Evolution of 3D-PDFs at Large- x_B and Generalized Loop Space

Igor O. Cherednikov

Universiteit Antwerpen





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What we can learn from the study of Wilson loops?

- Use of the idea of generalized loop space in the quantum field-theoretical description of the 3D-structure of the nucleon visible in high-energy hadron collisions
- Geometrical properties of the loop space can be utilized for understanding of the most general properties of the nonperturbative distribution of partons inside the nucleon
- Duality between equations of motion in the loop space and evolution of the 3D-parton densities

Ancient Greece

Aristotle: three divisions of human intellectual activity



- > physics: studies the causes of change of material things
- mathematics: addresses abstract quantity
- metaphysics: concerned with being as such

Ergo: physics is the learning of **evolution**

What are the Wilson Lines/Loops? Gauge-Invariant Hadronic Correlators

$$\mathcal{F}(k)_{\gamma} = \mathrm{F.T.} \langle h | \ \bar{\Psi}(z) \ \mathcal{W}_{\gamma}[z,0] \ \Psi(0) \ | h \rangle$$

Gauge invariance is guaranteed by the Wilson line

$$\mathcal{W}_{\gamma} = \mathcal{P} \; \exp\left[\pm ig \int_{0}^{z} d\zeta^{\mu} \mathcal{A}_{\mu}(\zeta)
ight]_{\gamma}$$

- Gauge invariance \rightarrow structure of Wilson lines
- Path dependence \rightarrow universality
- Singularities \rightarrow renormalization
- Factorization \rightarrow evolution

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3D Hadronic Correlators: Structure of Nucleon beyond the Collinear Approximation

@ [Belitsky, Ji, Yuan (2003); Boer, Mulders, Pijlman (2003)]

Generic 3D hadronic correlator with the light-like and transverse gauge links

 $\mathcal{F}(k^+, k_\perp; ext{scales}) \sim$ F.T. $\langle h | \bar{\Psi}(z) \ \mathcal{W}_{\gamma}[z^-, z_\perp; 0^-, 0_\perp] \Psi(0) | h \rangle$

 $\gamma \rightarrow \{ n \cup I_{\perp} \}$

Tree-level:

$$\mathcal{F}^{(0)}(k^+, k_{\perp}) = \delta(k^+ - p^+)\delta^{(2)}(k_{\perp})$$
$$\int d^2 k_{\perp} \mathcal{F}(k^+, k_{\perp}) = \mathcal{F}(k^+) = \text{collinear limit}$$
$$\mathcal{F}(k^+, \mu) = \int dz^- e^{-ik^+z^-} \langle h | \bar{\Psi}(z) \ \mathcal{W}_n[z^-, 0^-] \Psi(0)$$

Quantum corrections: \rightarrow emergent (light-cone/rapidity/overlapping) singularities \rightarrow problems with renormalization and evolution = , (=)

 $|h\rangle$

Singularities of Light-like Cusped Wilson Loops Generic light-like quadrilateral contour

@ [Alday, Maldacena (2007); Makeenko (2003); Korchemsky, Drummond, Sokatchev (2008); Alday et al. (2011); Beisert et al. (2012); Belitsky (2012)]



Hint: **duality** between 4-gluon planar scattering amplitude in $\mathcal{N} = 4$ SYM and the Wilson loop made up from four light-like segments:

$$x_i - x_{i+1} = \ell_i \to p_i$$

are equal to the external momenta of this 4-gluon amplitude. The IR evolution of the former is dual to the UV evolution of the latter: governed by the cusp anomalous dimension.

@ [Korchemsky, Radyushkin (1987)]

Singularities of Light-like Cusped Wilson Loops Generic light-like quadrilateral contour

Introduce shape differentiation operators:

$$S_{ij}\frac{\delta}{\delta S_{ij}} = (2\ell_i \cdot \ell_j)\frac{\partial}{\partial(2\ell_i \cdot \ell_j)} , \ S_{ij} = (\ell_i + \ell_j)^2$$

$$\left\langle \frac{\delta}{\delta \ln S} \right\rangle_{1} = S_{12} \frac{\delta}{\delta S_{12}} + S_{23} \frac{\delta}{\delta S_{23}}$$
$$\left\langle \frac{\delta}{\delta \ln S} \right\rangle_{2} = S_{23} \frac{\delta}{\delta S_{23}} + S_{34} \frac{\delta}{\delta S_{34}} , \text{ etc.}$$

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Singularities of Light-like Cusped Wilson Loops Generic Light-Like Quadrilateral Contour

@ [Alday, Maldacena (2007); Makeenko (2003); Korchemsky, Drummond, Sokatchev (2008); Alday et al. (2011); Beisert et al. (2012); Belitsky (2012)]



Figure : Quadrilateral contour γ with the light-like sides $\ell_i^2 = 0$; Examples of the shape variations generated by the shape differential operators $\left\langle \frac{\delta}{\delta \ln S} \right\rangle_1$ and $\left\langle \frac{\delta}{\delta \ln S} \right\rangle_2$.

Mathematics: Loop Space

Polyakov (1979); Makeenko, Migdal (1979, 1981); Kazakov, Kostov (1980); Brandt et al. (1981, 1982);
 Stefanis et al. (1989, 2003)]

Wilson loops as the (fundamental) gauge-invariant degrees of freedom:

$$egin{aligned} &\mathcal{W}^n_{\gamma_1,\dots\gamma_n} = \langle 0 | \mathcal{T} rac{1}{N_c} \mathcal{U}_{\gamma_1} \cdots rac{1}{N_c} \mathcal{U}_{\gamma_n} | 0
angle \ &\mathcal{U}_{\gamma_i} = \mathcal{P} \; \exp \left[ig \int_{\gamma_i} \, dz^\mu \mathcal{A}_\mu(z)
ight] \end{aligned}$$

The Wilson functionals obey the Makeenko-Migdal loop equations:

$$\partial^{
u} \frac{\delta}{\delta \sigma_{\mu
u}(x)} W^{1}_{\gamma} = N_{c}g^{2} \oint_{\gamma} dz^{\mu} \delta^{(4)}(x-z) W^{2}_{\gamma_{xz}\gamma_{zx}}$$

+ Mandelstam contraints

$$\sum a_i \mathcal{W}^{n_i} = 0$$

Equations are exact, but...

Loop Space Makeenko-Migdal approach

Area derivative:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\mathcal{U}_{\gamma} = \lim_{|\delta\sigma_{\mu\nu}(x)|\to 0} \frac{\mathcal{U}_{\gamma\delta\gamma_{x}} - \mathcal{U}_{\gamma}}{|\delta\sigma_{\mu\nu}(x)|}$$

Path derivative:

$$\partial_{\mu}\mathcal{U}(\gamma) = \lim_{|\delta x_{\mu}| \to 0} rac{\mathcal{U}_{\delta x_{\mu}^{-1} \gamma \delta x_{\mu}} - \mathcal{U}_{\gamma}}{|\delta x_{\mu}|}$$

Mandelstam formula:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)} \operatorname{Tr} \, \mathcal{U}_{\gamma} = ig \operatorname{Tr} \, \left[\mathcal{F}_{\mu\nu} \, \, \mathcal{U}_{\gamma} \right]$$

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Loop Space Makeenko-Migdal approach: issues

$$\partial^{
u} \frac{\delta}{\delta\sigma_{\mu\nu}(x)} W^{1}_{\gamma} = N_{c}g^{2} \oint_{\gamma} dz^{\mu} \delta^{(4)}(x-z)W^{2}_{\gamma_{xz}\gamma_{zx}}$$

The equation is exact and non-perturbative, but not closed and difficult to solve in general. Moreover:

- No information about cusps and other obstructions
- Wilson loops are functionals defined on the paths. But infinitesimal variation of a path doesn't necessarily yield infinitesimal variation of a functional

@ [ICh, Mertens (2014)]

Variational analysis in the loop space is by no means straightforward

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Loop Space Makeenko-Migdal approach: the Stokes theorem

$$\mathcal{W}_{\gamma} = \mathcal{W}^{(0)} + \mathcal{W}^{(1)} = 1 - \frac{g^2 C_F}{2} \oint_{\gamma} \oint_{\gamma} dz_{\mu} dz'_{\nu} D^{\mu\nu}(z - z') + O(g^4)$$
$$D^{\mu\nu}(z - z') = -g^{\mu\nu} \Delta(z - z')$$
$$\Delta(z - z') = \frac{\gamma(1 - \epsilon)}{4\pi^2} \frac{(\pi\mu^2)^{\epsilon}}{[-(z - z')^2 + i0]^{1 - \epsilon}}$$
$$\left\langle \frac{\delta}{\delta \ln S} \right\rangle \mathcal{W}_{\gamma} = \frac{g^2 C_F}{2} \left\langle \frac{\delta}{\delta \ln S} \right\rangle \oint_{\gamma} \oint_{\gamma} dz_{\lambda} dz'^{\lambda} \Delta(z - z') + O(g^4)$$

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Loop Space

Shape variations without the Stokes theorem

@ [ICh, Mertens, Van der Veken (2012,2013)]

$$\mathcal{W}_{\gamma}^{(1)} = \frac{g^2 C_F}{2} \frac{\gamma (1-\epsilon) (\pi \mu^2)^{\epsilon}}{4\pi^2} \cdot \sum_{i,j} (\ell_j^{\lambda} \ell_j^{\lambda}) \cdot \int_0^1 \int_0^1 \frac{d\tau d\tau'}{[-(x_i - x_j - \tau_i \ell_i + \tau_j \ell_j)^2 + i0]^{1-\epsilon}} \\ \left\langle \frac{\delta}{\delta \ln S} \right\rangle \mathcal{W}_{\gamma} = -\frac{\alpha_s N_c}{2\pi} \Gamma(1-\epsilon) (\pi \mu^2)^{\epsilon} \sum_{ij} \left\langle \frac{\delta}{\delta \ln S} \right\rangle_i (-S_{ij})^{\epsilon} \frac{1}{2} \int_0^1 \int_0^1 \frac{d\tau d\tau'}{[(1-\tau)\tau']^{1-\epsilon}} \\ \mu \frac{d}{d\mu} \left[\left\langle \frac{\delta}{\delta \ln S} \right\rangle \ln \mathcal{W}_{\gamma*} \right] = -\sum \gamma_{\text{cusp}}$$

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Intermediate Conclusions:

- Equations of motion in the Wilson Loop Space describe the reaction of the non-local functionals of the gauge fields to the shape variations of the paths in the underlying manifold
- In the subset of the cusped light-like paths, the geometrical evolution corresponds to the rapidity evolution
- Understanding evolution of the hadronic correlation functions via geometry of the WLS

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and evolution of transverse-distance dependent parton densities @ [ICh, Mertens, Taels, Van der Veken (2013); ICh (2014)]

Transverse-distance dependent PDFs

$$\mathcal{F}\left(x, b_{\perp}; P^{+}, n^{-}, \mu^{2}\right) = \int d^{2}k_{\perp} \, \mathrm{e}^{-ik_{\perp} \cdot b_{\perp}} \mathcal{F}\left(x, k_{\perp}; P^{+}, n^{-}, \mu^{2}\right) =$$

$$\int \frac{dz^-}{2\pi} e^{-ik^+z^-} \left\langle P \mid \bar{\psi}(z^-, b_\perp) \mathcal{U}_{n^-}^{\dagger}[z^-, b_\perp; \infty^-, b_\perp] \mathcal{U}_{n^-}^{\dagger}[\infty^-, b_\perp; \infty^-, \infty_\perp] \right\rangle$$

 $\mathcal{U}_{\boldsymbol{I}}[\infty^{-},\infty_{\perp};\infty^{-},\boldsymbol{0}_{\perp}]\mathcal{U}_{n^{-}}[\infty^{-},\boldsymbol{0}_{\perp};0^{-},\boldsymbol{0}_{\perp}] \ \psi(0^{-},\boldsymbol{0}_{\perp}) \ | \ \boldsymbol{P} \rangle$

$$\mathcal{U}_{\gamma} = \mathcal{P} \; \exp\left[-ig \oint_{\gamma} \mathcal{A}_{\mu}(z) \; dz^{\mu}
ight]$$

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and evolution of transverse-distance dependent parton densities

@ [ICh, Mertens, Taels, Van der Veken (2013); ICh (2014)]

The struck quark acquires almost all momentum of the nucleon: k_µ ≈ P_µ. Provided that the transverse component of the nucleon momentum is equal to zero, the transverse momentum of the quark k_⊥ is gained by the gluon interactions

@ [Bassetto, Ciafaloni, Marchesini (1983); Korchemsky, Marchesini (1993)]

A very fast moving quark with momentum k_μ can be considered as a classical particle with a (dimensionless) velocity parallel to the nucleon momentum P, so that the quark fields are replaced by the Mandelstam fields

$$\psi(0) = \mathcal{W}_{P}[\infty; 0] \ \Psi_{\mathrm{in-jet}}(0) \ , \ \bar{\psi}(z^{-}, z_{\perp}) = \bar{\Psi}_{\mathrm{in-jet}}(z) \ \mathcal{W}_{P}^{\dagger}[z; \infty]$$

 $\bar{\psi}_{\rm in-jet}, \psi_{\rm in-jet}$ — incoming-collinear jets in the initial and final states

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and evolution of transverse-distance dependent parton densities

@ [ICh, Mertens, Taels, Van der Veken (2013)]

Provided that almost all momentum of the nucleon is carried by the struck quark, real radiation can only be soft

$$q_\mu \sim (1-x) P_\mu$$

- Virtual gluons can be soft or collinear, collinear gluons can only be virtual, quark radiation is suppressed in the leading-twist
- ▶ Rapidity singularities stem only from the soft contributions: they are known to occur at small gluon momentum $q^+ \rightarrow 0$. Rapidity divergences are known to originate from the minus-infinite rapidity region, where gluons travel along the direction of the outgoing jet, not incoming-collinear
- Real contributions are UV-finite (in contrast to the integrated PDFs), but can contain rapidity singularities and a non-trivial x_Band b_⊥-dependence

Large-*x*_B Factorization

and evolution of transverse-distance dependent parton densities

Large- x_B factorization formula

$$\mathcal{F}(x, b_{\perp}; \mathcal{P}^+, \mu^2) = \mathcal{H}(\mu, \mathcal{P}^2) \times \Phi(x, b_{\perp}; \mathcal{P}^+, \mu^2)$$

- \mathcal{H} is x_B -independent, resums incoming-collinear partons
- Φ is the soft function

$$\Phi(x, b_{\perp}; P^+, \mu^2) = P^+ \int dz^- e^{-i(1-x)P^+z^-} \cdot \\ \times \langle 0 | \mathcal{W}_P^{\dagger}[z; -\infty] \mathcal{W}_{n^-}^{\dagger}[z; \infty] \mathcal{W}_{n^-}[\infty; 0] \mathcal{W}_P[0; \infty] | 0 \rangle$$

Rapidity and renormalzation-group evolution equations

$$\mu \frac{d}{d\mu} \ln \mathcal{F} \left(x, b_{\perp}; P^{+}, \mu^{2} \right) = \mu \frac{d}{d\mu} \ln \mathcal{H} (\mu^{2}) + \mu \frac{d}{d\mu} \ln \Phi(x, b_{\perp}; P^{+}, \mu^{2})$$

$$P^{+} \frac{\partial}{\partial P^{+}} \ln \mathcal{F} \left(x, b_{\perp}; P^{+}, \mu^{2} \right) = P^{+} \frac{\partial}{\partial P^{+}} \ln \Phi(x, b_{\perp}; P^{+}, \mu^{2})$$

and evolution of transverse-distance dependent parton densities

The soft function \mathcal{F} is a Fourier transform of an element of the (generalized) loop space. This fact enables us to consider the shape variations of this path, which are generated by the infinitesimal variations of the rapidity variable $\ln P^+$. The corresponding differential operator reads

$$\left\langle \frac{\delta}{\delta \ln S} \right\rangle_1 \sim P^+ \frac{\partial}{\partial P^+} \; ,$$

Collins-Soper-Sterman rapidity-independent kernel

$$\mu \frac{d}{d\mu} \left(P^+ \frac{\partial}{\partial P^+} \ln \mathcal{F} \right) = \mu \frac{d}{d\mu} \left(P^+ \frac{\partial}{\partial P^+} \ln \Phi \right) =$$
$$= -\sum_{\text{TDD}} \Gamma_{\text{cusp}}(\alpha_s) = \mu \frac{d}{d\mu} \mathcal{K}_{\text{CSS}}(\alpha_s)$$

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and rapidity evolution @ [ICh, Mertens (2014); ICh (2014)]

Operator-valued functional

$$\mathcal{U}_{\gamma^t} = \mathcal{P} \exp \left[ig \int\limits_0^t \mathcal{A}_{\mu}(x) \ \dot{\gamma}^{\mu} d\sigma
ight]_{\gamma}$$

$$x_\mu(\sigma) = \dot{\gamma}_\mu \sigma$$
 , $\sigma \in [0,1]$, $x_\mu(0) = x_\mu(1)$, $U_\gamma = U_{\gamma^1}$

Frechét logarithmic derivative

$$\mathcal{D}_V \; \mathcal{U}_\gamma = \mathcal{U}_\gamma \cdot \int_0^1 dt \; \mathcal{U}_{\gamma^t} \cdot \; \mathcal{F}_{\mu
u}(t) \; \left[V^\mu(t) \wedge \dot{\gamma}^
u(t)
ight] \cdot \mathcal{U}_{\gamma^t}^{-1}$$

 $\dot{\gamma}^{
u}(t)$ parametrizes the integration trajectory; $V^{\mu}(t)$ defines the direction of the variation

@ [ICh, Mertens (2014); ICh (2014)]

Local area variation (Makeenko-Migdal approach) and non-local $\ensuremath{\mathsf{Frech\acute{e}t}}$ variation



@ [ICh, Mertens (2014); ICh (2014)]

Non-local Frechét variation for the light-like rectangular contour

$$V^{\mu}(t) = (\ell_1^+, \ell_2^-, \mathbf{0}_{\perp})$$



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$$\left\langle \frac{\delta}{\delta \ln S} \right\rangle_1 \mathcal{W}_{\gamma} = D_V \mathcal{W}_{\gamma} \quad , \quad V^{\mu} = V_1^{\mu} + V_2^{\mu} = (\ell_1^+ \ , \ \ell_2^- \ , \ \mathbf{0}_{\perp})$$

$$\mu \frac{d}{d\mu} \left[D_V \ \mathcal{W}_{\gamma} \right] = \left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \left[D_V \ \mathcal{W}_{\gamma} \right] = -\sum \mathsf{\Gamma}_{\mathrm{cusp}}$$

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Outlook:

- Theoretical and phenomenological study of the three-dimensional structure of nucleons with the Wilson lines/loops formalism as the main instrument
- Mathematical structure of the loop space: gauge-invariant formulation of the TMDs in terms of the nucleon matrix elements; complete evolution of the TMDs from geometrical properties of the loop space
- Ultimate goal: field-theoretically motivated dynamical 3D-picture of the nucleon; the fundamental problem of the nucleons spin composition from the quark and gluon constituents

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