

# QCD Evolution of Sivers Asymmetry

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## - QCD Evolution of the Sivers Asymmetry

MGE, Ahmad Idilbi, Zhong-Bo Kang, Ivan Vitev. Phys.Rev. D89 (2014) 074013 [arXiv: 1401.5078]

## - A Unified Treatment of the QCD Evolution of All (Un-)Polarized TMD Functions: Collins Function as a Study Case

MGE, Ahmad Idilbi, Ignazio Scimemi. [arXiv: 1402.0869]

# Sivers Asymmetry: The Goal

- Improve our understanding of QCD (test theoretical tools: factorization, TMD framework,...)
- How? By testing one of the main predictions of the TMD formalism: sign change of Sivers function between SIDIS and DY processes.
- We want to extract it from SIDIS and make reliable predictions for DY.
- Distribution of unpolarized quarks inside a transversely polarized hadron:

$$f_{q/A^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = f_1^q(x, k_T^2) - f_{1T}^{\perp q}(x, k_T^2) \frac{(\hat{P} \times \mathbf{k}_\perp) \cdot \mathbf{S}_\perp}{M_A}$$

They include the soft function!

QCD invariant under P and T

$$f_{q/A^\uparrow}^{SIDIS}(x, \mathbf{k}_\perp, \vec{S}) = f_{q/A^\uparrow}^{DY}(x, \mathbf{k}_\perp, -\vec{S})$$

$$f_{1T}^{\perp q}(x, k_T^2)^{SIDIS} = -f_{1T}^{\perp q}(x, k_T^2)^{DY}$$

- Different Wilson lines in SIDIS and DY processes
- Soft function is universal



# The Object

- SIDIS cross-section with a transversely polarized hadron:

$$e(\ell) + A^\uparrow(P) \rightarrow e(\ell') + h(P_h) + X$$

$$\frac{d\sigma}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2) \left[ F_{UU} + \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)} \right]$$

- Sivers asymmetry is defined as:

$$A_{UT}^{\sin(\phi_h - \phi_s)} = \frac{F_{UT}^{\sin(\phi_h - \phi_s)}}{F_{UU}}$$

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- Factorization?
- Relation with Sivers function?
- Evolution?
- ...



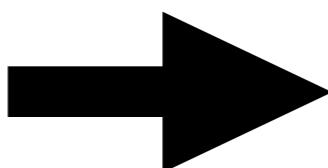
SIDIS factorization...

# SIDIS Factorization (1/3)

$$l(k) + N(P, S) \rightarrow l'(k') + h(P_h, S_h) + X(P_X)$$

- Applying the SCET machinery:

$$J_{\text{QCD}}^\mu = \sum_q e_q \bar{\psi} \gamma^\mu \psi$$



$$J_{\text{SCET}}^\mu = \sum_q e_q \bar{\xi}_{\bar{n}} \tilde{W}_{\bar{n}}^T \tilde{S}_{\bar{n}}^{T\dagger} \gamma^\mu S_n^T \tilde{W}_n^{T\dagger} \xi_n$$

T: MGE, Idilbi, Scimemi PRD'11

More details:  
MGE, Idilbi, Scimemi 1402.0869

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$$W^{\mu\nu} = H(Q^2/\mu^2) \frac{2}{N_c} \sum_q e_q \int d^2 \mathbf{k}_{n\perp} d^2 \mathbf{k}_{\bar{n}\perp} d^2 \mathbf{k}_{s\perp} \delta^{(2)}(\mathbf{q}_\perp + \mathbf{k}_{n\perp} - \mathbf{k}_{\bar{n}\perp} + \mathbf{k}_{s\perp}) \\ \times \text{Tr} [\Phi^{(0)}(x, \mathbf{k}_{n\perp}, S) \gamma^\mu \Delta^{(0)}(z, \mathbf{k}_{\bar{n}\perp}, S_h) \gamma^\nu] S(\mathbf{k}_{s\perp})$$

$$\Phi_{ij}^{(0)}(x, \mathbf{k}_{n\perp}, S) = \frac{1}{2} \int \frac{dy^- d^2 \mathbf{y}_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}y^- k_n^+ - \mathbf{y}_\perp \cdot \mathbf{k}_{n\perp})} \langle PS | \left[ \bar{\xi}_{nj} \tilde{W}_n^T \right] (0^+, y^-, \mathbf{y}_\perp) \left[ \tilde{W}_n^{T\dagger} \xi_{ni} \right] (0) | PS \rangle \\ \Delta_{ij}^{(0)}(z, \hat{\mathbf{P}}_{h\perp}, S_h) = \frac{1}{2} \int \frac{dy^+ d^2 \mathbf{y}_\perp}{(2\pi)^3} e^{i(\frac{1}{2}y^+ k_{\bar{n}}^- - \mathbf{y}_\perp \cdot \mathbf{k}_{\bar{n}\perp})} \\ \times \frac{1}{z} \sum_X \langle 0 | \left[ \tilde{W}_{\bar{n}}^{T\dagger} \xi_{\bar{n}i} \right] (y^+, 0^-, \mathbf{y}_\perp) | X; P_h S_h \rangle \langle X; P_h S_h | \left[ \bar{\xi}_{\bar{n}} \tilde{W}_{\bar{n}j}^T \right] (0) | 0 \rangle \\ S(\mathbf{k}_{s\perp}) = \int \frac{d^2 \mathbf{y}_\perp}{(2\pi)^2} e^{i\mathbf{y}_\perp \cdot \mathbf{k}_{s\perp}} \frac{1}{N_c} \langle 0 | \text{Tr} \left[ S_n^{T\dagger} \tilde{S}_{\bar{n}}^T \right] (0^+, 0^-, \mathbf{y}_\perp) \left[ \tilde{S}_{\bar{n}}^{T\dagger} S_n^T \right] (0) | 0 \rangle$$

- Pure vs. naive collinear (double counting)
- Matrix elements suffer from rapidity divergences

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# SIDIS Factorization (2/3)

- The soft function can be split (in *rapidity space*) to all orders as:

$$\ln \tilde{S} = \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s) \ln \left( \frac{\Delta^+ \Delta^-}{Q^2 \mu^2} \right)$$

$$\tilde{S} = \tilde{S}_+ \tilde{S}_-$$

$$\frac{dD}{d\ln\mu} = \Gamma_{\text{cusp}}$$



$$\ln \tilde{S}_- = \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left( \frac{(\Delta^-)^2}{\zeta_F \mu^2} \right)$$

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$$\zeta_F \zeta_D = Q^4$$

$$D \equiv \tilde{K} \quad \text{See John's talk}$$

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- To cancel rapidity divergences, TMDPDFs and TMDFFs are defined as:

$$F_{ij}(x, \mathbf{k}_{n\perp}, S; \zeta_F, \mu^2; \Delta^-) = \int d^2 \mathbf{b}_\perp e^{i \mathbf{b}_\perp \cdot \mathbf{k}_{n\perp}} \tilde{\Phi}_{ij}^{(0)}(x, \mathbf{b}_\perp, S; \mu^2; \Delta^-) \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-)$$

$$D_{ij}(z, \hat{\mathbf{P}}_{h\perp}, S_h; \zeta_D, \mu^2; \Delta^+) = \int d^2 \mathbf{b}_\perp e^{-i \mathbf{b}_\perp \cdot \mathbf{k}_{\bar{n}\perp}} \tilde{\Delta}_{ij}^{(0)}(z, \mathbf{b}_\perp, S_h; \mu^2; \Delta^+) \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+)$$

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- Straightforward to update the decomposition of TMDs given by [Boer, Mulders PRD'98] to include the soft function.

$$\zeta_{F,D} \neq \zeta = \frac{(2v \cdot P)^2}{v^2}$$

This story is regulator IN-dependent!!

# SIDIS Factorization (3/3)

- Fierzing, going to impact parameter space,...

$$\begin{aligned}
 W^{\mu\nu} = & H(Q^2/\mu^2) \frac{2}{N_c} \sum_f e_f \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{P}_{h\perp}/z} \\
 & \times \left[ \left( \tilde{F}_{f/N}^{[\gamma^+]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) \tilde{D}_{h/f}^{[\gamma^-]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) \right. \right. \\
 & + \tilde{F}_{f/N}^{[\gamma^+\gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) \tilde{D}_{h/f}^{[\gamma^-\gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) \Big) g_\perp^{\mu\nu} \\
 & \left. \left. + \tilde{F}_{f/N}^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) \tilde{D}_{h/f}^{[i\sigma^j-\gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) \left( g_{\perp i}^{\{\mu} g_j^{\nu\}} - g_{\perp ij} g_\perp^{\mu\nu} \right) \right] \right]
 \end{aligned}$$

Soft function included in F/D

$$F^{[\Gamma]} \equiv \frac{1}{2} \text{Tr} (\mathbf{F} \Gamma)$$

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$$\left. + \tilde{F}_{f/N}^{[\gamma^+\gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) \tilde{D}_{h/f}^{[\gamma^-\gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) \right) g_\perp^{\mu\nu}$$

$$\left. + \tilde{F}_{f/N}^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) \tilde{D}_{h/f}^{[i\sigma^{j-}\gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) \left( g_{\perp i}^{\mu} g_j^{\nu} - g_{\perp ij} g_\perp^{\mu\nu} \right) \right]$$

Soft function included in F/D

**Known at 3 loops!**

**Depends just on Q!**

$$F^{[\Gamma]} \equiv \frac{1}{2} \text{Tr} (\mathbf{F} \Gamma)$$

- TMDs contain perturbative information at large  $k_T$  (refactorization):

$$\tilde{T}(b_T; \zeta, \mu^2) = \left( \frac{\zeta b_T^2}{4e^{-2\gamma_E}} \right)^{-D(b_T; \mu)} \tilde{C}_T^\phi(b_T; \mu^2) \otimes t(\mu^2) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

Any TMD PDF/FF

Perturbative  
No divergences

Collinear distribution  
(PDF, FF, ETQS,...)

# Universal Evolution

- All the 8 TMDPDFs and 8 TMDFFs (leading twist) evolve with the same kernel:

$$\tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,f}, \mu_f^2) = \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{F,i}, \mu_i^2, \zeta_{F,f}, \mu_f^2)$$
$$\tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,f}, \mu_f^2) = \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{D,i}, \mu_i^2, \zeta_{D,f}, \mu_f^2)$$

$$\tilde{R}(b; \zeta_i, \mu_i^2, \zeta_f, \mu_f^2) = \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}} \gamma \left( \alpha_s(\bar{\mu}), \ln \frac{\zeta_f}{\bar{\mu}^2} \right) \right\} \left( \frac{\zeta_f}{\zeta_i} \right)^{-D(b_T; \mu_i)}$$

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Known at 3-loops!!

$$\ln \tilde{S} = \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s) \ln \left( \frac{\Delta^+ \Delta^-}{Q^2 \mu^2} \right)$$

The Soft function is universal!!

$$\frac{dD}{d\ln \mu} = \Gamma_{\text{cusp}}$$

$$D \equiv \tilde{K}$$

$$D(b_T; \mu_i) = D^R(b_T; \mu_i) \theta(b_{Tc} - b_T) + D^{NP}(b_T) \theta(b_T - b_{Tc})$$

See John's talk

Universal!!

MGE, Idilbi, Scimemi 1402.0869

# The Object (again)

- SIDIS cross-section with a transversely polarized hadron:

$$e(\ell) + A^\uparrow(P) \rightarrow e(\ell') + h(P_h) + X$$

$$\frac{d\sigma}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2) \left[ F_{UU} + \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)} \right]$$

$$F_{UU} = \frac{1}{2\pi} H^{DIS}(Q) \int_0^\infty db b J_0(P_{h\perp} b/z_h) \sum_q e_q^2 f_{q/A}(x_B, b; Q) D_{h/q}(z_h, b; Q)$$

$$F_{UT}^{\sin(\phi_h - \phi_s)} = -H^{DIS}(Q) \int \frac{d^2 b}{(2\pi)^2} e^{-i P_{h\perp} \cdot b / z_h} \hat{P}_{h\perp}^\alpha \sum_q e_q^2 f_{1T, \text{SIDIS}}^{\perp q(\alpha)}(x_B, b; Q) D_{h/q}(z_h, b; Q)$$

We have set:  $\mu^2 = \zeta_A = \zeta_h = Q^2$

$$f_{q/A}(x, b; Q) = \int d^2 k_\perp e^{-ik_\perp \cdot b} f_{q/A}(x, k_\perp^2; Q)$$

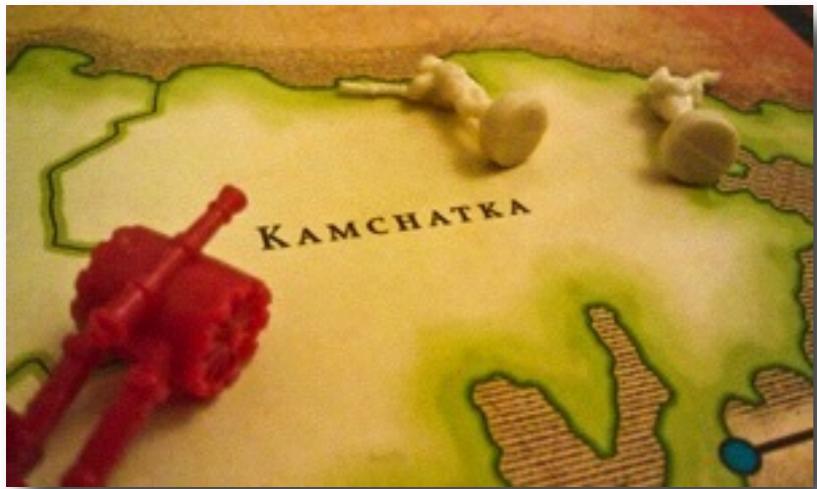
$$D_{h/q}(z, b; Q) = \frac{1}{z^2} \int d^2 p_T e^{-ip_T \cdot b / z} D_{h/q}(z, p_T^2; Q)$$

$$f_{1T}^{\perp q(\alpha)}(x, b; Q) = \frac{1}{M} \int d^2 k_\perp e^{-ik_\perp \cdot b} k_\perp^\alpha f_{1T}^{\perp q}(x, k_\perp^2; Q)$$

- Sivers asymmetry is defined as:

$$A_{UT}^{\sin(\phi_h - \phi_s)} = \frac{F_{UT}^{\sin(\phi_h - \phi_s)}}{F_{UU}}$$

# Sivers Asymmetry: The Strategy



1.-

Describe unpolarized DY+SIDIS with proper QCD Evolution and a universal non-perturbative model

- *W/Z boson production (CDF, D0, CMS)*
- *DY lepton-pair production at (E288, E605)*
- *SIDIS process (COMPASS, HERMES)*

2.-

Model Sivers function and fit it with SIDIS data and using previously extracted parameters

- *SIDIS process (JLab, COMPASS, HERMES)*

3.-

Flip sign to make predictions for planned measurements in DY

- *DY lepton-pair production (Fermilab, CERN)*
- *W boson production (RHIC)*

We perform the resummation of large logarithms at NLL accuracy

# Unpolarized DY+SIDIS: Perturbative

- The necessary perturbative ingredients at NLL are:

$$H^{DIS, \text{DY}}(Q^2, \mu^2) = 1 + \frac{\alpha_s C_F}{2\pi} \left[ 3 \ln \frac{Q^2}{\mu^2} - \ln^2 \frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6} + \pi^2 \right]$$

LO hard part

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LO hard part

$$F_{i/h}(x, b; \mu) = \sum_a \int_x^1 \frac{d\xi}{\xi} C_{i/a} \left( \frac{x}{\xi}, b; \mu \right) f_{a/h}(\xi, \mu) + \mathcal{O}(b \Lambda_{\text{QCD}})$$

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LO coefficients

$$f_{q/A}(x, b; c/b) = f_{q/A}(x, c/b) + \dots$$

$$D_{h/q}(z, b; c/b) = \frac{1}{z^2} D_{h/q}(z, c/b) + \dots$$

Natural scale:  $c/b$

$$c = 2e^{-\gamma_E}$$

# Unpolarized DY+SIDIS: Perturbative

$$\tilde{R}(b; c/b, Q) = \exp \left\{ - \int_{c/b}^Q \frac{d\mu}{\mu} \left( \Gamma_{\text{cusp}} \ln \frac{Q^2}{\mu^2} + \gamma^V \right) \right\} \left( \frac{Q^2}{(c/b)^2} \right)^{-D(b; c/b)}$$

$$\Gamma^{(1)} = C_F$$

$$\Gamma^{(2)} = \frac{C_F}{2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f \right]$$

$$\gamma^{V(1)} = -\frac{3}{2} C_F$$

$$D^{(1)} = \frac{C_F}{2} \ln \frac{\mu^2 b^2}{c^2} \quad \rightarrow \quad D^{(1)}(b; \mu = c/b) = 0$$

# Unpolarized DY+SIDIS: Perturbative

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We take into account the thresholds and integrate analytically

$$\Gamma^{(1)} = C_F$$

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$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0 x} - \frac{\beta_1}{\beta_0^3} \frac{\ln x}{x^2} + \frac{\beta_1^2}{\beta_0^5} \frac{\ln^2 x - \ln x - 1}{x^3} + \frac{\beta_2}{\beta_0^4} \frac{1}{x^3} + \dots$$

$$x = \ln \frac{\mu^2}{\Lambda_{\text{QCD}}^2}$$

For NLL

# Unpolarized DY+SIDIS: Non-Perturbative

- Thus, the perturbative part of the TMD PDF/FF is written at NLL as:

$$F_{\text{pert}}(x, b; Q) = f(x, c/b) \exp \left\{ - \int_{c/b}^Q \frac{d\mu}{\mu} \left( \Gamma_{\text{cusp}} \ln \frac{Q^2}{\mu^2} + \gamma^V \right) \right\} \quad 1/b \gg \Lambda_{\text{QCD}}$$

- We need to separate and parameterize the non-perturbative large  $b$  contributions:

$$f_{q/A}(x, b; Q) = f_{q/A}(x, c/b_*) e^{-\textcolor{red}{g_1^{\text{pdf}}} b^2} \exp \left\{ - \int_{c/b_*}^Q \frac{d\mu}{\mu} \left( \Gamma_{\text{cusp}} \ln \frac{Q^2}{\mu^2} + \gamma^V \right) \right\} \left( \frac{Q^2}{Q_0^2} \right)^{-\frac{1}{4} \textcolor{red}{g_2} b^2}$$

$$D_{h/q}(z, b; Q) = \frac{1}{z^2} D_{h/q}(z, c/b_*) e^{-\textcolor{red}{g_1^{\text{ff}}} b^2} \exp \left\{ - \int_{c/b_*}^Q \frac{d\mu}{\mu} \left( \Gamma_{\text{cusp}} \ln \frac{Q^2}{\mu^2} + \gamma^V \right) \right\} \left( \frac{Q^2}{Q_0^2} \right)^{-\frac{1}{4} \textcolor{red}{g_2} b^2}$$

$$Q_0^2 = 2.4 \text{ GeV}^2$$

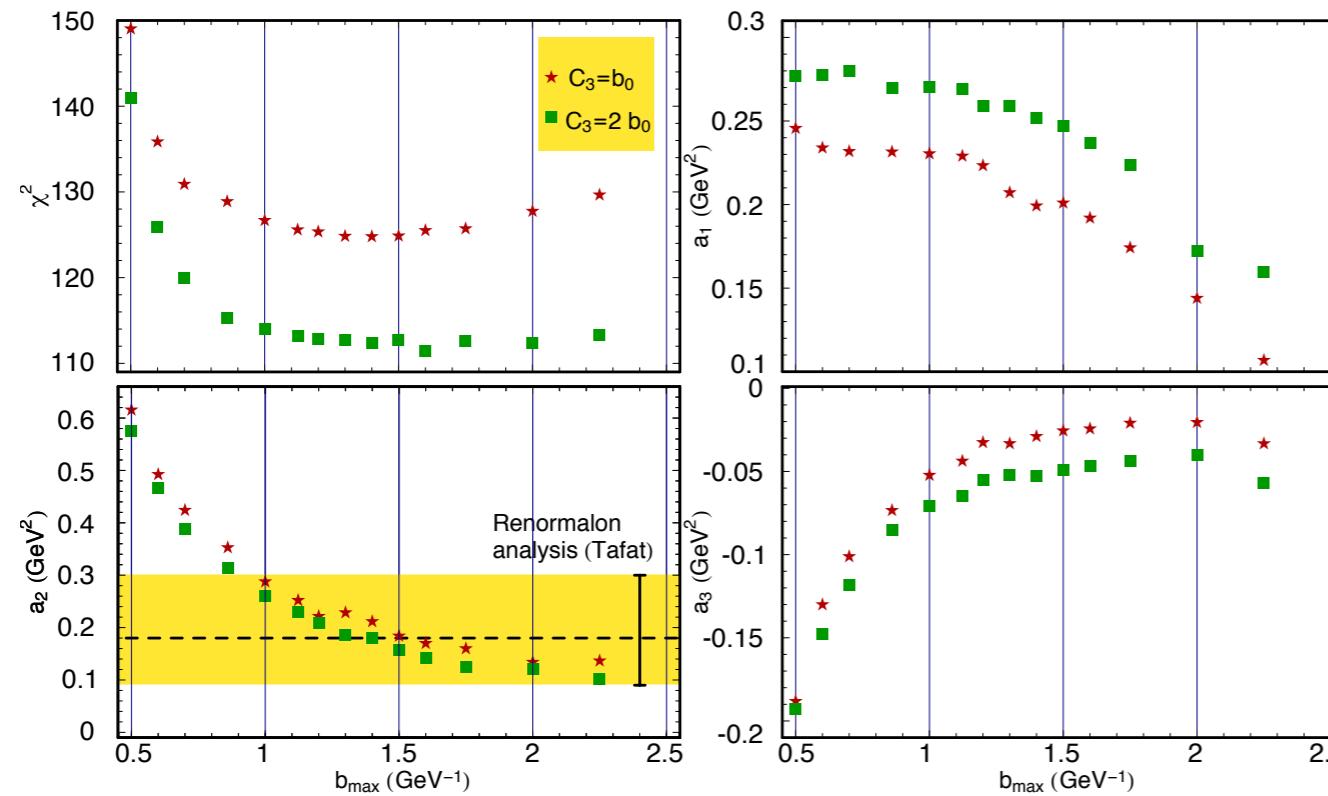
We use here the  $b^*$  prescription proposed by CSS

$$b_* = \frac{b}{\sqrt{1 + (b/b_{\max})^2}}$$

We use simple gaussian NP input  
No x/z dependence  
No flavor-dependence  
 $g_2$  is universal

# Unpolarized DY+SIDIS: Non-Perturbative

- From [Konychev, Nadolsky PLB'06]:



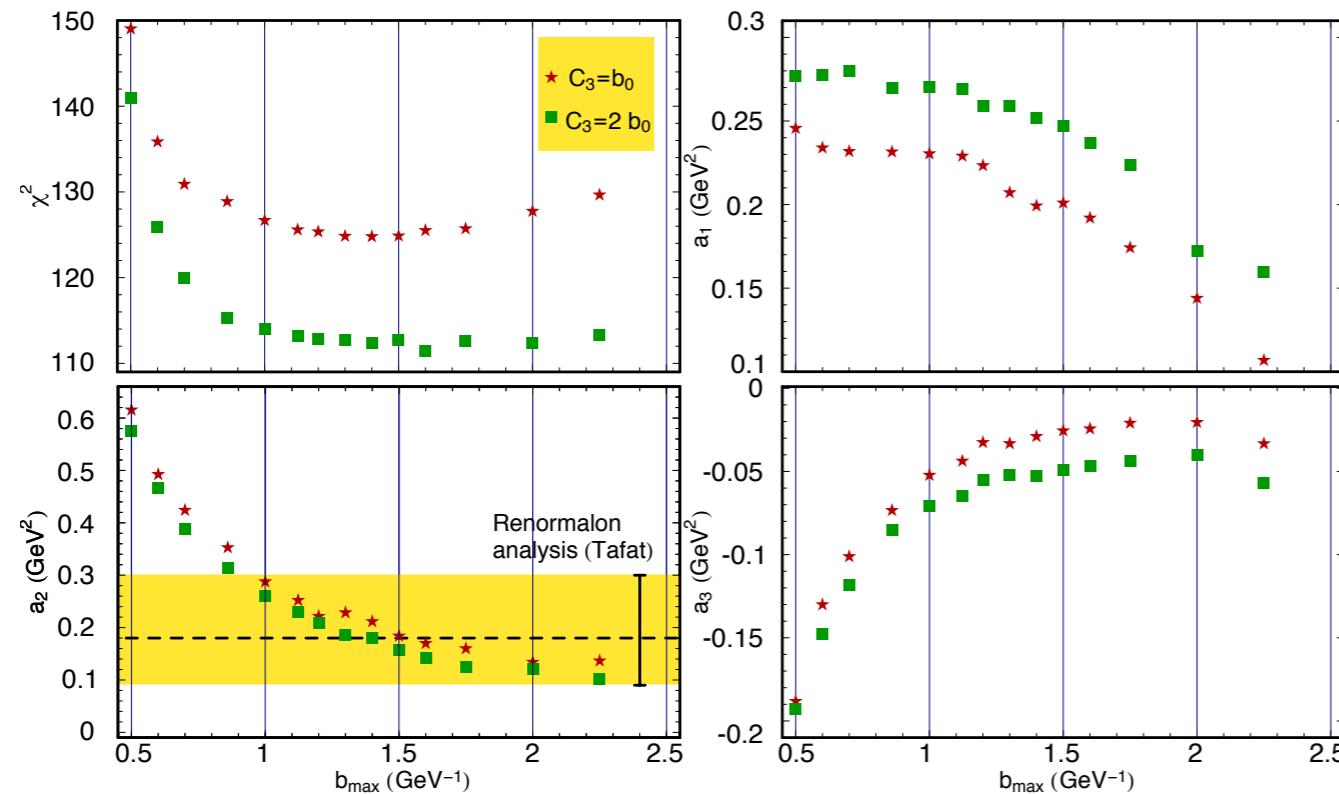
$$b_{\max} = 1.5 \text{ GeV}^{-1}$$

$$g_2 = 0.184 \pm 0.018 \text{ GeV}^2$$

Extracted from DY and W/Z

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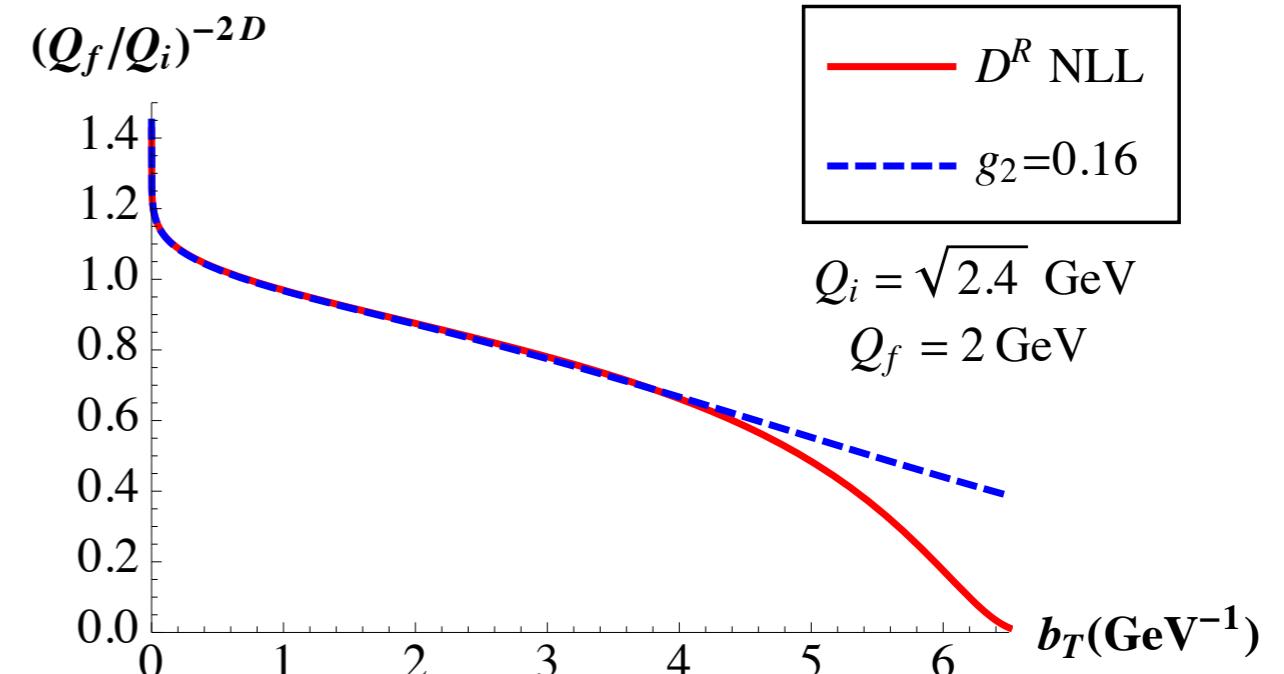
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Extracted from DY and W/Z

- We choose  $g_2=0.16$  for  $b_{\max}=1.5$ :

$$\left(\frac{Q_f}{Q_i}\right)^{-2D^R(b; Q_i)}$$

$$\left(\frac{Q_f}{Q_i}\right)^{-2\left[D(b^*; \mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\mu} \Gamma_{\text{cusp}} + \frac{1}{4} g_2 b^2\right]}$$



# Unpolarized DY+SIDIS: Non-Perturbative

- Regarding the intrinsic widths:

Good description of  
HERMES data

$$\begin{aligned}\langle k_{\perp}^2 \rangle_{Q_0} &= 0.25 - 0.44 \text{ GeV}^2 \\ \langle p_T^2 \rangle_{Q_0} &= 0.16 - 0.20 \text{ GeV}^2\end{aligned}$$

Schweitzer, Teckentrup, Metz PRD'10  
Torino group

$$g_1^{\text{pdf}} = \frac{\langle k_{\perp}^2 \rangle_{Q_0}}{4}$$

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$$\begin{aligned}\langle k_{\perp}^2 \rangle_{Q_0} &= 0.38 \text{ GeV}^2 \\ \langle p_T^2 \rangle_{Q_0} &= 0.19 \text{ GeV}^2 \\ g_2 &= 0.16 \text{ GeV}^2 \\ b_{\max} &= 1.5 \text{ GeV}^{-1}\end{aligned}$$

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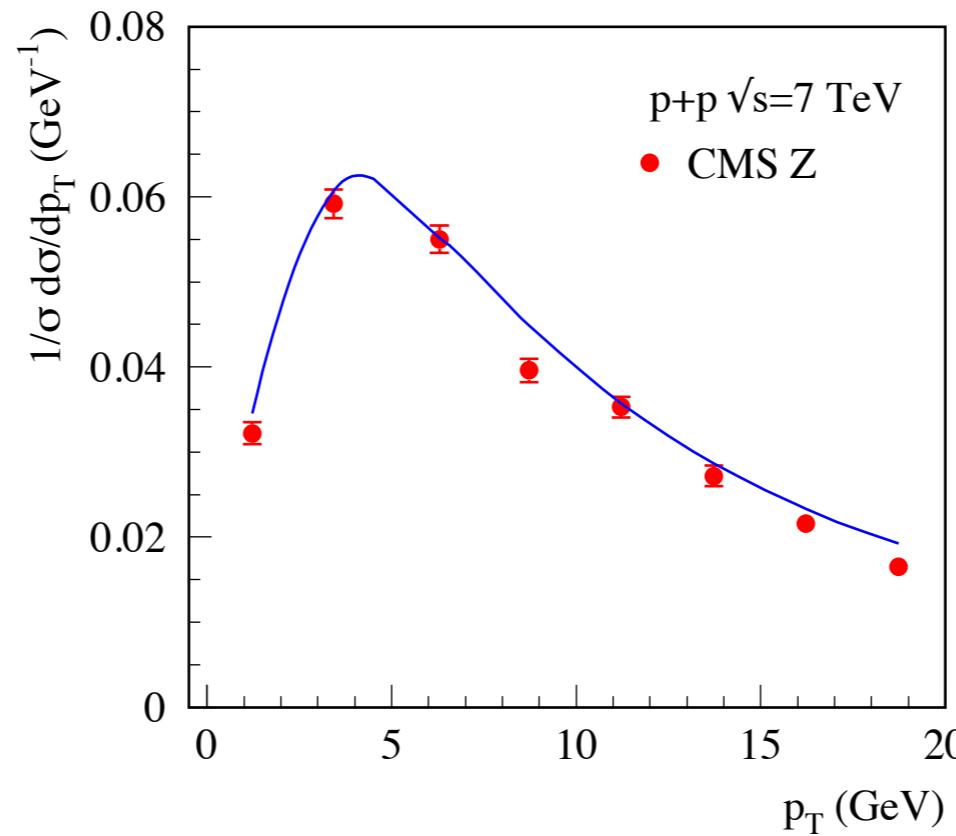
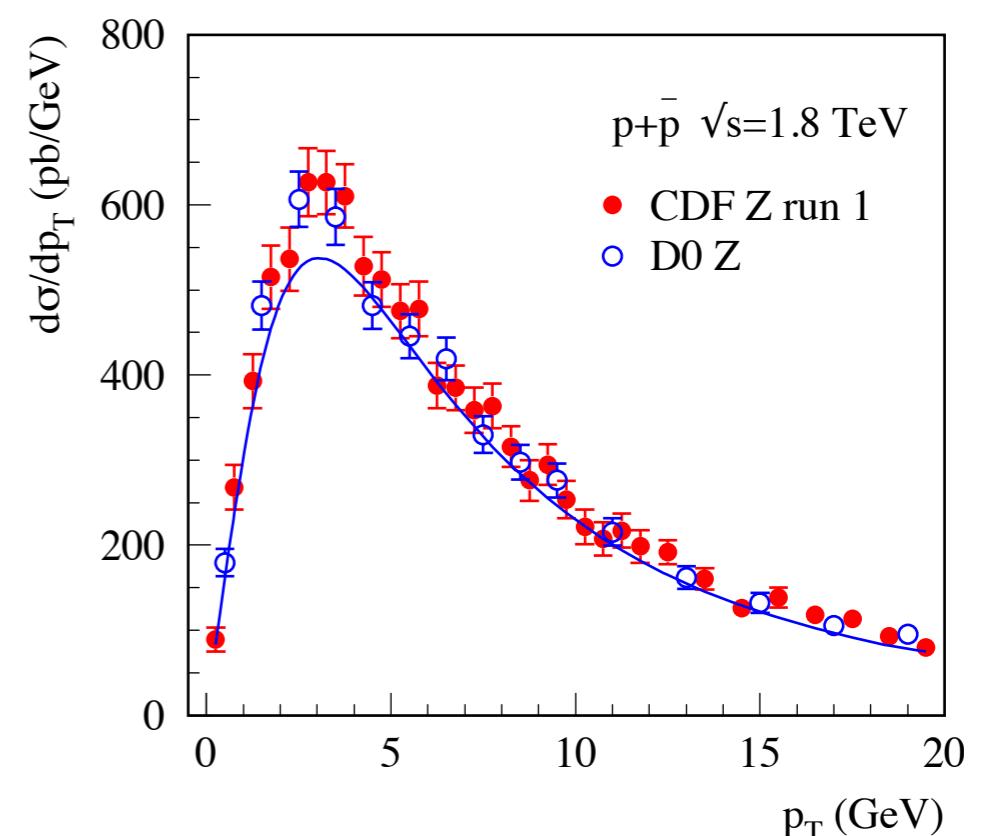
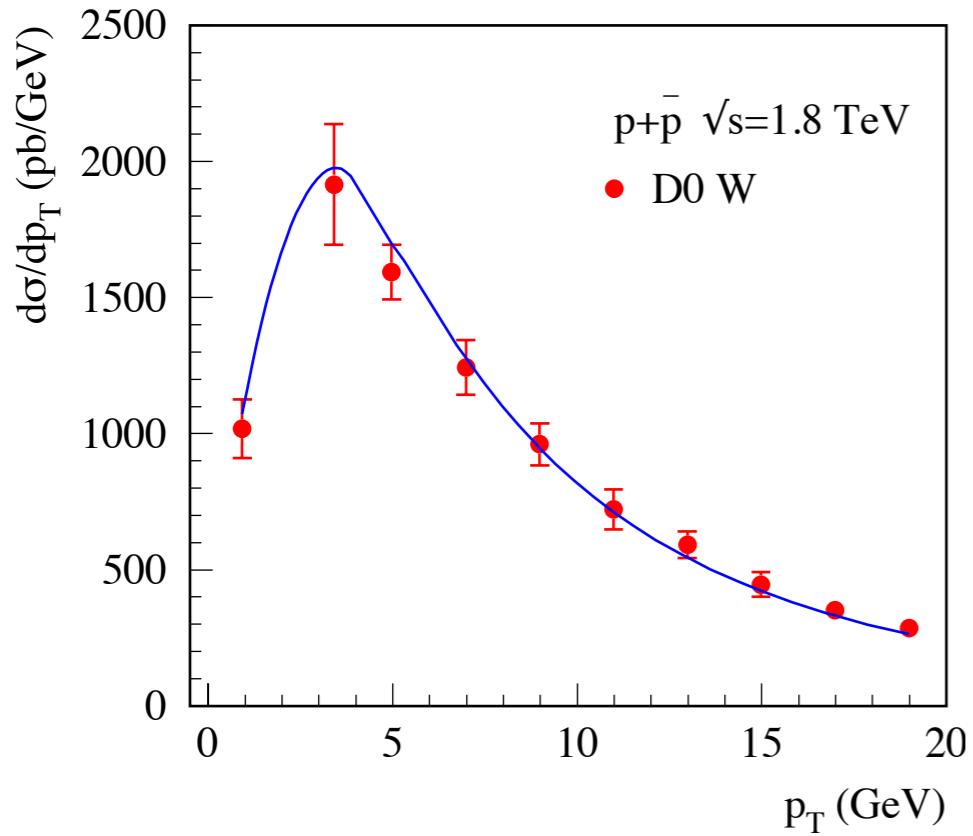
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Plots...



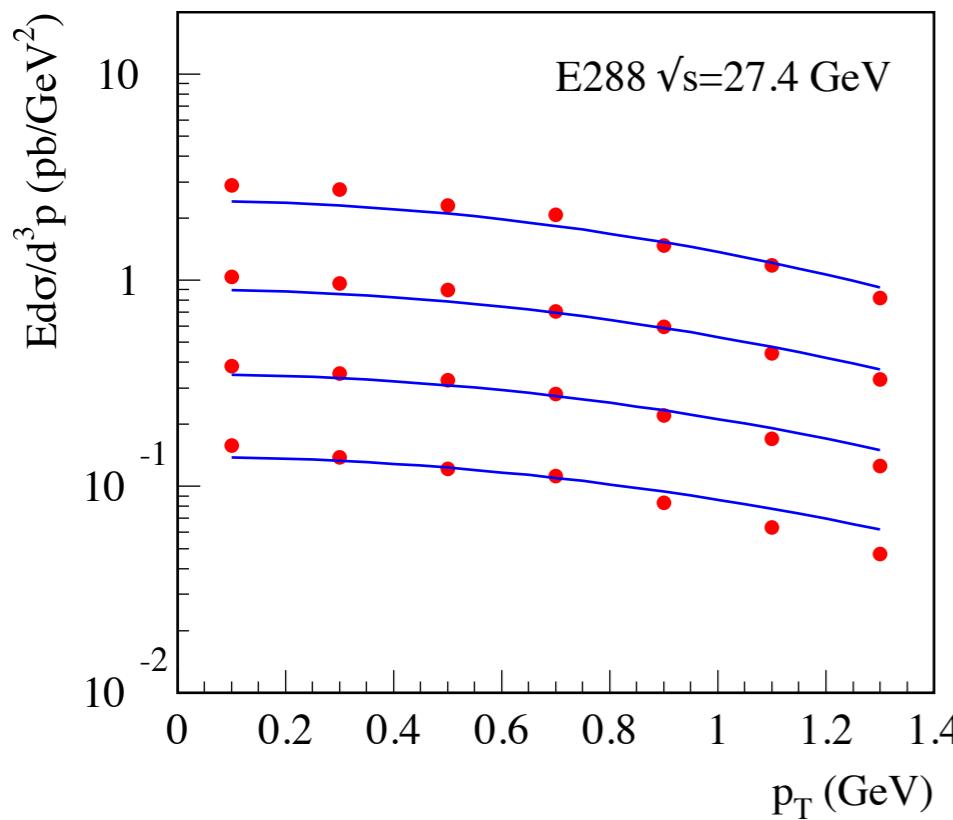
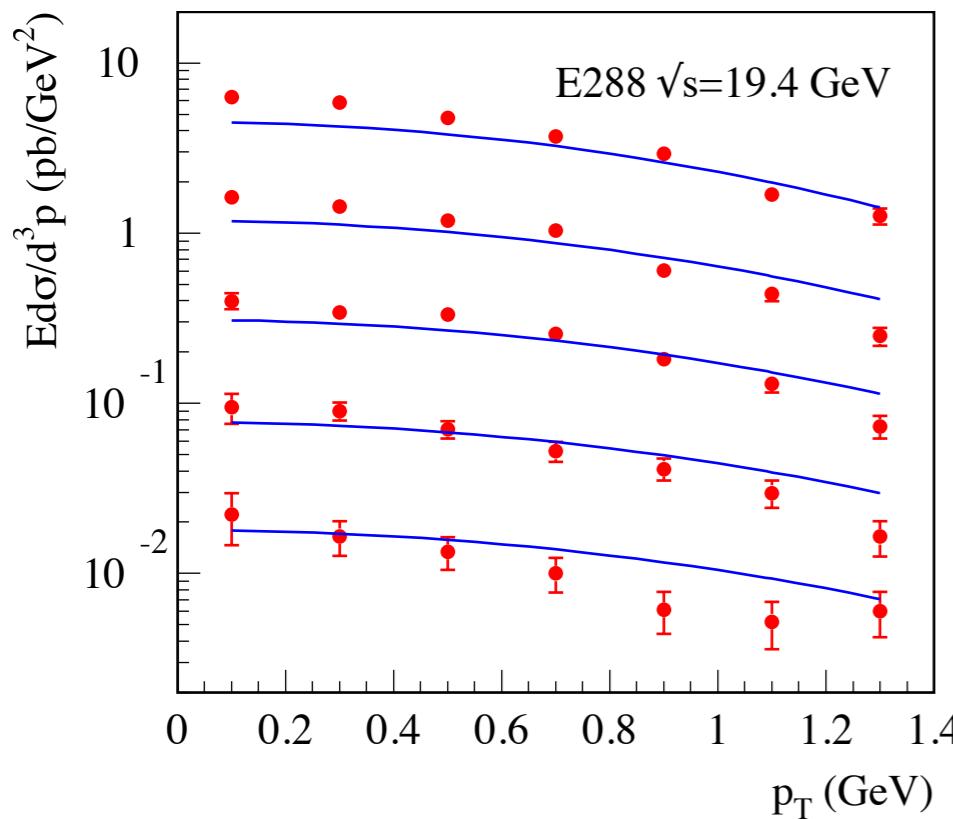
# Unpolarized DY+SIDIS: Plots (1/4)

- W/Z boson production at Tevatron (D0 and CDF) and LHC (CMS)

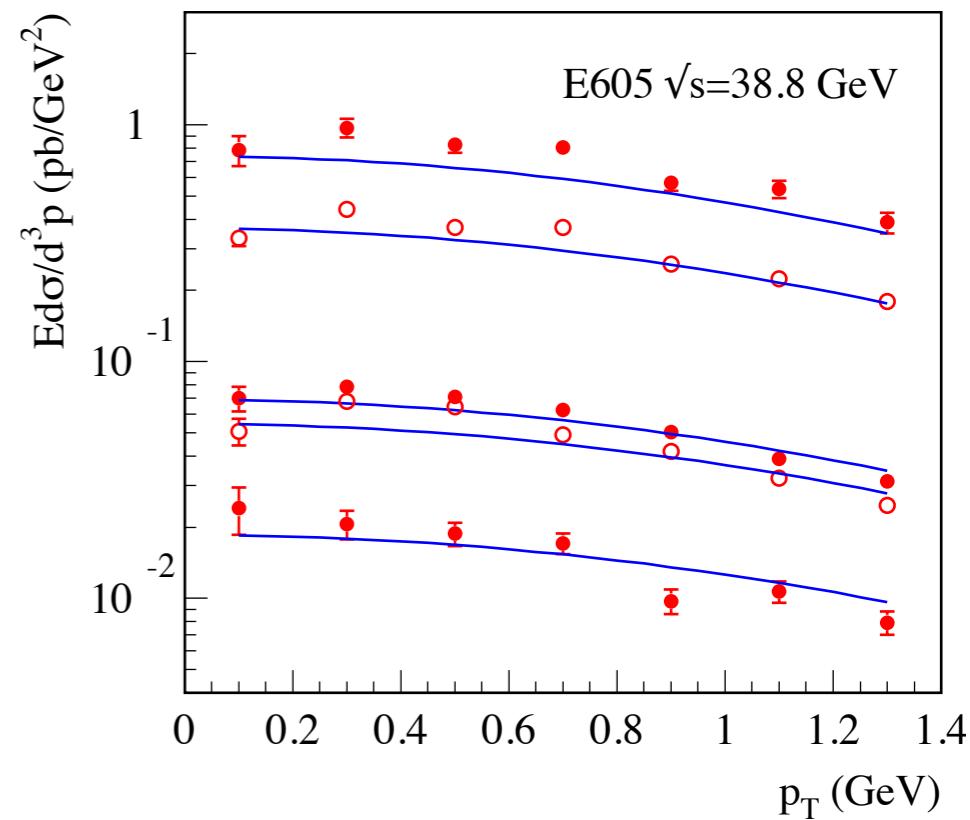
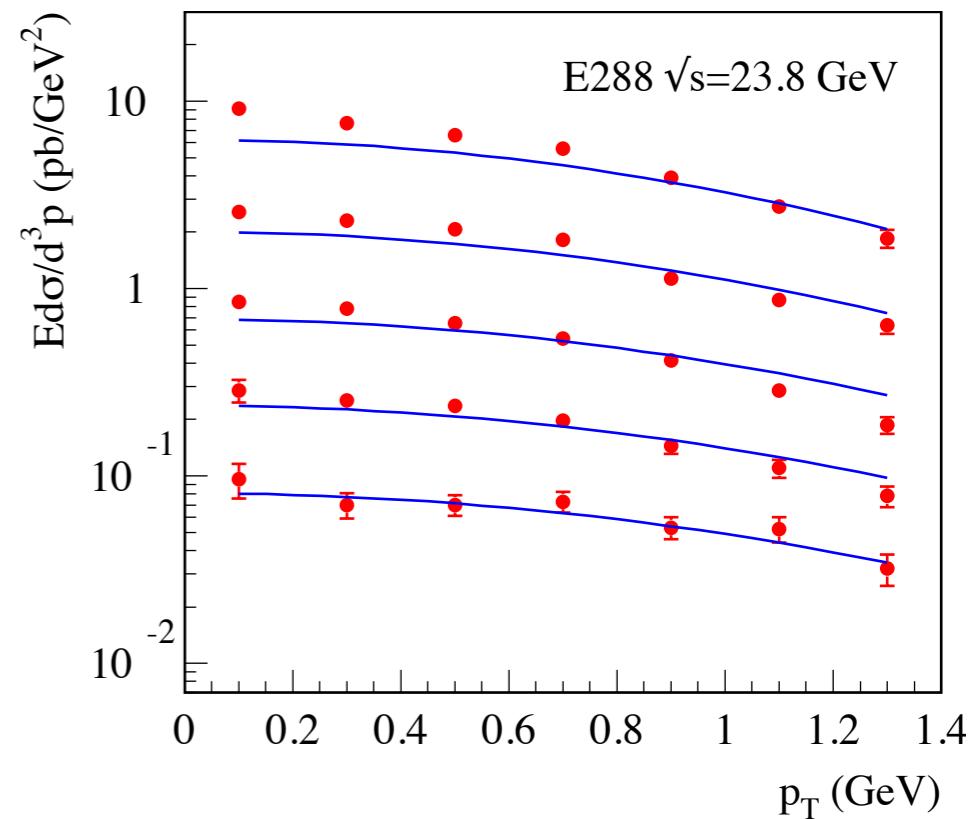


# Unpolarized DY+SIDIS: Plots (2/4)

- DY lepton-pair production at Fermilab (E288 and E605)

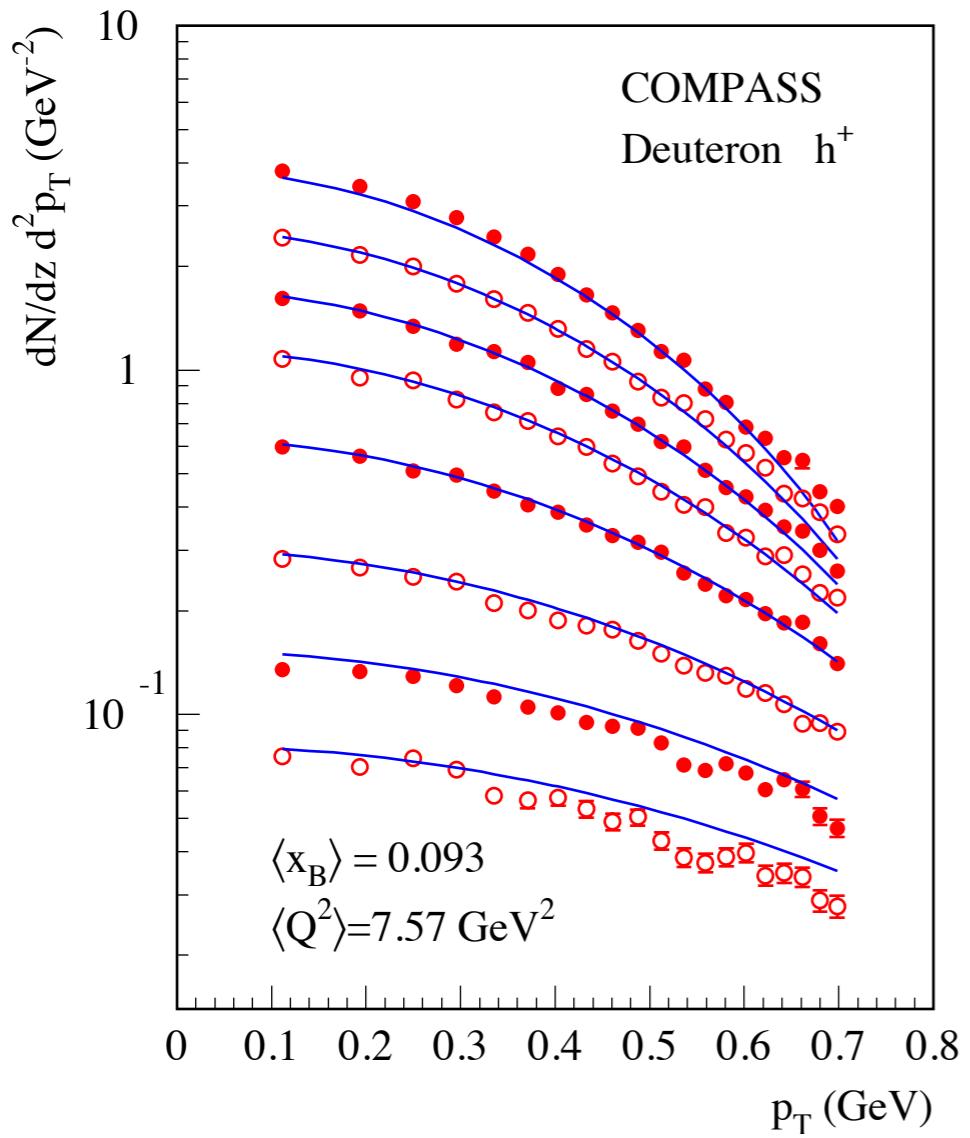


Data from top to  
bottom  
correspond to  
different  
invariant mass  $Q$ .

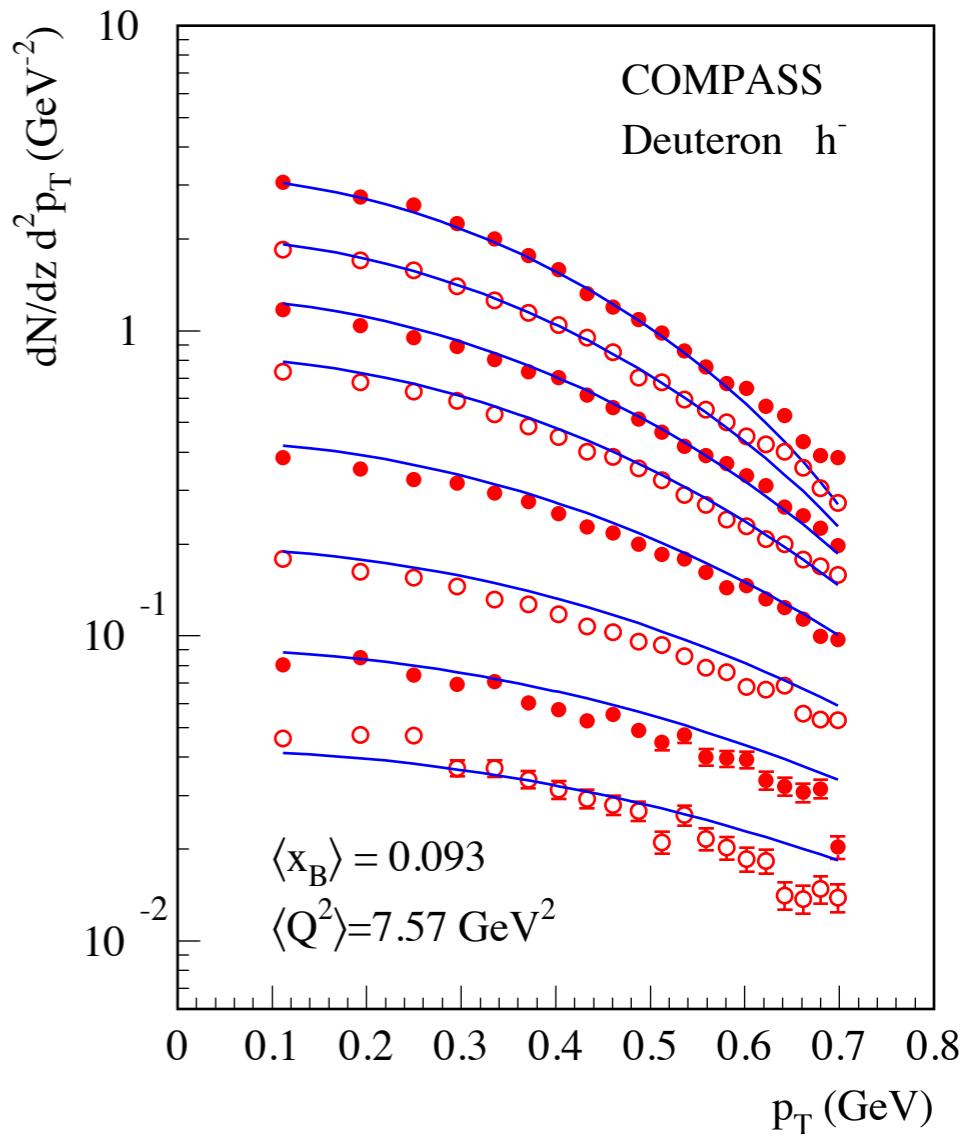


# Unpolarized DY+SIDIS: Plots (3/4)

- SIDIS Multiplicities at CERN (COMPASS)

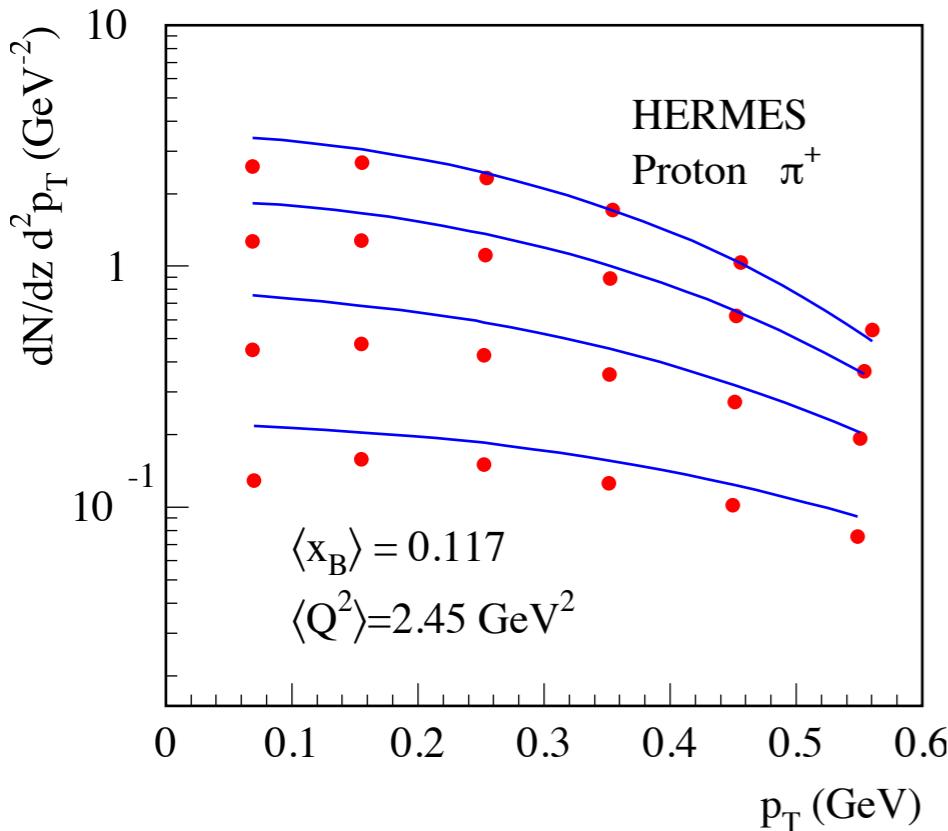


Data from top to  
bottom correspond  
to different  $z$  regions

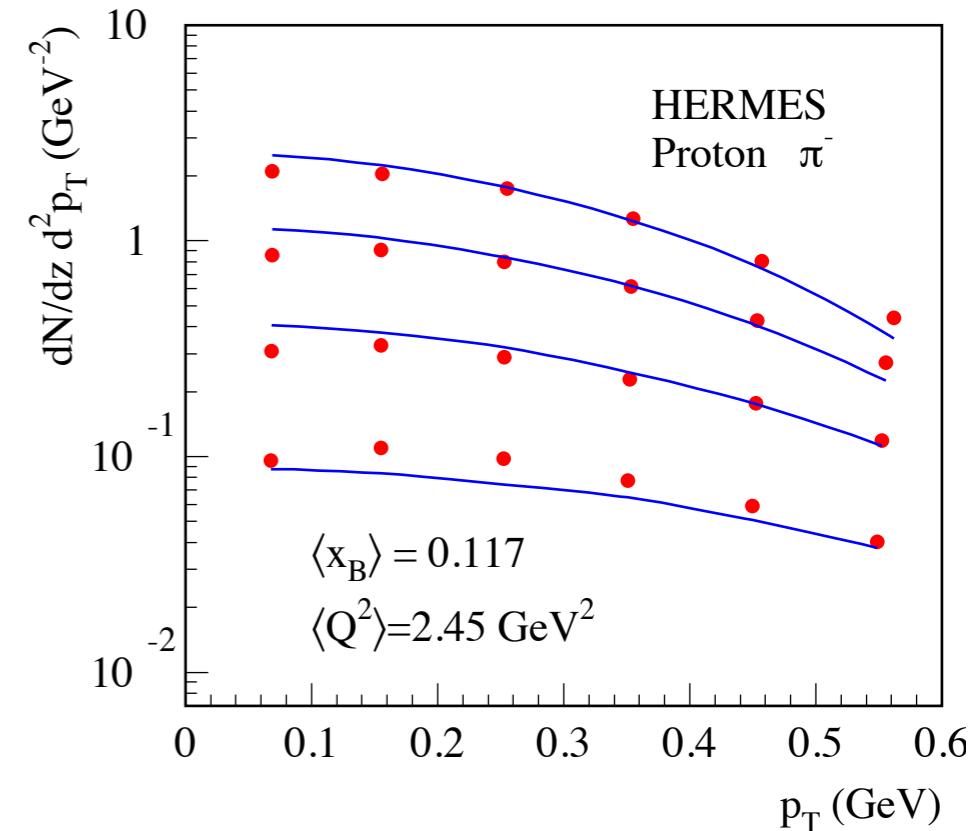


# Unpolarized DY+SIDIS: Plots (4/4)

- SIDIS Multiplicities at DESY (HERMES)

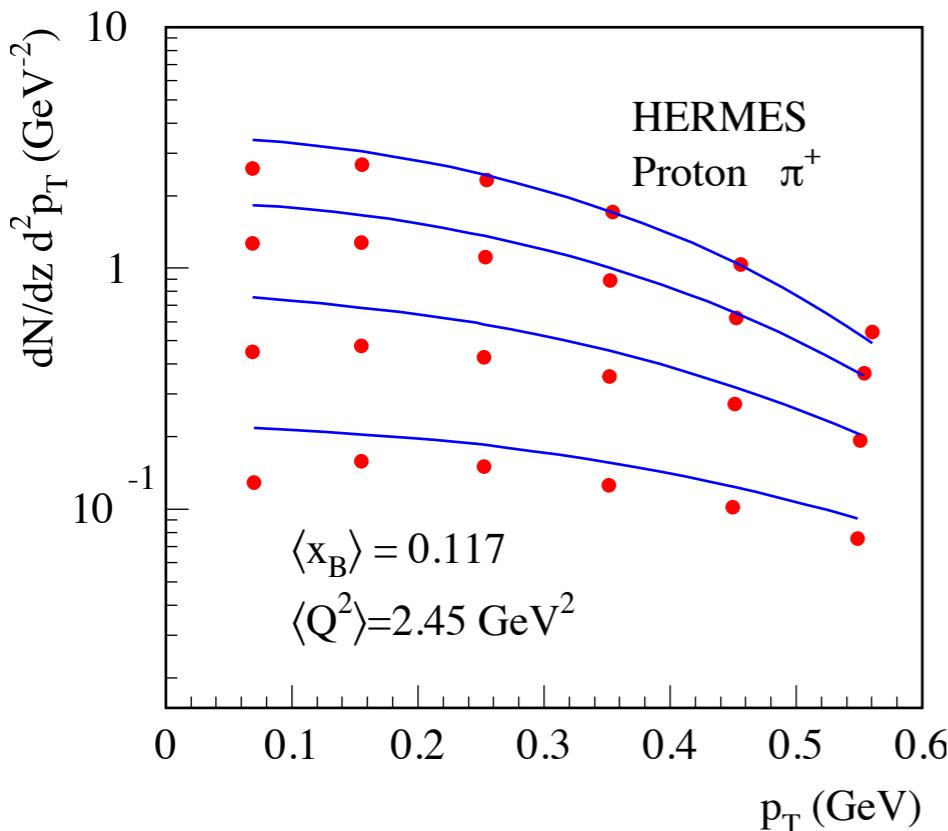


**Data from top to bottom correspond to different z regions**

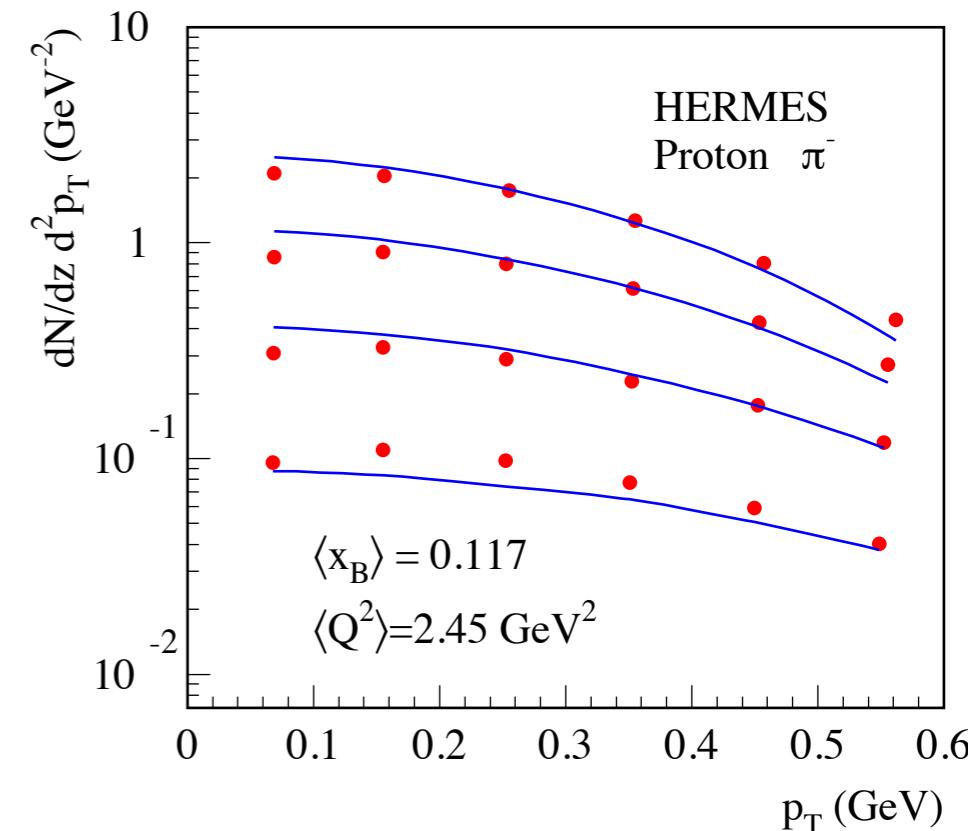


# Unpolarized DY+SIDIS: Plots (4/4)

- SIDIS Multiplicities at DESY (HERMES)



**Data from top to bottom correspond to different z regions**



We have a good description of SIDIS+DY data



# Sivers Function: Parameterization

- Sivers function is OPEd onto the Qiu-Sterman matrix element:

$$\tilde{f}_{1T f/P}^{\alpha SIDIS}(z, b_T; \zeta_F, \mu) = \sum_j \left( \frac{i b_\perp^\alpha}{2} \right) \int_z^1 \frac{d\hat{x}_1 d\hat{x}_2}{\hat{x}_1 \hat{x}_2} \tilde{C}_{f/j}(z/\hat{x}_1, z/\hat{x}_2, b_T; \zeta_F, \mu) T_{F j/P}(\hat{x}_1, \hat{x}_2; \mu) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

$$\tilde{f}_{1T f/P}^{\alpha SIDIS}(z, b_T; c/b_T) = \left( \frac{i b_\perp^\alpha}{2} \right) T_{F f/P}(z, z; c/b_T) + \dots$$

$\mu^2 = \zeta_F$

- With evolution and non-perturbative model:

$$\begin{aligned} \tilde{f}_{1T f/P}^{\alpha SIDIS}(z, b; Q) &= \left( \frac{i b^\alpha}{2} \right) T_{F f/P}(z, z; c/b_*) e^{-g_1^{\text{sivers}} b^2} \\ &\times \exp \left\{ - \int_{c/b_*}^Q \frac{d\mu}{\mu} \left( \Gamma_{\text{cusp}} \ln \frac{Q^2}{\mu^2} + \gamma_V \right) \right\} \left( \frac{Q^2}{Q_0^2} \right)^{-\frac{1}{4} g_2 b^2} \end{aligned}$$

$Q_0^2 = 2.4 \text{ GeV}^2$

- We parameterize the Qiu-Sterman function as:

$$T_{q,F}(x, x, \mu) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1-x)^{\beta_q} f_{q/A}(x, \mu)$$

Kouvaris, Qiu, Vogelsang, Yuan PRD'06

# Sivers Function: Fit

- We use the parameters obtained from the analysis of the unpolarized DY+SIDIS:

$$\begin{aligned}\langle k_{\perp}^2 \rangle_{Q_0} &= 0.38 \text{ GeV}^2 \\ \langle p_T^2 \rangle_{Q_0} &= 0.19 \text{ GeV}^2 \\ g_2 &= 0.16 \text{ GeV}^2 \\ b_{\max} &= 1.5 \text{ GeV}^{-1}\end{aligned}$$

- We fit 11 parameters, chi2/dof=1.3:

$\chi^2/d.o.f. = 1.3$			
$\alpha_u$	$= 1.051^{+0.192}_{-0.180}$	$\alpha_d$	$= 1.552^{+0.303}_{-0.275}$
$\alpha_{\text{sea}}$	$= 0.851^{+0.307}_{-0.305}$	$\beta$	$= 4.857^{+1.534}_{-1.395}$
$N_u$	$= 0.106^{+0.011}_{-0.009}$	$N_d$	$= -0.163^{+0.039}_{-0.046}$
$N_{\bar{u}}$	$= -0.012^{+0.018}_{-0.020}$	$N_{\bar{d}}$	$= -0.105^{+0.043}_{-0.060}$
$N_s$	$= 0.103^{+0.548}_{-0.604}$	$N_{\bar{s}}$	$= -1.000 \pm 1.757$
$\langle k_{s\perp}^2 \rangle$	$= 0.282^{+0.073}_{-0.066} \text{ GeV}^2$		

$$g_1^{\text{sivers}} = \frac{\langle k_{s\perp}^2 \rangle}{4}$$

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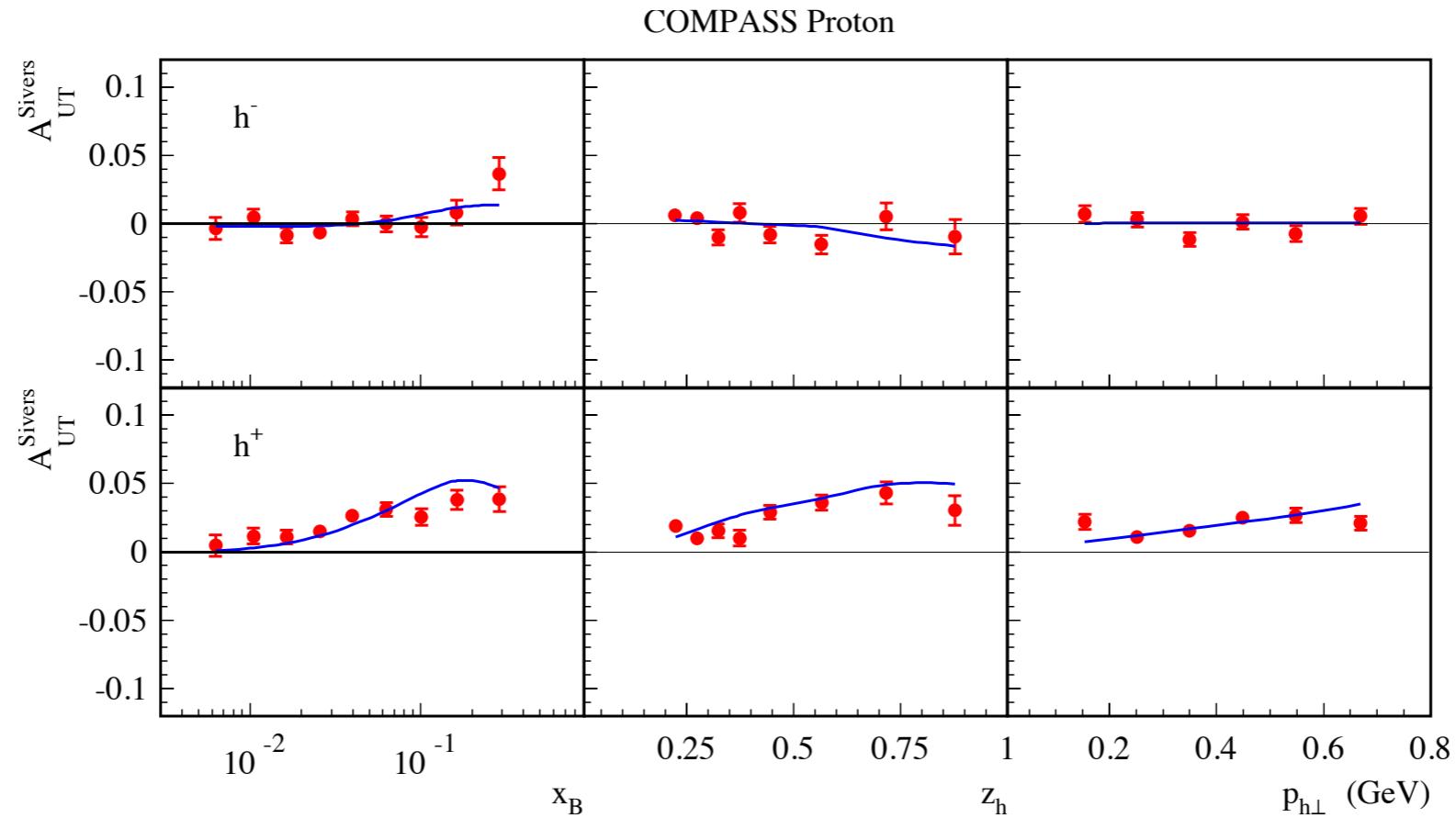
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Plots...

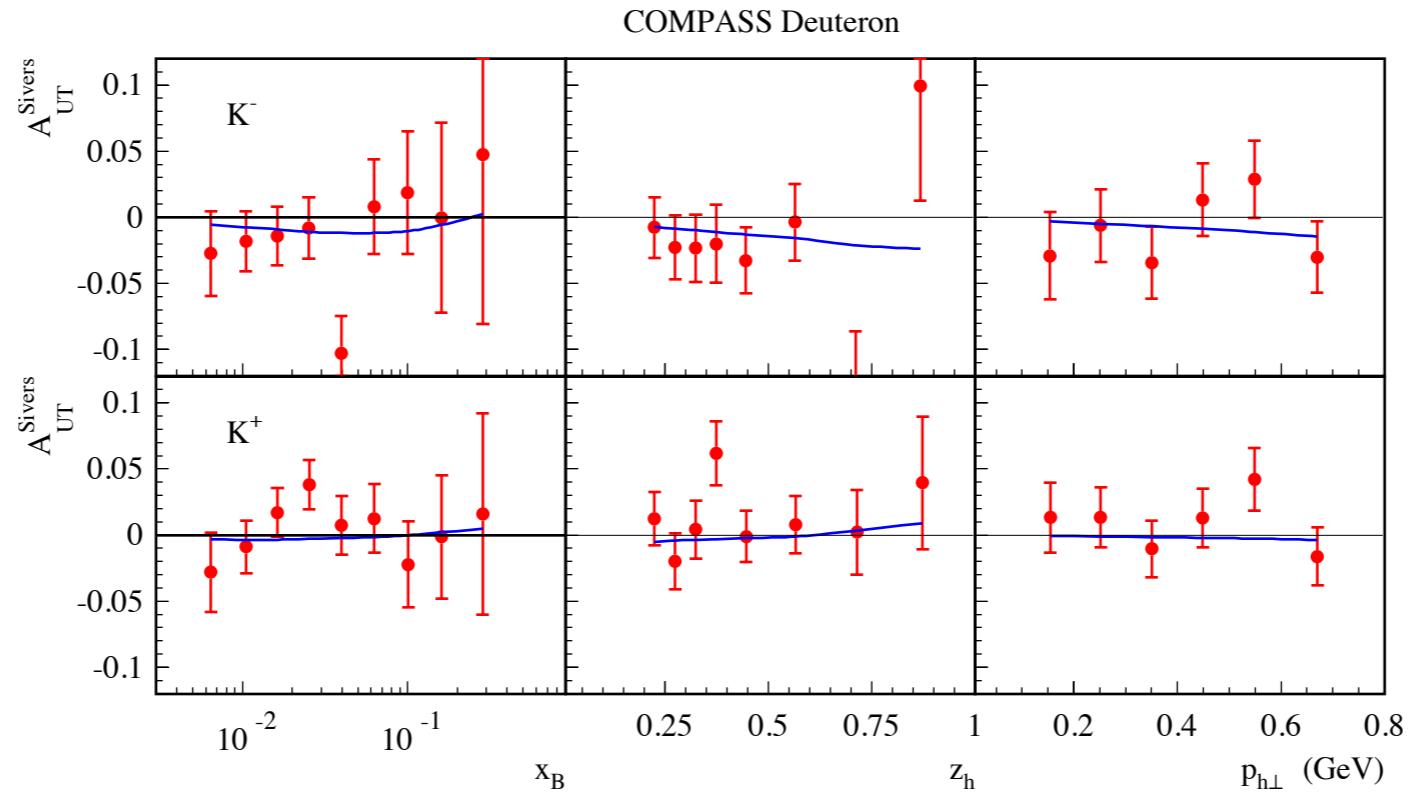
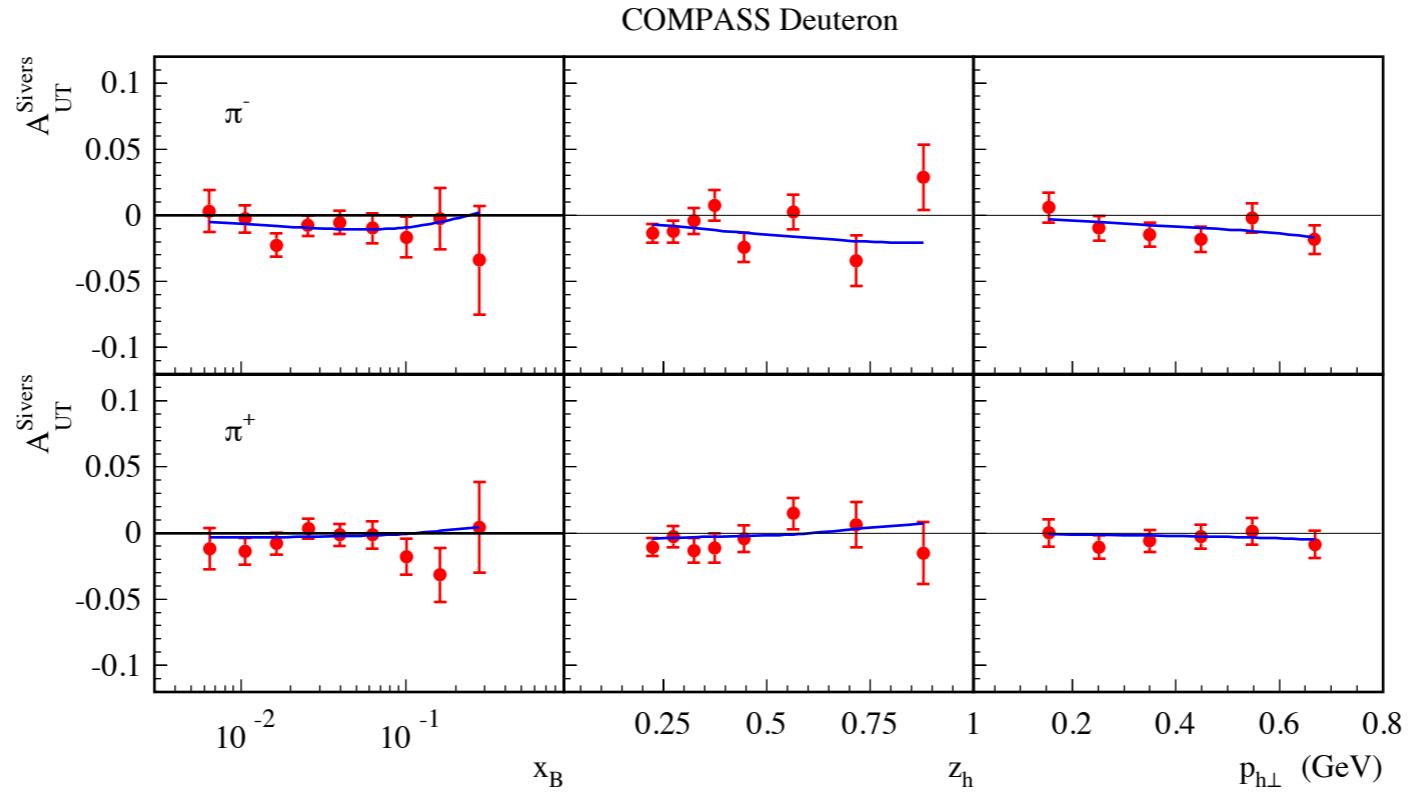
# Sivers Asymmetry: Fit

- COMPASS proton target



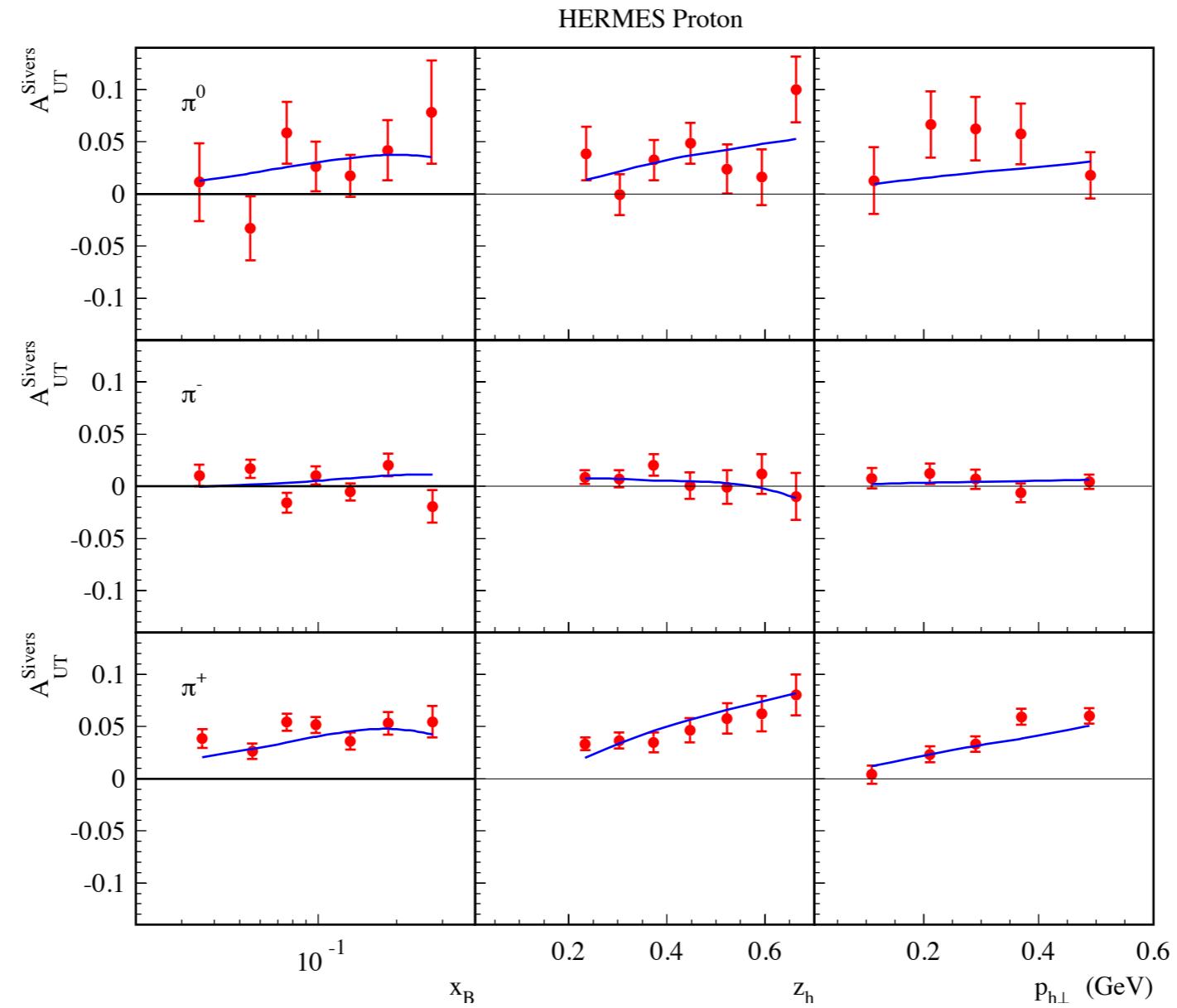
# Sivers Asymmetry: Fit

- COMPASS deuteron target



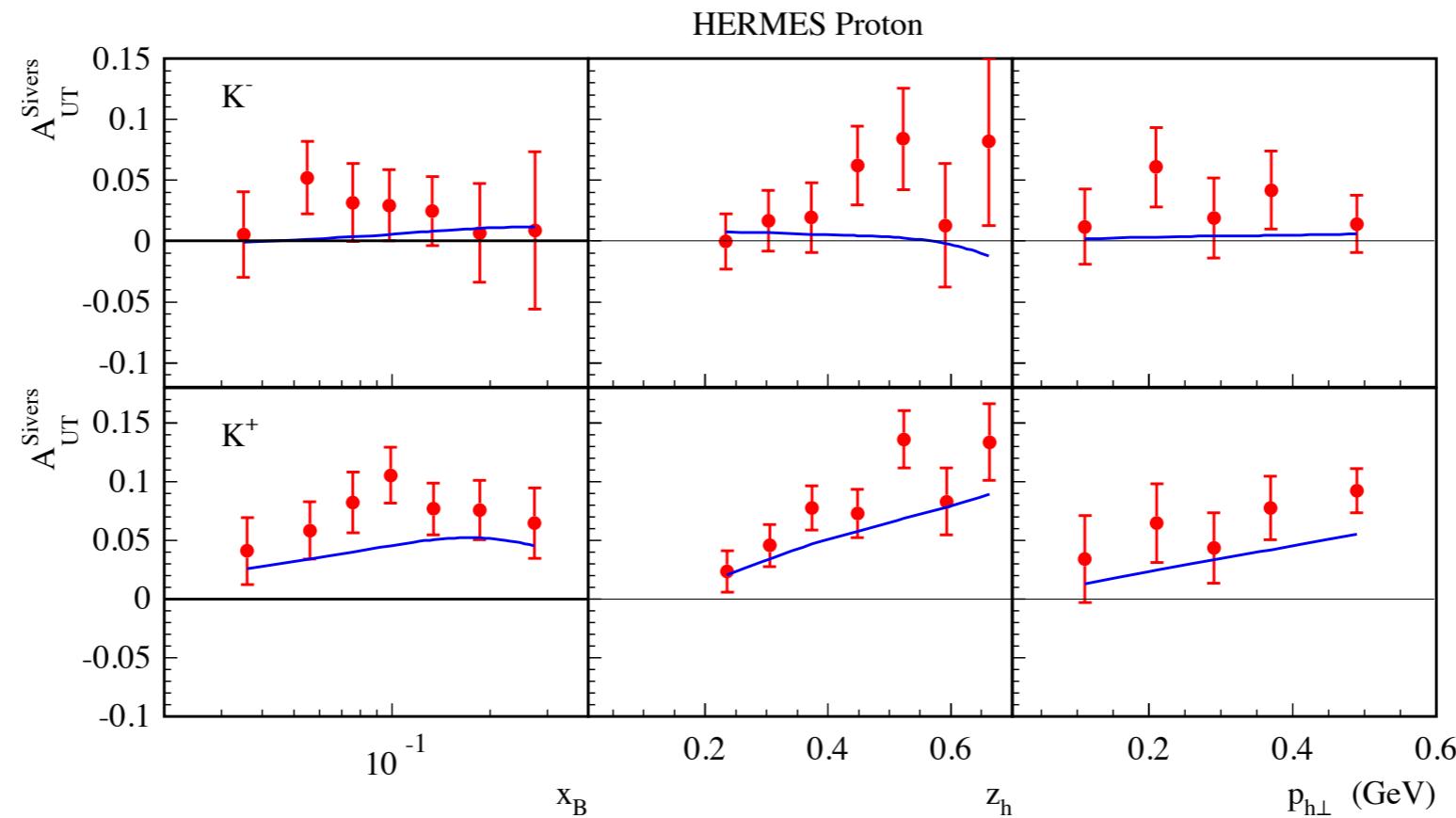
# Sivers Asymmetry: Fit

- HERMES proton target



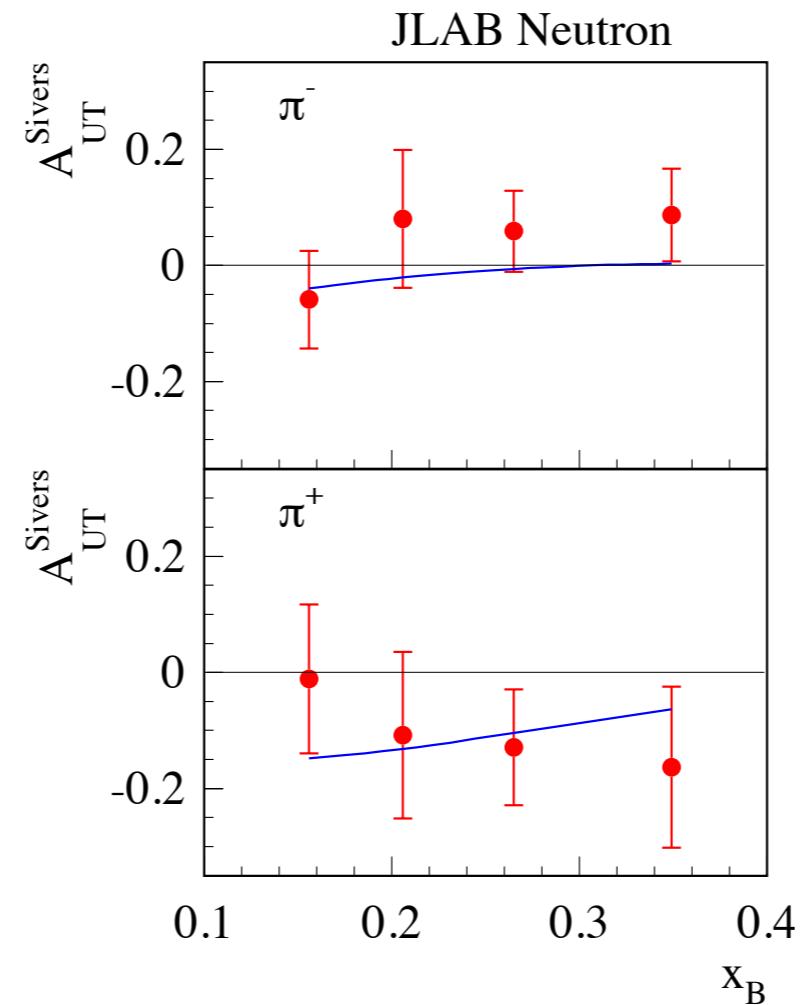
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- HERMES proton target



# Sivers Asymmetry: Fit

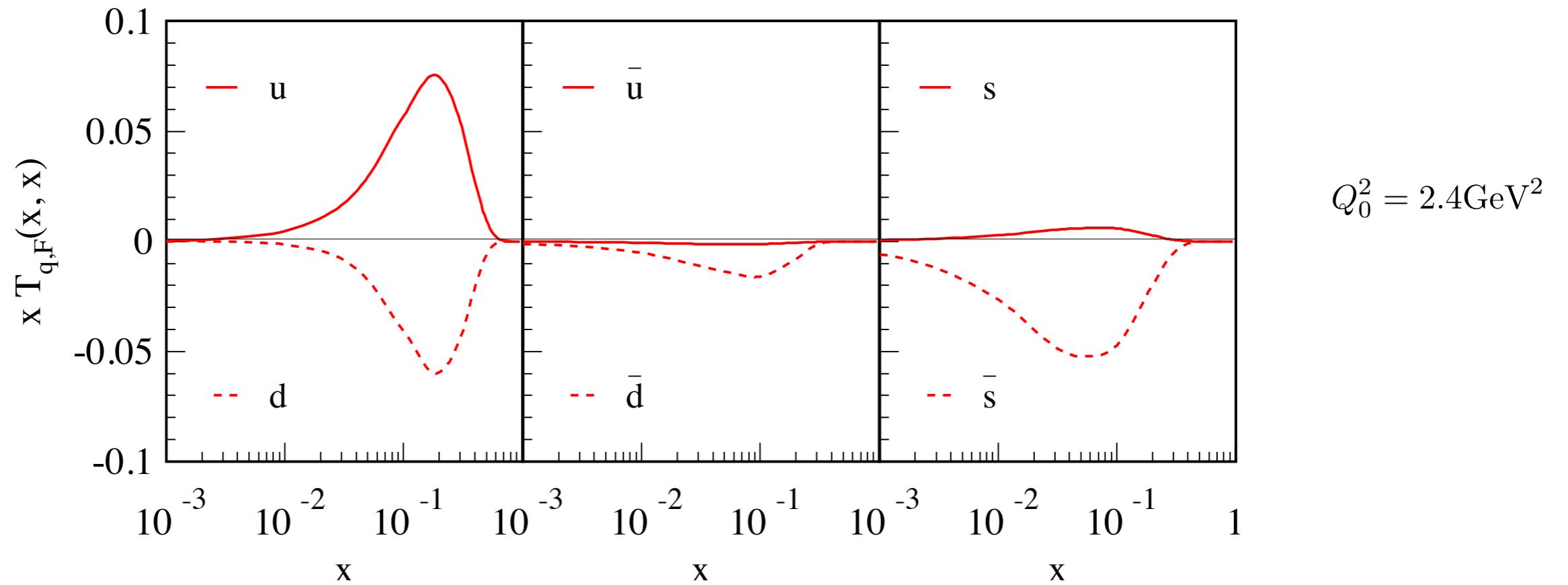
- JLAB neutron target



# Sivers Asymmetry: Fit

- Qiu-Sterman function:

$$T_{q,F}(x, x, \mu) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1 - x)^{\beta_q} f_{q/A}(x, \mu)$$

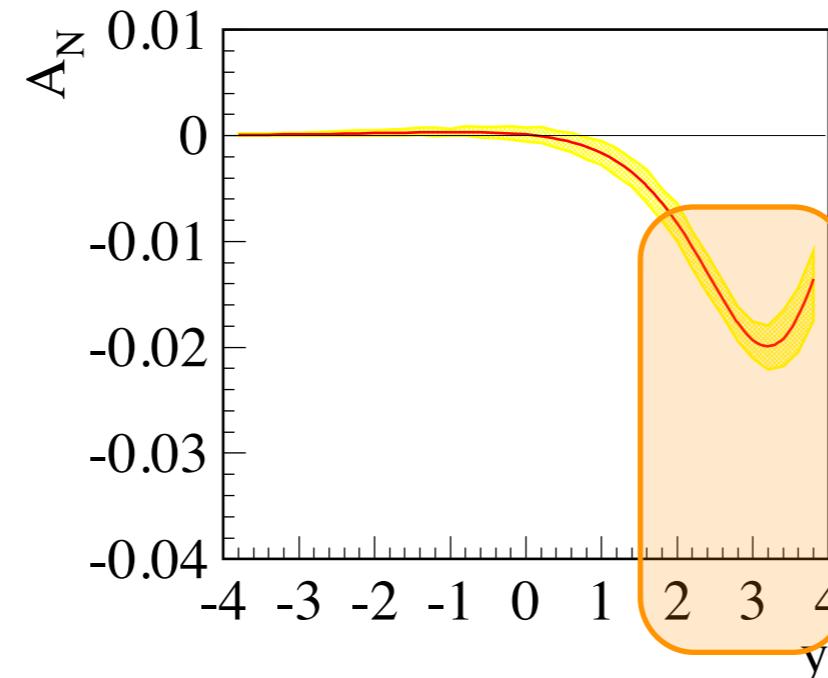


- SIDIS data only constrain **u** and **d** quark flavours. Sea quarks are not constrained.

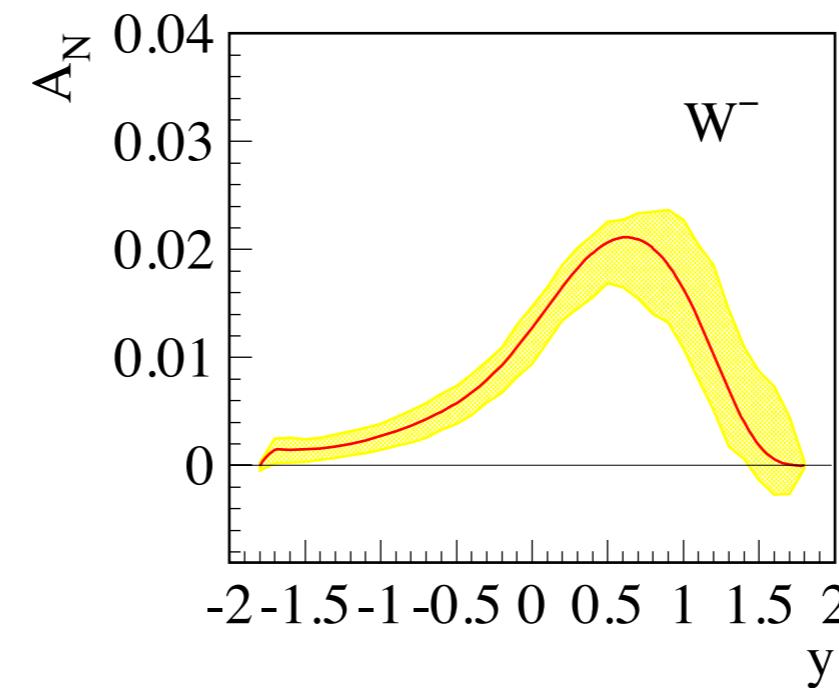
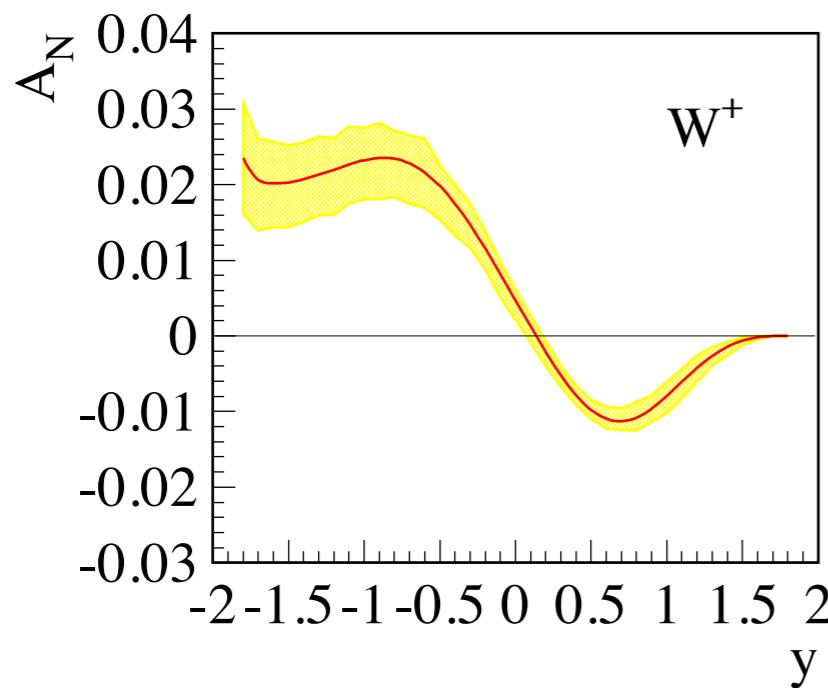
# Sivers Asymmetry: Predictions for DY

- DY at RHIC: proton(polarized)-proton at  $\sqrt{s}=510\text{GeV}$

**Integrate:**  
pT: [0,1]GeV  
Q: [4,9]GeV



**Forward rapidity region**  
**Prediction: 2%-3%**

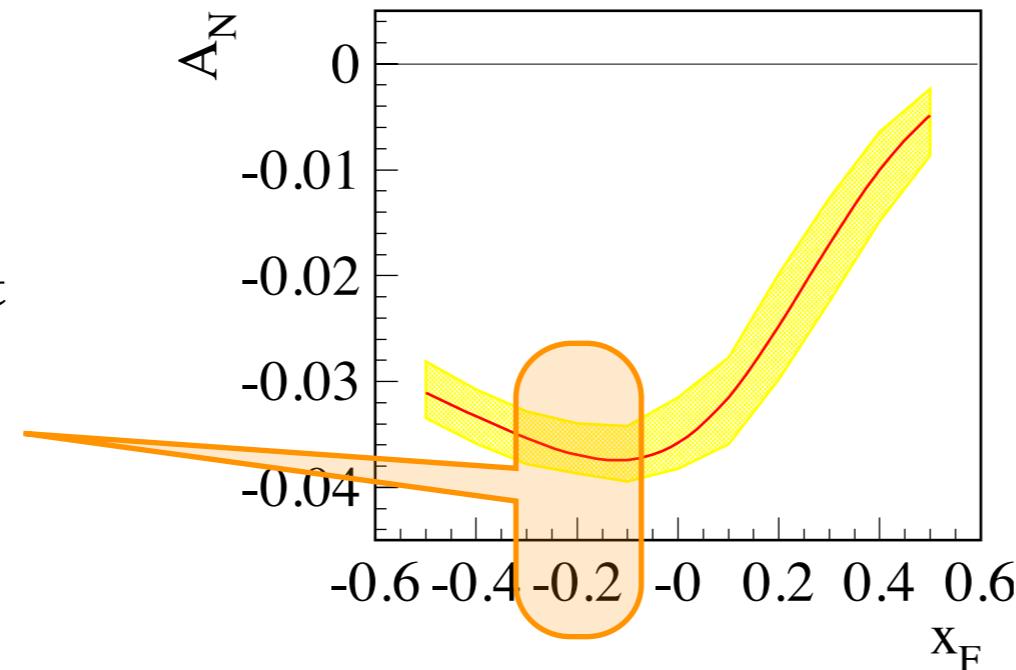


**Prediction: 2%-3%**  
**Integrate:**  
pT: [0,3]GeV

# Sivers Asymmetry: Predictions for DY

- DY at CERN (COMPASS): 190GeV Pi- beam to scatter on polarized proton target.  $\sqrt{s}=18.9\text{GeV}$ .

**Projected measurement**  
at  $x_F = -0.2$   
**Prediction:** 3%-4%



$$x_F = x_a - x_b$$

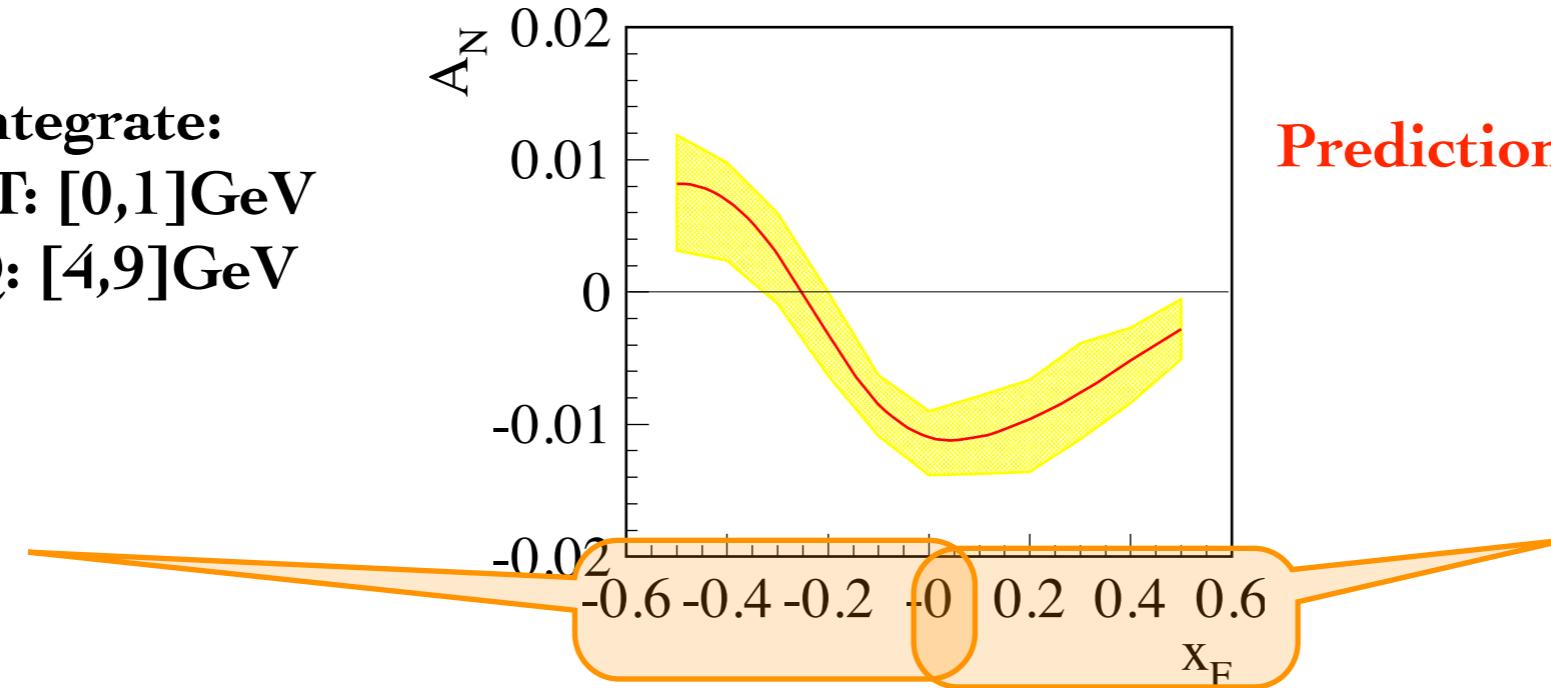
Polarized hadron

Integrate:  
 $p_T: [0,1]\text{GeV}$   
 $Q: [4,9]\text{GeV}$

- DY at Fermilab: 120GeV proton beam to scatter on proton target. Either polarized beam or polarized target.  $\sqrt{s}=15.1\text{GeV}$ .

Integrate:  
 $p_T: [0,1]\text{GeV}$   
 $Q: [4,9]\text{GeV}$

Polarized target



**Prediction:** 1%-2%

Polarized beam

# Conclusions & Outlook

- TMDs should contain the soft function to be well-defined.
- The evolution of all the 8 TMDPDFs and 8 TMDFFs is driven by the same evolution kernel.
- We know the perturbative ingredients to get resummation up to NNLL.
- We found a simple non-perturbative model that can describe reasonably well all SIDIS, DY and W/Z boson data.
- Using that model and proper QCD evolution, we fit the Sivers asymmetry in SIDIS and made reliable predictions for DY and W boson to be compared with coming measurements (RHIC, CERN (COMPASS), Fermilab).
- Improvements: NLL to NNLL, different models, and different ways to avoid the Landau pole.

[See Stefano's talk](#)

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[See Stefano's talk](#)



# Back up slides

# Unpolarized DY+SIDIS

- SIDIS process:

$$e(\ell) + A(P) \rightarrow e(\ell') + h(P_h) + X$$

- We measure the hadron multiplicity:

$$\frac{dN}{dz_h d^2 P_{h\perp}} = \frac{d\sigma}{dx_B dQ^2 dz_h d^2 P_{h\perp}} \Big/ \frac{d\sigma}{dx_B dQ^2}$$

$$\frac{d\sigma}{dx_B dQ^2 dz_h d^2 P_{h\perp}} = \frac{\sigma_0^{\text{DIS}}}{2\pi} H^{\text{DIS}}(Q) \sum_q e_q^2 \int_0^\infty db b J_0(P_{h\perp} b/z_h) f_{q/A}(x_B, b; Q) D_{h/q}(z_h, b; Q)$$

$$\frac{d\sigma}{dx_B dQ^2} = \sigma_0^{\text{DIS}} \sum_q e_q^2 f_{q/A}(x_B, Q)$$

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$$\frac{d\sigma}{dx_B dQ^2} = \sigma_0^{\text{DIS}} \sum_q e_q^2 f_{q/A}(x_B, Q)$$

Well-defined TMDs

Depends just on  $Q$

Collins 11

MGE, Idilbi, Scimemi JHEP'12, PLB'13

See John's talk

We have set:  $\mu^2 = \zeta_A = \zeta_h = Q^2$

# Unpolarized DY+SIDIS

- Drell-Yan process:

$$A(P_A) + B(P_B) \rightarrow [\gamma^* \rightarrow] \ell^+ \ell^-(y, Q, p_\perp) + X$$

$$\frac{d\sigma}{dQ^2 dy d^2 p_\perp} = \frac{\sigma_0^{\text{DY}}}{2\pi} H^{\text{DY}}(Q) \sum_q e_q^2 \int_0^\infty db b J_0(p_\perp b) f_{q/A}(x_a, b; Q) f_{\bar{q}/B}(x_b, b; Q)$$

- W/Z boson production:

$$A(P_A) + B(P_B) \rightarrow W/Z(y, p_\perp) + X$$

$$\frac{d\sigma^W}{dy d^2 p_\perp} = \frac{\sigma_0^W}{2\pi} H^{\text{DY}}(Q) \sum_{q,q'} |V_{qq'}|^2 \int_0^\infty db b J_0(q_\perp b) f_{q/A}(x_a, b; Q) f_{q'/B}(x_b, b; Q)$$

$$\frac{d\sigma^Z}{dy d^2 p_\perp} = \frac{\sigma_0^Z}{2\pi} H^{\text{DY}}(Q) \sum_q (V_q^2 + A_q^2) \int_0^\infty db b J_0(q_\perp b) f_{q/A}(x_a, b; Q) f_{q'/B}(x_b, b; Q)$$