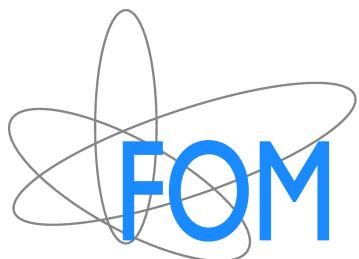


Color entanglement in hadronic processes for TMD PDFs

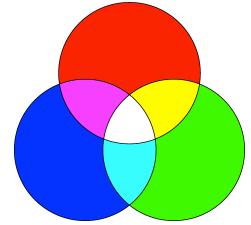
Maarten Buffing

QCD Evolution workshop

May 12, 2014



Content

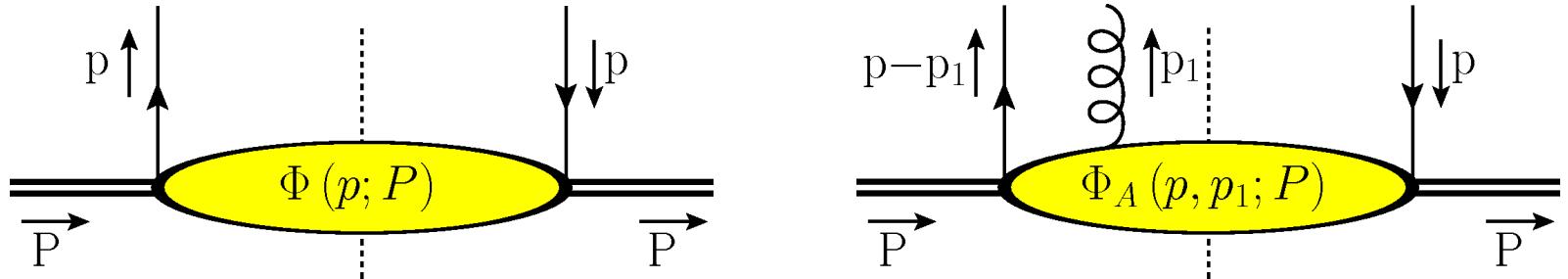


- Part I: individual correlators
 - TMDs
 - Transverse momentum weighting
- Part II: multi hadron processes
 - Color entanglement for Drell-Yan
- Summary and conclusions

Work in collaboration with Piet Mulders and Asmita Mukherjee

Part I: Individual correlators

- Quark correlators can be written as matrix elements



$$\Phi_{ij}(p; P) = \Phi_{ij}(p|p) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle$$

$$\Phi_{A;ij}^\alpha(p - p_1|p) = \int \frac{d^4\xi d^4\eta}{(2\pi)^8} e^{i(p-p_1) \cdot \xi + ip_1 \cdot \eta} \langle P | \bar{\psi}_j(0) A^\alpha(\eta) \psi_i(\xi) | P \rangle$$

- The field combination is non-local

TMD PDFs

- Distribution functions could be described in terms of parton distribution functions (TMDs)
- Depending on polarization(s), different contributions are required

$$\Phi^{[U]}(x, p_T) = \left(f_1^{[U]}(x, p_T^2) + i h_1^{\perp[U]}(x, p_T^2) \frac{p'_T}{M} + \dots \right) \frac{P}{2}$$

		quark polarization		
		U	L	T
nucleon pol.	U	f_1		h_1^\perp
	L		g_{1L}	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	h_{1T}, h_{1T}^\perp

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T-odd

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quark polarization

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_{1T}^\perp

Process dependent

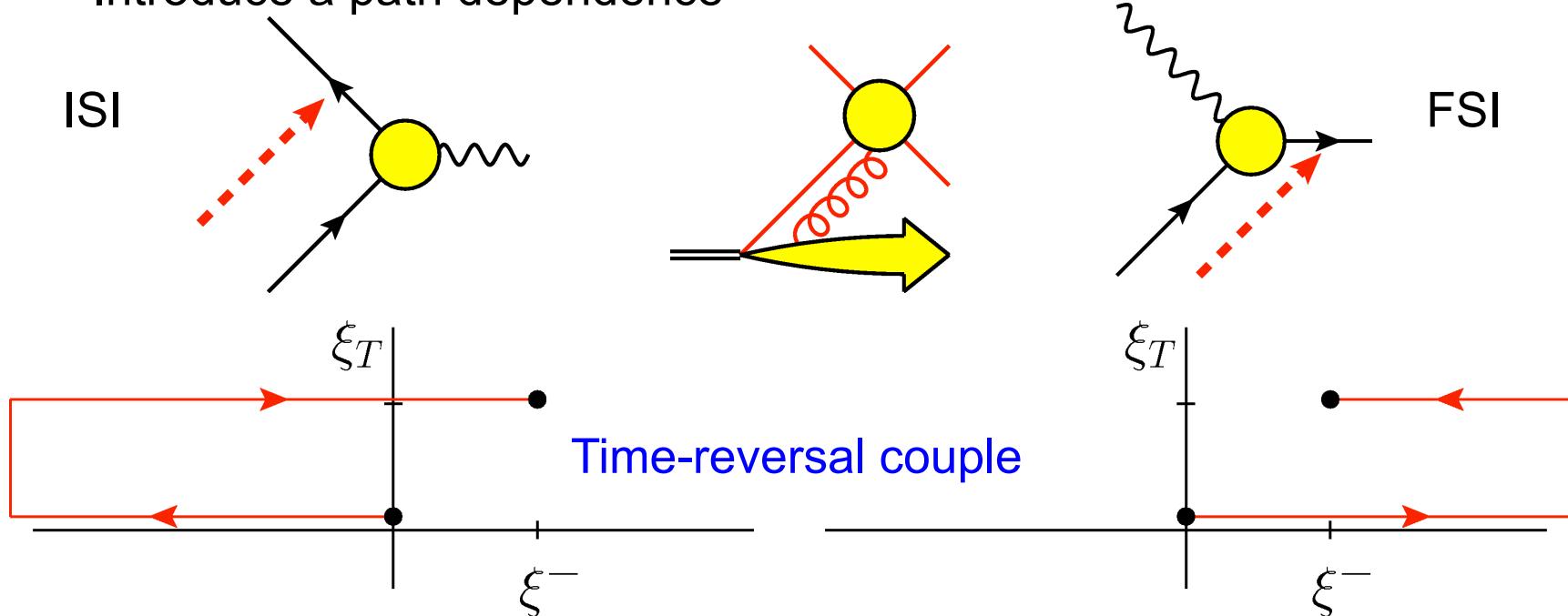
Pretzelosity is T-even
and process dependent

Gauge invariance for quark TMDs

- Gauge links make the nonlocal combinations of fields gauge invariant

$$U_{[0,\xi]} = \mathcal{P} \exp \left(-ig \int_0^\xi ds^\mu A_\mu \right)$$

- Introduce a path dependence



Mellin moments

- Collinear functions

$$\Phi^q(x) = \int \frac{d(\xi \cdot P)}{2\pi} e^{ip \cdot \xi} \langle P | \bar{\psi}(0) U_{[0,\xi]}^{[n]} \psi(\xi) | P \rangle_{\xi \cdot n = \xi_T = 0}$$

$$\begin{aligned} x^{N-1} \Phi^q(x) &= \int \frac{d(\xi \cdot P)}{2\pi} e^{ip \cdot \xi} \langle P | \bar{\psi}(0) (i\partial^n)^{N-1} U_{[0,\xi]}^{[n]} \psi(\xi) | P \rangle_{\xi \cdot n = \xi_T = 0} \\ &= \int \frac{d(\xi \cdot P)}{2\pi} e^{ip \cdot \xi} \langle P | \bar{\psi}(0) U_{[0,\xi]}^{[n]} (iD^n)^{N-1} \psi(\xi) | P \rangle_{\xi \cdot n = \xi_T = 0} \end{aligned}$$

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

$$\Phi^{(N)} = \langle P | \bar{\psi}(0) (iD^n)^{N-1} \psi(\xi) | P \rangle_{\xi \cdot n = \xi_T = 0}$$

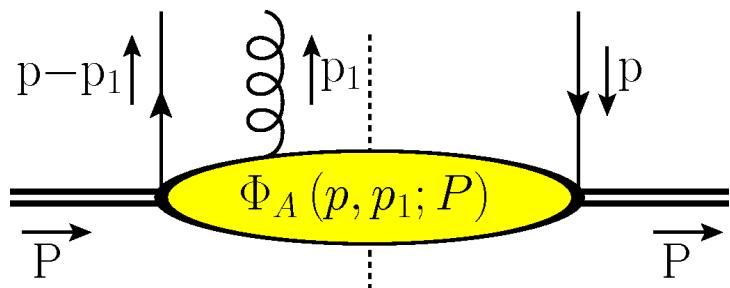
Transverse moments

- For TMD functions one can consider transverse moments

$$p_T^\alpha \Phi^{[U]}(x, p_T; n) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P | \bar{\psi}(0) i\partial_T^\alpha U_{[0,\xi]} \psi(\xi) | P \rangle_{\xi \cdot n = 0}$$

- Due to transverse directions, partonic operators show up. For the simplest gauge links one finds

$$i\partial_T^\alpha U_{[0,\xi]}^{[\pm]} = U_{[0,\xi]}^{[\pm]} \left(iD_T^\alpha(\xi) - gA_T^\alpha(\xi) \pm G^{n\alpha}(\xi) \right)$$



$$\begin{aligned} \Phi_D(x) &= \int dx_1 \Gamma_D^\alpha(x - x_1, x_1 | x) \\ \Phi_A(x) &= \text{PV} \int dx_1 \frac{1}{x_1} \Gamma_F^{n\alpha}(x - x_1, x_1 | x) \\ \tilde{\Phi}_\partial(x) &= \Phi_D^\alpha(x) - \Phi_A^\alpha(x) \\ \Phi_G(x) &= \pi \Phi_F^{n\alpha}(x, 0 | x) \end{aligned}$$

Transverse moments

- For a general gauge link

$$\int d^2 p_T p_T^\alpha \Phi^{[U]}(x, p_T; n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]} \Phi_G^\alpha(x)$$

↑

T-even

↑

T-odd (gluonic pole or ETQS m.e.)

$$\tilde{\Phi}_\partial(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$$

$$\Phi_G(x) = \pi \Phi_F^{n\alpha}(x, 0|x)$$

Transverse moments

- For a general gauge link

$$\int d^2 p_T p_T^\alpha \Phi^{[U]}(x, p_T; n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]} \Phi_G^\alpha(x)$$



T-even

T-odd (gluonic pole or ETQS m.e.)

$$\tilde{\Phi}_\partial(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$$

$$\Phi_G(x) = \pi \Phi_F^{n\alpha}(x, 0|x)$$

- Behavior of TMDs under time reversal symmetry and rank can be used to identify their corresponding matrix element

$$i \frac{p_T}{M} \cancel{h}_1^{\perp[U]}(x, p_T^2) \longrightarrow \text{T-odd \& rank 1} \longrightarrow \text{part of } \Phi_G(x)$$

$$- \frac{p_T \cdot S_T}{M} \gamma_5 \cancel{g}_{1T}^{[U]}(x, p_T^2) \longrightarrow \text{T-even \& rank 1} \longrightarrow \text{part of } \tilde{\Phi}_\partial(x)$$

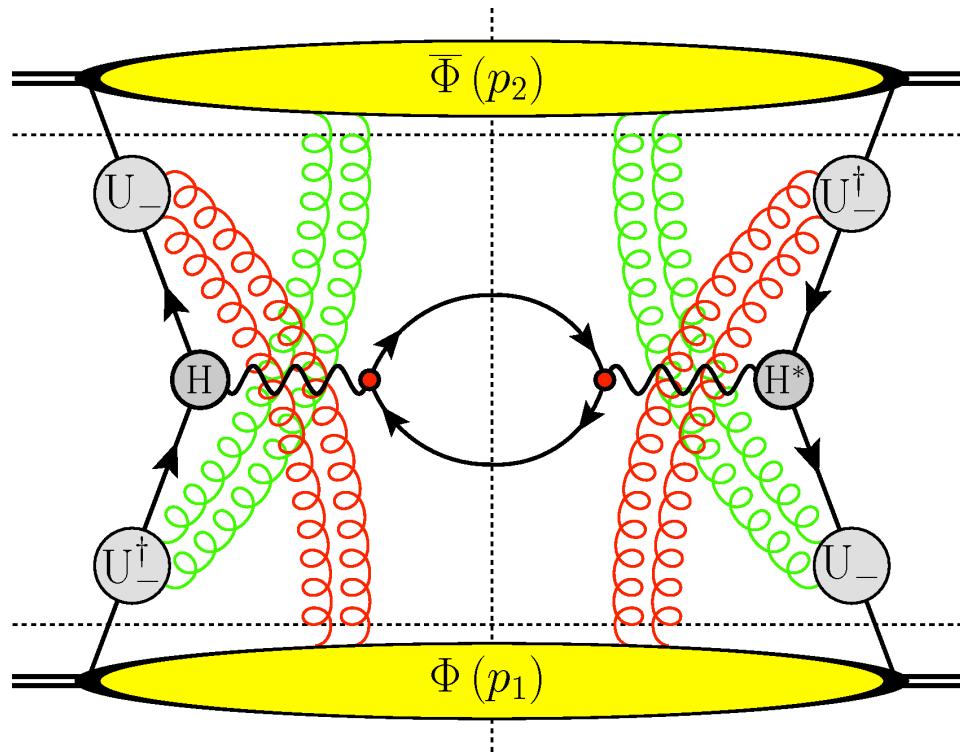
Transverse moments

- Generalization for quarks

# GPs	RANK		
	0	1	2
0	$\Phi(x, p_T^2)$	$\tilde{\Phi}_\partial$	$\tilde{\Phi}_{\partial\partial}$
1		$C_G^{[U]} \Phi_G$	$C_G^{[U]} \tilde{\Phi}_{\{\partial G\}}$
2			$C_{GG,c}^{[U]} \Phi_{GG,c}$

# GPs	RANK OF TMD PDFs FOR QUARKS		
	0	1	2
0	f_1^q, g_1^q, h_1^q	$g_{1T}^q, h_{1L}^{\perp q}$	$h_{1T}^{\perp q(A)}$
1		$f_{1T}^{\perp q}, h_1^{\perp q}$	
2			$h_{1T}^{\perp q(Bc)}$

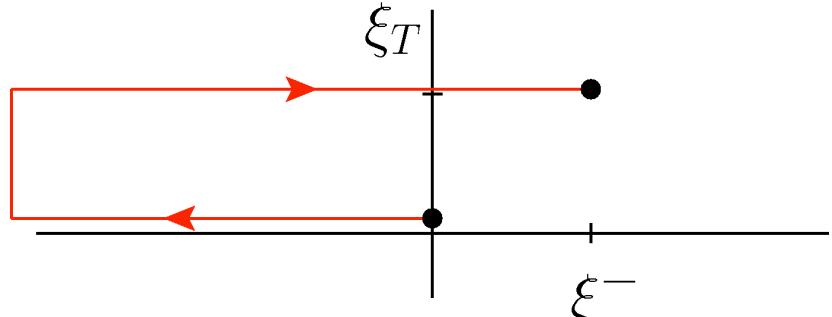
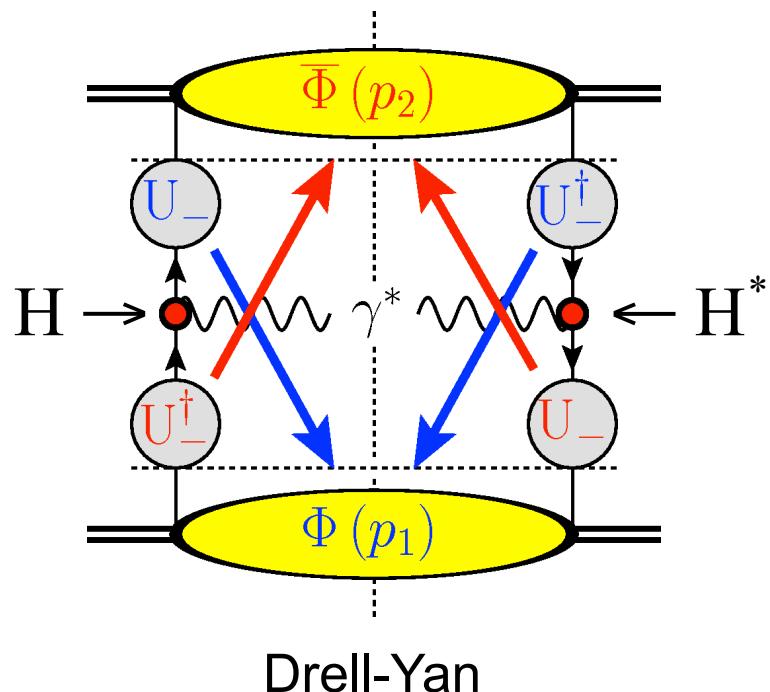
Part II: multi hadron processes



- One has to take care of gauge links from both the correlators!

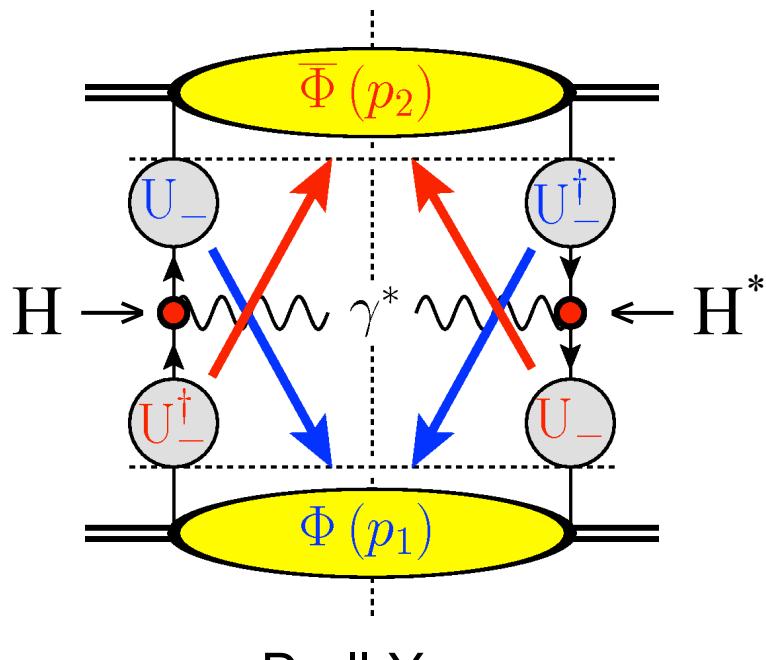
Color entanglement in DY

- Incorporate gluons from both hadrons

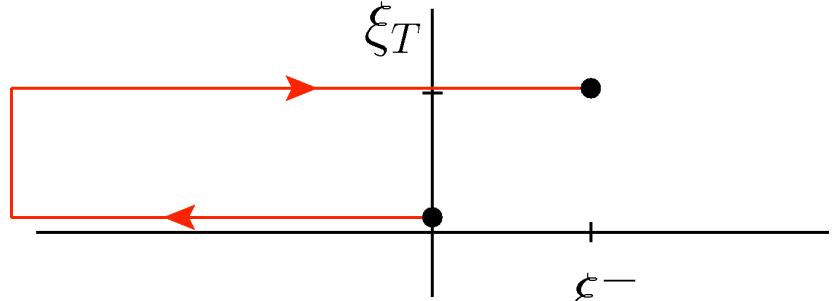


Color entanglement in DY

- Incorporate gluons from both hadrons



Drell-Yan



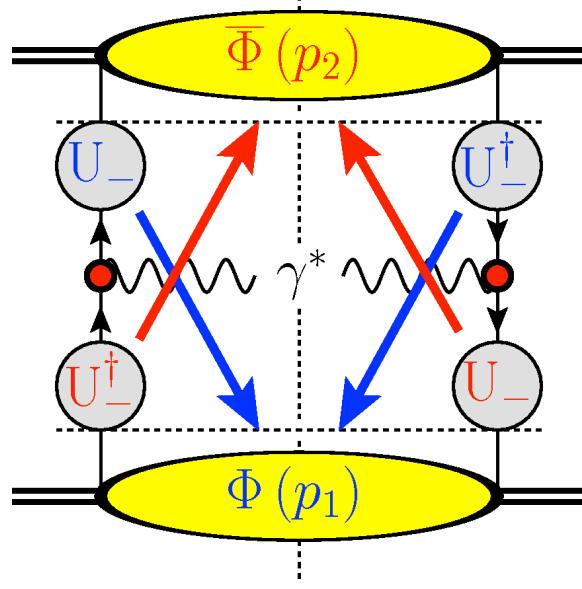
$$d\sigma_{\text{DY}} \propto \frac{1}{N_c} \Phi^{[-]}(x_1, p_{1T}) H^* \bar{\Phi}^{[-\dagger]}(x_2, p_{2T}) H$$

$$d\sigma_{\text{DY}} = \text{Tr}_c \left[U_-^\dagger(p_2) \Phi(x_1, p_{1T}) U_-(p_2) H^* U_-^\dagger(p_1) \bar{\Phi}(x_2, p_{2T}) U_-(p_1) H \right]$$

Color entanglement in DY

- Reminder:

$$i\partial_T^\alpha U_{[0,\xi]}^{[\pm]} = U_{[0,\xi]}^{[\pm]} \left(iD_T^\alpha(\xi) - gA_T^\alpha(\xi) \pm G^{n\alpha}(\xi) \right)$$

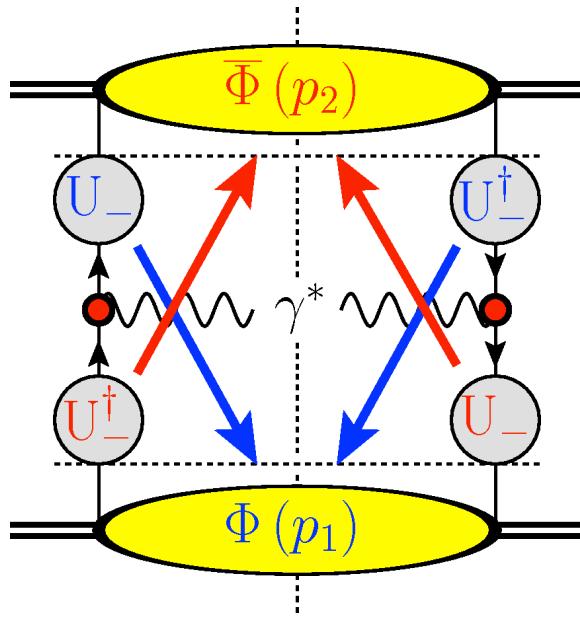


- For transverse weightings in both correlators:

$$\begin{aligned} p_{1T}^\alpha p_{2T}^\beta d\sigma_{\text{DY}} = \text{Tr}_c & \left[U_-^\dagger [p_2] \Phi(x_1, p_{1T}) U_- [p_2] \left(iD_T^\beta [p_2] - gA_T^\beta [p_2] - G^{n\beta} [p_2] \right) H^* \right. \\ & \times U_-^\dagger [p_1] \left(iD_T^\alpha [p_1] - gA_T^\alpha [p_1] - G^{n\alpha} [p_1] \right) \bar{\Phi}(x_2, p_{2T}) U_- [p_1] H \left. \right] \end{aligned}$$

- Gluons forming the gauge links have to be accounted for

Color entanglement in DY



- For transverse weightings in both correlators:

$$p_{1T}^\alpha p_{2T}^\beta d\sigma_{\text{DY}} = \text{Tr}_c \left[U_-^\dagger [p_2] \Phi(x_1, p_{1T}) U_- [p_2] \left(iD_T^\beta [p_2] - gA_T^\beta [p_2] - G^{n\beta} [p_2] \right) H^* \times U_-^\dagger [p_1] \left(iD_T^\alpha [p_1] - gA_T^\alpha [p_1] - G^{n\alpha} [p_1] \right) \bar{\Phi}(x_2, p_{2T}) U_- [p_1] H \right]$$

- Gluons forming the gauge links have to be accounted for

- For double gluonic pole term:

$$\text{Tr}_c [T^a T^b T^a T^b]$$

- Naively:

$$\text{Tr}_c [T^a T^a T^b T^b]$$

Color entanglement in DY

- Color factor

$$\frac{\text{Tr}_c [T^b T^a T^b T^a]}{\text{Tr}_c [T^a T^a] \text{Tr}_c [T^b T^b]} = -\frac{1}{N_c^2 - 1} \frac{1}{N_c}$$

- Naive color factor

$$\frac{\text{Tr}_c [T^a T^a T^b T^b]}{\text{Tr}_c [T^a T^a] \text{Tr}_c [T^b T^b]} = \frac{1}{N_c}$$

Implications for the actual cross section:

- Naively

$$d\sigma_{h_1^\perp h_1^\perp \text{ in DY}} = \frac{1}{N_c} h_1^\perp(x_1, p_{1T}) h_1^\perp(x_2, p_{2T}) H^* H$$

- Reality:

$$d\sigma_{h_1^\perp h_1^\perp \text{ in DY}} = -\frac{1}{N_c^2 - 1} \frac{1}{N_c} h_1^\perp(x_1, p_{1T}) h_1^\perp(x_2, p_{2T}) H^* H$$

Drell-Yan results

- Generalization

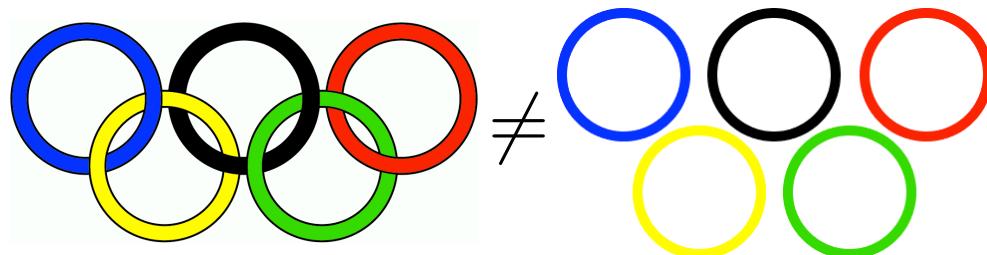
R_G for $\bar{\Phi}^{[-\dagger]}$	R_G for $\Phi^{[-]}$		
R_G for $\bar{\Phi}^{[-\dagger]}$	0	1	2
0	1	1	1
1	1	$-\frac{1}{N_c^2 - 1}$	$\frac{N_c^2 + 2}{(N_c^2 - 2)(N_c^2 - 1)}$
2	1	$\frac{N_c^2 + 2}{(N_c^2 - 2)(N_c^2 - 1)}$	$\frac{3N_c^4 - 8N_c^2 - 4}{(N_c^2 - 2)^2(N_c^2 - 1)}$

- This affects:
 - Double Boer-Mulders
 - Double Sivers
 - Combinations of Pretzelosity with Boer-Mulders/Sivers/Pretzelosity

$$\sigma(x_1, x_2, q_T) \sim \frac{1}{N_c} f_{R_{G1} R_{G2}}^{[U_1, U_2]} \Phi^{[U_1]}(x_1, p_{1T}) \otimes \bar{\Phi}^{[U_2]}(x_2, p_{2T}) \hat{\sigma}(x_1, x_2)$$

Summary and conclusions

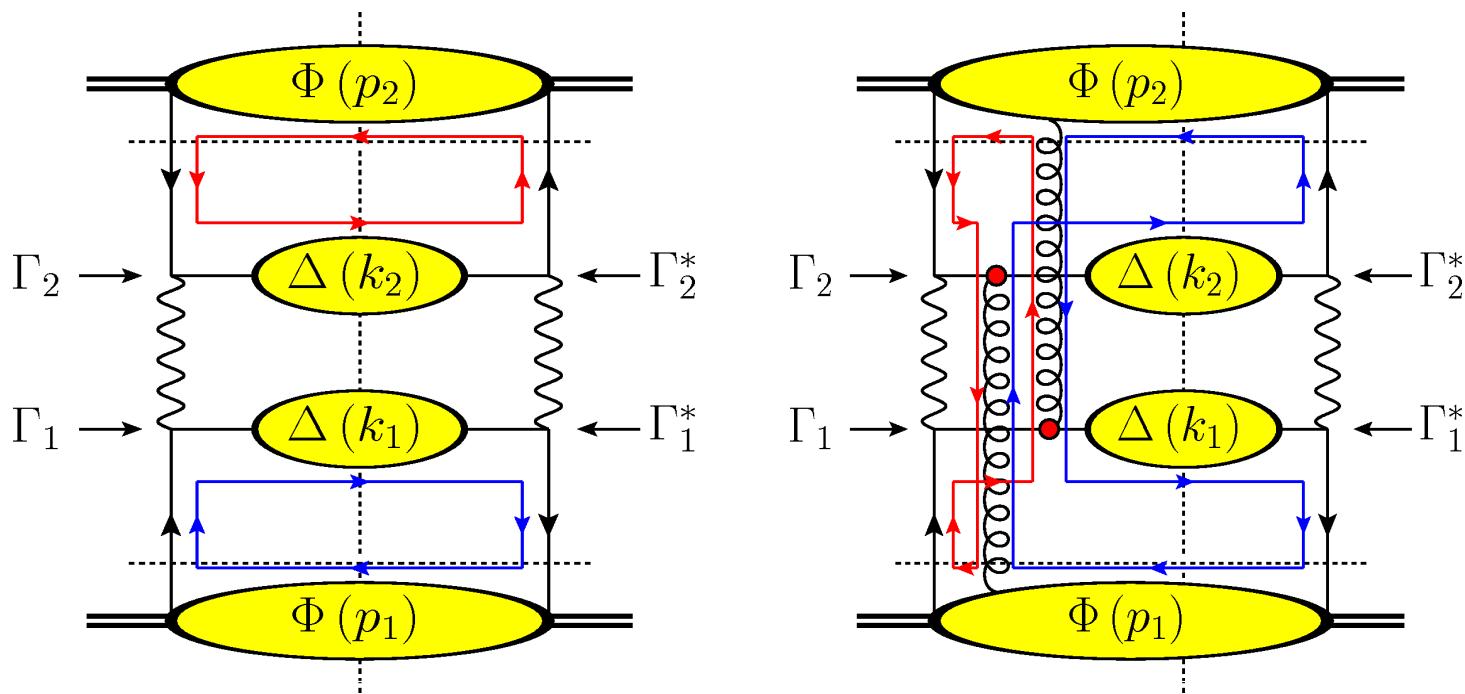
- Color flow is important
- Due to gauge links, color is entangled in even the simplest hadronic processes
- Due to entanglement, cross sections have other color factors than one might naively expect



Backup slides

Multiple correlators at once

- Color entanglement of the process



- Gluons forming the gauge links have to be accounted for