TMD evolution of Collins asymmetries in e^+e^- annihilation and SIDIS (preliminary)

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Outlines

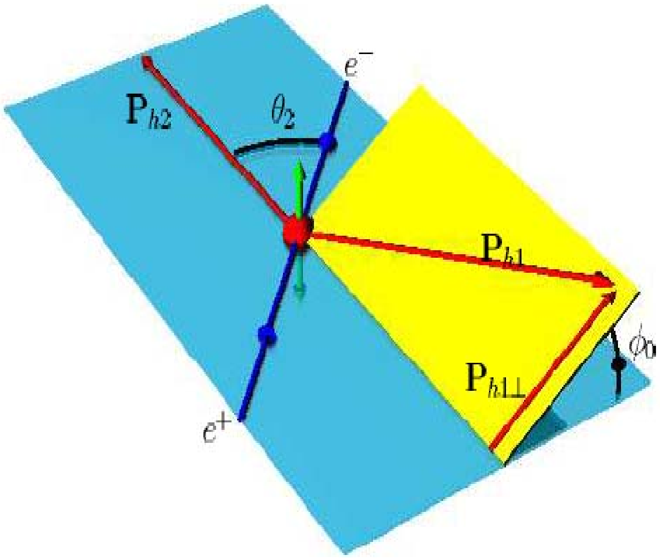
- Energy evolution factor in TMD factorization
- Nonperturbative Sudakov factor fitting from Drell-Yan and SIDIS processes
- Collins asymmetry in e^+e^- annihilation and SIDIS
- Summary

Collins asymmetries in e^+e^- annihilation

For the process

$$e^+ + e^- \to H_1 + H_2 + X$$

The cross section can be written as



$$\frac{d\sigma}{dz_{h1}dz_{h2}d^2P_{h\perp}d\theta} = \frac{2\pi N_c\alpha^2}{4Q^2}\left[\left(1+\cos^2\theta\right)Z_{uu} + \sin^2\theta\left(2\hat{e}_x^\alpha\hat{e}_x^\beta - g_\perp^{\alpha\beta}\right)Z_{collins}^{\alpha\beta}\right]$$

$\Downarrow$ $\cos(2\phi_0)$

Boer, Jakob, Mulders, 1998
Boer, 2001, 2009

Energy evolution from TMD factorization

(Ji-Ma-Yuan TMD factorization)

In the TMD factorization, in the small transverse momentum region

$$\widetilde{Z}_{uu} = D(z_1, b_\perp, \zeta_1; \mu) D(z_2, b_\perp, \zeta_2; \mu) H_{uu}^{e^+e^-}(Q;\mu) S(b_\perp, \rho; \mu) ,$$

$$\widetilde{Z}_{\rm collins}^{\alpha\beta} = \widetilde{H}_1^{\perp\alpha}(z_1, b_\perp, \zeta_1; \mu) \widetilde{H}_1^{\perp\beta}(z_2, b_\perp, \zeta_2; \mu) H_{\rm collins}^{e^+e^-}(Q;\mu) S(b_\perp, \rho; \mu)$$

$\zeta^2/\rho = Q^2 \times z^2$

$P_{h\perp}/z$

$$D(z_2, b_\perp, b_\perp; \mu) \times e^{\int_{b_\perp}^{\zeta_2} \gamma(\mu) \frac{d\mu}{\mu}}$$

Z_{uu} and $Z_{collins}$ satisfy CSS evolution equation

$$\frac{\partial}{\partial \ln Q^2} \widetilde{Z}_{uu}(Q;b) = \left(K(b,\mu) + G(Q,\mu)\right) \widetilde{Z}_{uu}(Q;b)$$

At one-loop order

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu) = -\frac{\alpha_s C_F}{\pi}\left(\ln\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$

Substituting the above result into the evolution equation, and taking into account the running effects in K(b, μ)

$$\widetilde{Z}_{uu}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}(Q,b)}\Sigma_{i,j}\hat{C}_{qi}^{(e^+e^-)} \otimes D_{i/A}(z_1,1/b_*)\hat{C}_{qj}^{(e^+e^-)} \otimes D_{j/B}(z_2,1/b_*)$$

$$\widetilde{Z}^{\alpha\beta}_{\text{collins}}(Q;b) = \left(\frac{-ib_\perp^\alpha}{2}\right)\left(\frac{-ib_\perp^\beta}{2}\right) e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}^T(Q,b)}$$
$$\times\Sigma_{i,j}\Delta\hat{C}_{qi}^{\text{collins}(e^+e^-)} \otimes D_{i/A}^{(3)}(z_1,1/b_*)\hat{C}_{qj}^{\text{collins}(e^+e^-)} \otimes D_{j/B}^{(3)}(z_2,1/b_*) ,$$

Where S_{pert} is universal to Drell-Yan, SIDIS and ee to di-hadron

$$S_{pert}(Q,b_*) = \int_{c_0/b_*}^{Q} \frac{d\bar{\mu}}{\bar{\mu}}\left[A\ln\frac{Q^2}{\bar{\mu}^2} + B\right]$$

b_* prescription (CSS, 85) in S_{pert}

$$b \Rightarrow b_* = b/\sqrt{1+b^2/b_{max}^2}\ , \quad b_{max} < 1/\Lambda_{QCD}$$

- Sudakov factor
  - There are two parts in the Sudakov factor

$$\mathcal{S}_{sud} \Rightarrow \mathcal{S}_{pert}(Q;b_*) + S_{NP}(Q;b)$$

  - the nonperturbative part

$$\begin{aligned} S_{NP}(Q,b) &= \boxed{g_2(b)\ln Q} + \boxed{g_1}(b;z_1,z_2) \\ S_{NP}^T(Q,b) &= \boxed{g_2(b)\ln Q} + \boxed{g_1^T}(b;z_1,z_2) \end{aligned}$$

Q dependence always satisfies CSS equation.

$g_2(b)$ is universal in Drell-Yan, SIDIS, and $e^+e^- \to hh$

- Gaussian assumptions are usually made for $g_1(b)$ and $g_2(b)$ (BLNY 2002):

$$S_{NP} = g_1 b^2 + g_2 b^2 \ln\left(Q/3.2\right) + g_1 g_3 b^2 \ln(100 x_1 x_2)$$

- However, this assumption does not work for SIDIS and Drell-Yan simultaneously in the range of Q^2~(3—100)GeV^2

Sun,Yuan,1308.5003

- Our model

$$S_{NP} = g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda \right)$$

This term is from

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2}$$

$\mu = c_0/b^*$

Fitting Drell-Yan processes

$$S_{NP} = g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda\right)$$

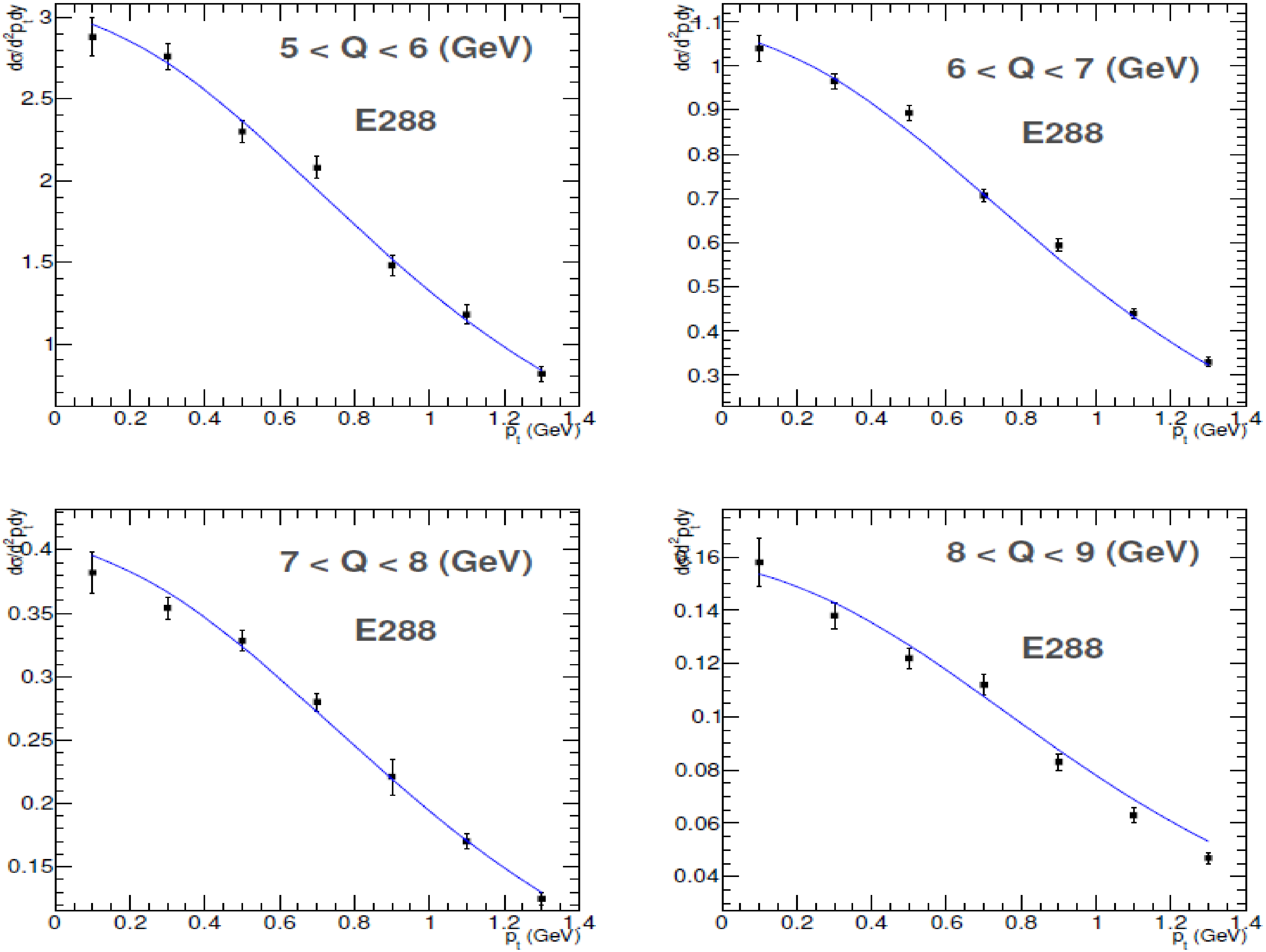
Parameter	SIYY fit
g_1	0.158
g_2	0.935
g_3	0.040
E288 (28 points)	$N_{fit} = 0.90$ $\chi^2 = 59$
E605 (35 points)	$N_{fit} = 0.95$ $\chi^2 = 63$
Tevatron (30 points)	$N_{fit} = 1.01$ $\chi^2 = 38$
χ^2	160
χ^2/DOF	1.84

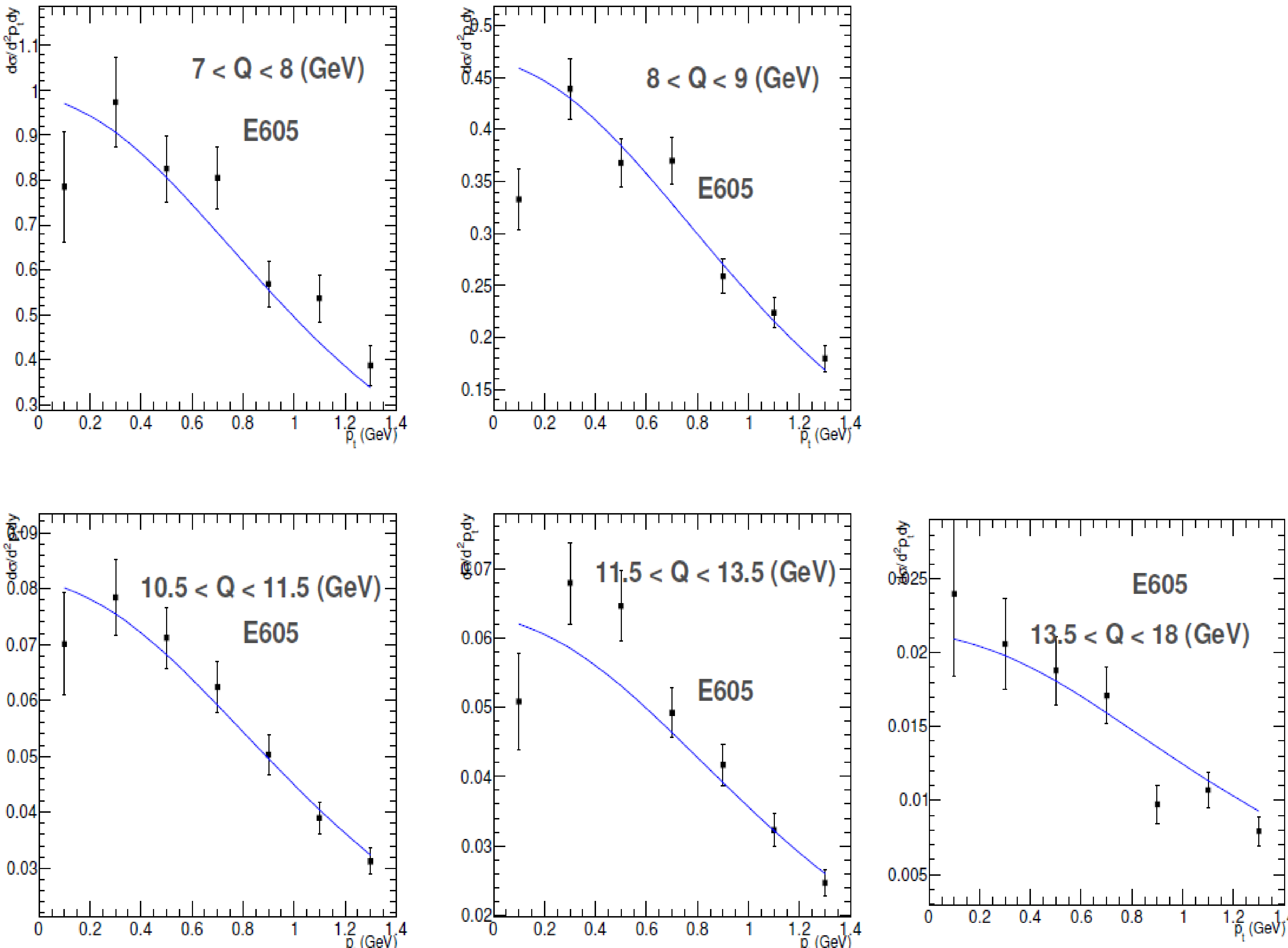
A new non-perturabtive Sudakov factor is used. Where x_0=0.01, Q_0^2=2.4GeV^2, and λ =0.2

g_1 , g_2 and g_3 are free parameters

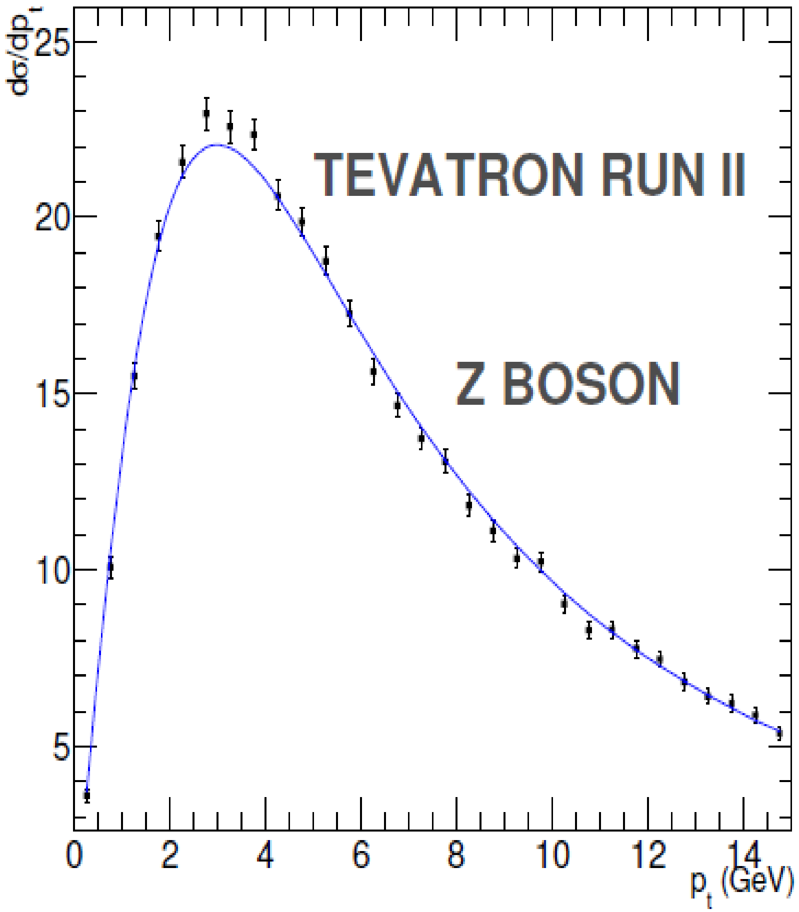
In our fit, we choose b_{max}=1.5GeV^{-1}

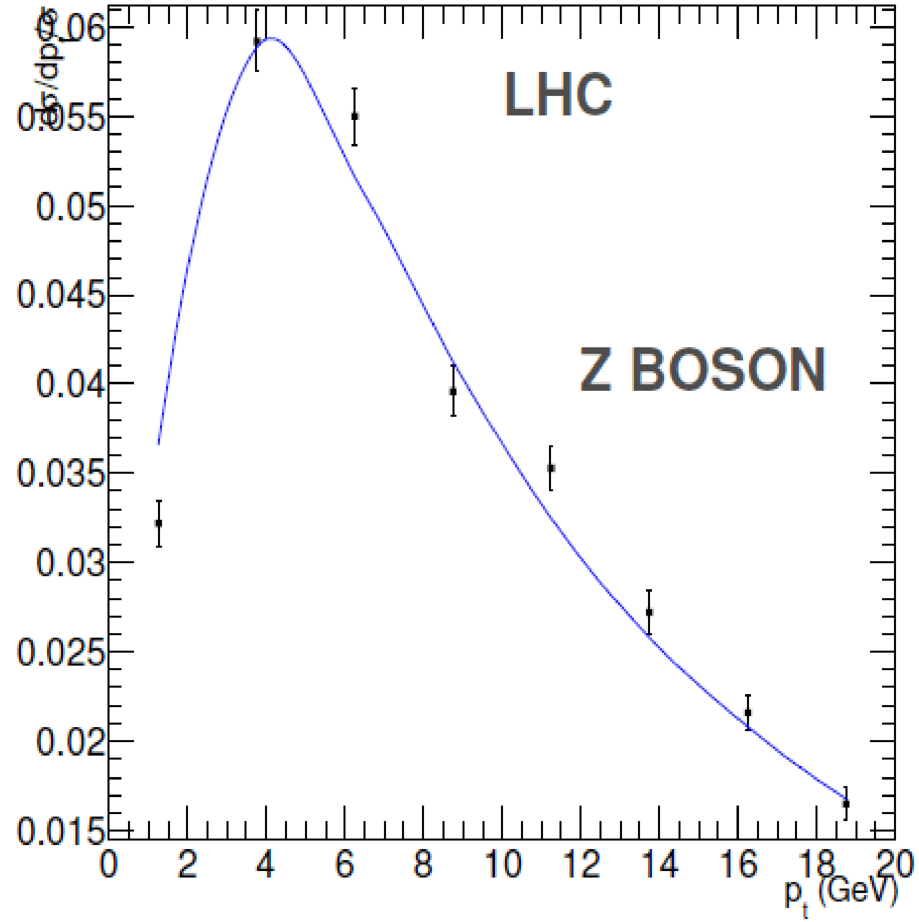
Y piece is included


5 < Q < 6 (GeV)
E288
6 < Q < 7 (GeV)
E288
7 < Q < 8 (GeV)
E288
8 < Q < 9 (GeV)
E288
dσ/d²p_t dy
p_t (GeV)



7 < Q < 8 (GeV)
E605
8 < Q < 9 (GeV)
E605
10.5 < Q < 11.5 (GeV)
E605
11.5 < Q < 13.5 (GeV)
E605
E605
13.5 < Q < 18 (GeV)
dσ/d²p_t dy
p_t (GeV)

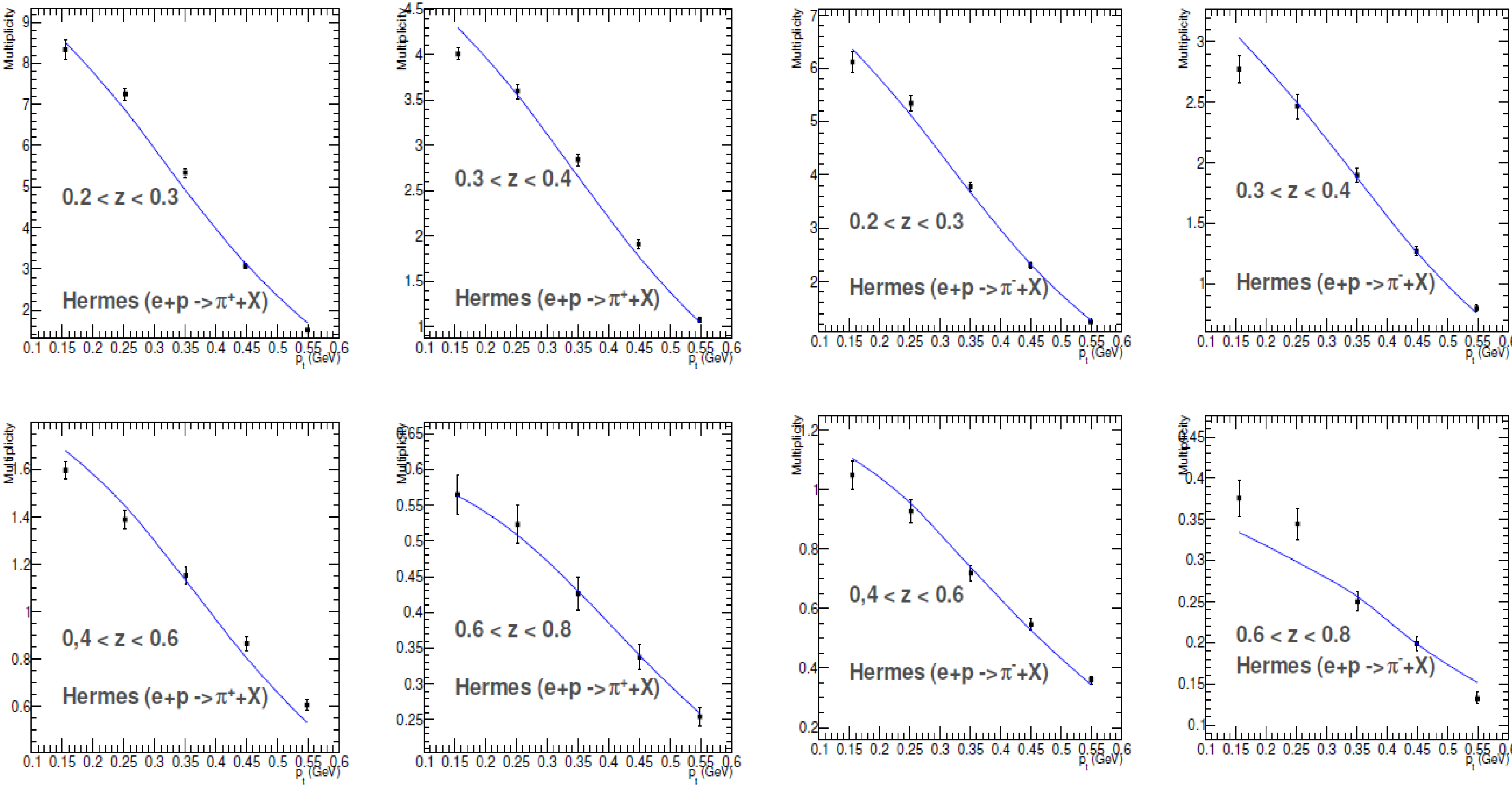




For Z boson production $A^{(2)}$ is very important

SIDIS at HERMES

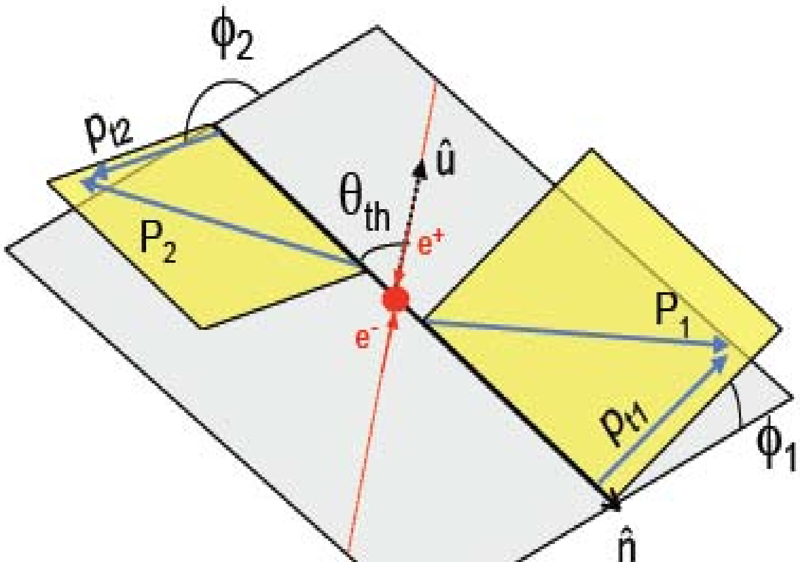
$Q^2 = 3GeV^2$



- So we have found a good NP form factor which can describe the data for the Q^2 from 3 to10000GeV^2

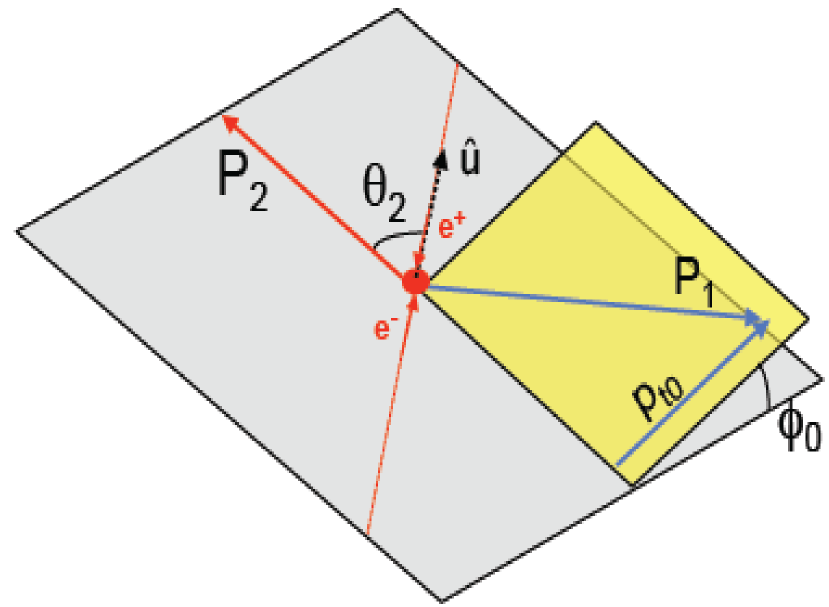
- Now we can study the TMD evolution for the Collins asymmetries in e^+e^- annihilation and SIDIS

Collins asymmetries in $e^+e^- \rightarrow hh+X$ from BELLE and BABAR



Thrust axis

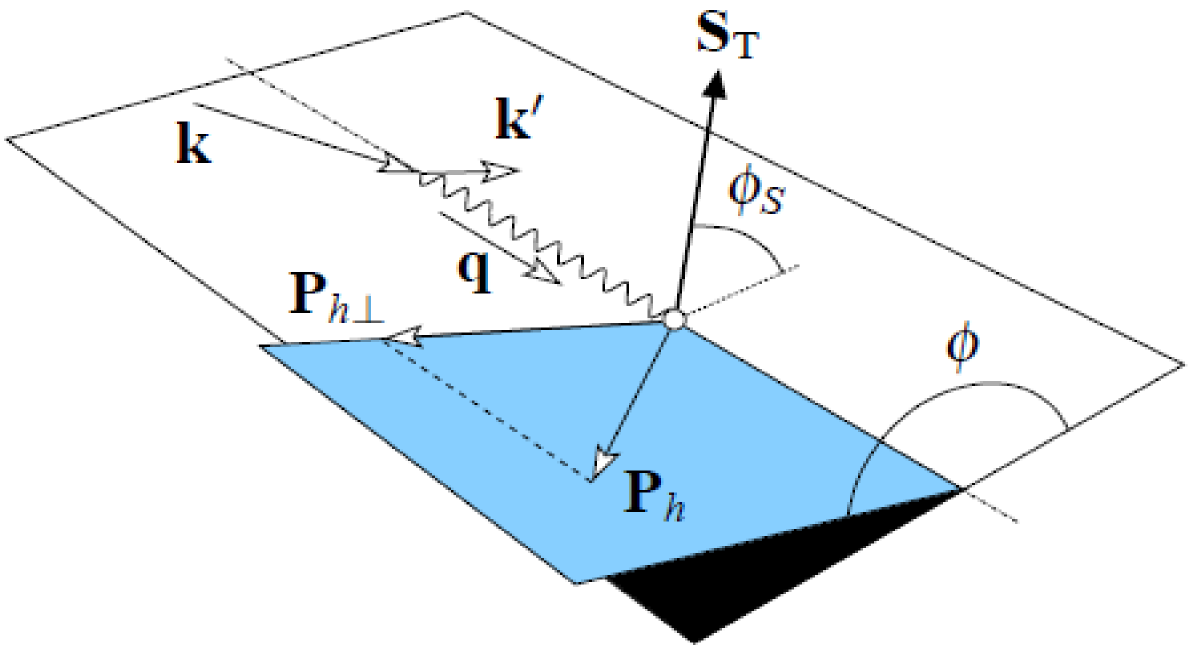
A_{12}



A_0

The Collins asymmetries is proportional to $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$

Collins asymmetries in SIDIS at HERMES and COMPASS



$$A_{UT}^{\sin(\phi_h+\phi_S)} = 2\,\frac{\int d\phi_h\, d\phi_S\,[d\sigma^{\uparrow} - d\sigma^{\downarrow}]\,\sin(\phi_h+\phi_S)}{\int d\phi_h\, d\phi_S\,[d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

The Collins asymmetries is proportional to $\sin(\phi+\phi_S)$

Globe Fitting of Collins functions

Collins function

$$\hat{H}_{\pi^+/u}(z,\mu^2=2.4\mathrm{GeV}^2) = N_u z^{a_u}(1-z)^{b_u} D_{\pi^+/u}(z,\mu^2=2.4\mathrm{GeV}^2)$$
$$\hat{H}_{\pi^+/d}(z,\mu^2=2.4\mathrm{GeV}^2) = N_d z^{a_d}(1-z)^{b_d} D_{\pi^+/d}(z,\mu^2=2.4\mathrm{GeV}^2)$$

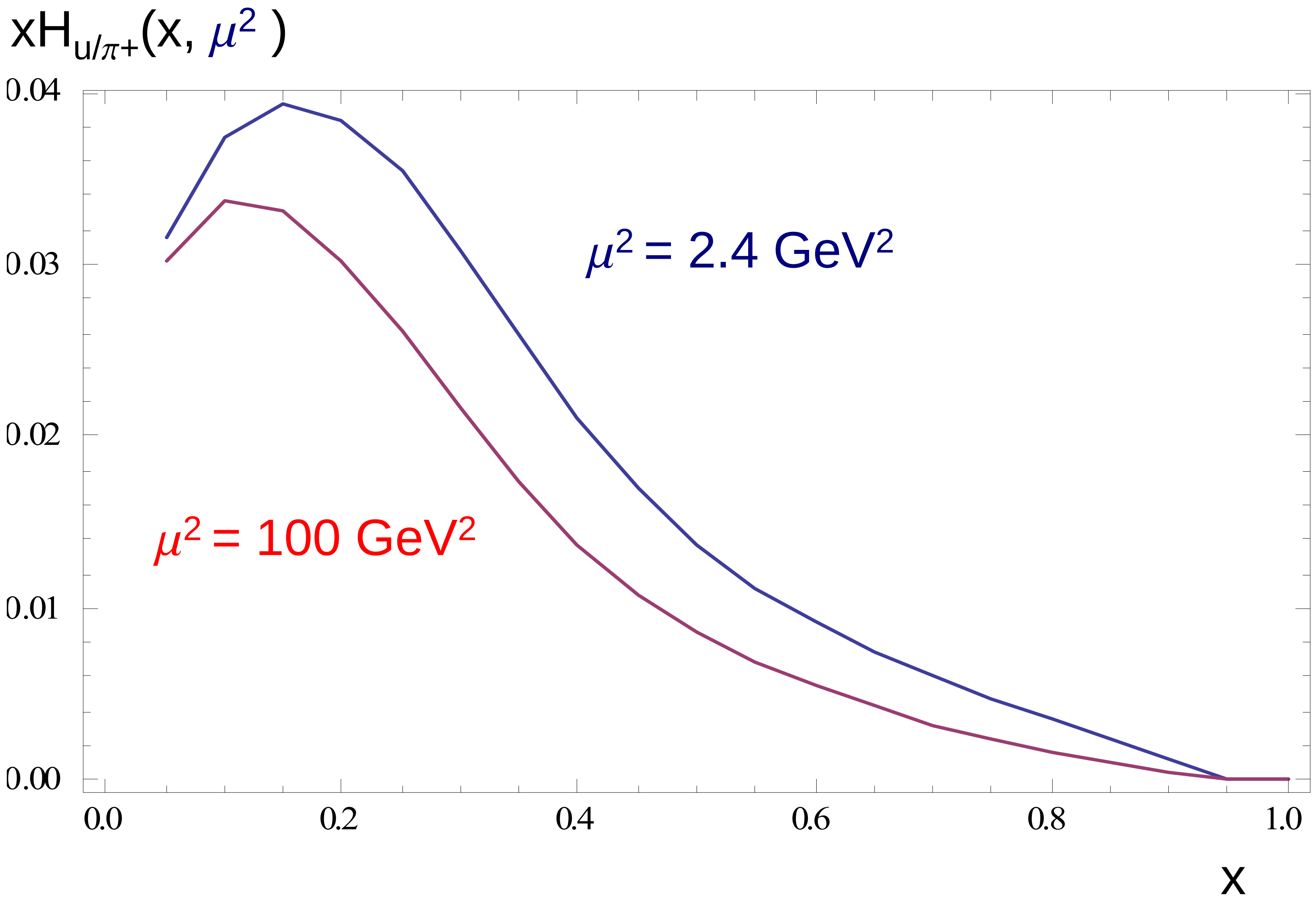
Then we solve DGLAP evolution equation to get the distributions at the scale of $1/b_*$.

quark transversity distributions

$$\hat{q}^t_u(z,\mu^2=2.4\mathrm{GeV}^2) = N^t_u z^{a^t_u}(1-z)^{b^t_u} f_u(z,\mu^2=2.4\mathrm{GeV}^2)$$
$$\hat{q}^t_d(z,\mu^2=2.4\mathrm{GeV}^2) = N^t_d z^{a^t_d}(1-z)^{b^t_d} f_d(z,\mu^2=2.4\mathrm{GeV}^2)$$

nonperturbative factor:

$$S^{DIS}_{NPcollins} = g_q \ln(b/b_*)\ln(Q/Q_0) + g_0 b^2 + (g_h - g_c) b^2/z_h^2 + g_3 b^2 (x/x_0)^\lambda$$
$$S^{e^+e^-}_{NPcollins} = g_q \ln(b/b_*)\ln(Q/Q_0) + (g_h - g_c) b^2 \left(1/z_{h1}^2 + 1/z_{h2}^2\right) ,$$

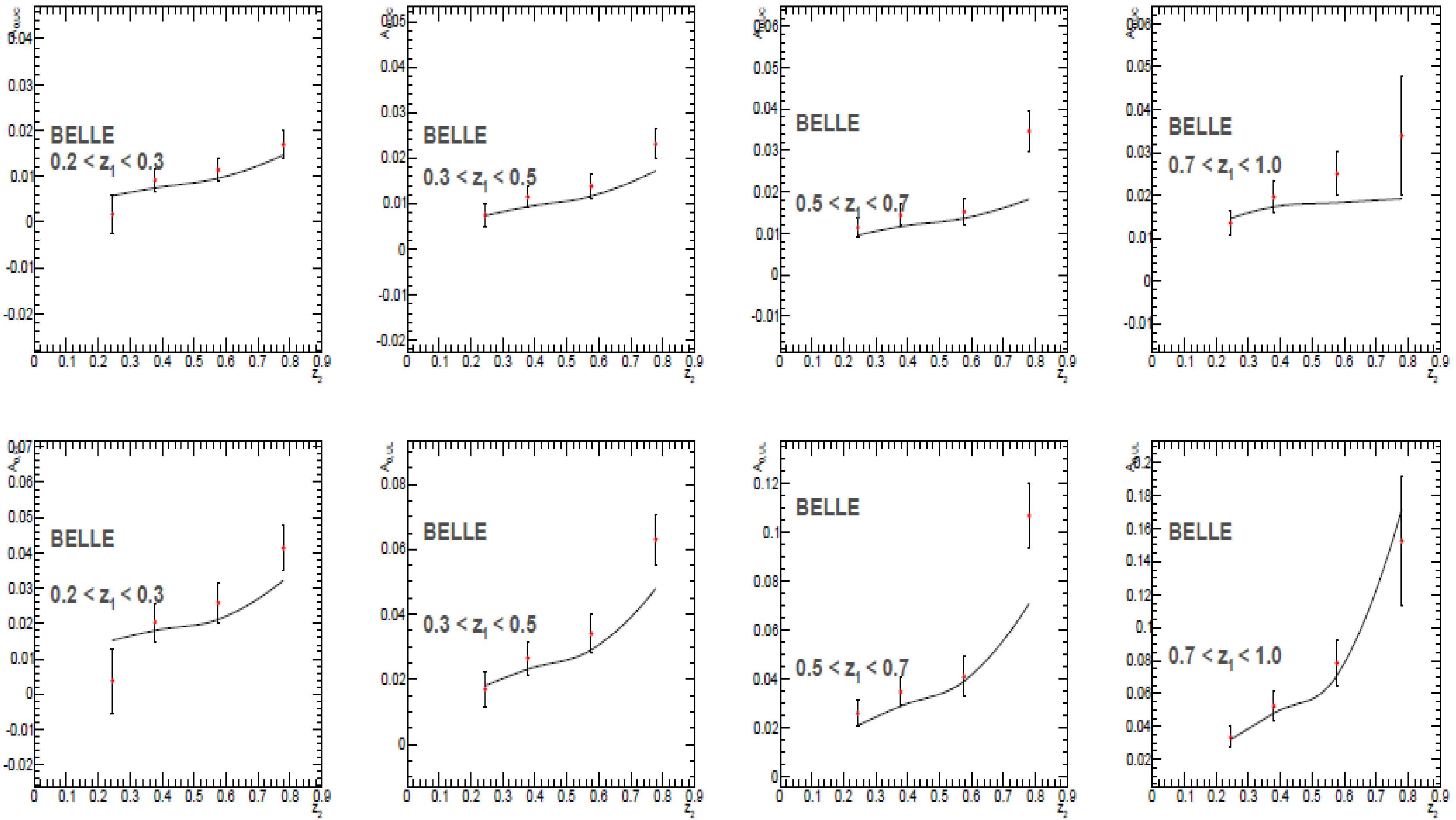


$xH_{u/\pi+}(x, \mu^2)$
0.04
0.03
0.02
0.01
0.00
0.0
0.2
0.4
0.6
0.8
1.0
x
$\mu^2 = 2.4\ GeV^2$
$\mu^2 = 100\ GeV^2$

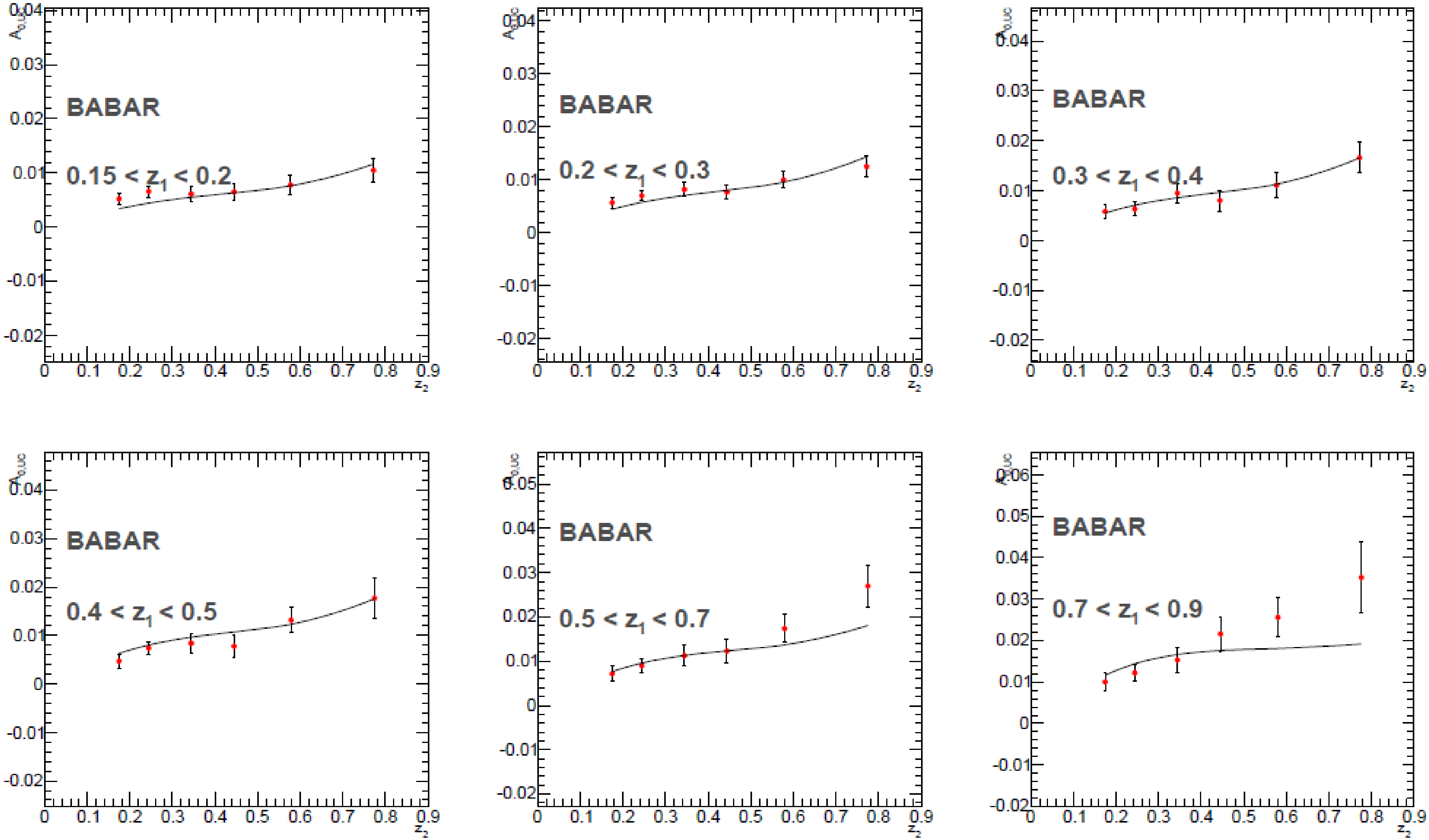
The values of parameters for our best fitting

NO.	NAME	VALUE	ERROR	SIZE	DERIVATIVE
1	gc	2.18260e-02	1.12393e-03	1.48687e-04	-2.03740e+00
2	Nu	7.81212e-02	5.78840e-03	2.41211e-06	-1.52947e+02
3	Nd	-2.02106e-01	1.50742e-02	8.36614e-06	7.19575e+01
4	au	2.95074e-05	3.51199e-02	1.15434e-03	4.96392e-01
5	ad	1.53385e+00	9.13030e-02	4.37936e-05	1.11582e+01
6	bu	2.76803e-01	9.60465e-02	1.12180e-04	7.20488e+00
7	bd	7.72477e-04	8.23850e-02	8.53659e-04	1.10266e+00
8	Nu_t	7.24232e-01	2.73277e-01	1.92943e-04	1.87945e+00
9	Nd_t	-2.99997e+00	3.83250e+00	1.05435e-02	6.75250e-03
10	au_t	9.43028e-01	1.60234e-01	1.12450e-04	-4.53609e+00
11	ad_t	9.87548e-01	6.32205e-02	1.34950e-04	-3.67556e+00
12	bu_t	2.32996e-04	7.44004e-01	4.49400e-03	6.33734e-02

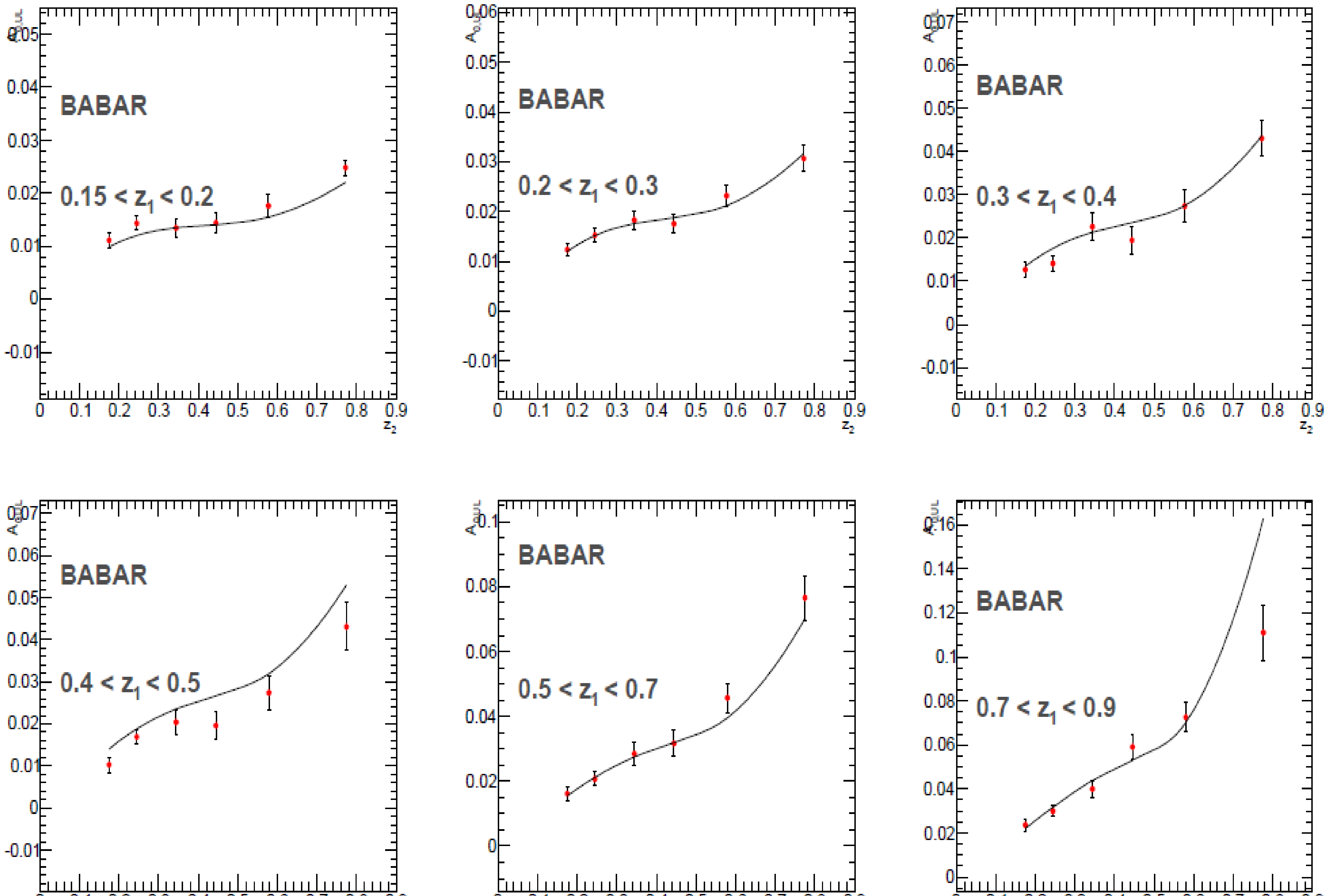
Where $\mathcal{X}^2 \approx 227$ vs 252 data points (preliminary)



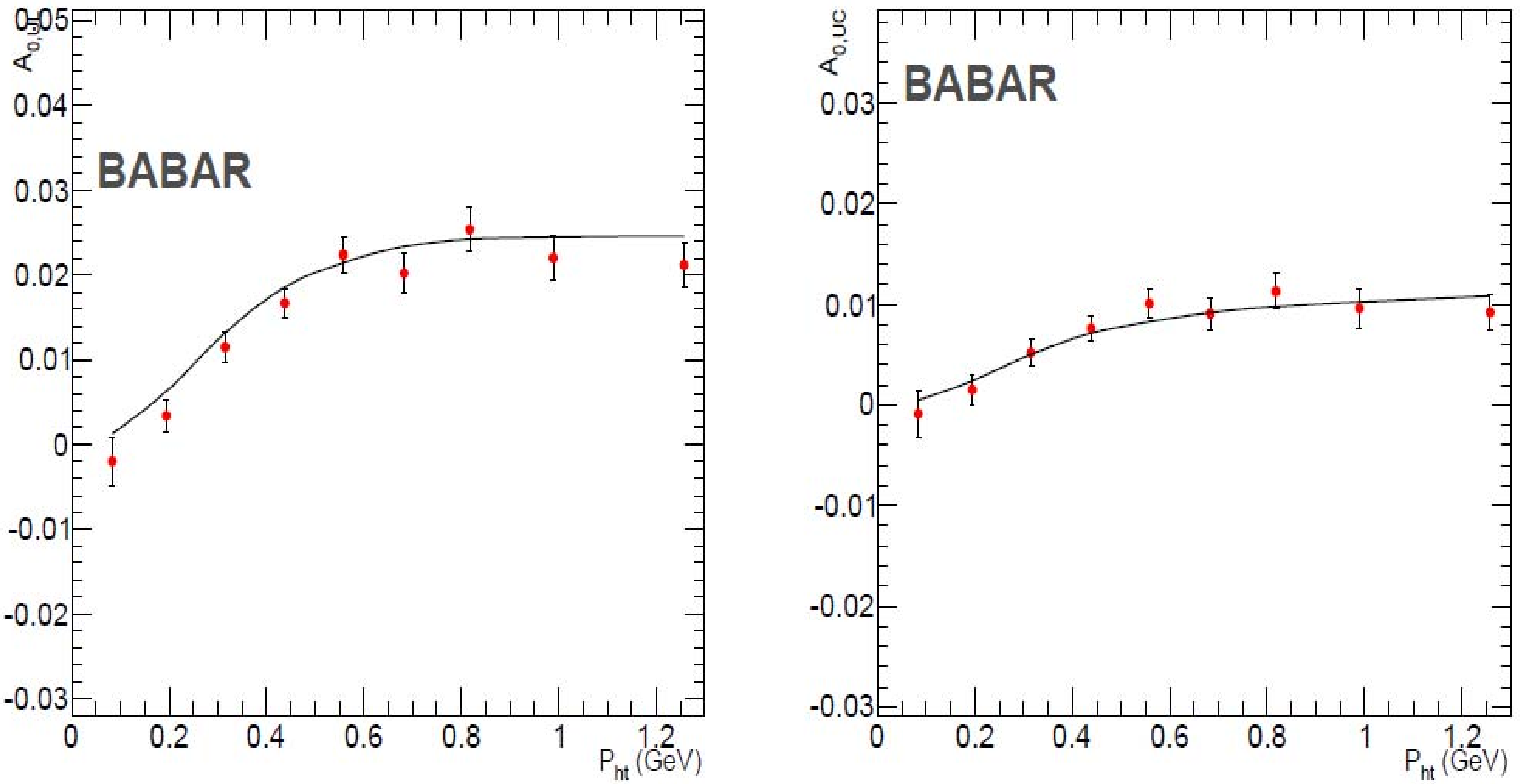
Our fitting result for BELLE.



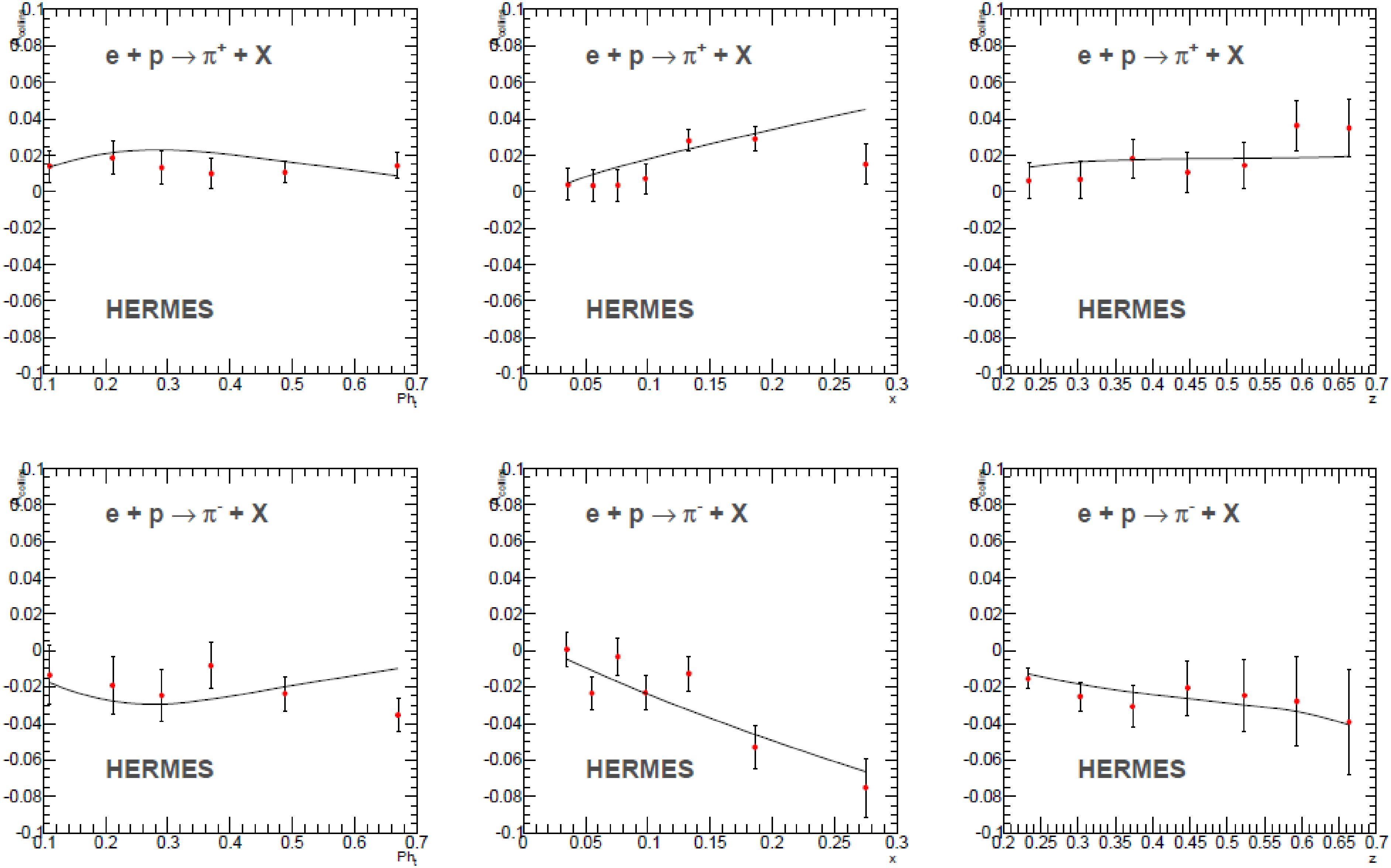
Our fitting result for BABAR.



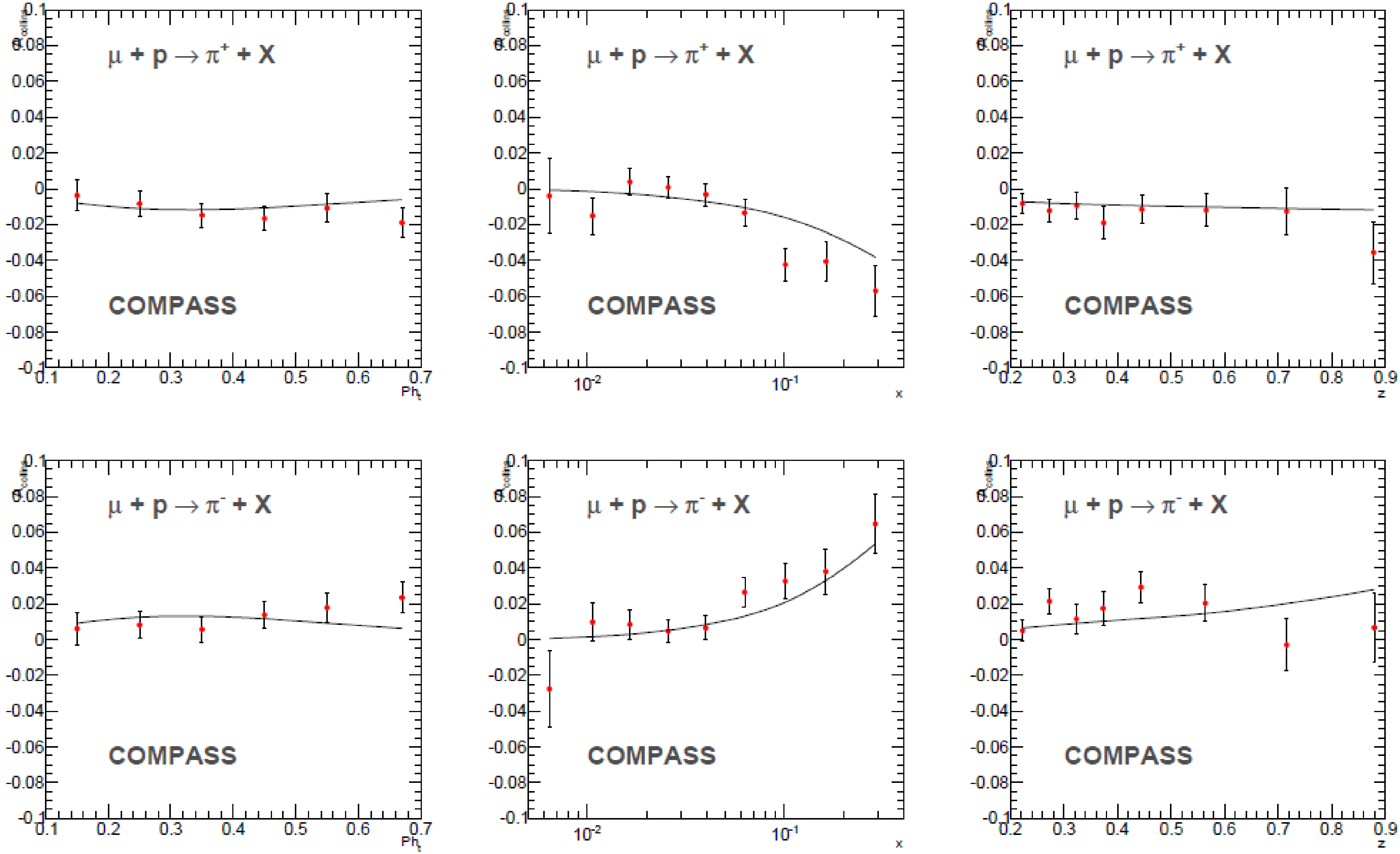
A_0^{UL}
BABAR
$0.15 < z_1 < 0.2$
BABAR
$0.2 < z_1 < 0.3$
BABAR
$0.3 < z_1 < 0.4$
BABAR
$0.4 < z_1 < 0.5$
BABAR
$0.5 < z_1 < 0.7$
BABAR
$0.7 < z_1 < 0.9$
z_2

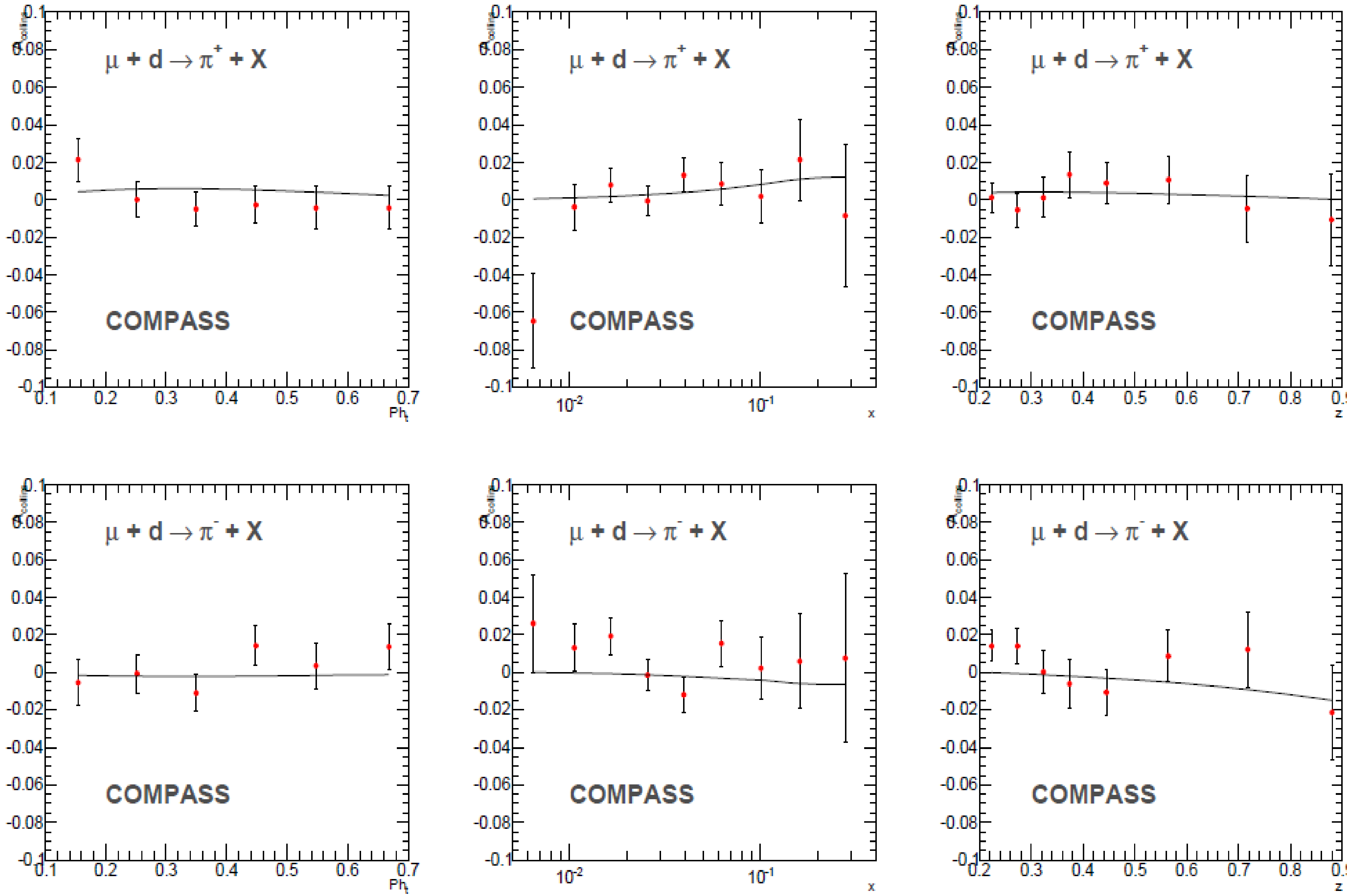


Pt distribution from BABAR



e + p → π+ + X
HERMES
Ph_t
x
z
e + p → π- + X
HERMES
Ph_t
x
z


μ + p → π⁺ + X
COMPASS
Ph_t
μ + p → π⁺ + X
COMPASS
x
μ + p → π⁺ + X
COMPASS
z
μ + p → π⁻ + X
COMPASS
Ph_t
μ + p → π⁻ + X
COMPASS
x
μ + p → π⁻ + X
COMPASS
z


μ + d → π⁺ + X
COMPASS
μ + d → π⁺ + X
COMPASS
μ + d → π⁺ + X
COMPASS
μ + d → π⁻ + X
COMPASS
μ + d → π⁻ + X
COMPASS
μ + d → π⁻ + X
COMPASS
Ph_t
x
z
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
0.1 0.2 0.3 0.4 0.5 0.6 0.7
10^-2 10^-1
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Summary

- TMD evolution is studied for the Collins effects in e^+e^- annihilation and SIDIS

- Collins functions fitted from the existing data at BELLE , BABAR, COMPASS and HERMES with CSS resummation scheme

- It is still an open question to get correct nonperturbative Sudakov factor form

Thank you very much!

- Fit the Collins function from BELLE and BARBAR data
- Energy dependence follows the CSS resummation formulism

$$D_q(z_1, C_0/b)$$

$$\widetilde{Z}_{uu}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}(Q,b)} \Sigma_{i,j} \hat{C}_{qi}^{(e^+e^-)} \otimes D_{i/A}(z_1) \hat{C}_{qj}^{(e^+e^-)} \otimes D_{j/B}(z_2') ,$$

$$\widetilde{Z}^{\alpha\beta}_{\rm collins}(Q;b) = \left(\frac{-ib_\perp^\alpha}{2}\right)\left(\frac{-ib_\perp^\beta}{2}\right) e^{-\mathcal{S}_{pert}(Q^2,b_*)-S^T_{NP}(Q,b)}$$

$$\times \Sigma_{i,j} \Delta \hat{C}_{qi}^{\rm collins(e^+e^-)} \otimes D_{i/A}^{(3)} \hat{C}_{qj}^{\rm collins(e^+e^-)} \otimes D_{j/B}^{(3)} ,$$

$$\hat{H}_{1q}(z_{h1}, C_0/b)$$

- then, we can predict the Collins effect at BEPC

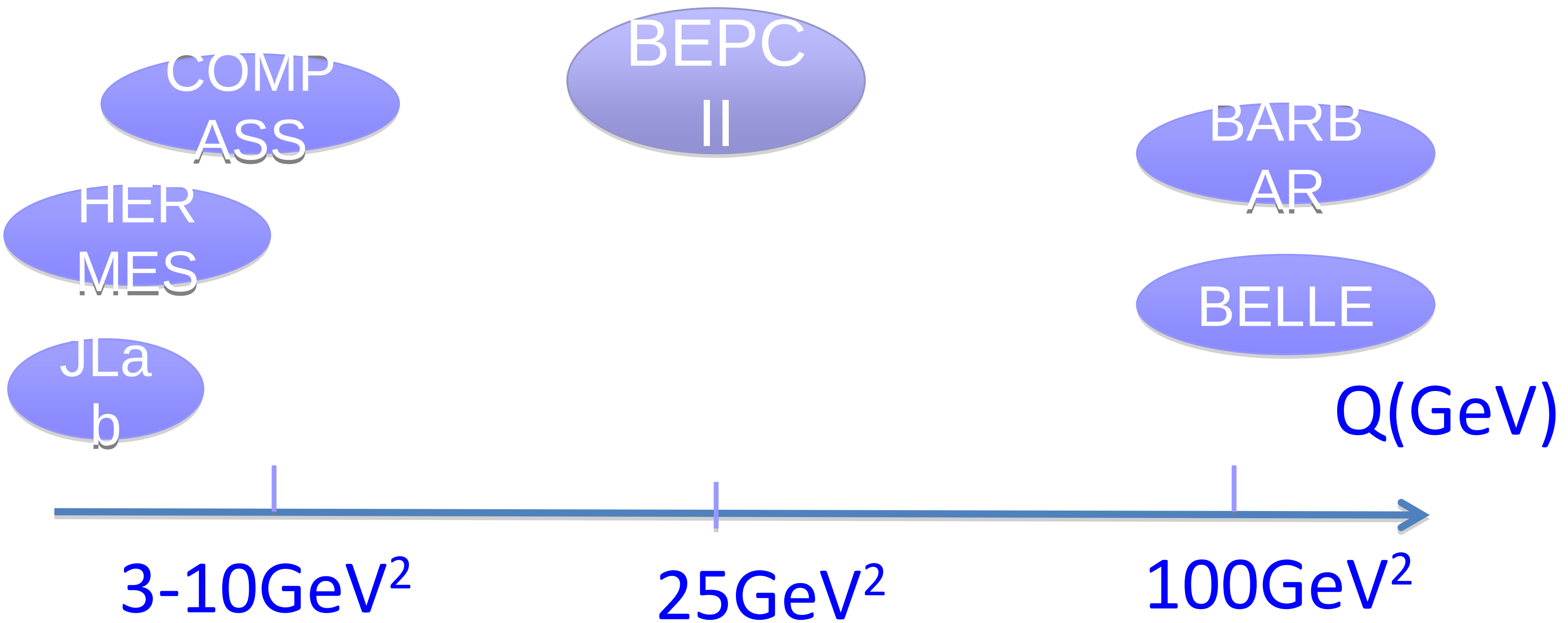
- Gaussian assumptions are usually made for $g_1(b)$ and $g_2(b)$ (BLNY 2002):

$$S_{NP}^{DIS} = g_q b^2 \ln(Q/Q_0) + g_0 b^2 + g_h b^2/z_h^2 \ ,$$
$$S_{NP}^{DY} = g_q b^2 \ln(Q/Q_0) + 2g_0 b^2 \ ,$$
$$S_{NP}^{e^+e^-} = g_q b^2 \ln(Q/Q_0) + g_h b^2 \left(1/z_{h1}^2 + 1/z_{h2}^2\right)$$

- However, these assumptions do not work for SIDIS and Drell-Yan simultaneously in the range of Q^2~(3—100)GeV^2
  - in particular, for Q_0^2=2.4GeV^2, fitting Drell-Yan data leads to a negative g_0

Sun,Yuan,1308.5003

Q^2 for all these experiments



Importance

- Reliable determination of the Collins functions.

- Study the QCD evolution effects
  - By theory
  - By experiments

Substituting the above result into the evolution equation, and taking into account the running effects in K(b, μ)

CSS ⇒ $\text{Exp}[\int_{c_0/b}^{Q}\frac{d\bar\mu}{\bar\mu}\left[A\ln\frac{Q^2}{\bar\mu^2}+B\right]]$

$$\tilde{Z}_{uu}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}^{e^+e^-}(Q,b)}\Sigma_q D_q(z_1,C_0/b_*)D_{\bar q}(z_2,C_0/b_*) \;,$$

$C_0 = 2\,e^{-\gamma} \approx 1$

$$\tilde{Z}^{\alpha\beta}_{\rm collins}(Q;b) = \left(\frac{-ib_\perp^\alpha}{2}\right)\left(\frac{-ib_\perp^\beta}{2}\right)e^{-\mathcal{S}_{pert}(Q^2,b_*)-S^{e^+e^-}_{\rm collins}(Q,b)}\Sigma_q\hat{H}_{1q}(z_{h1},C_0/b_*)\hat{H}_{1\bar q}(z_{h2},C_0/b_*)$$

$$e^{-\mathcal{S}_{pert}(Q^2,b_*)-S^{e^+e^-}_{\rm collins}(Q,b)}$$

$S^{e^+e^-}_{\rm collins}(Q,b)$ → NP part

perturbative part:

$$\mathcal{S}_{pert}(Q,b_*) = \int_{c_0/b_*}^{Q}\frac{d\bar\mu}{\bar\mu}\left[A\ln\frac{Q^2}{\bar\mu^2}+B\right]$$

S_{pert} is universal

where $A=C_F\times\alpha_s(\bar\mu)/\pi$, $B=3/2\times\alpha_s(\bar\mu)/\pi$

b_* prescription (CSS, 85) in S_{pert}

$$b \Rightarrow b_* = b/\sqrt{1+b^2/b_{max}^2}\;,\quad b_{max} < 1/\Lambda_{QCD}$$

Globe Fitting of Collins functions

$$\hat{H}_{\pi^+/u}(z) = N_u z^{a_u}(1-z)^{b_u} D_{\pi^+/u}(z)$$
$$\hat{H}_{\pi^+/d}(z) = N_d z^{a_d}(1-z)^{b_d} D_{\pi^+/d}(z)$$

nonperturbative factor:

$$S^{e^+e^-}_{\text{collins}}(Q,b) = g_q \ln(Q/Q_0)\ln(b/b^*) + (g_h - g_c)b^2\left(1/z_{h1}^2 + 1/z_{h2}^2\right)$$

Minuit package is uesd

$\mathcal{X}^2 \approx 120$ vs 122 data points

NO.	NAME	VALUE	ERROR
1	g_c	1.97919e-02	1.79702e-03
2	Nu	1.00000e+01	1.68748e+01
3	Nd	-1.53931e+00	1.22775e-01
4	a_u	7.96969e+00	4.45691e-01
5	a_d	1.43211e+00	7.83941e-02
6	b_u	1.18983e+00	6.43297e-02
7	b_d	7.20196e-09	1.94833e-01

Energy Evolution (Ji-Ma-Yuan TMD factorization)

At the small transverse momentum, the TMD factorization

$$\widetilde{Z}_{uu} = D(z_1, b_\perp, \zeta_1; \mu) D(z_2, b_\perp, \zeta_2; \mu) H_{uu}^{e^+e^-}(Q;\mu) S(b_\perp, \rho; \mu) \,,$$

$$\widetilde{Z}_{\rm collins}^{\alpha\beta} = \widetilde{H}_1^{\perp\alpha}(z_1, b_\perp, \zeta_1; \mu) \widetilde{H}_1^{\perp\beta}(z_2, b_\perp, \zeta_2; \mu) H_{\rm collins}^{e^+e^-}(Q;\mu) S(b_\perp, \rho; \mu)$$

Q_t ↔ $b_\perp$

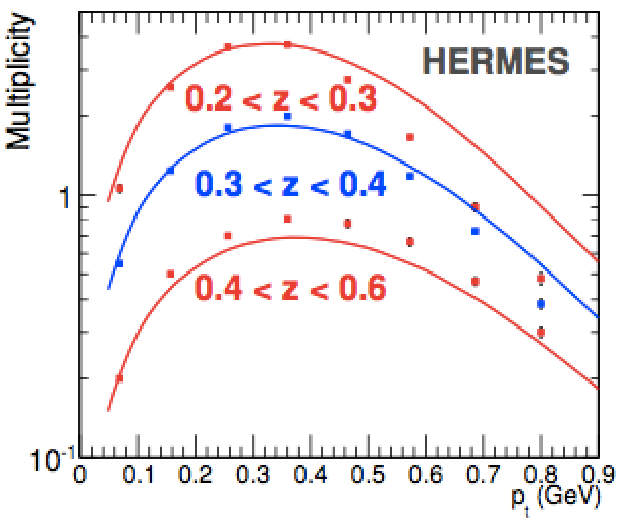
factorization scale → μ

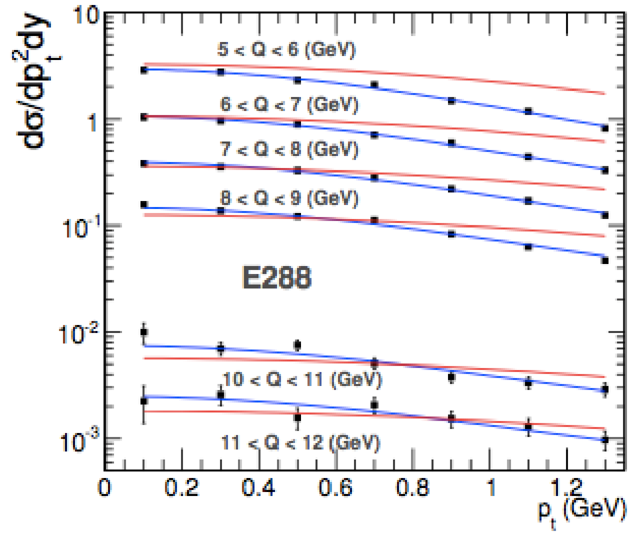
Z_{uu} and $Z_{collins}$ satisfy CSS evolution equation

$$\frac{\partial}{\partial \ln Q^2} \widetilde{Z}_{uu}(Q;b) = \left(K(b,\mu) + G(Q,\mu)\right) \widetilde{Z}_{uu}(Q;b)$$

At one-loop order

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu) = -\frac{\alpha_s C_F}{\pi}\left(\ln\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$





CT10 and DSS are used here, so are our other fittings.

Sun,Yuan, 1304.5037, 1308.5003

- Sun-Yuan (1308.5003) has shown that direct integration of the evolution kernel from low to high Q can describe both Drell-Yan and SIDIS data
  - This suggests that Log(b) maybe a good choice for $g_2(b)$.

- Besides Collins effect, higher order QCD correction can also contribute $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$ terms

$$
\begin{aligned}
R_{12} &= \frac{N(\phi_1+\phi_2)}{\langle N_{12}\rangle} \\
&\propto \Bigg[(1+\cos^2\theta)\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2) + \sin^2\theta\cos(\phi_1+\phi_2)\Big[\sum_q e_q^2 f(H_1^{\perp}(z_1)\overline{H}_1^{\perp}(z_2)) \\
&+ \boxed{C\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)}\Big]\Bigg] \cdot \Bigg[(1+\cos^2\theta)\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)\Bigg]^{-1} \\
&= 1 + \frac{\sin^2}{1+\cos^2\theta}\cos(\phi_1+\phi_2)\left[\frac{\sum_q e_q^2 f(H_1^{\perp}(z_1)\overline{H}_1^{\perp}(z_2))}{\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)} + \boxed{C}\right] .
\end{aligned}
$$

Valence quarks go to pion

$$N^U(\phi) = \frac{d\sigma(e^+e^- \to \pi^\pm\pi^\mp X)}{d\Omega dz_1 dz_2} \propto \frac{5}{9} D^{\text{fav}}(z_1)\overline{D}^{\text{fav}}(z_2) + \frac{7}{9} D^{\text{dis}}(z_1)\overline{D}^{\text{dis}}(z_2)$$

Sea quarks go to pion

$$N^L(\phi) = \frac{d\sigma(e^+e^- \to \pi^\pm\pi^\pm X)}{d\Omega dz_1 dz_2} \propto \frac{5}{9} D^{\text{fav}}(z_1)\overline{D}^{\text{dis}}(z_2) + \frac{5}{9} D^{\text{dis}}(z_1)\overline{D}^{\text{fav}}(z_2) + \frac{2}{9} D^{\text{dis}}(z_1)\overline{D}^{\text{dis}}(z_2)$$

$$N^C(\phi) = \frac{d\sigma(e^+e^- \to \pi\pi X)}{d\Omega dz_1 dz_2} = N^U(\phi) + N^L(\phi) \propto \frac{5}{9}[D^{\text{fav}}(z_1) + D^{\text{dis}}(z_1)][\overline{D}^{\text{fav}}(z_2) + \overline{D}^{\text{dis}}(z_2)] + \frac{4}{9} D^{\text{dis}}(z_1)\overline{D}^{\text{dis}}(z_2)$$

By a double ratio: $\frac{R^U_\alpha}{R^L_\alpha} := \frac{N^U_\alpha(\beta_\alpha)/\langle N^U_\alpha \rangle}{N^L_\alpha(\beta_\alpha)/\langle N^L_\alpha \rangle}, (\alpha = 0, 12)$

$$\frac{R^U_{12}}{R^L_{12}} = 1 + \cos(\phi_1 + \phi_2)\frac{\sin^2\theta}{1 + cos^2\theta}\left\{ \frac{f\left(H_1^{\perp,fav}\overline{H}_2^{\perp,fav} + H_1^{\perp,dis}\overline{H}_2^{\perp,dis}\right)}{\left(D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}\right)} - \frac{f\left(H_1^{\perp,fav}\overline{H}_2^{\perp,dis}\right)}{\left(D_1^{fav}\overline{D}_2^{dis}\right)} \right\}$$

$\Downarrow$

A^{UL}

Similarly, we can also get A^{UC} from the ratio R^U/R^C

$$A^{UL} \sim \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \frac{\pi\langle k_{tC}^2\rangle}{4M^2} \left[\frac{H_1^{fav}\overline{H}_2^{fav} + H_1^{dis}\overline{H}_2^{dis}}{D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}} - \frac{H_1^{fav}\overline{H}_2^{dis} + H_1^{dis}\overline{H}_2^{fav}}{D_1^{fav}\overline{D}_2^{dis} + D_1^{dis}\overline{D}_2^{fav}} \right]$$

$$A^{UC} \sim \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \frac{\pi\langle k_{tC}^2\rangle}{4M^2} \left[\frac{H_1^{fav}\overline{H}_2^{fav} + H_1^{dis}\overline{H}_2^{dis}}{D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}} - \frac{\left(H_1^{fav} + H_1^{dis}\right)\left(\overline{H}_2^{fav} + \overline{H}_2^{dis}\right)}{\left(D_1^{fav} + D_1^{dis}\right)\left(\overline{D}_2^{fav} + \overline{D}_2^{dis}\right)} \right]$$