QCD Evolution 2014 Workshop at Santa Fe, NM (May 12 – 16, 2014)

QCD Factorization and PDFs from Lattice QCD Calculation

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- ✧ *Observation + Motivation*
- ✧ *Our proposal*
- ✧ *Case study*
- ✧ *Summary and outlook*

Based on work done with

Tomomi Ishikawa, Yan-Qing Ma, Shinsuke Yoshida, …

arXiv:1404.6860, …

Global QCD analyses – a successful story

- World data with "Q" > 2 GeV

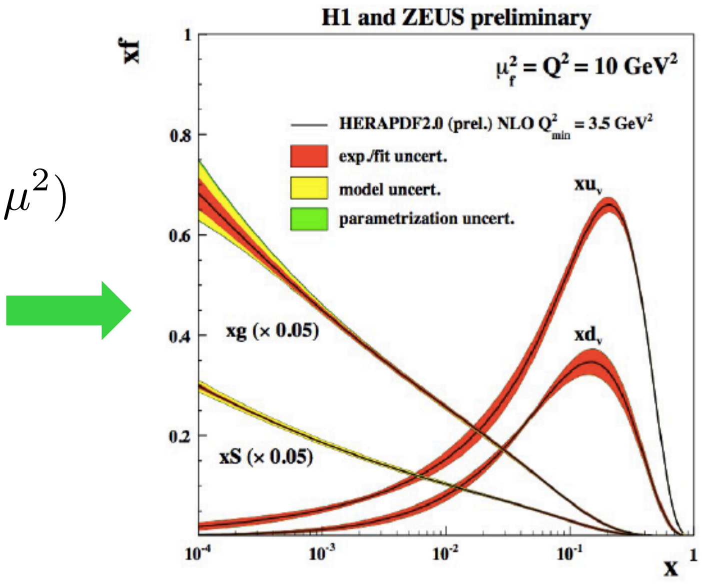
  + Factorization:

DIS: $$F_2(x_B,Q^2) = \Sigma_f C_f(x_B/x,\mu^2/Q^2) \otimes f(x,\mu^2)$$

H-H: $$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

+ DGLAP Evolution:

$$\frac{\partial f(x,\mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x',\mu^2)$$



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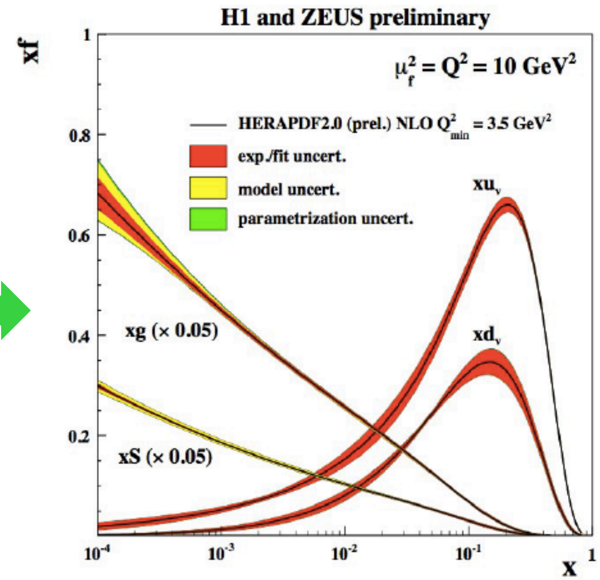
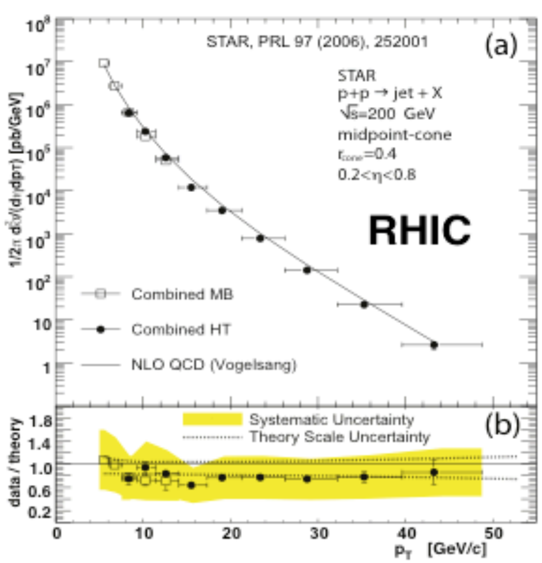
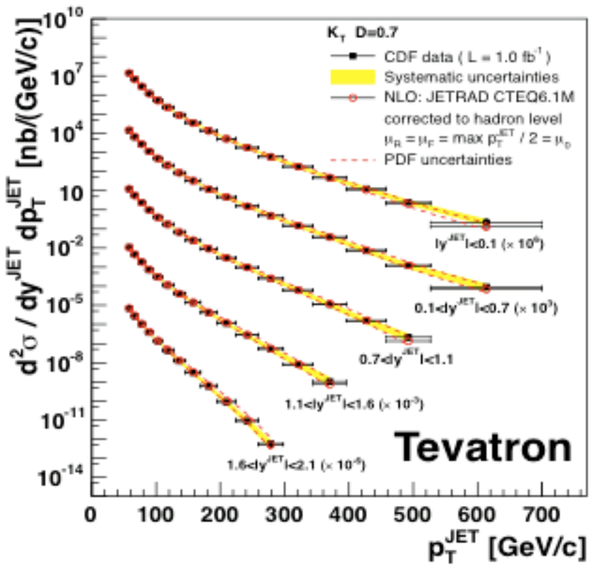
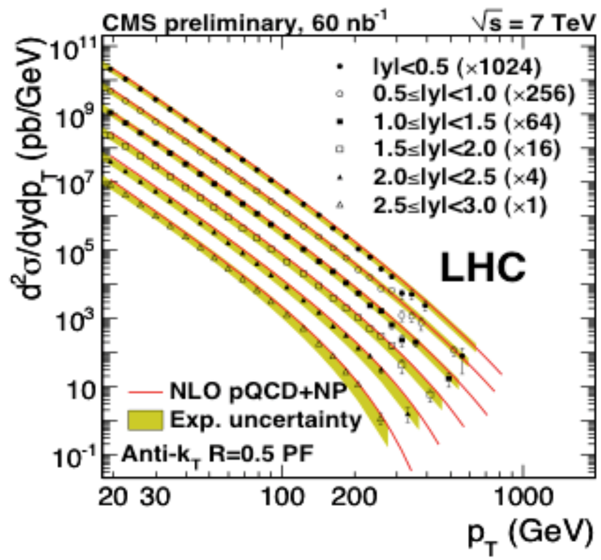
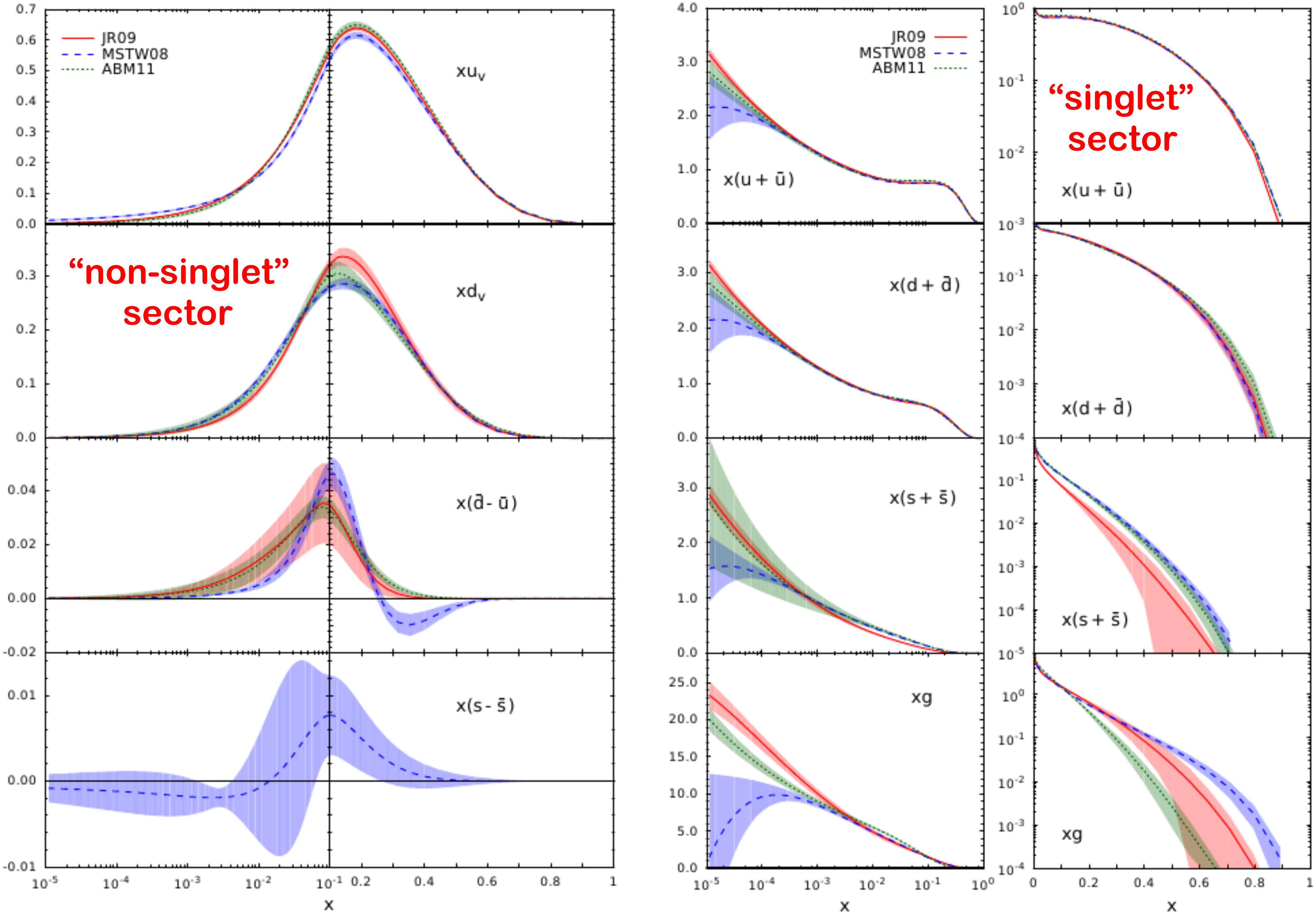








Uncertainties of PDFs



PDFs at large x

❑ **Testing ground for hadron structure at x → 1:**

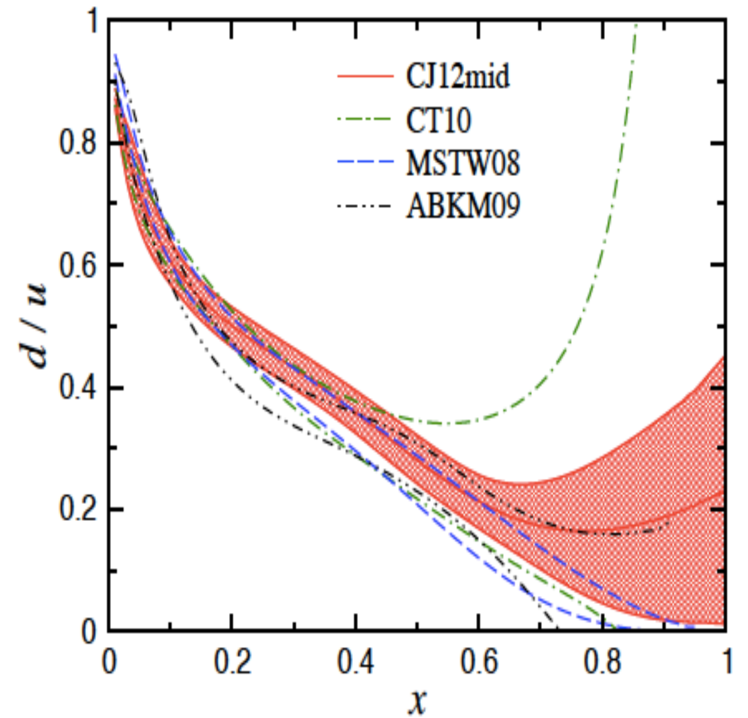
✧ $d/u \to 1/2$ **SU(6) Spin-flavor symmetry**

✧ $d/u \to 0$ **Scalar diquark dominance**

✧ $d/u \to 1/5$ **pQCD power counting**

✧ $d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2}$ **Local quark-hadron duality**

$\approx \; 0.42$



PDFs at large x

- Testing ground for hadron structure at x → 1:

$\diamond\ d/u \to 1/2$	**SU(6) Spin-flavor symmetry**	$\diamond\ \Delta u/u \to 2/3$ $\Delta d/d \to -1/3$
$\diamond\ d/u \to 0$	**Scalar diquark dominance**	$\diamond\ \Delta u/u \to 1$ $\Delta d/d \to -1/3$
$\diamond\ d/u \to 1/5$	**pQCD power counting**	$\diamond\ \Delta u/u \to 1$ $\Delta d/d \to 1$
$\diamond\ d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$	**Local quark-hadron duality**	$\diamond\ \Delta u/u \to 1$ $\Delta d/d \to 1$

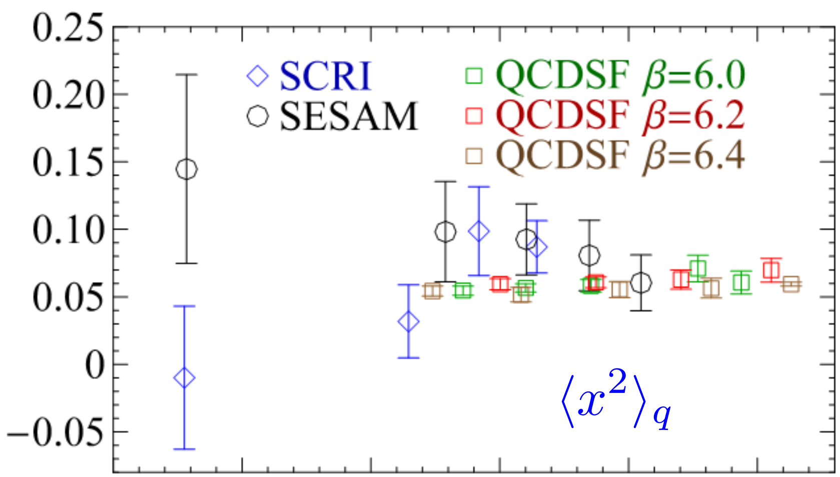
Can lattice QCD help?

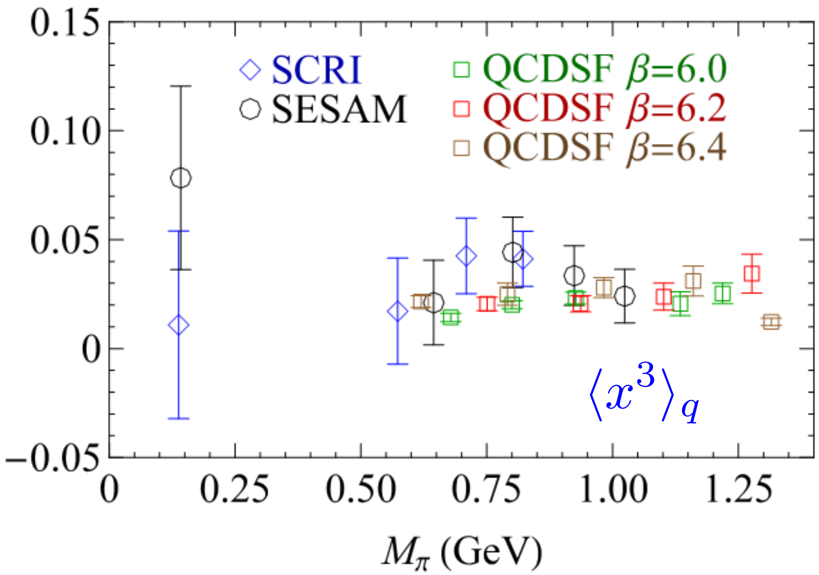
PDFs from lattice

- Moments of PDFs – matrix elements of local operators

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2)$$

- Works, but, hard and limited moments:





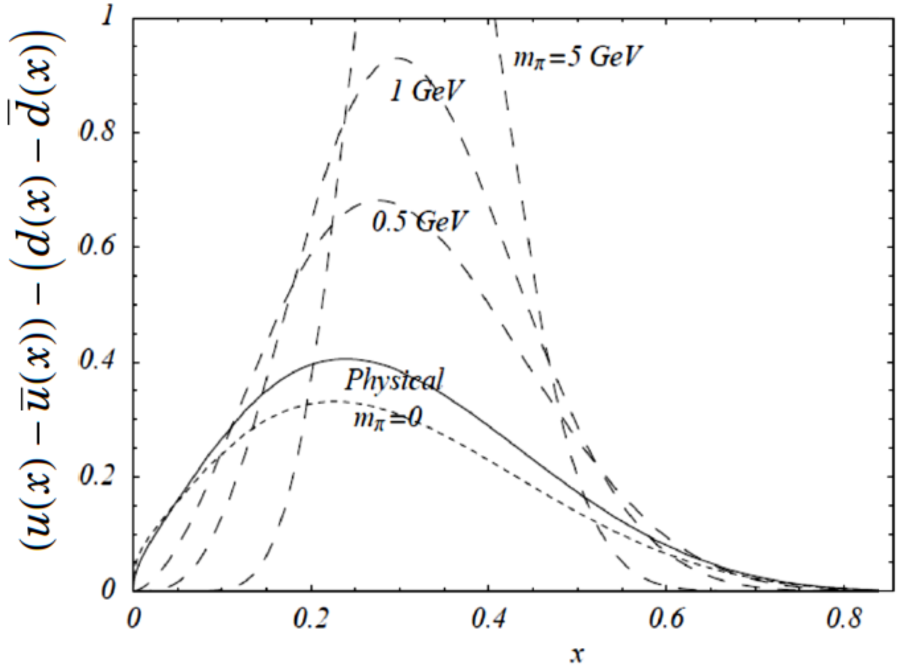
Dolgov et al., hep-lat/0201021

Gockeler et al., hep-ph/0410187

PDFs from lattice

- How to get x-dependent PDFs with a limited moments?
  - Assume a smooth functional form with some parameters
  - Fix the parameters with the lattice calculated moments

$$xq(x) = a\,x^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma\,x)$$



W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

Cannot distinguish valence quark contribution from sea quarks

Ji's new idea

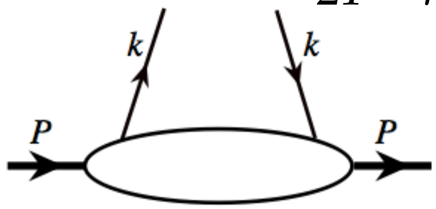
Ji, arXiv:1305. 1539

- "Quasi" quark distribution (spin-averaged):

$$\tilde{f}_q(\tilde{x},\tilde{\mu}^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- Cut-vertex notation:

$$\longleftarrow \frac{\gamma\cdot n_z}{2P\cdot n_z}\,\delta\left(x-\frac{k\cdot n_z}{P\cdot n_z}\right)\frac{d^4k}{(2\pi)^4}$$



- Features:

  - Quark fields separated along the z-direction – not boost invariant!
  - Perturbatively UV power divergent: quark $\propto \mu/P_z$ gluon $\propto (\mu/P_z)^2$
  - This distribution can be calculated using standard lattice method
  - quasi-PDFs ➔ *normal PDFs* *when* P_z ➔ ∞
  - Matching: $$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

“Quasi-PDFs” have no parton interpretation

❑ Normal PDFs conserve parton momentum:

$$M = \sum_q \left[\int_0^1 dx\, x f_q(x) + \int_0^1 dx\, x f_{\bar{q}}(x) \right] + \int_0^1 dx\, x f_g(x)$$
$$= \sum_q \int_{-\infty}^{\infty} dx\, x f_q(x) + \frac{1}{2} \int_{-\infty}^{\infty} dx\, x f_g(x)$$
$$= \frac{1}{2(P^+)^2} \langle P | T^{++}(0) | P \rangle = \text{constant}$$

$T^{\mu\nu}$

Energy-momentum tensor

❑ “Quasi-PDFs” do not conserve “parton” momentum:

$$\widetilde{\mathcal{M}} = \sum_q \left[\int_0^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \int_0^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_{\bar{q}}(\tilde{x}) \right] + \int_0^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x})$$
$$= \sum_q \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \frac{1}{2} \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x})$$
$$= \frac{1}{2(P_z)^2} \langle P | \left[T^{zz}(0) - g^{zz}(\ldots) \right] | P \rangle \neq \text{constant}$$

Note: “Quasi-PDFs” are not boost invariant

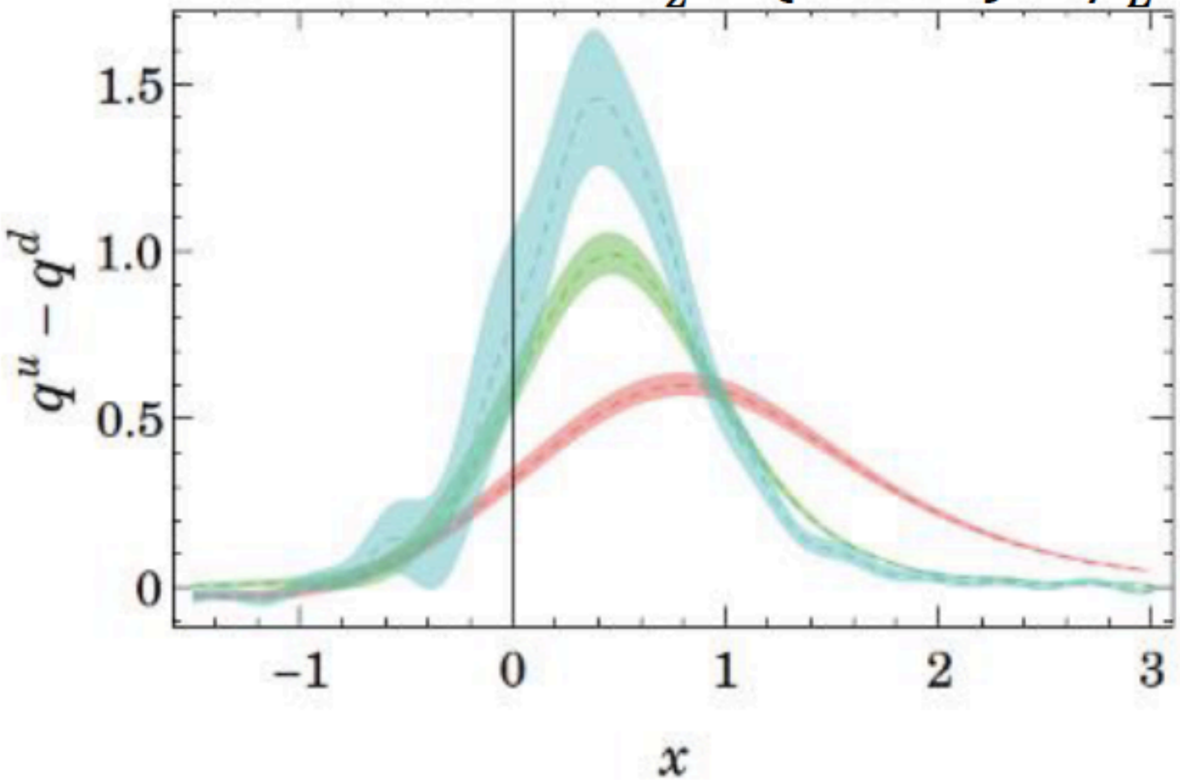
The first try

Lin *et al.*, arXiv:1402.1462

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\, \gamma_z \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

Work out leading-P_z corrections

Observation

- Lattice QCD calculates "single" hadron matrix elements:

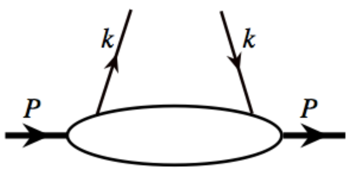
$$\langle 0|\, \mathcal{O}(\overline{\psi},\psi,A)\, |0\rangle = \frac{1}{Z}\int \mathcal{D}A\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS(\overline{\psi},\psi,A)}\mathcal{O}(\overline{\psi},\psi,A)$$

$$\sum_{P'} |P'\rangle\langle P'| \quad \sum_{P} |P\rangle\langle P|$$

$$\longrightarrow \quad \langle P_z|\mathcal{O}(\overline{\psi},\psi,A)|P_z\rangle$$

With an Euclidean time

- Collinear divergence (CO) from the region when $k_T \rightarrow 0$:



$$\propto \;\; \text{F.T. of } \langle P_z|\mathcal{O}(\overline{\psi},\psi)|P_z\rangle$$

Same CO divergence regardless Minkowski or Euclidean time

- PDFs should cover all leading power CO divergences of

$$\text{F.T. of } \langle P_z|\mathcal{O}(\overline{\psi},\psi,A)|P_z\rangle$$

"Single" hadron matrix elements with a large momentum scale

Our proposal - a "slightly" modified idea

Ma and Qiu, arXiv:1404.6860

❑ **Not try to go the light-cone – Less ambiguous:**

– Lattice "cross sections" @ Finite P_z + QCD factorization

✧ Identify *single-hadron* "cross sections", *calculable* in Lattice QCD, and *factorizable* into a convolution with normal PDFs

$$\widetilde{\sigma}^i(\tilde{x}, \tilde{Q}^2, \{\tilde{\nu}\}) = \Sigma_f\, C_f^i(\tilde{x}/x, \mu^2/\tilde{Q}^2, \{\tilde{\nu}\}) \otimes f(x, \mu^2) + \mathcal{O}\left[\frac{1}{\tilde{Q}^\alpha}\right] \qquad (1)$$

e.g. $\widetilde{\sigma}^i(\tilde{x}, \tilde{\mu}^2, P_z) \;\Rightarrow\; \widetilde{f}^i(\tilde{x} = k_z/P_z, \tilde{\mu}^2, P_z)$ Ji's quasi-PDFs

$\widetilde{\sigma}^i(\tilde{x}, \tilde{\mu}^2, P_z) \;\propto\; \langle P_z | \mathcal{O}(\psi, A) | P_z \rangle$

Need a large scale if $\mathcal{O}(\psi, A)$ is made of conserved currents

KEY: all order factorization into the Normal PDFs with $\tilde{\mu}^2 \sim (\tilde{x} P_z)^2 \gg \Lambda^2_{\rm QCD}$

✧ Calculate the Lattice "cross sections" in Lattice QCD – the LHS of (1)

"Measure" high energy scattering cross sections on Lattice

✧ Global analysis of Lattice "cross sections" – to extract the normal PDFs

Note: $\tilde{x}, \tilde{\mu}, \{\tilde{\nu}\}$ are finite parameters: "rapidity", "hard scale", …

Lattice "cross section"

- **Definition:** $\widetilde{\sigma} \;\propto\; \text{F.T. of } \langle P_z | \mathcal{O}(\overline{\psi}, \psi, A) | P_z \rangle$
  - ✧ Calculable in lattice QCD with an Euclidean time, "E"
  - ✧ Its continuum limit is UV and IR safe perturbatively
  - ✧ All CO divergences can be factorized into the normal PDFs with perturbatively calculable hard coefficient functions

$$\tilde{\sigma}_{\rm E}^{\rm Lat}(\tilde{x}, 1/a, P_z)| \xleftrightarrow{\;Z\;} \tilde{\sigma}_{\rm E}(\tilde{x}, \tilde{\mu}^2, P_z)$$

$$\Updownarrow$$

$$\tilde{\sigma}_{\rm M}(\tilde{x}, \tilde{\mu}^2, P_z) \xleftrightarrow{\;\mathcal{C}\;} f_{i/h}(x, \mu^2)$$

"Collision energy" $P_z \sim$ "$\sqrt{s}$" *"rapidity"* $\tilde{x} \sim$ "y"

"Hard momentum transfer" $1/a \sim \tilde{\mu} \sim$ "Q"

- **Factorization:**

$$\tilde{\sigma}_{\rm E}^{\rm Lat}(\tilde{x}, \frac{1}{a}, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \widetilde{C}_i(\frac{\tilde{x}}{x}, \frac{1}{a}, \mu^2, P_z)$$

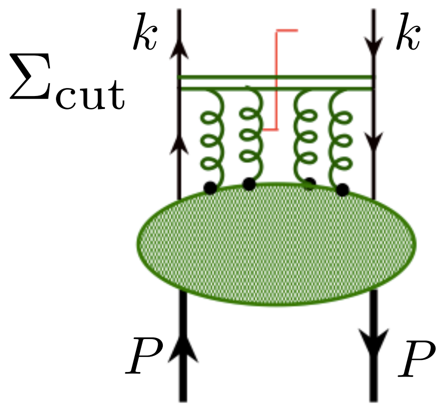
Case study – factorization of quasi-PDFs

Ma and Qiu, arXiv:1404.6860

- The "Quasi-quark" distribution, as an example:

$$\tilde{q}(\tilde{x},\tilde{\mu}^2,P_z) = \int \frac{dy_z}{4\pi} e^{i\tilde{x}P_z y_z}\langle P|\overline{\psi}(y_z)\gamma_z \exp\left\{-ig\int_0^{y_z} dy_z' A_z(y_z')\right\}\psi(0)|P\rangle$$

✧ Feynman diagram representation: $\Phi_{n_z}^{(f,a)}(\{\xi_z,0\}) = \Phi_{n_z}^{\dagger(f,a)}(\{\infty,\xi_z\})\,\Phi_{n_z}^{(f,a)}(\{\infty,0\})$



$$\longleftarrow \frac{\gamma_z}{2P_z}\,\delta\left(\tilde{x}-\frac{k_z}{P_z}\right)$$

$$P_z \sim \text{“}\sqrt{s}\text{”} \sim \tilde{\mu}/\tilde{x}$$

$$P^2 \sim 0$$

✧ Like PDFs, it is IR finite

✧ Unlike PDFs, it is linear UV divergent (quadratic UV divergent for gluon)

Potential trouble! - show power UV div. decouple from Log UV of PDFs

✧ CO divergence is the same as that of normal PDFs

Show to all orders in perturbation theory

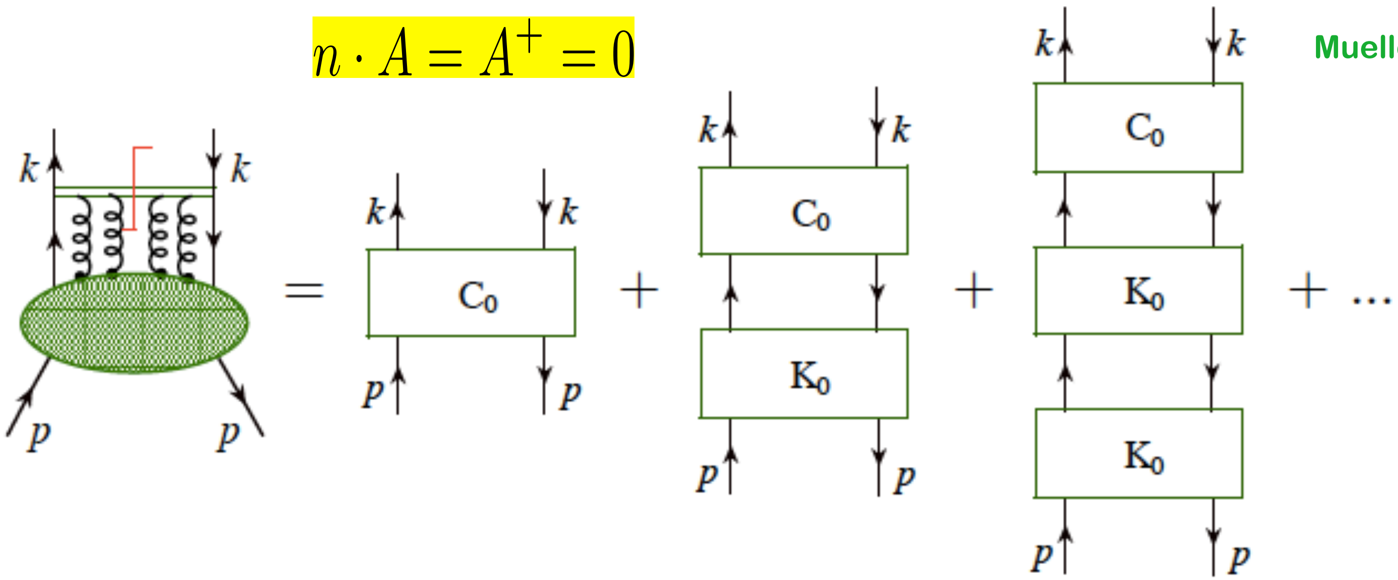
All order QCD factorization of CO divergence

Ma and Qiu, arXiv:1404.6860

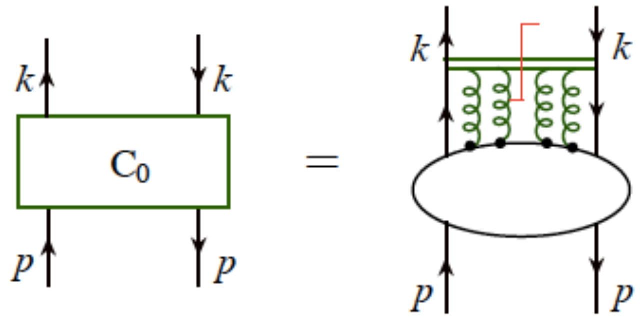
- Generalized ladder decomposition in a physical gauge

$n \cdot A = A^+ = 0$

Mueller, PRD 1974



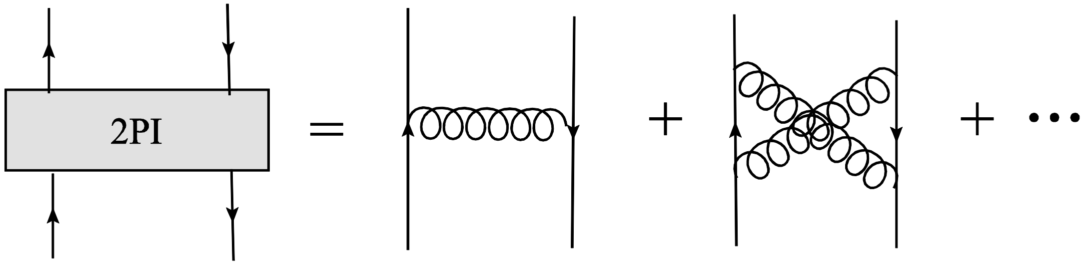
- $C_0, \ K_0$:2PI kernels
  - ✧ Only process dependence:
  - ✧ 2PI are finite in a physical gauge



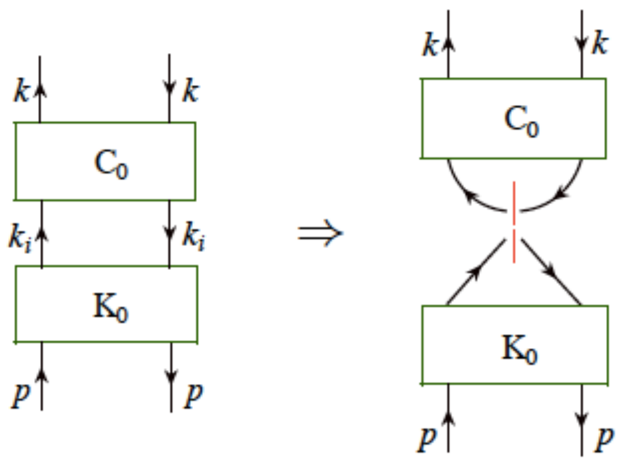
Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

All order QCD factorization of CO divergence

- 2PI kernels – Diagrams:



- Ordering in virtuality: $P^2 \ll k^2 \lesssim \tilde{\mu}^2$



$\leftarrow \frac{1}{2}\gamma \cdot p$

$\leftarrow \frac{\gamma \cdot n}{2p \cdot n}\,\delta\left(x_i - \frac{k_i \cdot n}{p \cdot n}\right)$ + power suppressed

Cut-vertex for normal quark distribution
Logarithmic UV and CO divergence

- Renormalized kernel - parton PDF:

$$K \equiv \int d^4k_i\,\delta\left(x_i - \frac{k^+}{p^+}\right) \mathrm{Tr}\left[\frac{\gamma \cdot n}{2p \cdot n}\, K_0\, \frac{\gamma \cdot p}{2}\right] + \mathrm{UVCT}$$

All order QCD factorization of CO divergence

- Projection operator for CO divergence:

$$\widehat{\mathcal{P}}\,K$$ Pick up the logarithmic CO divergence of *K*

- Factorization of CO divergence:

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K^i + \mathrm{UVCT}$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\widehat{\mathcal{P}})K\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\widehat{\mathcal{P}})K\right]^i\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$= \left[C_0 \frac{1}{1-(1-\widehat{\mathcal{P}})K}\right]_{\mathrm{ren}} \left[\frac{1}{1-\widehat{\mathcal{P}}K}\right]$$

Normal Quark distribution

All CO divergence of quasi-quark distribution

CO divergence free coefficient

$$\Rightarrow \quad \tilde{\sigma}_{\mathrm{M}}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_i \int_0^1 \frac{dx}{x}\, f_{i/h}(x,\mu^2)\, \mathcal{C}_i\!\left(\frac{\tilde{x}}{x},\tilde{\mu}^2,\mu^2,P_z\right)$$

One-loop example: quark →quark

Ma and Qiu, arXiv:1404.6860

- **Expand the factorization formula:**

$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

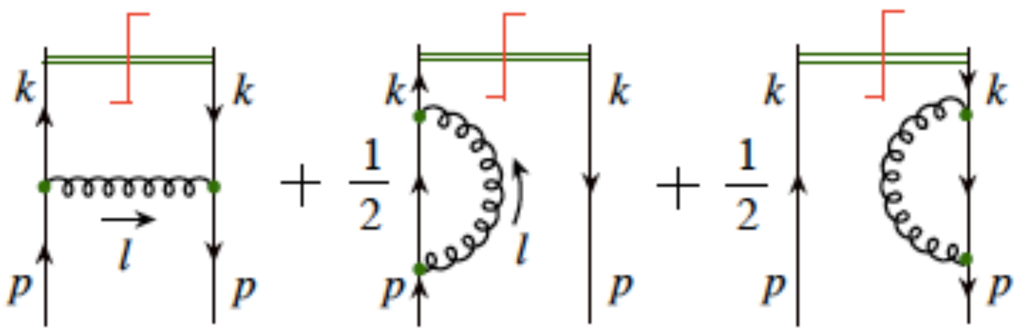
$$\Rightarrow \quad \mathcal{C}^{(1)}_{q/q}(t, \tilde{\mu}^2, \mu^2, P_z) = \tilde{f}^{(1)}_{q/q}(t, \tilde{\mu}^2, P_z) - f^{(1)}_{q/q}(t, \mu^2)$$

- **Feynman diagrams:**

Same diagrams for both

$\tilde{f}_{q/q}$ and $f_{q/q}$

But, in different gauge



- **Gauge choice:**

$n_z \cdot A = 0$ for $\tilde{f}_{q/q}$ $\qquad$ $n \cdot A = 0$ for $f_{q/q}$

Gluon propagator:

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2\, l^\alpha l^\beta}{l_z^2}$$

with $n_z^2 = -1$

One-loop "quasi-quark" distribution in a quark

Ma and Qiu, arXiv:1404.6860

❑ Real + virtual contribution:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F \frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta(1-\tilde{x}-y)-\delta(1-\tilde{x})\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$
$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

❑ Cancelation of CO divergence:

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for $0 < y < 1$, which is the same as the divergence of the normal quark distribution – necessary!

❑ UV renormalization:

Different treatment for the upper limit of l_z^2 integration - "scheme"

Here, a UV cutoff is used – other scheme is discussed in the paper

One-loop coefficient functions

Ma and Qiu, arXiv:1404.6860

- MS scheme for $f_{q/q}(x,\mu^2)$:

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}} = \left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ + \left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}+\frac{\mathrm{Sgn}(t)\Lambda_t}{\Lambda_t+|t|}\right.$$
$$\left. -\frac{1+t^2}{1-t}\left[\mathrm{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\mathrm{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t = \sqrt{\tilde{\mu}^2/P_z^2+t^2}-|t|$, $\mathrm{Sgn}(t)=1$ if $t\geq 0$, and -1 otherwise.

- Generalized "+" description:

$$t = \tilde{x}/x$$

$$\int_{-\infty}^{+\infty} dt\left[g(t)\right]_N h(t) = \int_{-\infty}^{+\infty} dt\, g(t)\,[h(t)-h(1)]$$

For a testing function $h(t)$

- Explicit verification of the factorization at one-loop:

Coefficient functions for all partonic channels are IR safe and finite!

$$\mathcal{C}^{(1)}_{i/j}(t,\tilde{\mu}^2,\mu,P_z) \quad \text{with } i,j=q,\bar{q},g$$

From Lattice "x-sections" to PDFs

❑ **BNL – RBRC efforts:**

$$\tilde{\sigma}_{\rm E}^{\rm Lat}(\tilde{x}, 1/a, P_z^2) \longleftrightarrow \tilde{\sigma}_{\rm E}(\tilde{x}, \tilde{\mu}^2, P_z^2)$$

$$\Updownarrow$$

$$\tilde{\sigma}_{\rm M}(\tilde{x}, \tilde{\mu}^2, P_z^2) \longleftrightarrow f_{\rm M}(x, \mu^2)$$

Case study
Factorization of
Quasi-PDFs
Ma, et al.

On-going
Lattice perturbation theory

❑ **To do list:**

- Identify more "good" "lattice cross sections"
- Extend to GPDs (effectively collinear factorization), and TMDs (operators with two momentum or position scales)
- …

Summary and outlook

- "lattice cross sections" = hadronic matrix elements that are calculable in Lattice QCD and factorizable in QCD factorization, without requiring $P_z \rightarrow \infty$ (a parameter)

  e.g. $\widetilde{\sigma}^i(\tilde{x}, \tilde{\mu}^2, \{\tilde{\nu}\}) \Rightarrow \widetilde{f}^i(\tilde{x} = k_z/P_z, \tilde{\mu}^2, P_z)$ Ji's quasi-PDFs

- Extract PDFs by global analysis of data on "Lattice cross sections". Same should work for other distributions

- Conservation of difficulties – complementarity:
  - High energy scattering experiments
    - – less sensitive to large x parton distribution/correlation
  - "Lattice factorizable cross sections"
    - – more suited for large x partons

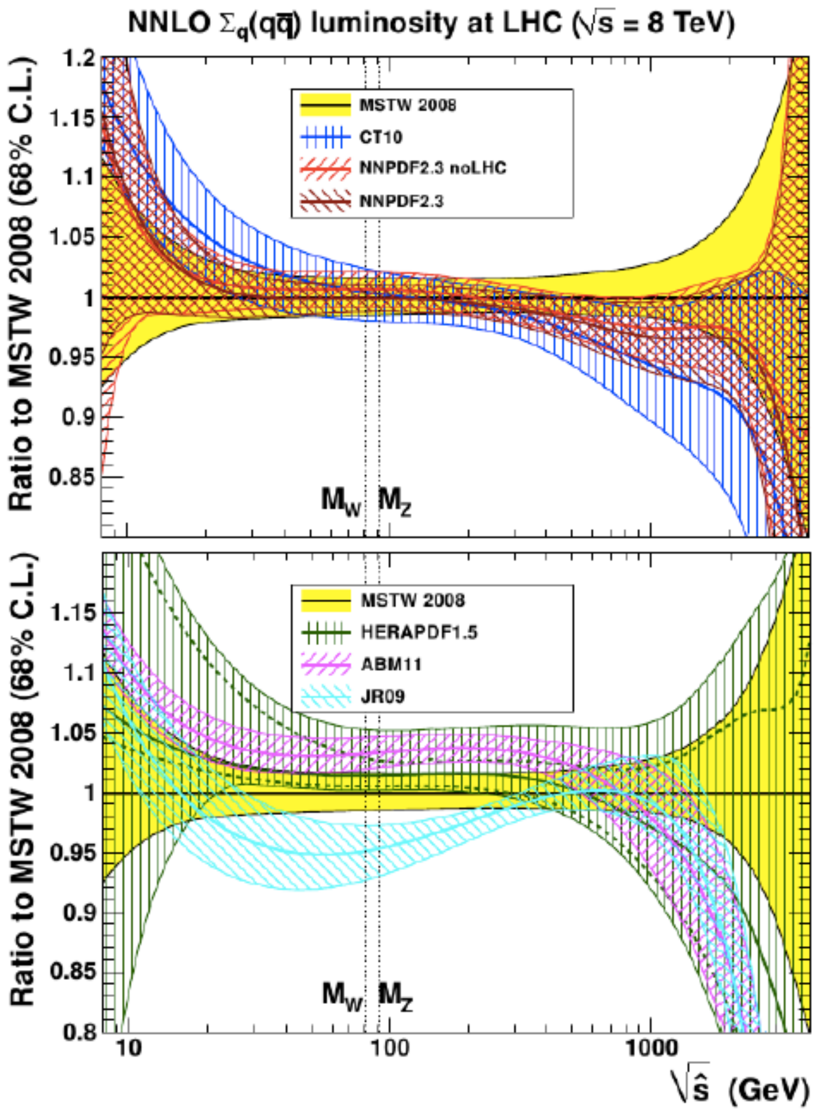
- Lattice QCD can calculate PDFs, but, more works are needed!

Thank you!

BACKUP SLIDES

Partonic luminosities

q - qbar



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