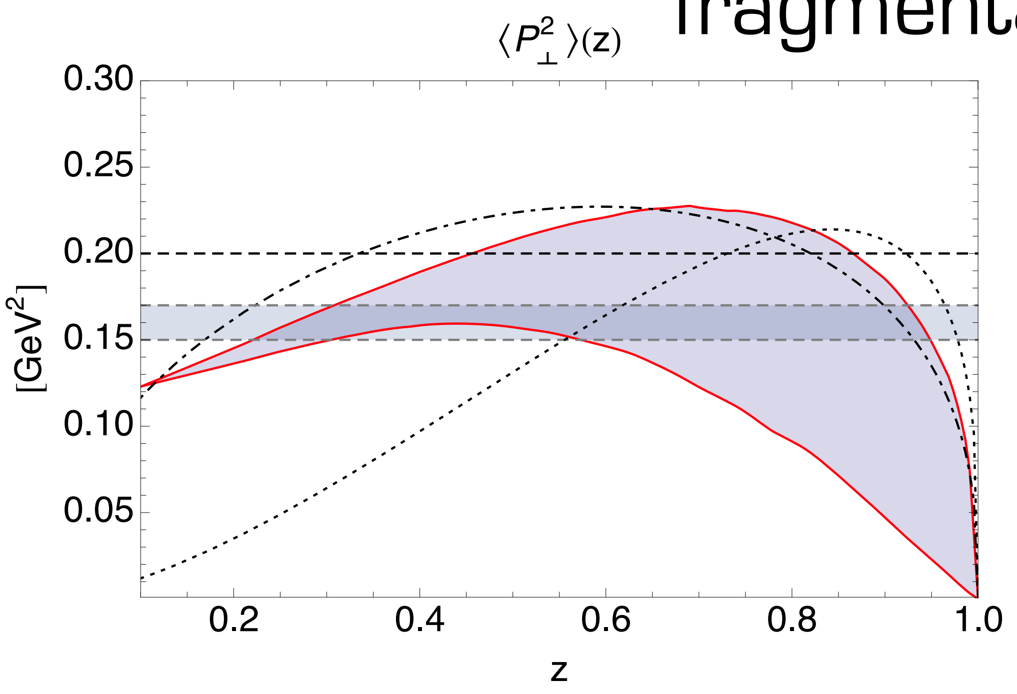
Transverse momentum in PDFs

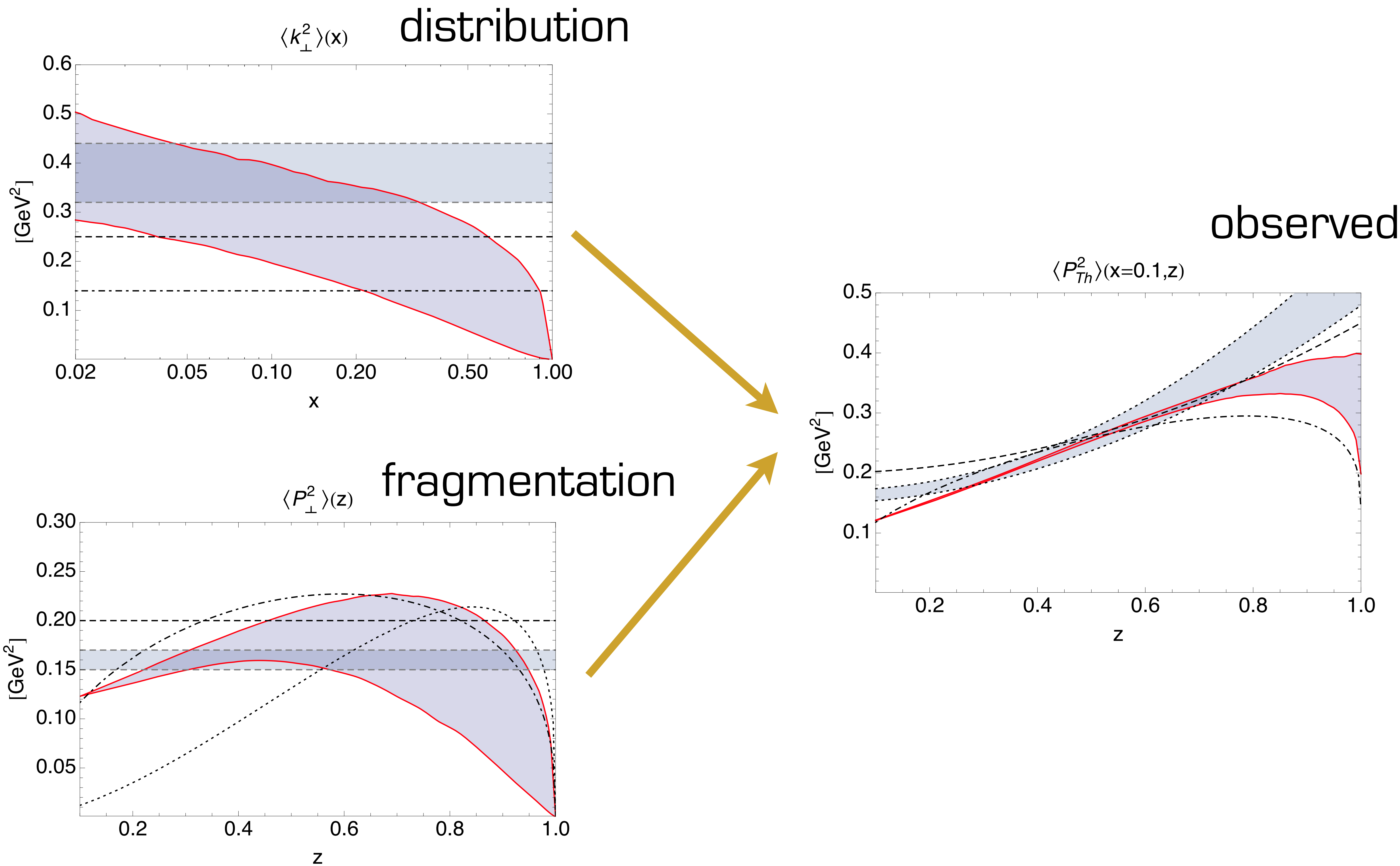
Strong anticorreleation between distribution and fragmentation

distribution



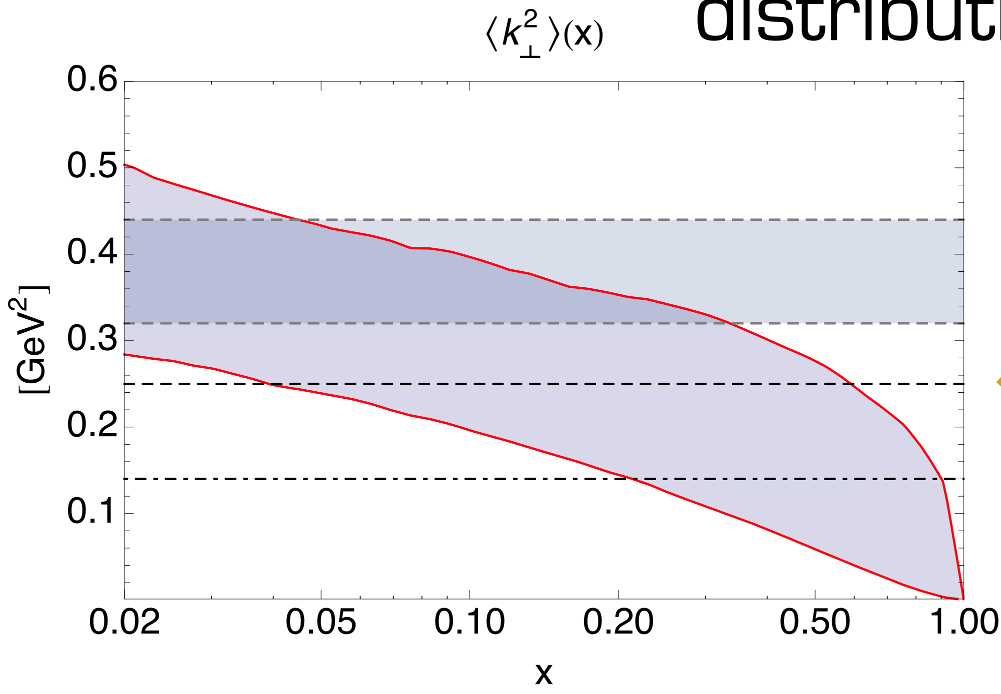
fragmentation



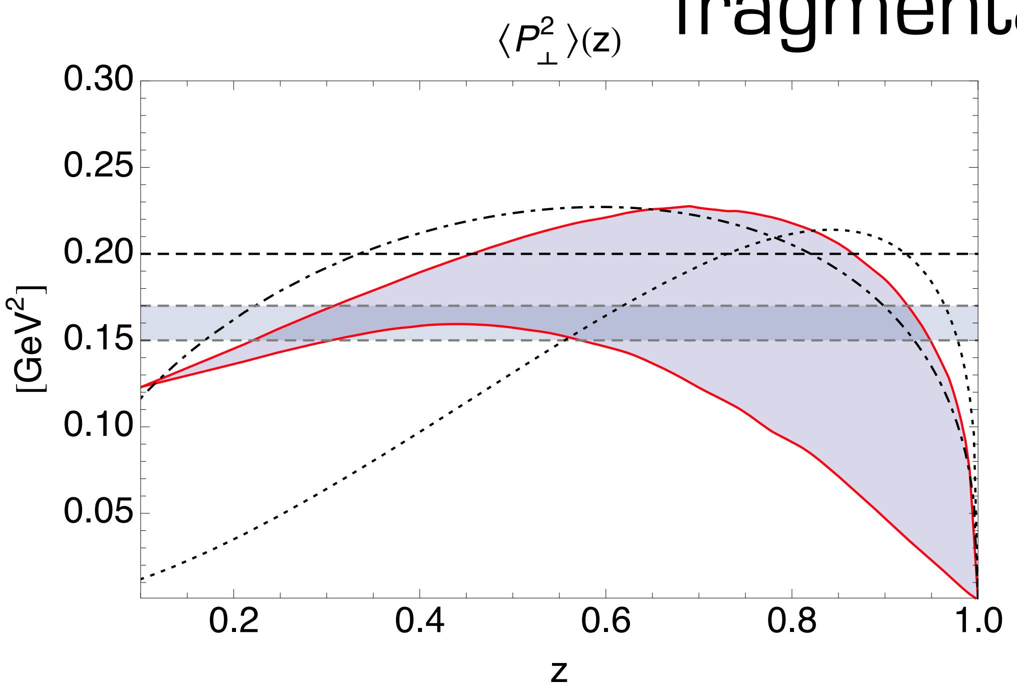


distribution
⟨k⊥²⟩(x)
[GeV²]
x
fragmentation
⟨P⊥²⟩(z)
[GeV²]
z
observed
⟨P_Th²⟩(x=0.1,z)
[GeV²]
z

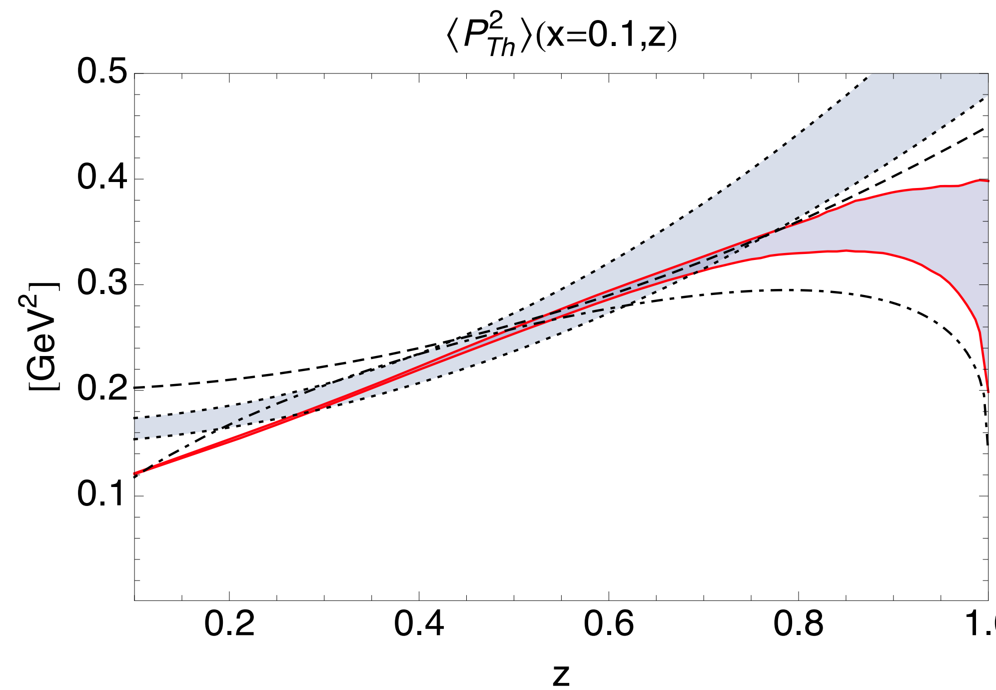
distribution



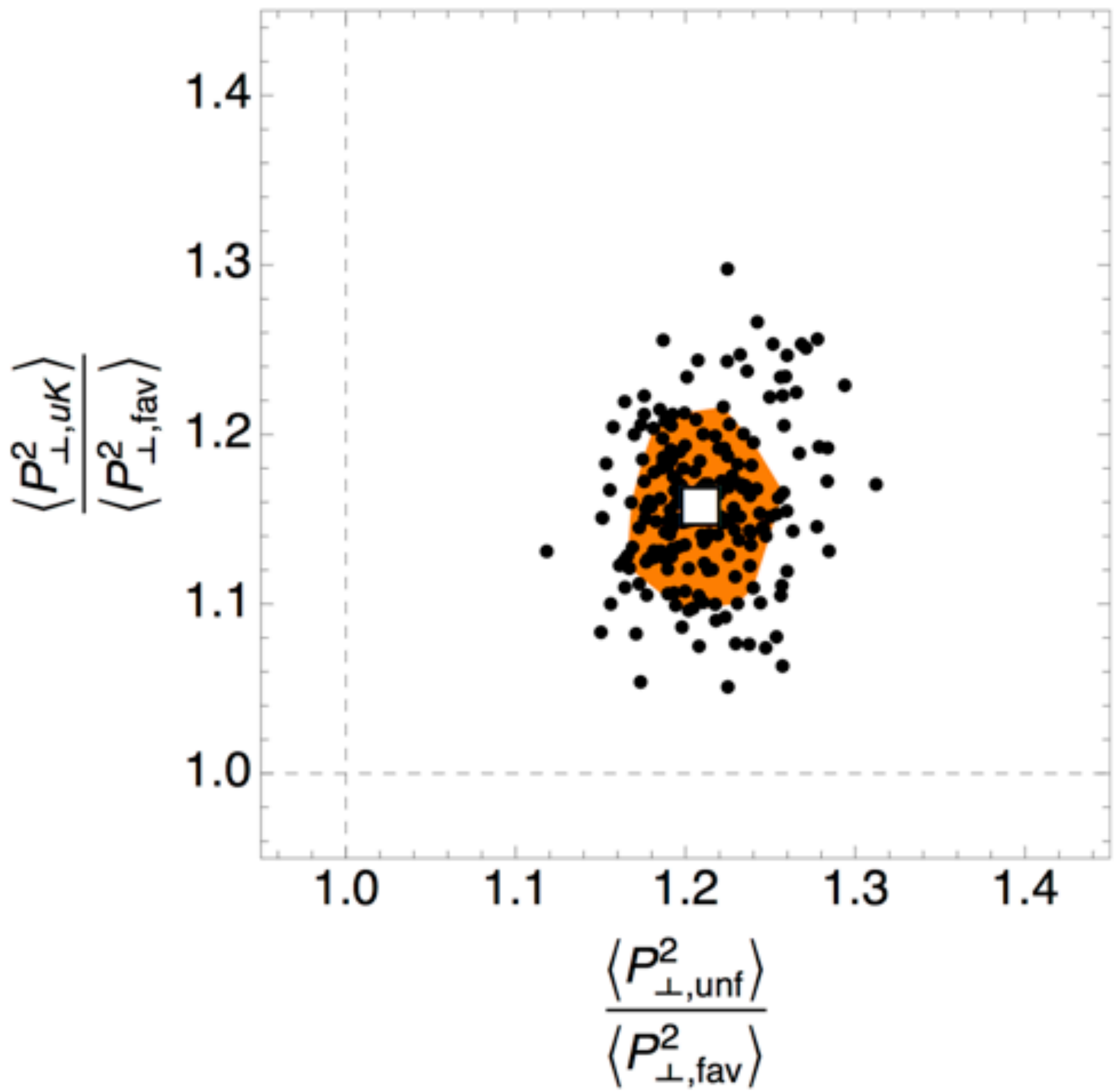
fragmentation



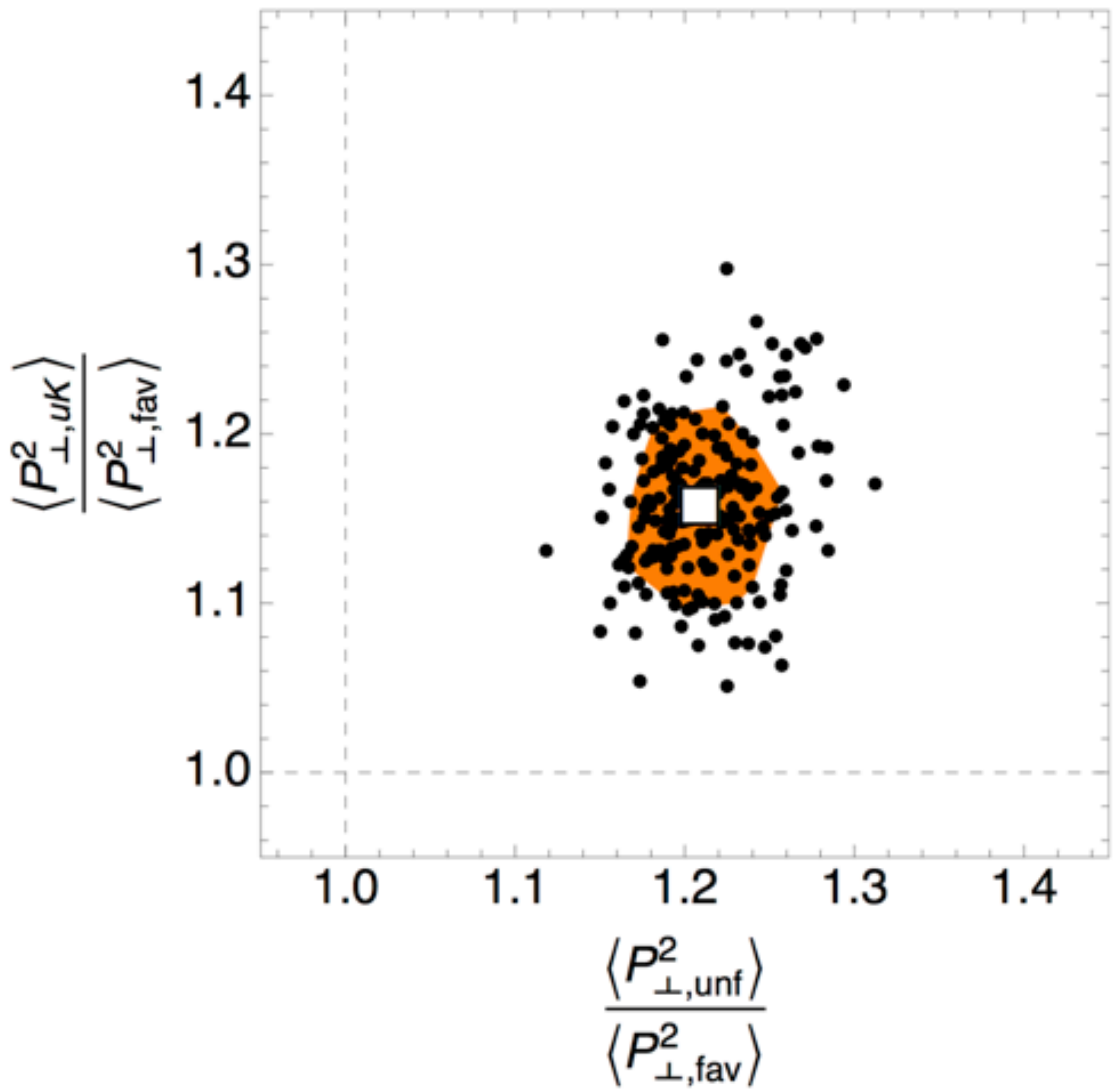
observed



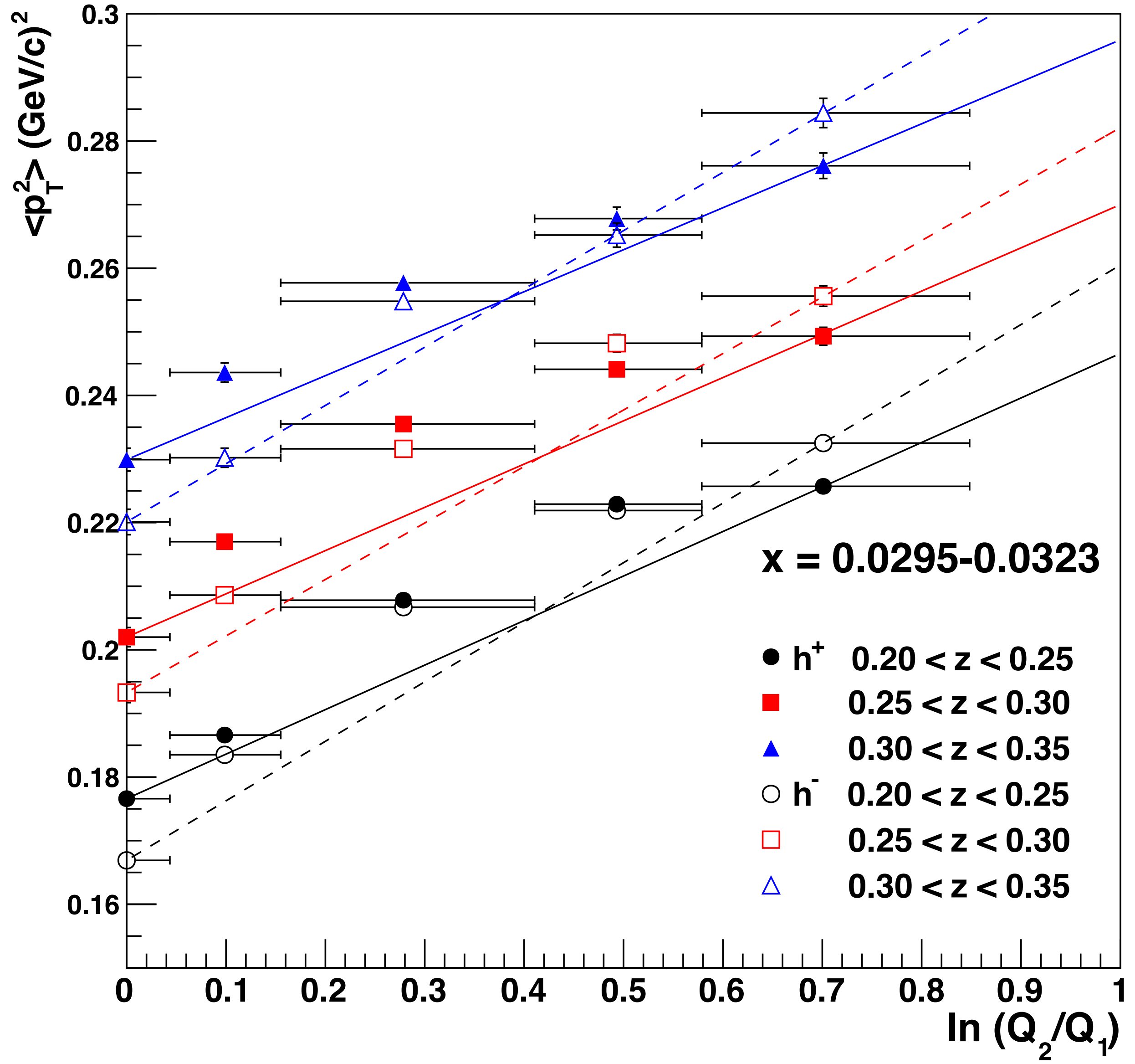
Difficult to pin down the x and z dependence



1.4
1.3
1.2
1.1
1.0
1.0
1.1
1.2
1.3
1.4
$\frac{\langle P^2_{\perp,uK} \rangle}{\langle P^2_{\perp,\mathrm{fav}} \rangle}$
$\frac{\langle P^2_{\perp,\mathrm{unf}} \rangle}{\langle P^2_{\perp,\mathrm{fav}} \rangle}$

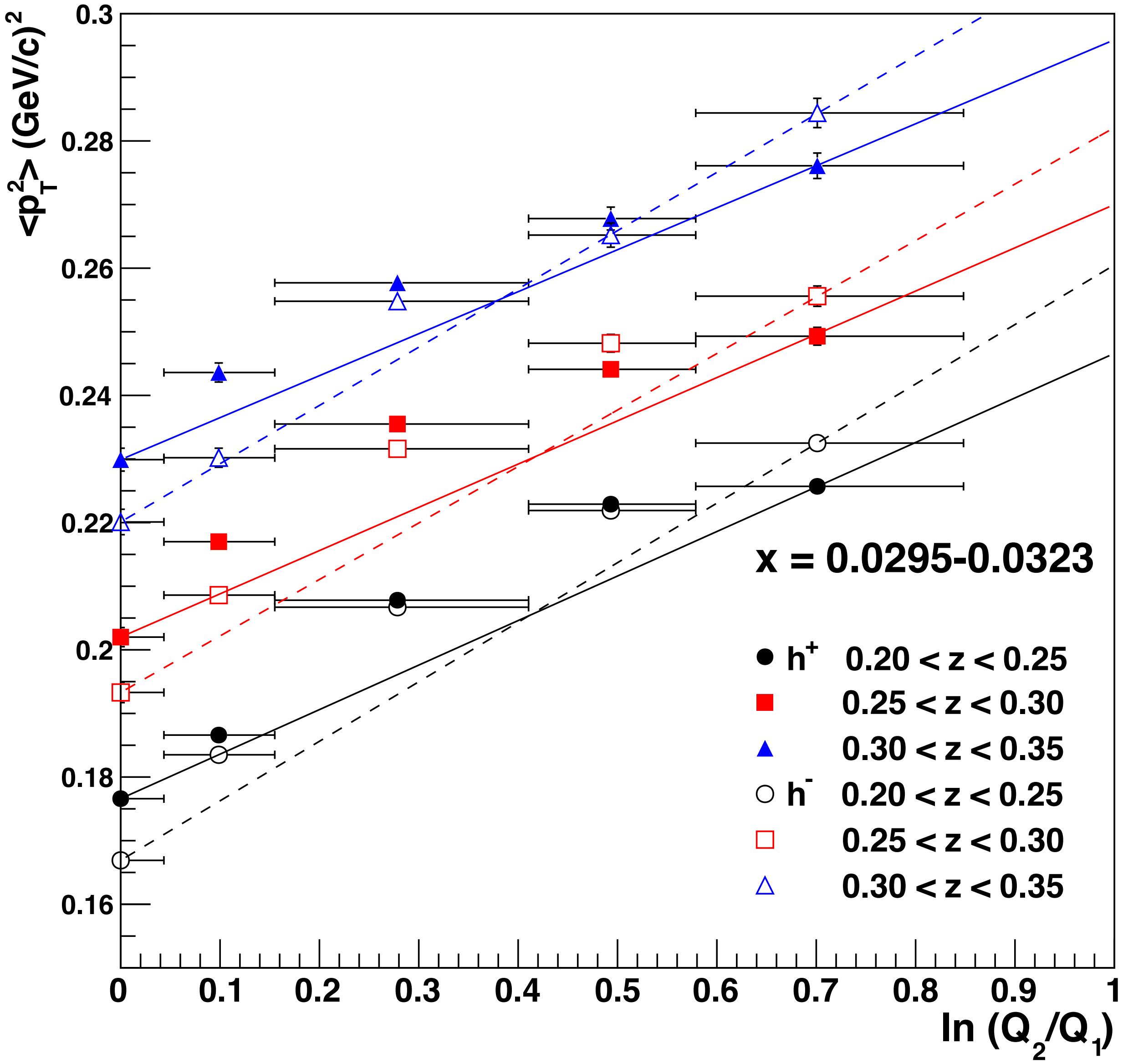


Significant evidence that pion-unfavored and kaon fragmentation functions are wider than pion-favored. Little sensitivity to strange.



Aidala, Field, Gamberg, Rogers: arXiv:1401.2654

see also Anselmino, Boglione, Gonzalez, Melis, Prokudin, arXiv:1312.6261



Aidala, Field, Gamberg, Rogers: arXiv:1401.2654

see also Anselmino, Boglione, Gonzalez, Melis, Prokudin, arXiv:1312.6261

Little sensitivity to evolution

The solution is...

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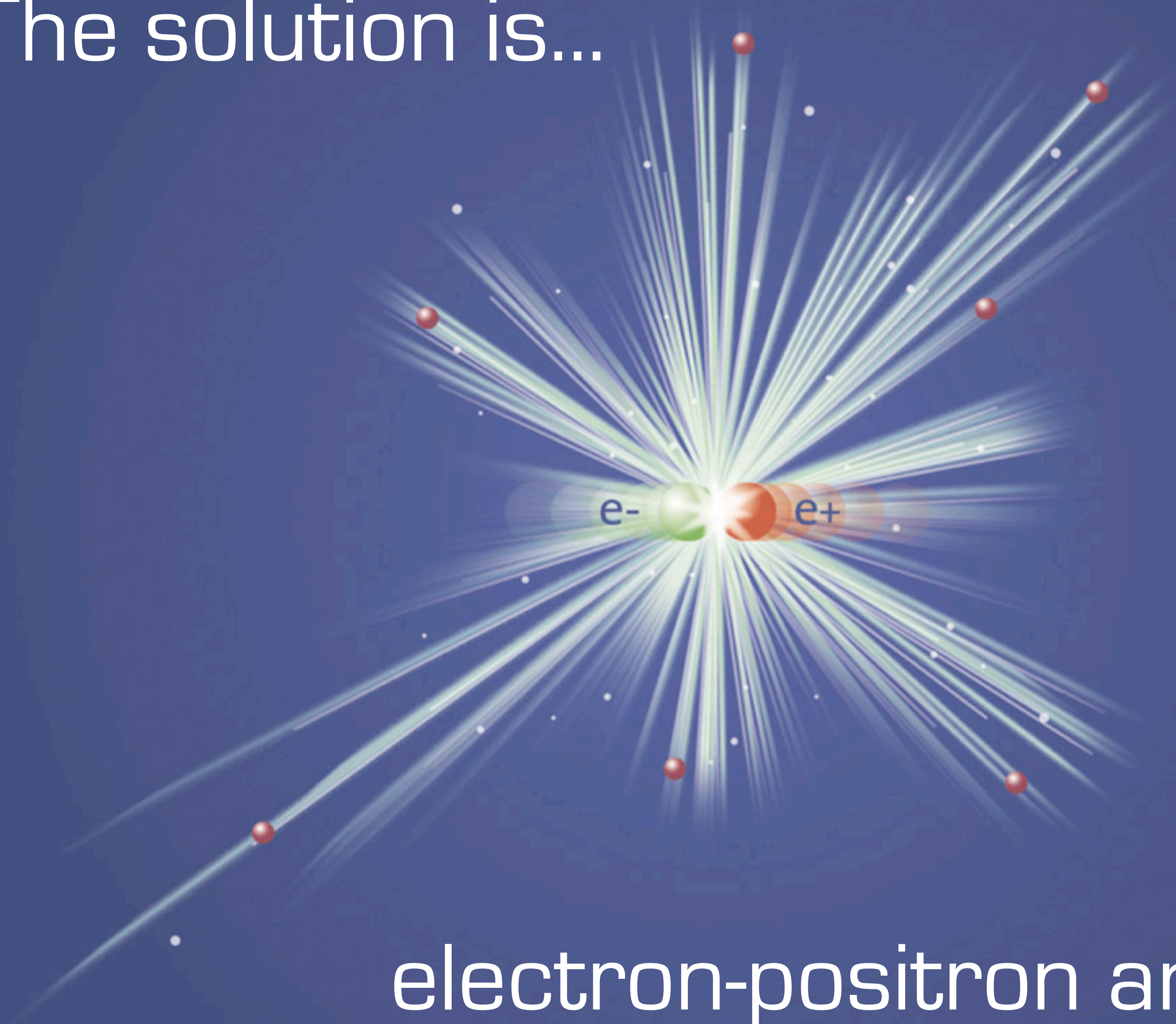
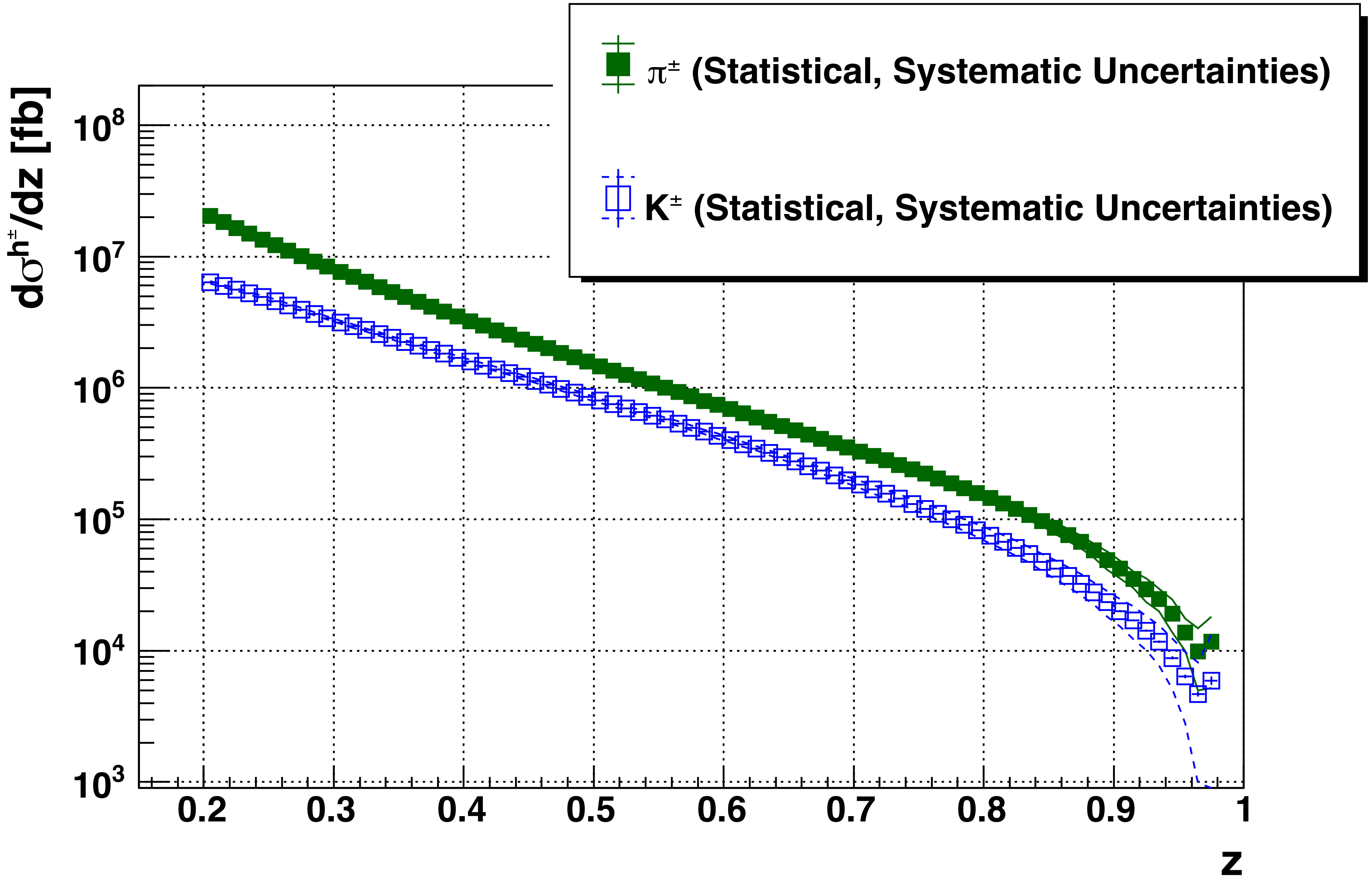


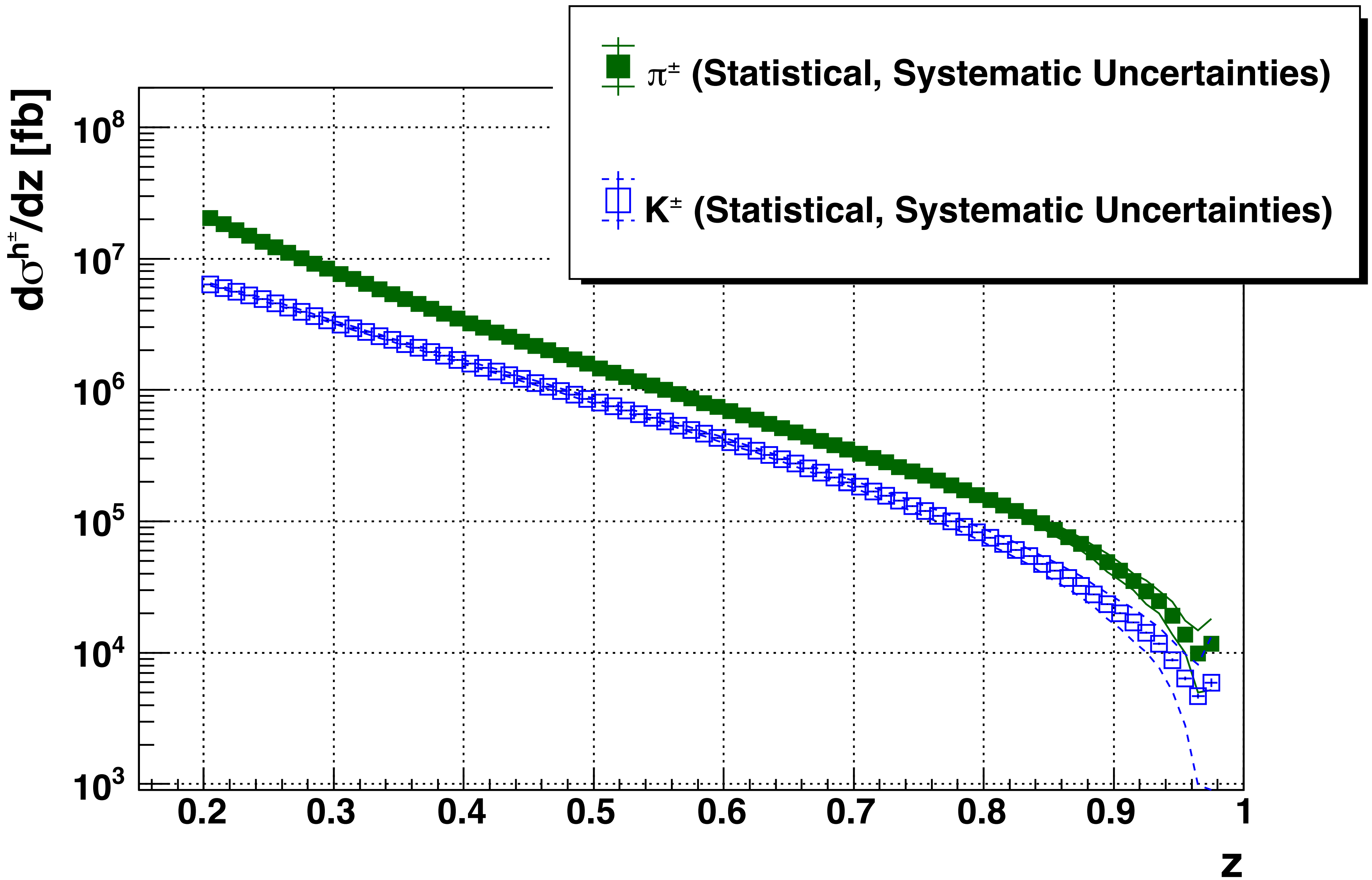
electron-positron annihilation

Collinear cross sections



Leitgab et al., PRL 111 (03), see talk by A. Vossen

Collinear cross sections



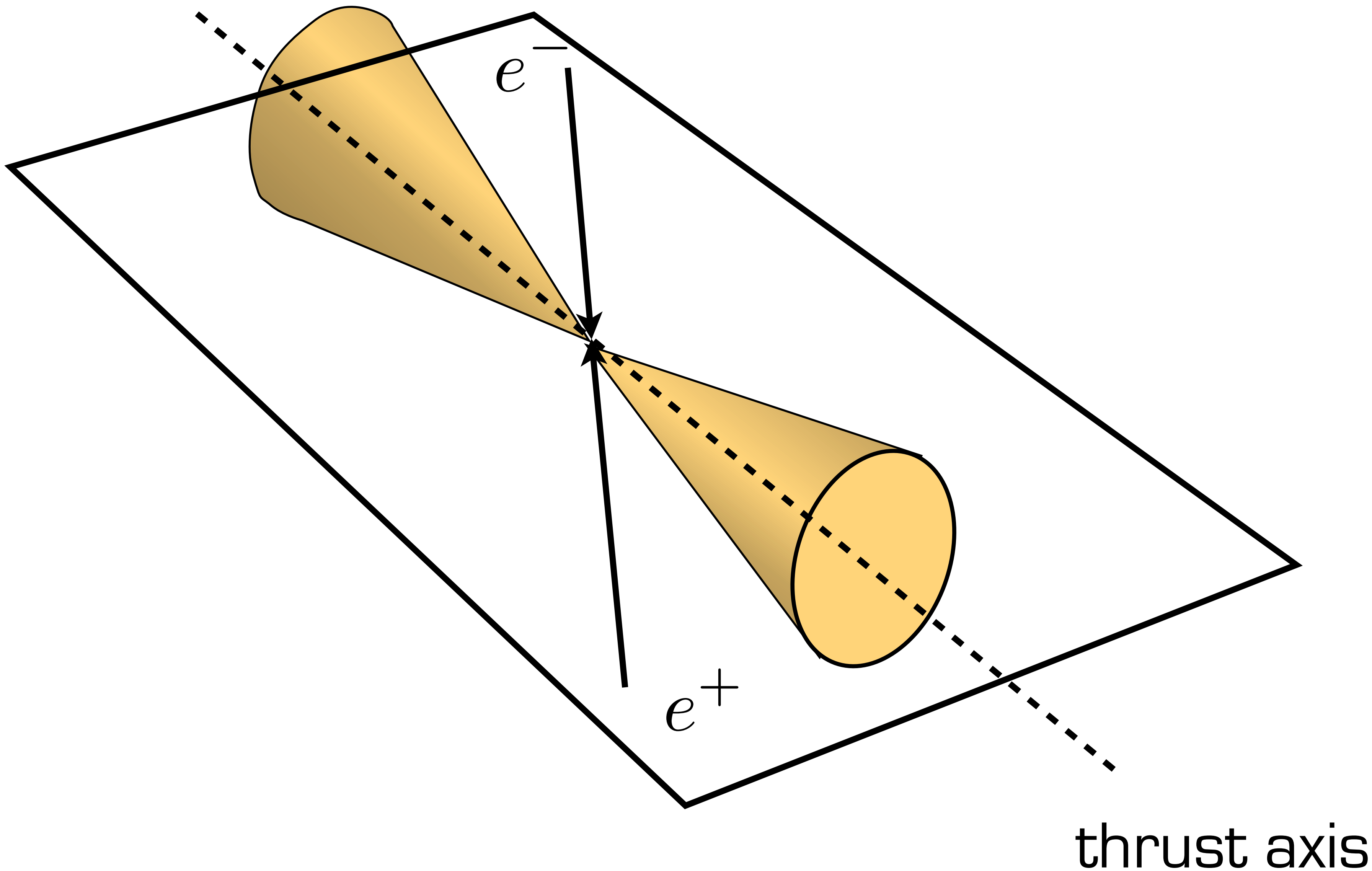
Leitgab et al., PRL 111 (03), see talk by A. Vossen

Errors of the order of 2% to 4%

Theoretical framework

Transverse-momentum dependence

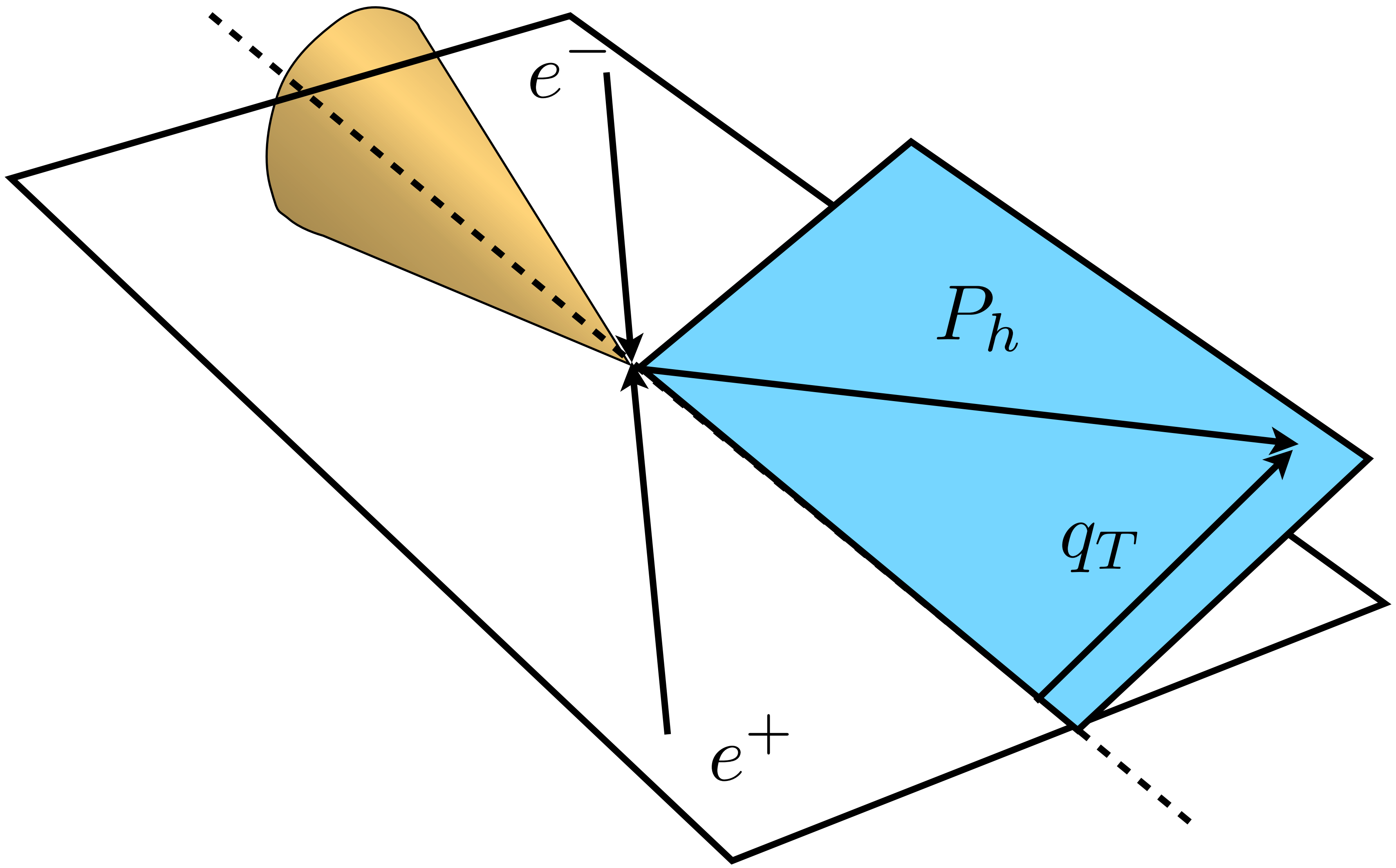
$$e^+e^- \to h \text{ jet } X$$



Boer, Jakob, Mulders, NPB504 (97)

Transverse-momentum dependence

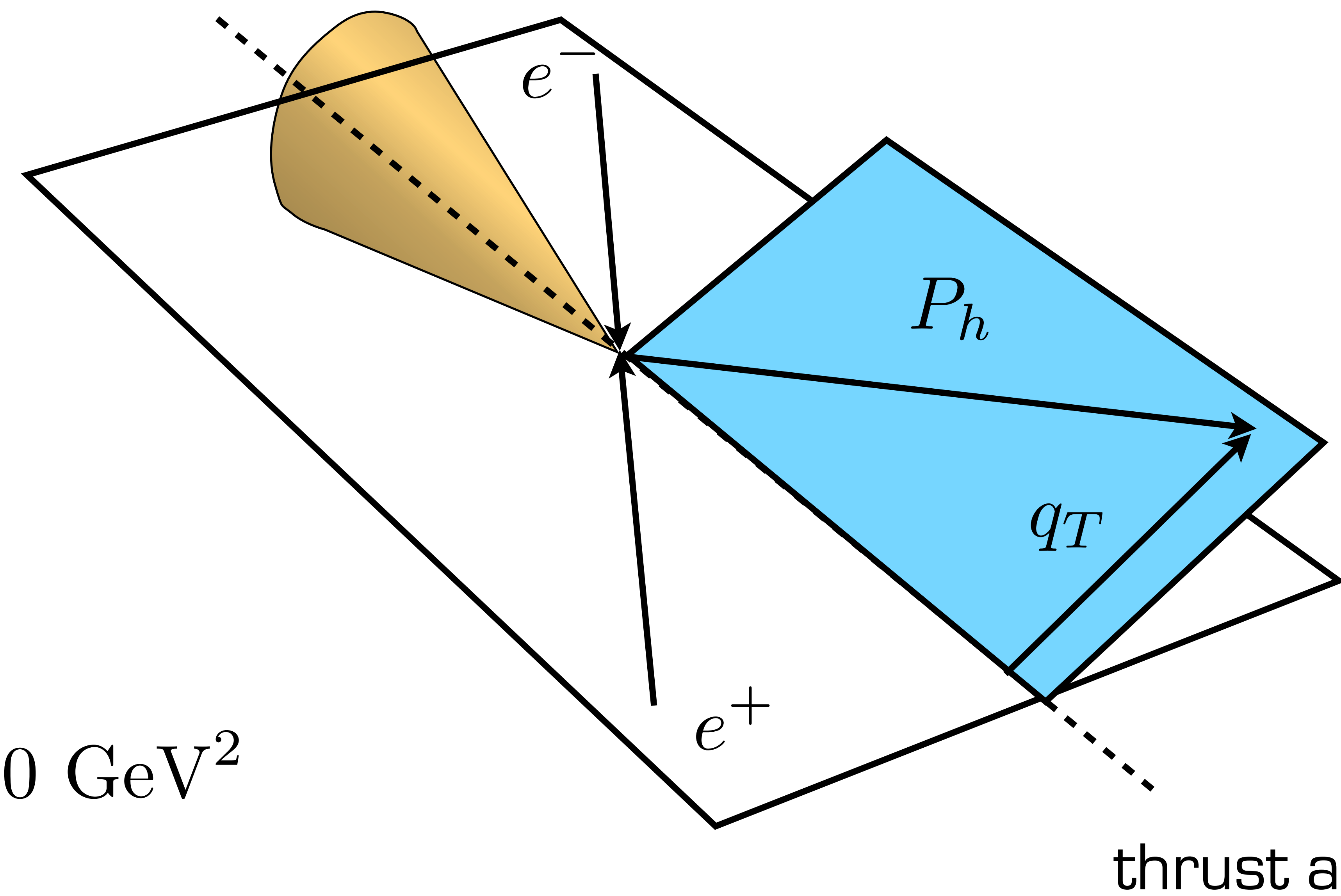
$e^+e^- \to h \text{ jet } X$



Boer, Jakob, Mulders, NPB504 (97)

Transverse-momentum dependence

$e^+e^- \to h$ jet X



$Q^2 \sim 100\ \mathrm{GeV}^2$

$q_T^2 \ll Q^2$

Boer, Jakob, Mulders, NPB504 (97)

Cross section

$$\frac{d\sigma}{dz\,dy\,dq_T^2} = \frac{12\pi^2\alpha^2}{Q^2} A(y) \sum_q e_q^2\, z^2 D_1^{q\to h}(z, q_T^2)$$

Cross section

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$$\frac{d\sigma}{dz\,dy\,dq_T^2} = \frac{6\pi\alpha^2}{Q^2} A(y) \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) z^2 \tilde{D}_1^{q\to h}(z, b_T^2)$$

in bT space

Cross section

$$\frac{d\sigma}{dz dy dq_T^2} = \frac{12\pi^2\alpha^2}{Q^2} A(y) \sum_q e_q^2 \, z^2 D_1^{q\to h}(z, q_T^2)$$

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in bT space

$$\frac{d\sigma}{dz dy dq_T^2} = \frac{6\pi\alpha^2}{Q^2} A(y) \sum_q e_q^2 \mathcal{H}(Q^2;\mu^2) \int_0^\infty db_T b_T J_0(q_T b_T) z^2 \tilde{D}_1^{q\to h}(z, b_T^2;\mu^2)$$
$$+ Y(Q^2, q_T^2)$$

with QCD corrections

Evolved TMDs

$$\widetilde{D}_1^a(z, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes D_1^i\big)(z, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{D}_{\rm NP}^a(z, b_T)$$

see talk by J. Collins

Evolved TMDs

$$\widetilde{D}_1^a(z, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes D_1^i\big)(z, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{D}_{\rm NP}^a(z, b_T)$$

collinear FF

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collinear FF

pQCD

see talk by J. Collins

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$$\widetilde{D}_1^a(z, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes D_1^i\big)(z, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{D}_{\mathrm{NP}}^a(z, b_T)$$

collinear FF

pQCD

nonperturbative part
of evolution

see talk by J. Collins

Evolved TMDs

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collinear FF

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nonperturbative part of evolution

nonperturbative part of TMD

see talk by J. Collins

Evolved TMDs

$$\widetilde{D}_1^a(z,b_T;\mu^2) = \sum_i \big(\tilde{C}_{a/i}\otimes D_1^i\big)(z,b_*;\mu_b)e^{\tilde{S}(b_*;\mu_b,\mu)}e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\hat{D}^a_{\mathrm{NP}}(z,b_T)$$

collinear FF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

$$b_* \equiv \frac{b_T}{\sqrt{1+b_T^2/b_{\max}^2}} \qquad \mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_*$$

see talk by J. Collins

Formula at NLL

$$\widetilde{D}_1^a(z, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes D_1^i\big)(z, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{D}_{\rm NP}^a(z, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_* \qquad\qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Formula at NLL

$$\widetilde{D}_1^a(z, b_T; \mu^2) = \sum_i \overbrace{\left(\tilde{C}_{a/i} \otimes D_1^i\right)(z, b_*; \mu_b)}^{D_1^a(z;\mu_b)} e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{D}_{\mathrm{NP}}^a(z, b_T)$$

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$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_* \qquad\qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Formula at NLL

$$D_1^a(z;\mu_b)$$

$$\widetilde{D}_1^a(z,b_T;\mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes D_1^i\right)(z,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{D}_{\mathrm{NP}}^a(z,b_T)$$

$$\exp\left\{-\int_{\mu_b=b_0/b_\star}^{\mu} \frac{d\mu'}{\mu'}\left[\Gamma_{\mathrm{cusp}}\ln\left(\frac{\mu^2}{\mu'^2}\right)+\gamma^V\right]\right\} \qquad e^{-\frac{g_2}{2}b_T^2\ln\frac{\mu}{\mu_0}}$$

$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_* \qquad b_* \equiv \frac{b_T}{\sqrt{1+b_T^2/b_{\max}^2}}$$

Formula at NLL

$$D_1^a(z;\mu_b) \qquad\qquad e^{-\frac{\langle \boldsymbol{P}^2_{\perp,a\to h}\rangle}{4} b_T^2}$$

$$\widetilde{D}_1^a(z,b_T;\mu^2) = \sum_i \boxed{\left(\tilde{C}_{a/i}\otimes D_1^i\right)(z,b_*;\mu_b)}\,\boxed{e^{\tilde{S}(b_*;\mu_b,\mu)}}\,\boxed{e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}}\,\boxed{\hat{D}^a_{\mathrm{NP}}(z,b_T)}$$

$$\exp\left\{-\int_{\mu_b=b_0/b_\star}^{\mu}\frac{d\mu'}{\mu'}\left[\Gamma_{\mathrm{cusp}}\ln\left(\frac{\mu^2}{\mu'^2}\right)+\gamma^V\right]\right\} \qquad\qquad e^{-\frac{g_2}{2}b_T^2\ln\frac{\mu}{\mu_0}}$$

$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_* \qquad\qquad b_* \equiv \frac{b_T}{\sqrt{1+b_T^2/b_{\max}^2}}$$

Formula at NLL

"intrinsic k_T parameters"

$$e^{-\frac{\langle \boldsymbol{P}^2_{\perp, a\to h}\rangle}{4} b_T^2}$$

$$D_1^a(z;\mu_b)$$

$$\widetilde{D}_1^a(z, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes D_1^i\right)(z, b_*; \mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{D}^a_{\rm NP}(z, b_T)$$

$$\exp\left\{-\int_{\mu_b = b_0/b_\star}^{\mu} \frac{d\mu'}{\mu'}\left[\Gamma_{\rm cusp}\ln\left(\frac{\mu^2}{\mu'^2}\right) + \gamma^V\right]\right\}$$

$$e^{-\frac{g_2}{2} b_T^2 \ln\frac{\mu}{\mu_0}}$$

$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_*$$

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Formula at NLL

"intrinsic k_T parameters"

$$D_1^a(z;\mu_b)$$

$$e^{-\frac{\langle \boldsymbol{P}^2_{\perp,a\to h}\rangle}{4} b_T^2}$$

$$\widetilde{D}_1^a(z,b_T;\mu^2) = \sum_i \big(\tilde{C}_{a/i}\otimes D_1^i\big)(z,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{D}^a_{\rm NP}(z,b_T)$$

$$\exp\left\{-\int_{\mu_b=b_0/b_*}^{\mu}\frac{d\mu'}{\mu'}\left[\Gamma_{\rm cusp}\ln\left(\frac{\mu^2}{\mu'^2}\right)+\gamma^V\right]\right\}$$

$$e^{-\frac{g_2}{2} b_T^2 \ln\frac{\mu}{\mu_0}}$$

"evolution parameters"

$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_* \qquad\qquad b_* \equiv \frac{b_T}{\sqrt{1+b_T^2/b_{\max}^2}}$$

Consistency with our fit

$$\widetilde{D}_1^a(z, b_T; \mu_0) = D_1^a(z; \mu_b) e^{-\frac{\langle P_{\perp,a}^2 \rangle}{4} b_T^2}$$

Consistency with our fit

$$\widetilde{D}_1^a(z, b_T; \mu_0) = D_1^a(z; \mu_b) e^{-\frac{\langle P_\perp^2, a\rangle}{4} b_T^2}$$

$$D_1^a(z; \mu_0) e^{-\frac{\langle P_\perp^2, a\rangle}{4} b_T^2}$$

Consistency with our fit

$$\widetilde{D}_1^a(z, b_T; \mu_0) = D_1^a(z; \mu_b) e^{-\frac{\langle P_\perp^2, a\rangle}{4} b_T^2}$$

?

$$D_1^a(z; \mu_0) e^{-\frac{\langle P_\perp^2, a\rangle}{4} b_T^2}$$

Consistency with our fit

$$\widetilde{D}_1^a(z, b_T; \mu_0) = D_1^a(z; \mu_b) e^{-\frac{\langle P_\perp^2, a\rangle}{4} b_T^2}$$

?

$$D_1^a(z; \mu_0) e^{-\frac{\langle P_\perp^2, a\rangle}{4} b_T^2}$$

The two expressions are almost the same at small kT (where the SIDIS data are) if

$$\frac{2e^{-\gamma_E}}{b_{\max}} \sim \mu_0$$

Numerical results

Assumptions

Assumptions

$$\left\langle \boldsymbol{P}^2_{\perp,a\to h} \right\rangle (z) = \left\langle \hat{\boldsymbol{P}}^2_{\perp,a\to h} \right\rangle \frac{(z^\beta + \delta)\ (1-z)^\gamma}{(\hat{z}^\beta + \delta)\ (1-\hat{z})^\gamma} \qquad \text{where } \left\langle \hat{\boldsymbol{P}}^2_{\perp,a\to h} \right\rangle \equiv \left\langle \boldsymbol{P}^2_{\perp,a\to h} \right\rangle (\hat{z}),\ \text{ and } \hat{z} = 0.5$$

z-dependent width

Assumptions

$$\left\langle \boldsymbol{P}^2_{\perp,a\to h}\right\rangle(z) = \left\langle \hat{\boldsymbol{P}}^2_{\perp,a\to h}\right\rangle \frac{(z^\beta+\delta)\ (1-z)^\gamma}{(\hat{z}^\beta+\delta)\ (1-\hat{z})^\gamma}$$

where $\left\langle \hat{\boldsymbol{P}}^2_{\perp,a\to h}\right\rangle \equiv \left\langle \boldsymbol{P}^2_{\perp,a\to h}\right\rangle(\hat{z}),$ and $\hat{z}=0.5$

z-dependent width

$$\begin{gathered}
\left\langle \boldsymbol{P}^2_{\perp,u\to\pi^+}\right\rangle = \left\langle \boldsymbol{P}^2_{\perp,\bar{d}\to\pi^+}\right\rangle = \left\langle \boldsymbol{P}^2_{\perp,\bar{u}\to\pi^-}\right\rangle = \left\langle \boldsymbol{P}^2_{\perp,d\to\pi^-}\right\rangle \equiv \left\langle \boldsymbol{P}^2_{\perp,\mathrm{fav}}\right\rangle, \\
\left\langle \boldsymbol{P}^2_{\perp,u\to K^+}\right\rangle = \left\langle \boldsymbol{P}^2_{\perp,\bar{u}\to K^-}\right\rangle \equiv \left\langle \boldsymbol{P}^2_{\perp,uK}\right\rangle, \\
\left\langle \boldsymbol{P}^2_{\perp,\bar{s}\to K^+}\right\rangle = \left\langle \boldsymbol{P}^2_{\perp,s\to K^-}\right\rangle \equiv \left\langle \boldsymbol{P}^2_{\perp,sK}\right\rangle, \\
\left\langle \boldsymbol{P}^2_{\perp,\text{all others}}\right\rangle \equiv \left\langle \boldsymbol{P}^2_{\perp,\mathrm{unf}}\right\rangle.
\end{gathered}$$

simplified flavor dependence

Assumptions

$$\left\langle \boldsymbol{P}_{\perp,a\to h}^2 \right\rangle(z) = \left\langle \hat{\boldsymbol{P}}_{\perp,a\to h}^2 \right\rangle \frac{(z^\beta + \delta)\ (1-z)^\gamma}{(\hat{z}^\beta + \delta)\ (1-\hat{z})^\gamma} \qquad \text{where } \left\langle \hat{\boldsymbol{P}}_{\perp,a\to h}^2 \right\rangle \equiv \left\langle \boldsymbol{P}_{\perp,a\to h}^2 \right\rangle(\hat{z}), \ \text{ and } \hat{z} = 0.5$$

z-dependent width

$$\begin{gathered}
\left\langle \boldsymbol{P}_{\perp,u\to\pi^+}^2 \right\rangle = \left\langle \boldsymbol{P}_{\perp,\bar{d}\to\pi^+}^2 \right\rangle = \left\langle \boldsymbol{P}_{\perp,\bar{u}\to\pi^-}^2 \right\rangle = \left\langle \boldsymbol{P}_{\perp,d\to\pi^-}^2 \right\rangle \equiv \left\langle \boldsymbol{P}_{\perp,\text{fav}}^2 \right\rangle, \\
\left\langle \boldsymbol{P}_{\perp,u\to K^+}^2 \right\rangle = \left\langle \boldsymbol{P}_{\perp,\bar{u}\to K^-}^2 \right\rangle \equiv \left\langle \boldsymbol{P}_{\perp,uK}^2 \right\rangle, \\
\left\langle \boldsymbol{P}_{\perp,\bar{s}\to K^+}^2 \right\rangle = \left\langle \boldsymbol{P}_{\perp,s\to K^-}^2 \right\rangle \equiv \left\langle \boldsymbol{P}_{\perp,sK}^2 \right\rangle, \\
\left\langle \boldsymbol{P}_{\perp,\text{all others}}^2 \right\rangle \equiv \left\langle \boldsymbol{P}_{\perp,\text{unf}}^2 \right\rangle.
\end{gathered}$$

simplified flavor dependence

we have in total 7 free parameters for the TMD FFs

Parameters

$\langle \hat{\boldsymbol{P}}^2_{\perp,\mathrm{fav}} \rangle$ [GeV^2]	$\langle \hat{\boldsymbol{P}}^2_{\perp,\mathrm{unf}} \rangle$ [GeV^2]	$\langle \hat{\boldsymbol{P}}^2_{\perp,sK} \rangle$ [GeV^2] (random)	$\langle \hat{\boldsymbol{P}}^2_{\perp,uK} \rangle$ [GeV^2]	β	δ	γ
0.15 ± 0.04	0.19 ± 0.04	0.19 ± 0.04	0.18 ± 0.05	1.43 ± 0.43	1.29 ± 0.95	0.17 ± 0.09

Parameters

$\langle \hat{\boldsymbol{P}}^2_{\perp,\text{fav}} \rangle$ [GeV^2]	$\langle \hat{\boldsymbol{P}}^2_{\perp,\text{unf}} \rangle$ [GeV^2]	$\langle \hat{\boldsymbol{P}}^2_{\perp,sK} \rangle$ [GeV^2] (random)	$\langle \hat{\boldsymbol{P}}^2_{\perp,uK} \rangle$ [GeV^2]	β	δ	γ
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we are not using the mean values, but the 200 replicas

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we are not using the mean values, but the 200 replicas

Three different choices for the evolution parameters

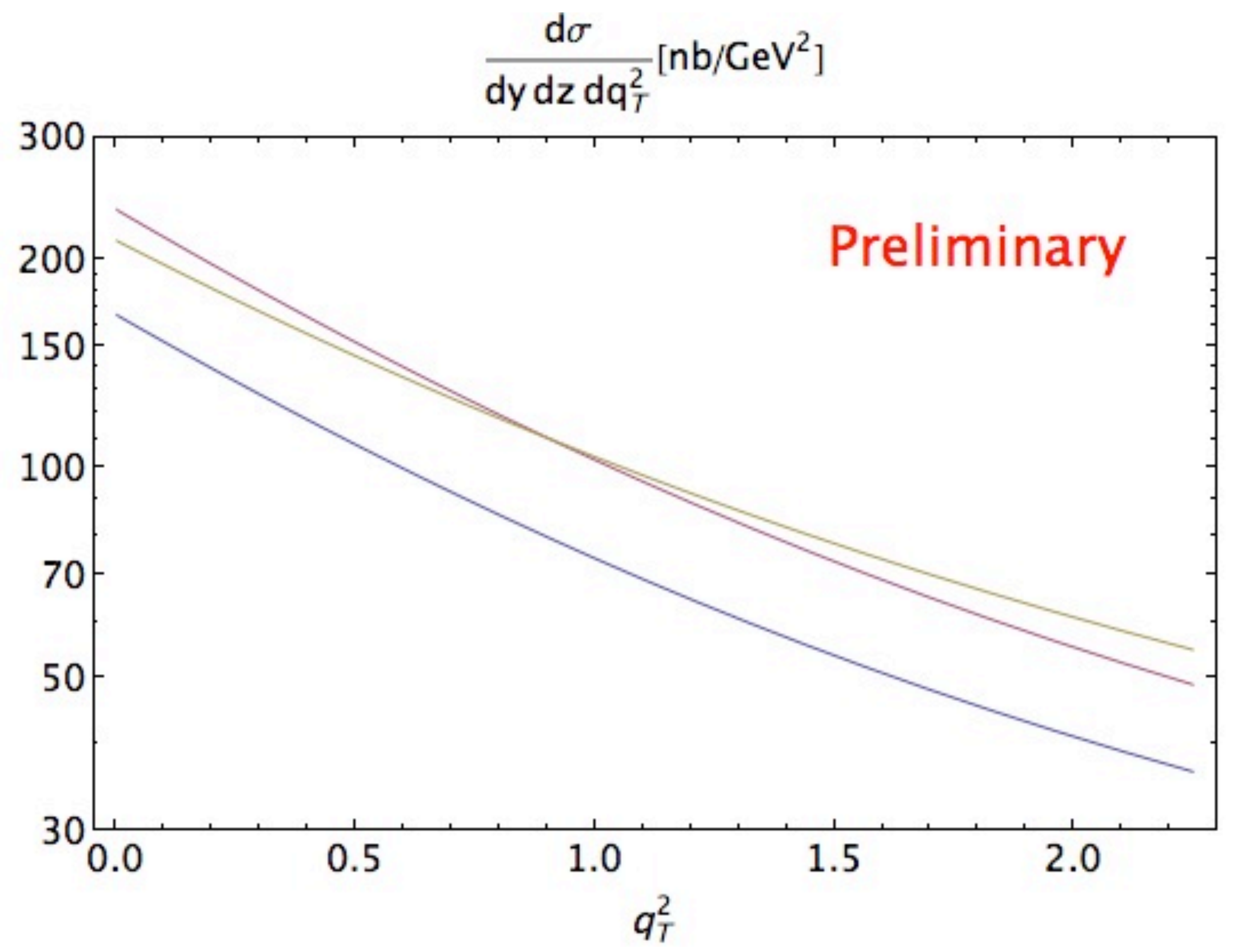
b_{max}=1.5 GeV^{-1}, g_2=0.18

b_{max}=1.0 GeV^{-1}, g_2=0.41

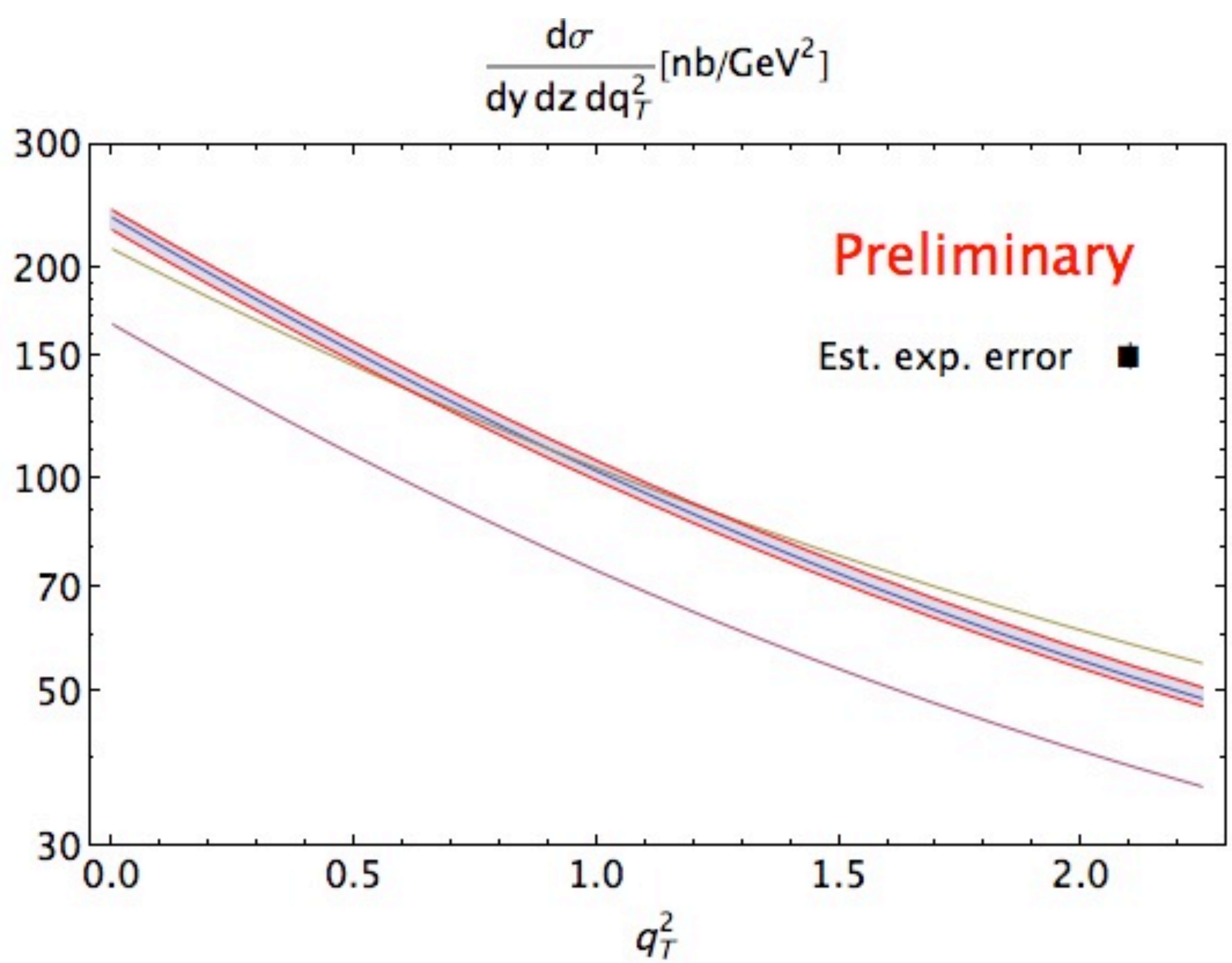
b_{max}=0.5 GeV^{-1}, g_2=0.64

Effect of theoretical accuracy

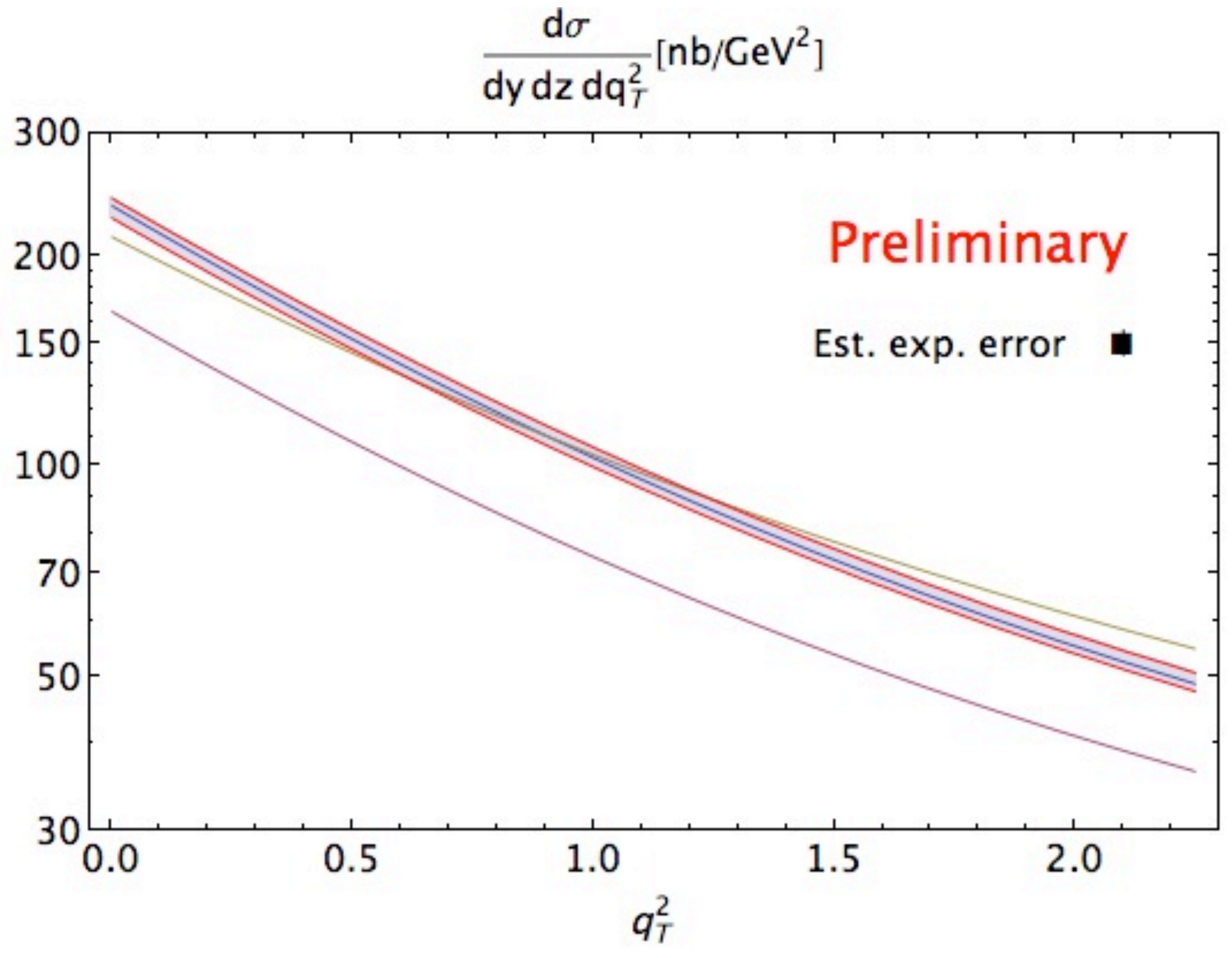
unless otherwise specified, plots are shown for y=0.2, z=0.2



Effect of theoretical accuracy

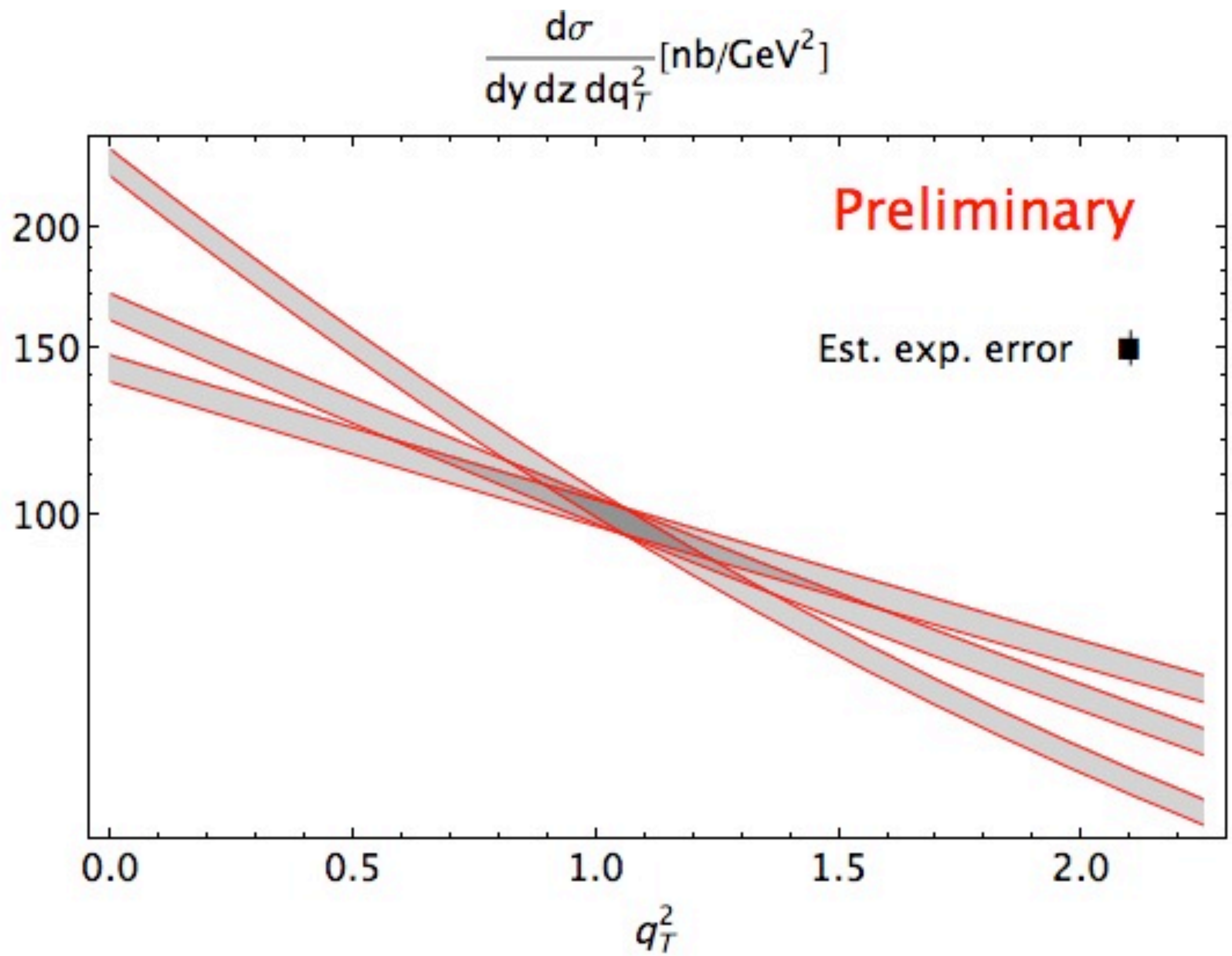


Effect of theoretical accuracy



at least NLL accuracy is required

Sensitivity to evolution parameters



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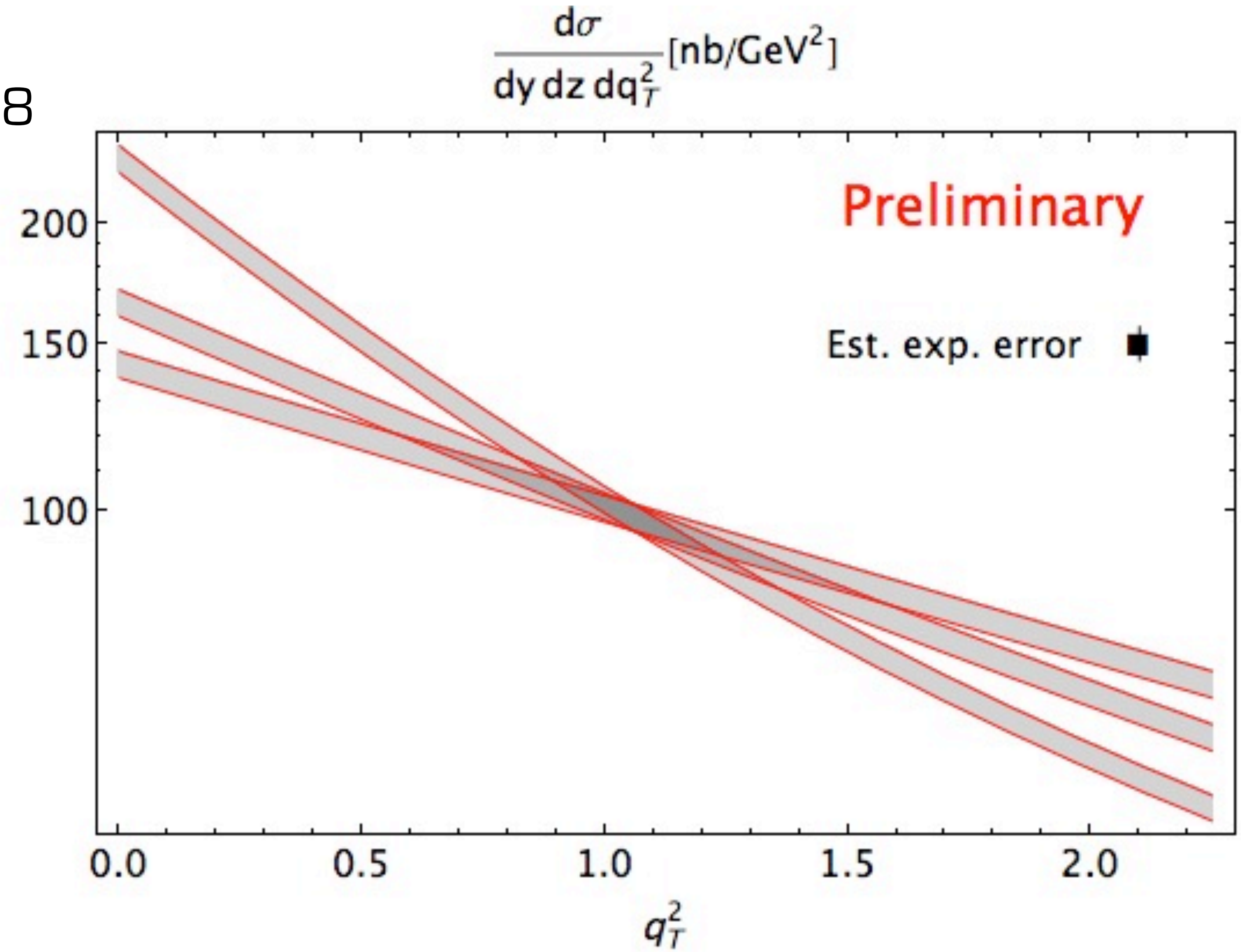

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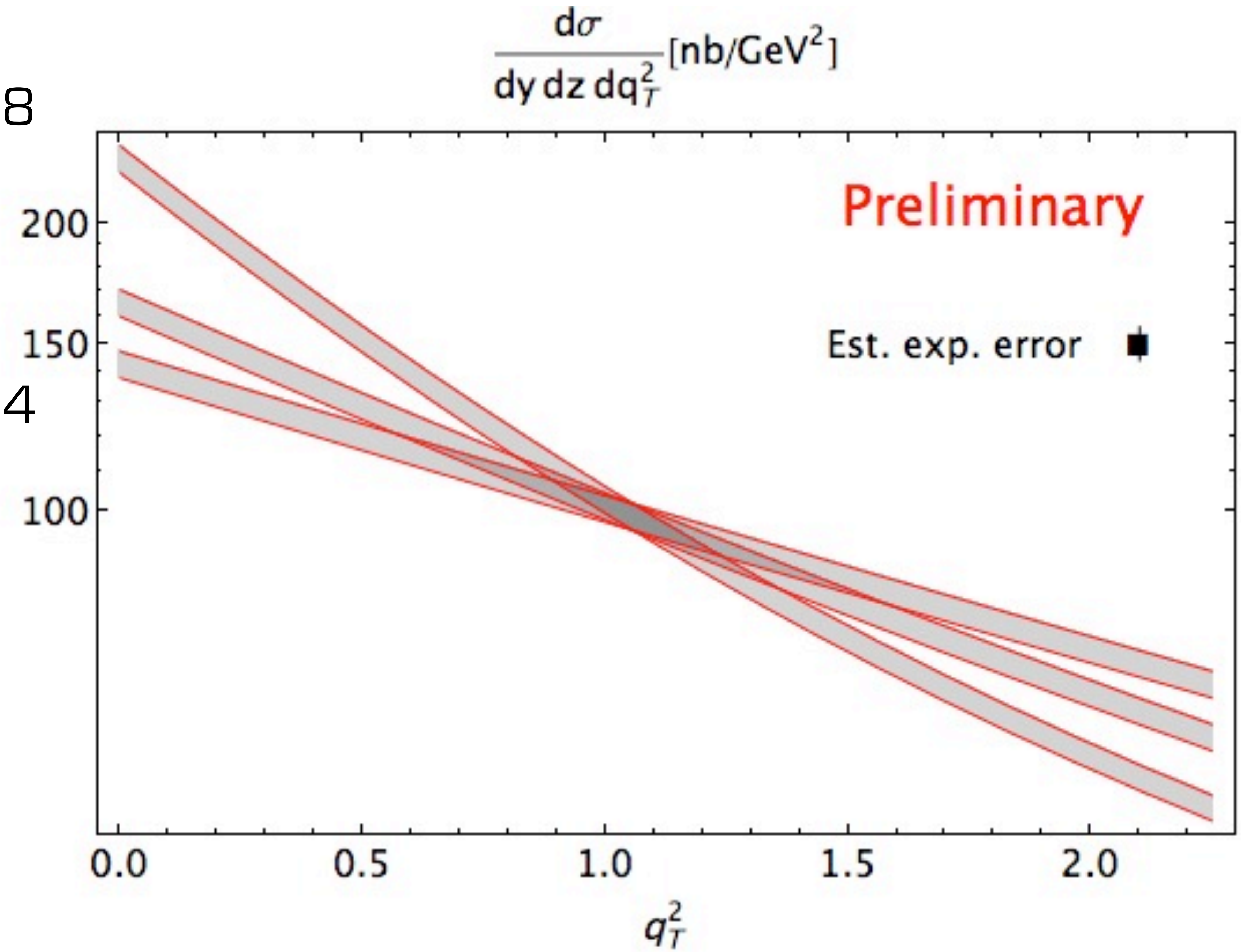

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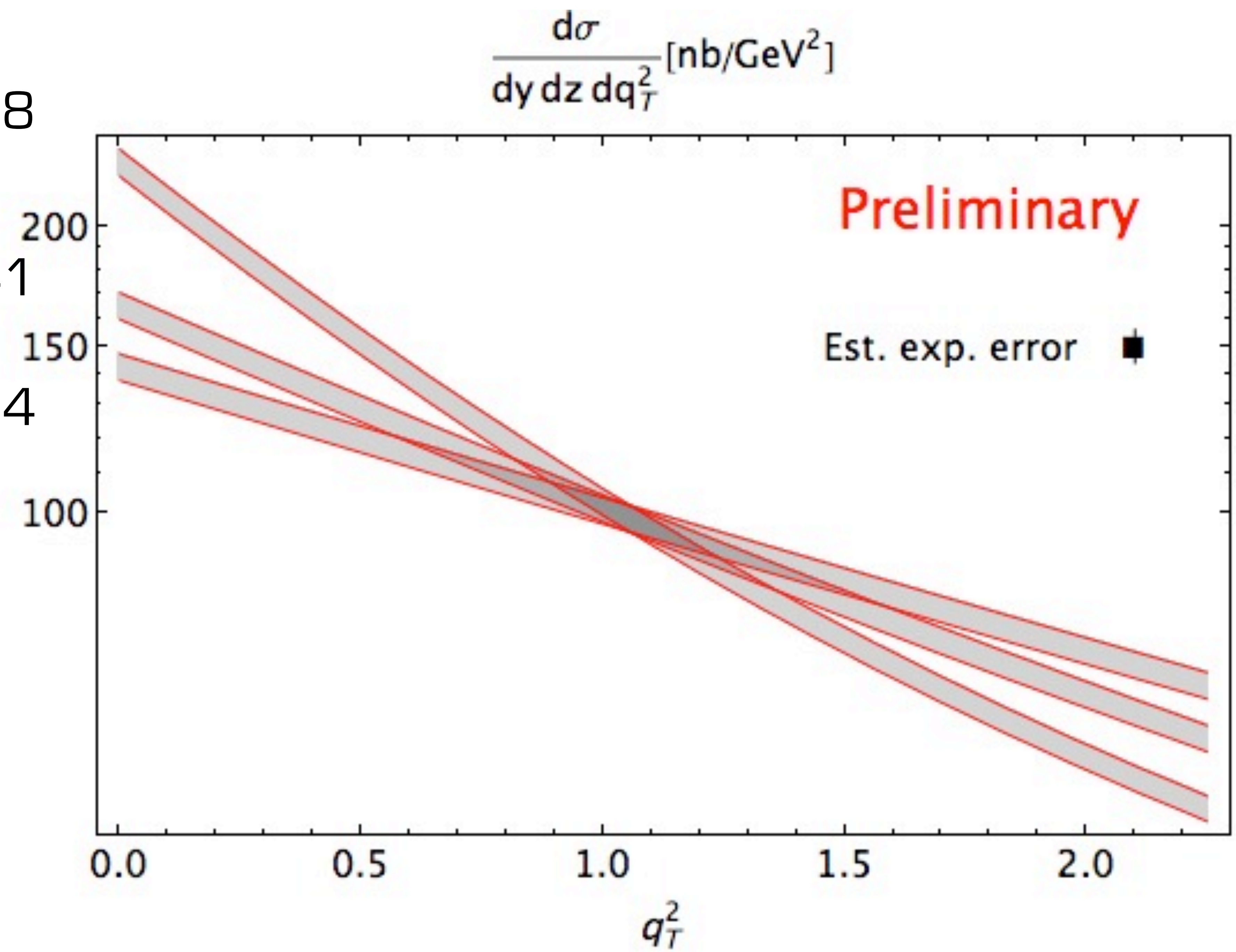

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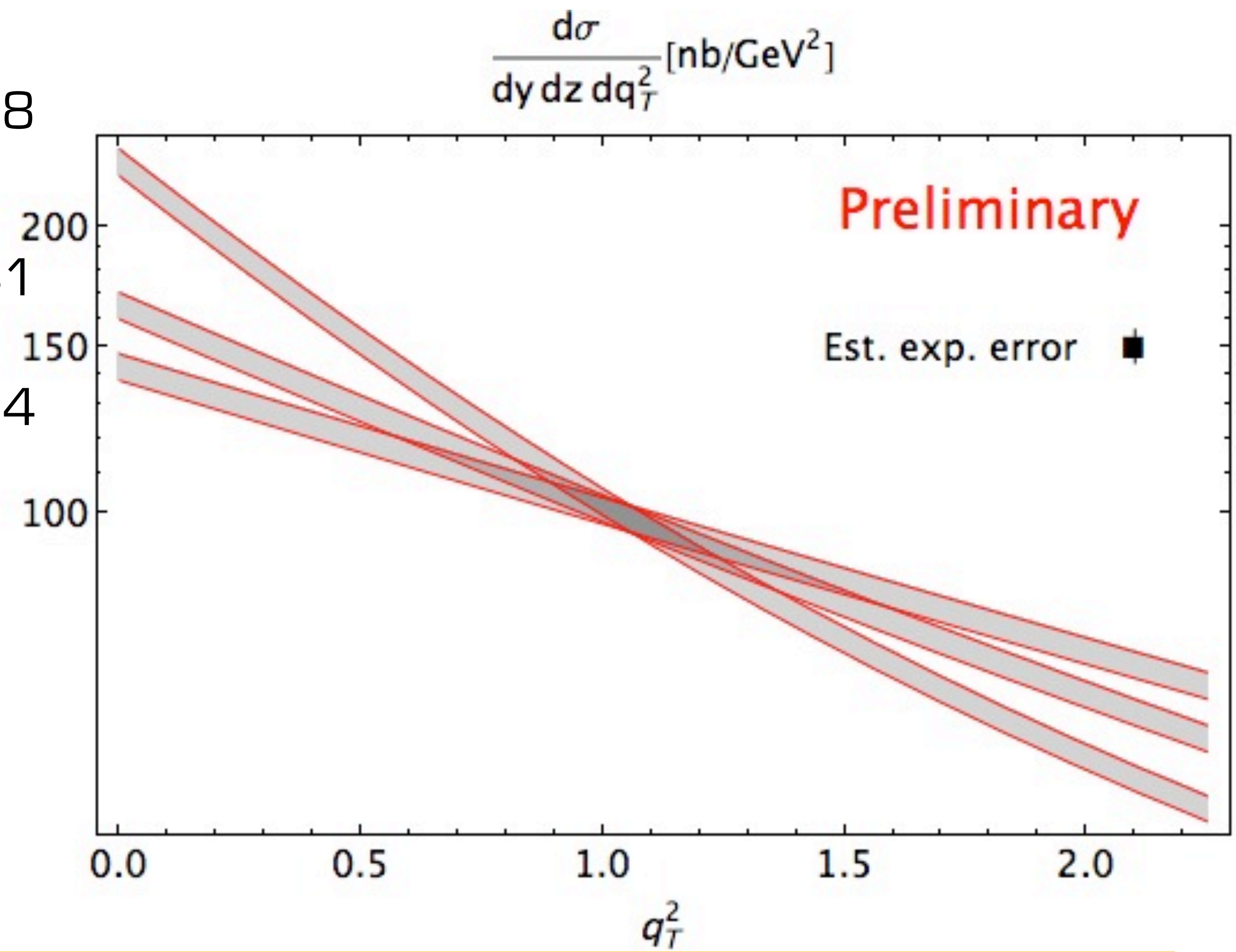
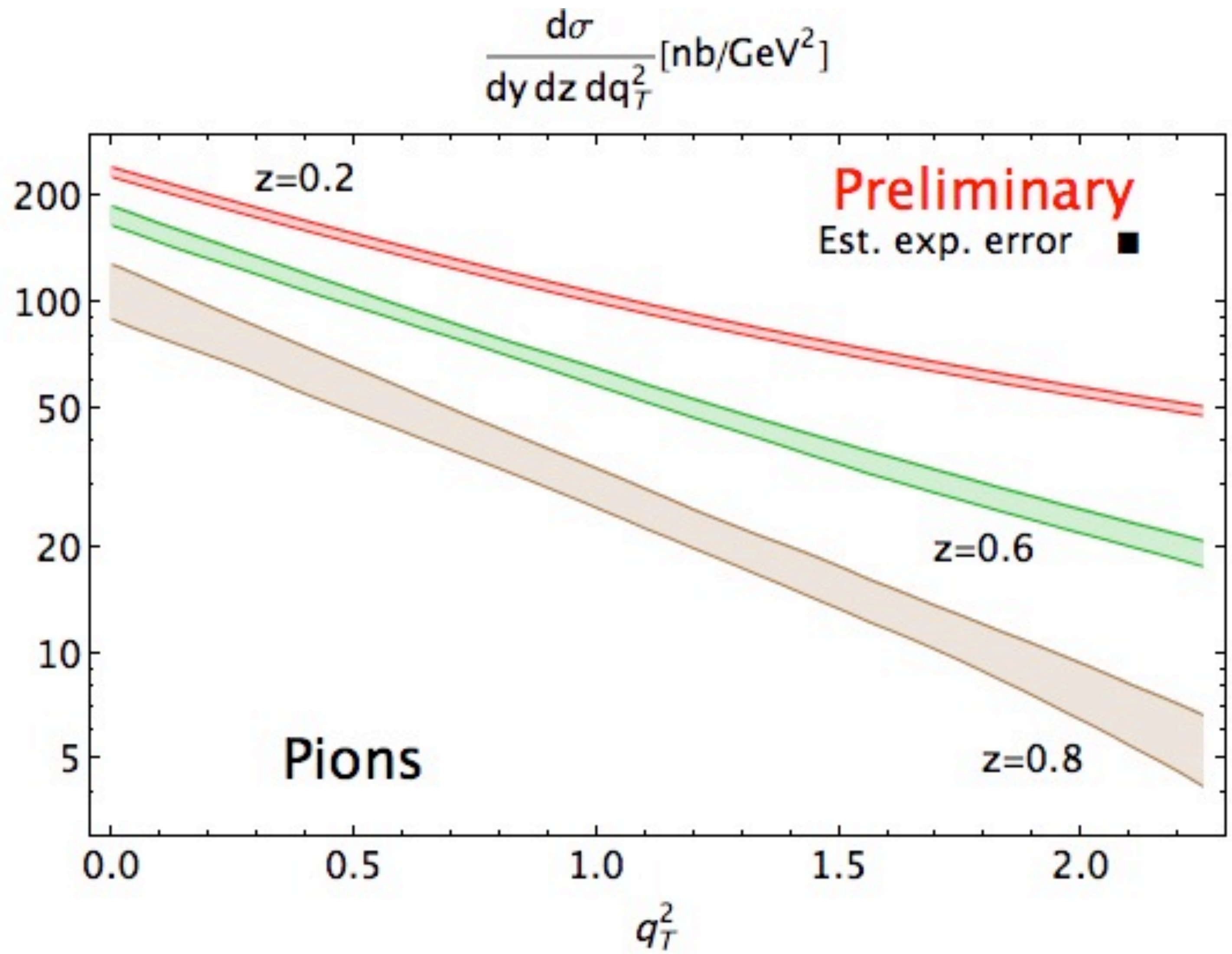
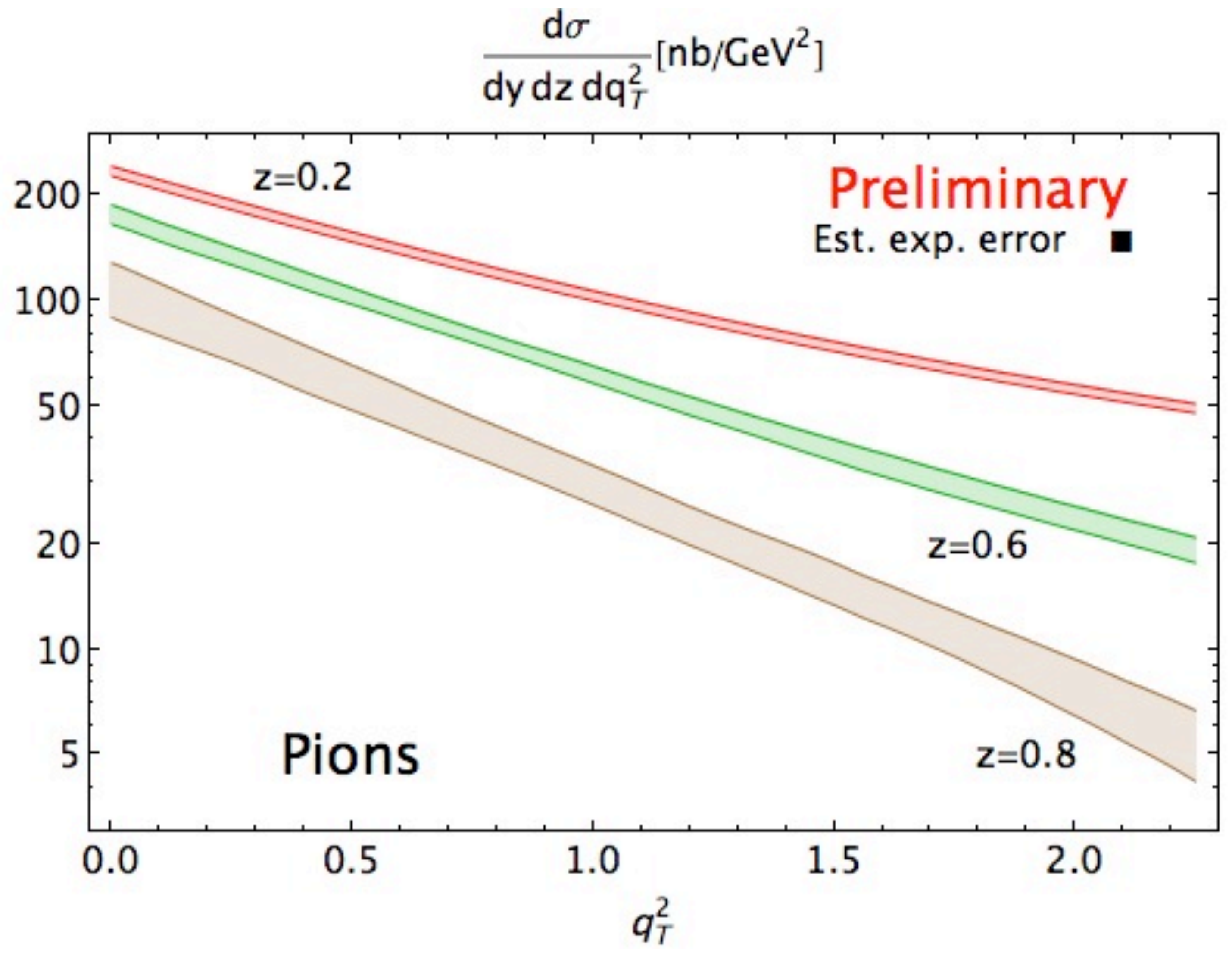


Data can significantly constrain the evolution parameters

Nonperturbative parameters

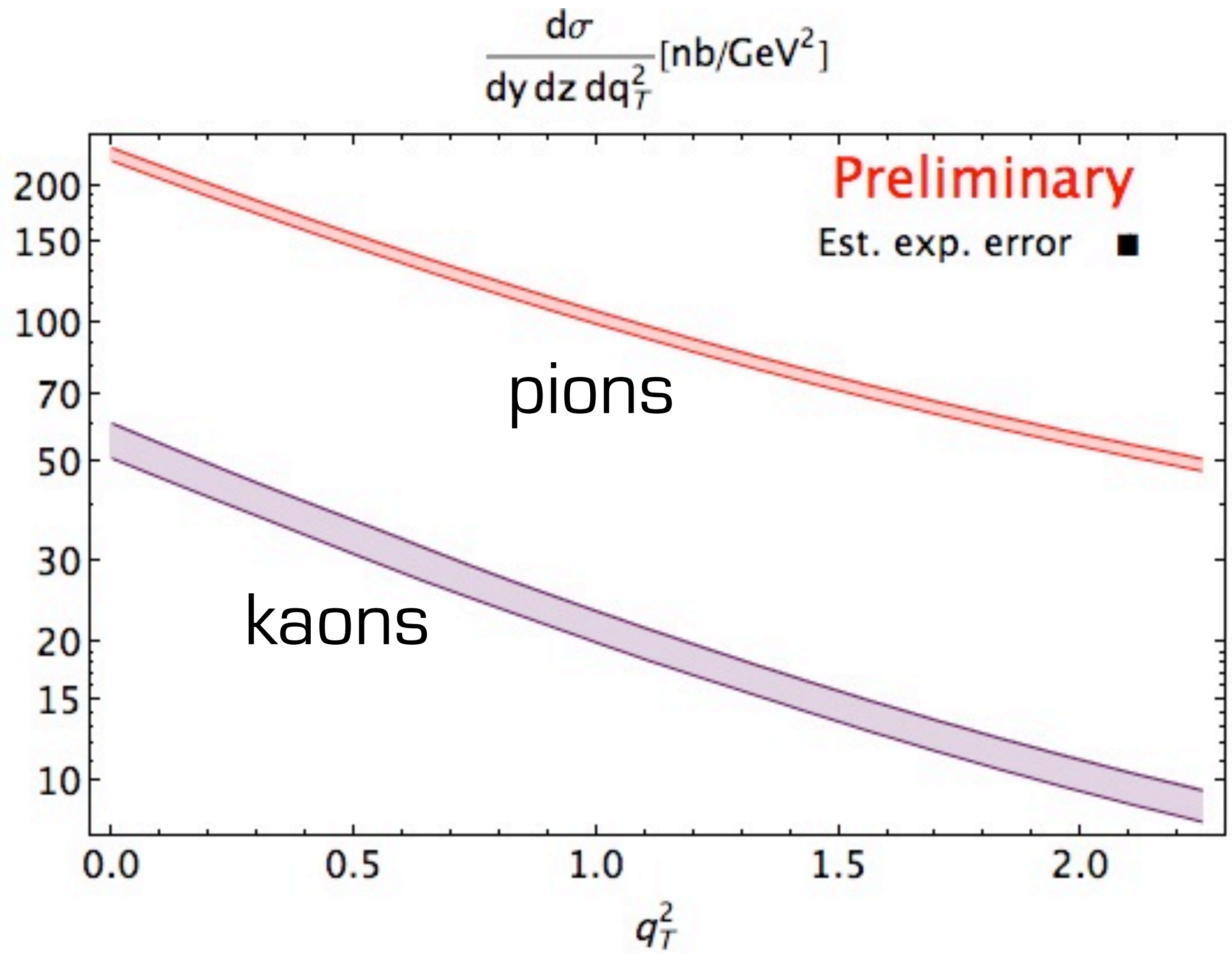


Nonperturbative parameters

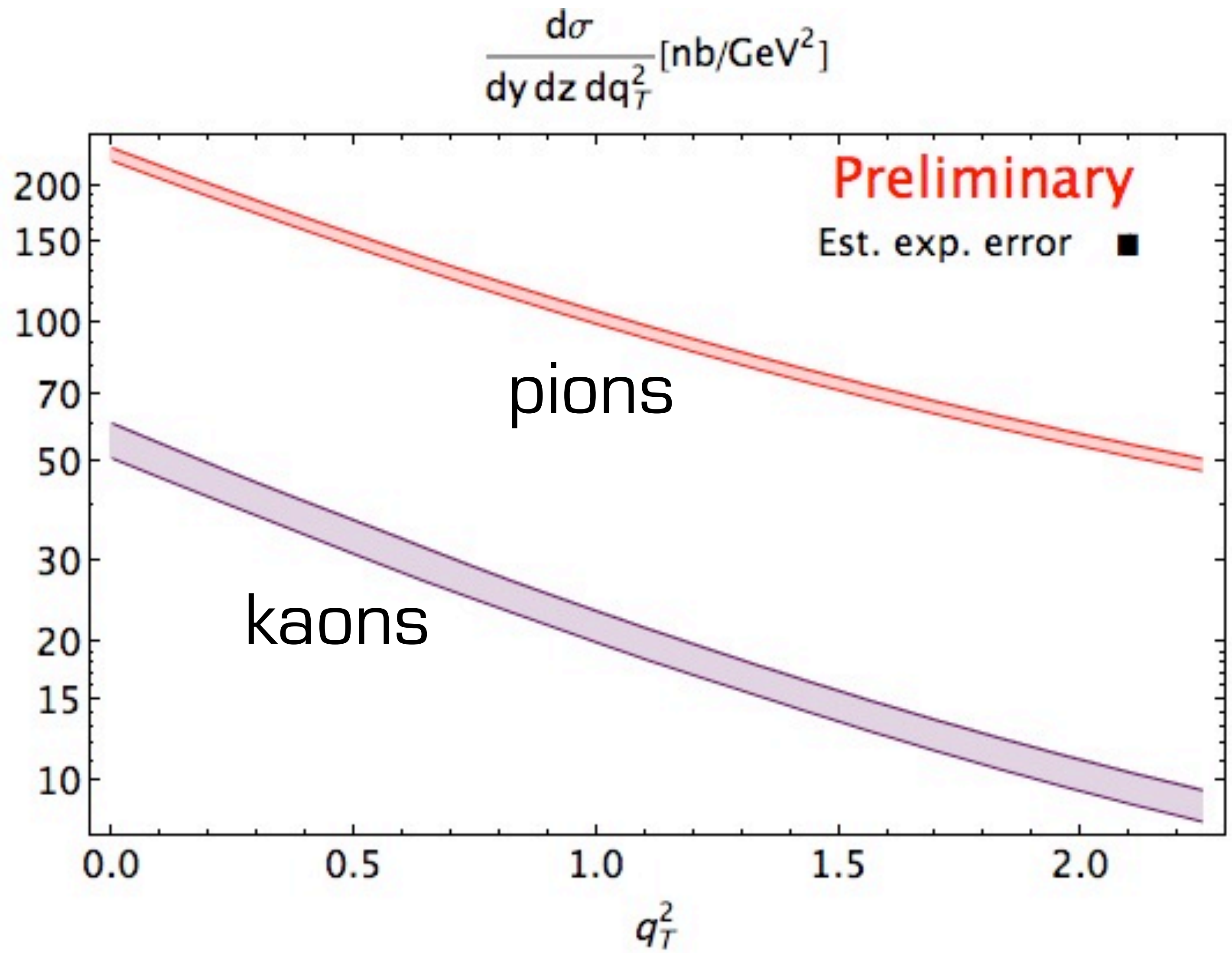


data should constrain the nonperturbative parameters, especially in the high-z region

Flavor dependence

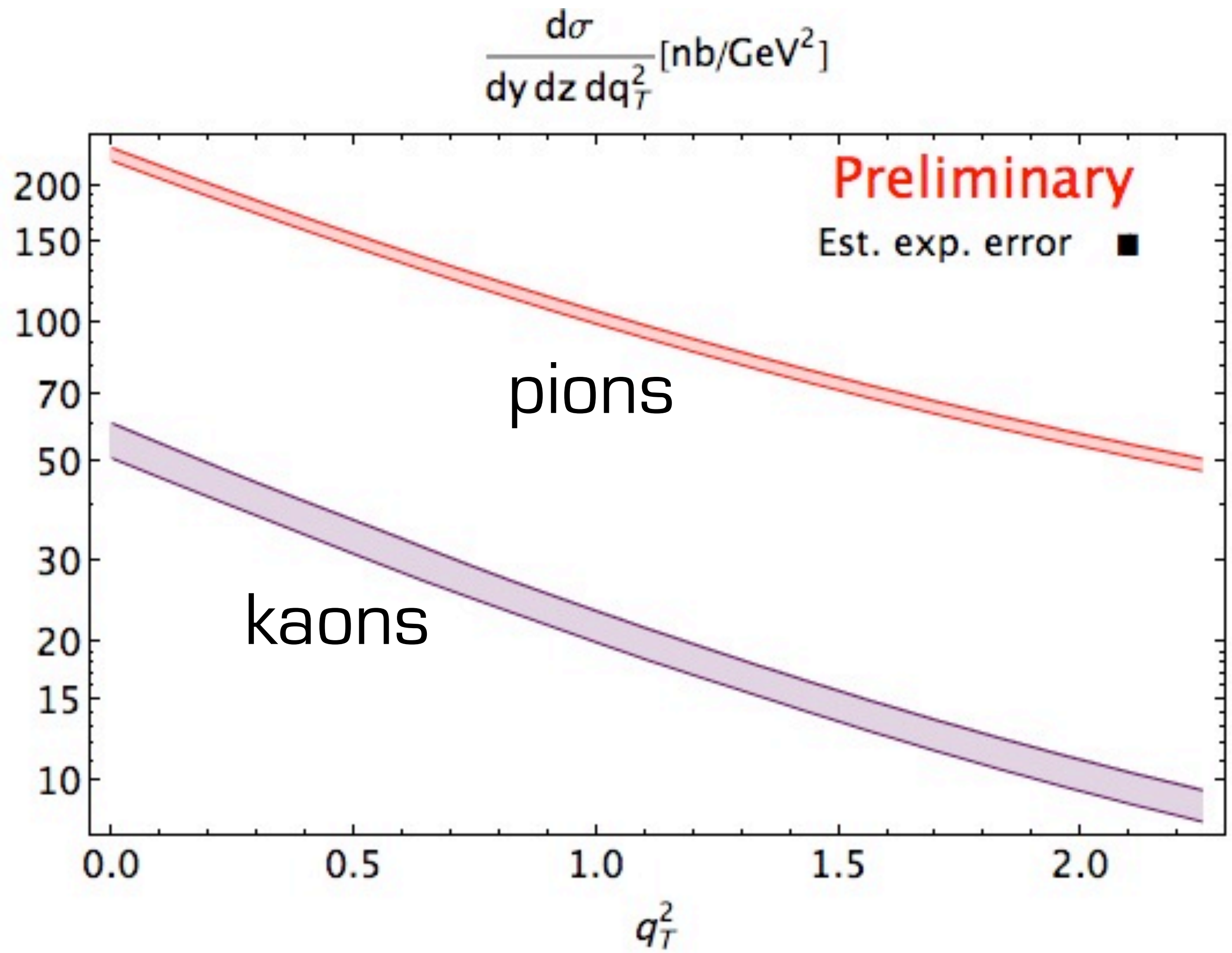


Flavor dependence



pions $$\left[\frac{4}{9}D_1^{u\to\pi^+} + \frac{1}{9}D_1^{\bar{d}\to\pi^+}\right] + \left[\frac{4}{9}D_1^{\bar{u}\to\pi^+} + \frac{1}{9}\left(D_1^{d\to\pi^+} + D_1^{s\to\pi^+} + D_1^{\bar{s}\to\pi^+}\right)\right]$$

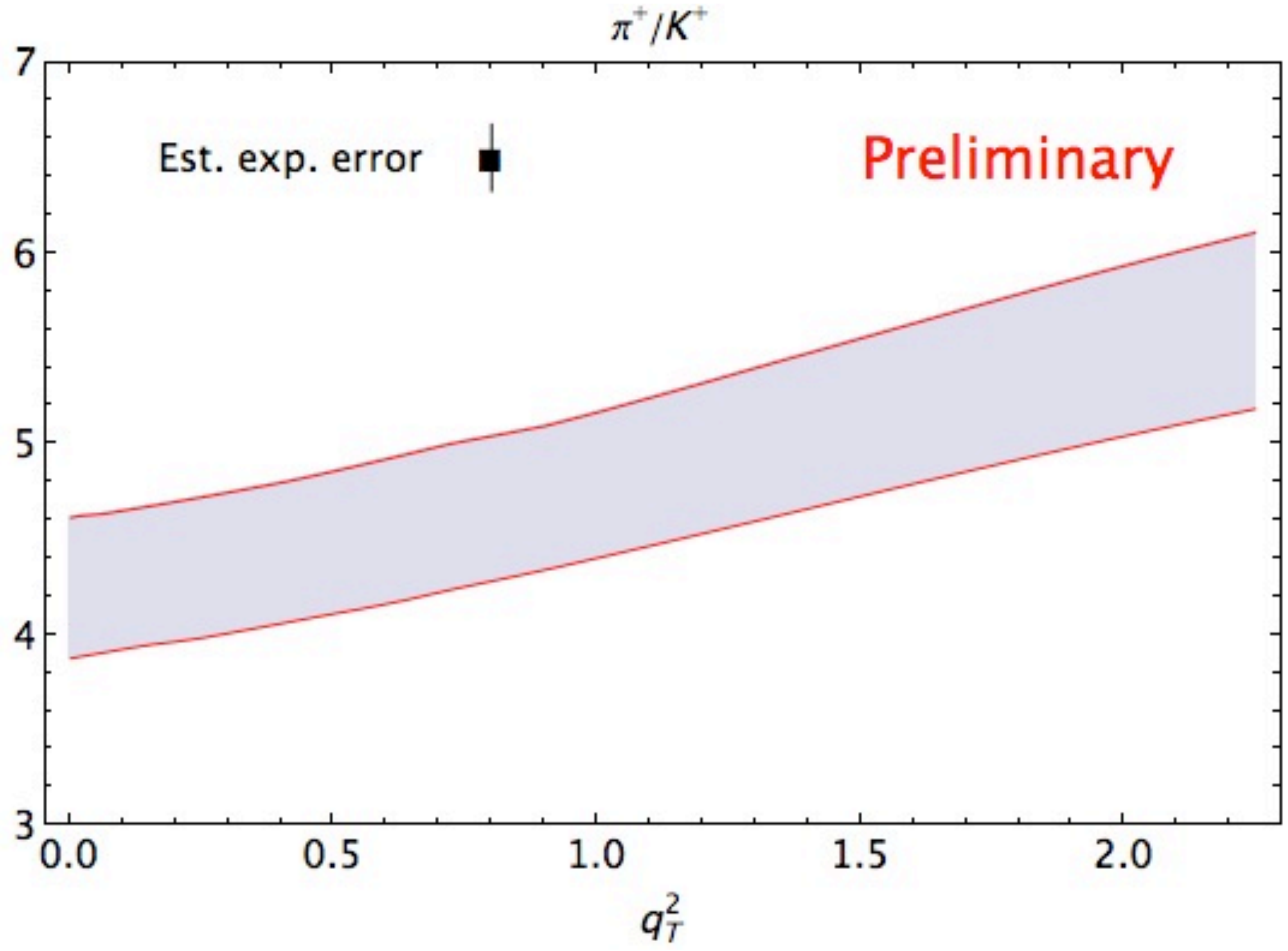
Flavor dependence



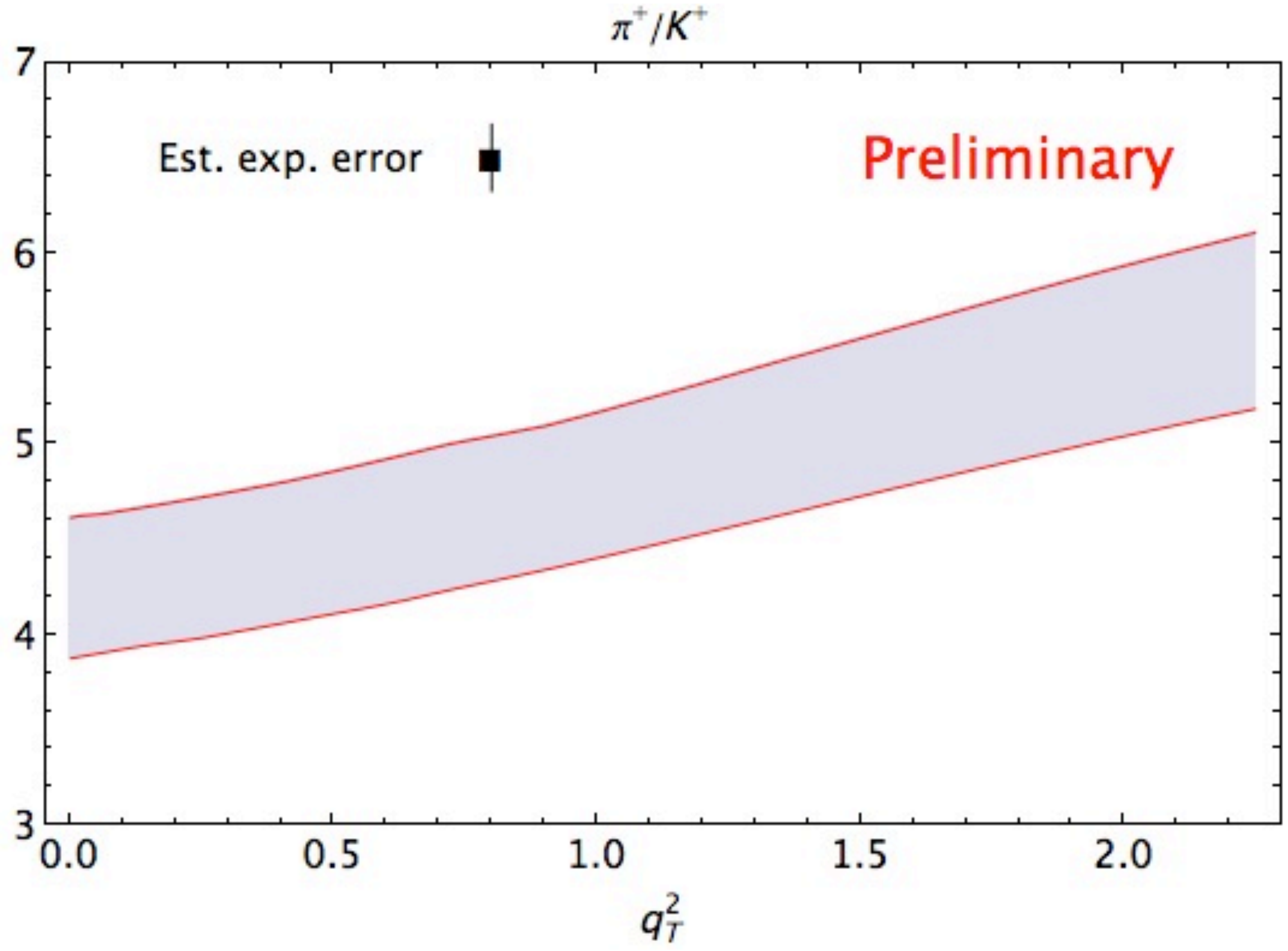
pions $$\left[\frac{4}{9}D_1^{u\to\pi^+}+\frac{1}{9}D_1^{\bar{d}\to\pi^+}\right]+\left[\frac{4}{9}D_1^{\bar{u}\to\pi^+}+\frac{1}{9}\left(D_1^{d\to\pi^+}+D_1^{s\to\pi^+}+D_1^{\bar{s}\to\pi^+}\right)\right]$$

kaons $$\left[\frac{4}{9}D_1^{u\to K^+}+\frac{1}{9}D_1^{\bar{s}\to K^+}\right]+\left[\frac{4}{9}D_1^{\bar{u}\to K^+}+\frac{1}{9}\left(D_1^{d\to K^+}+D_1^{\bar{d}\to K^+}+D_1^{s\to K^+}\right)\right]$$

Flavor dependence: ratios



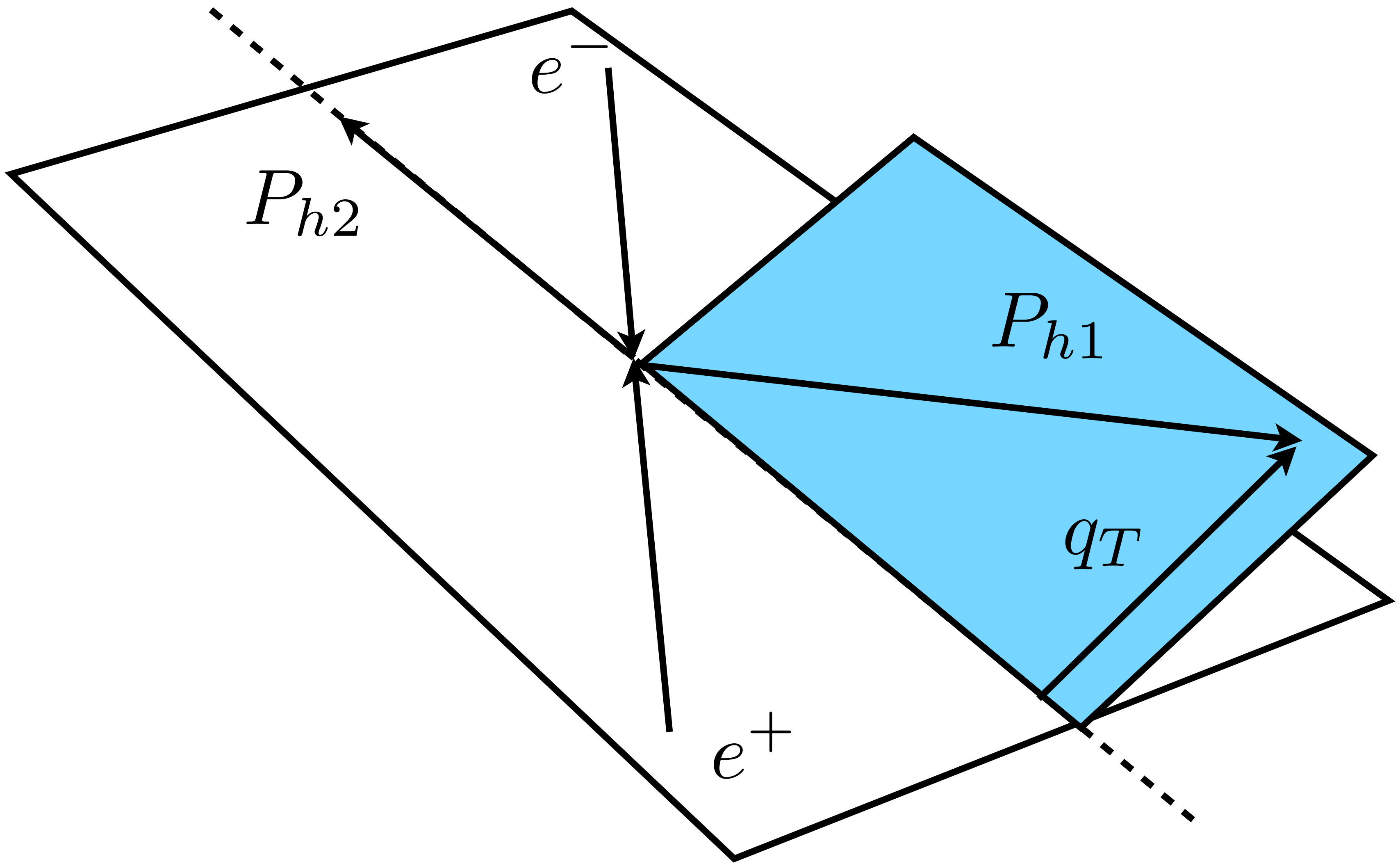
Flavor dependence: ratios



The ratio of pions and kaons should be sensitive to flavor differences in the TMD FFs.

Next steps

$e^+e^- \to h_1 h_2 X$



$$\frac{d\sigma}{dz_1 dz_2 dy dq_T^2} = \frac{6\pi\alpha^2}{Q^2} A(y) \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) z^2 \tilde{D}_1^{q\to h}(z_1, b_T^2) z_2^2 \tilde{D}_1^{\bar{q}\to h}(z_2, b_T^2)$$

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Conclusions

- Electron-positron data at 100 GeV^2 can be extremely valuable to **test evolution** and **pin down evolution parameters**
- They are needed to determine the **nonperturbative parameters** of TMD fragmentation functions
- They are useful to **constrain flavor dependence** of the TMD fragmentation functions
- Indirectly, the knowledge of TMD fragmentation functions will help constraining TMD distribution functions

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