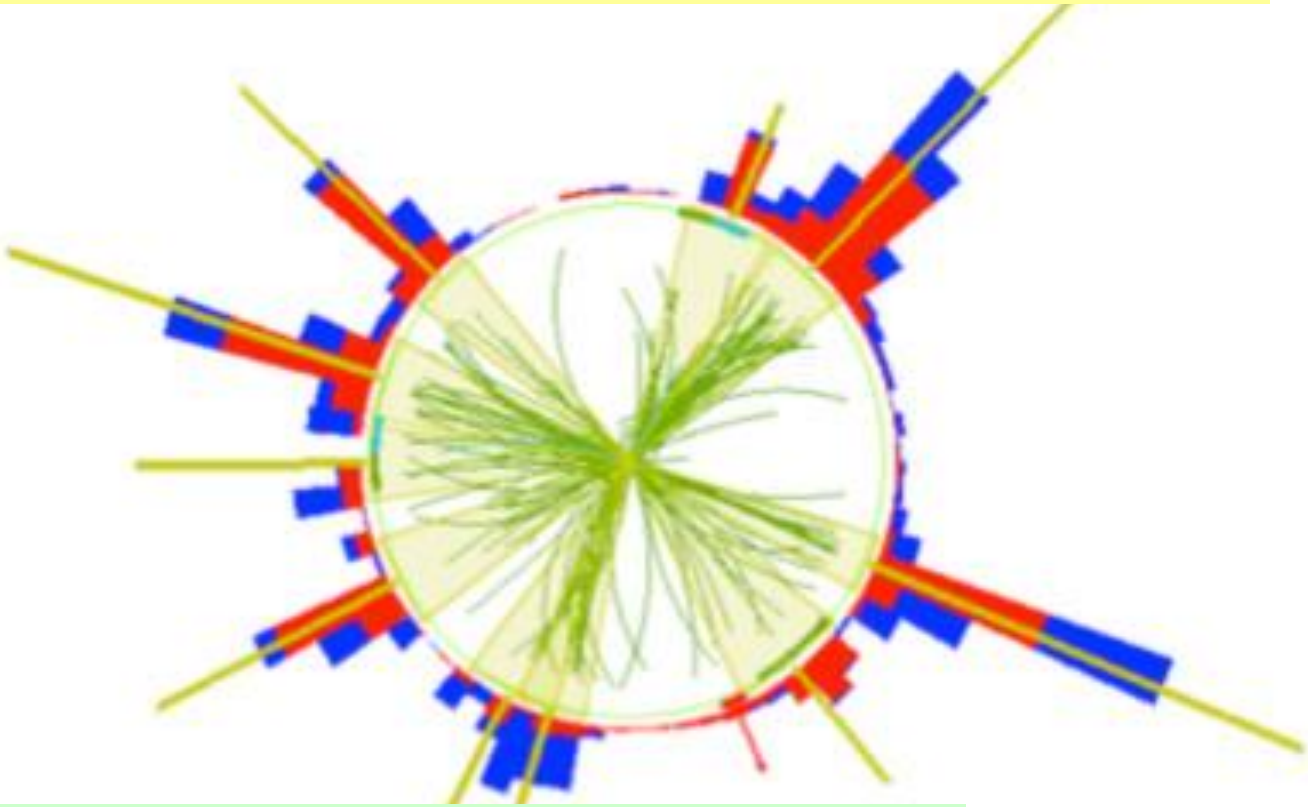
Perturbative QCD for the LHC



Lance Dixon

QCD Evolution Workshop – Santa Fe

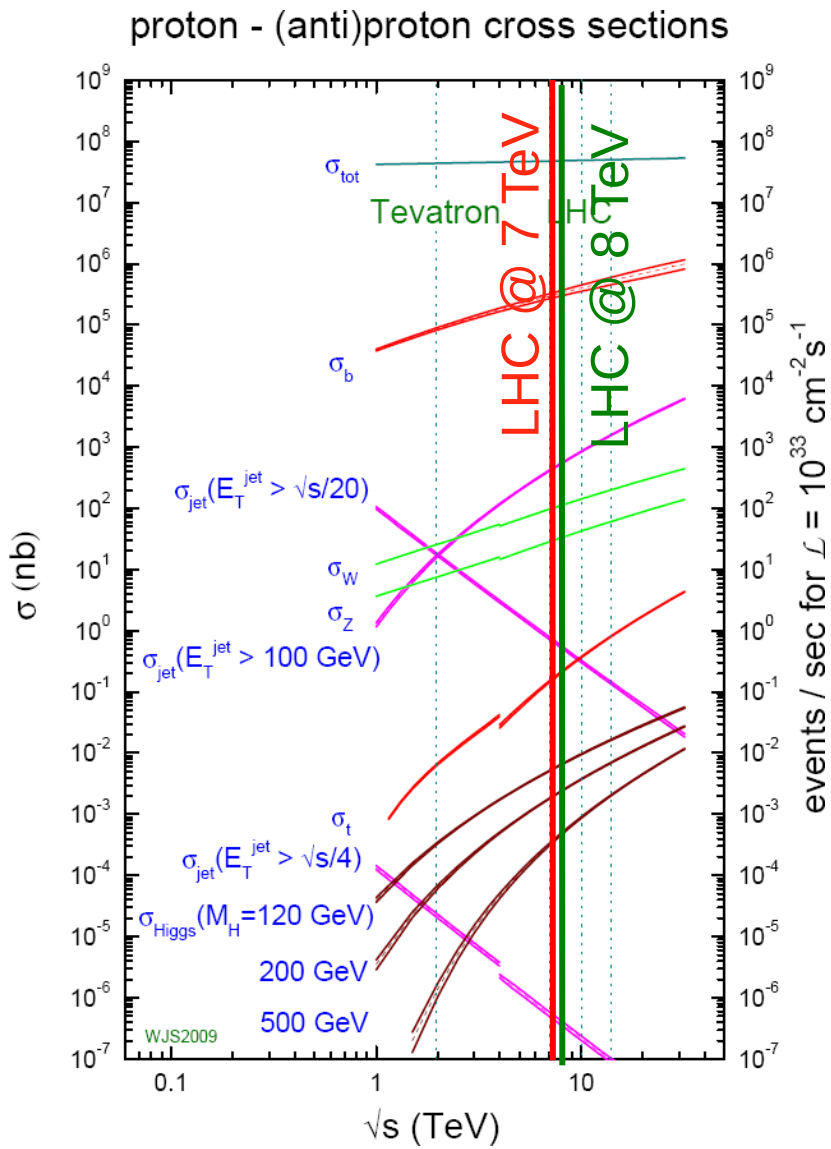
May 15, 2014

SLAC
NATIONAL ACCELERATOR LABORATORY

Outline

- Motivation
- NLO
- NNLO
- Conclusions

LHC Data Dominated by Jets

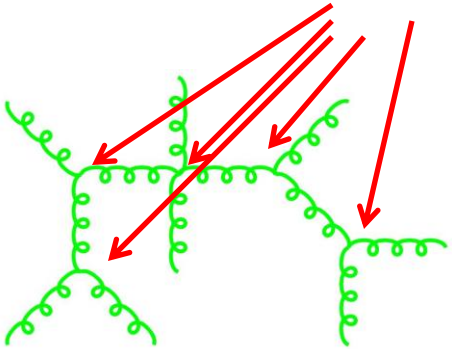


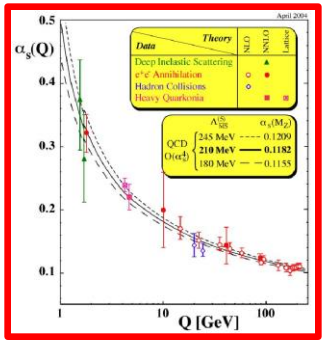
- Need precise (NLO) predictions for a wide variety of processes, often with high jet multiplicity, in order to cover the broad scope of LHC Standard Model measurements and search strategies.
- In some cases, NLO accuracy is not sufficient, need NNLO.
- In one case – σ_{Higgs} – N^3LO would be very nice

LO uncertainty increases with number of jets

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO





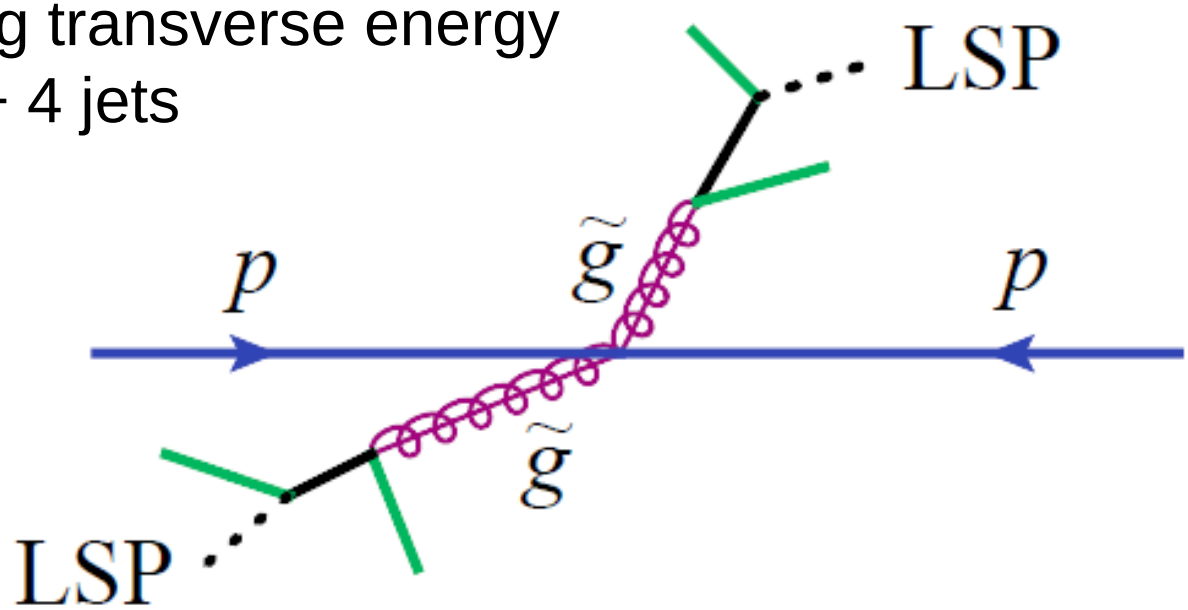
Uncertainty brought under much better control with NLO corrections: ~ 50% or more → ~ 15-20%

NLO required for quantitative control of multi-jet final states

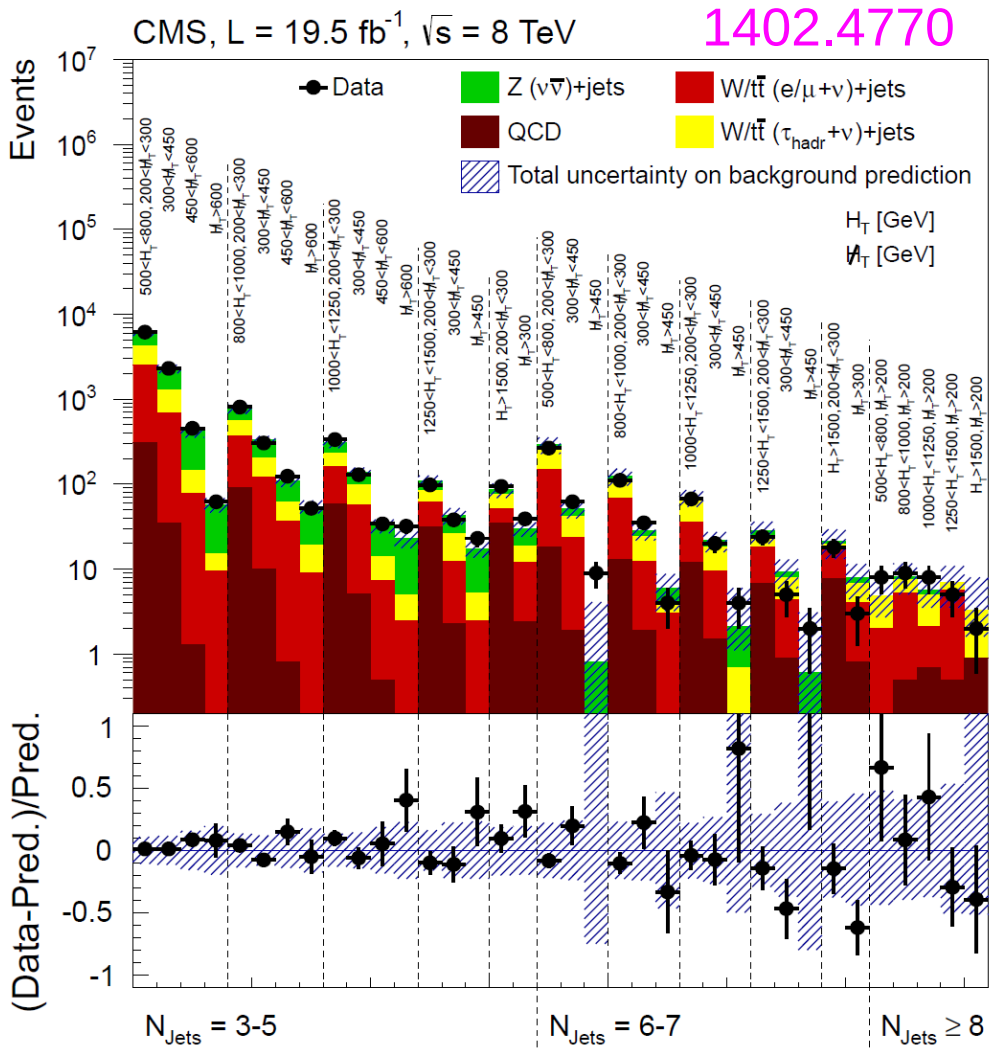
Why care about high multiplicity final states? For searches for SUSY, etc.

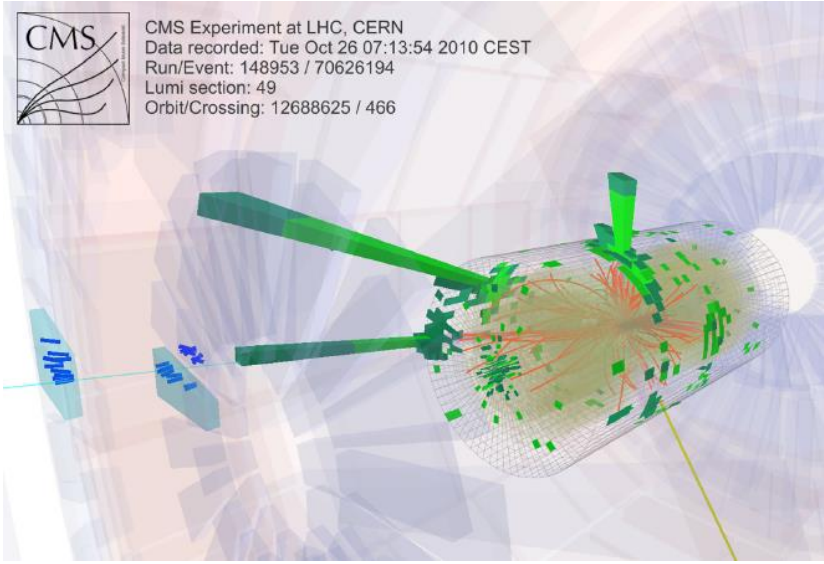
Classic SUSY signature:
Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus multiple jets

→ Missing transverse energy
MET + 4 jets



CMS search in MET+jets





Irreducible background:

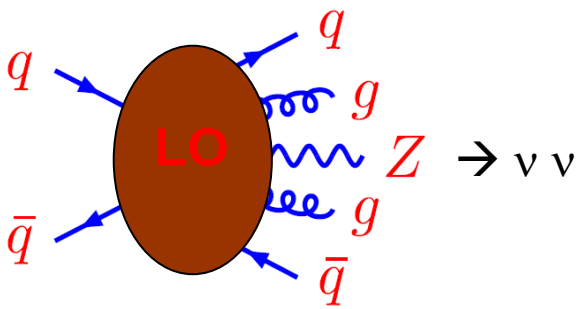
MET + jets from

pp → Z + jets,

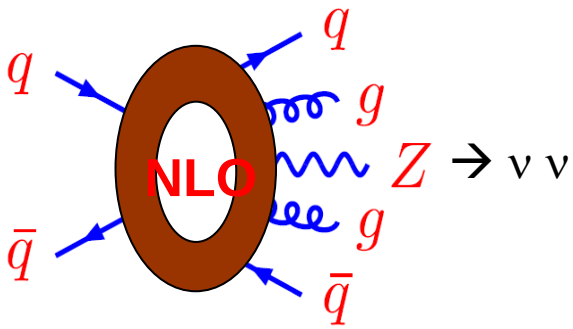
$Z \rightarrow \nu\bar{\nu}$

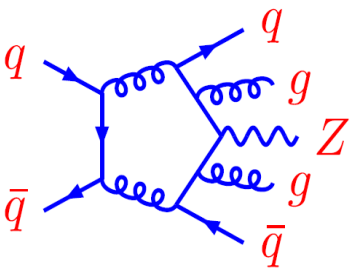
NLO theme: pushing to high multiplicity

Until recently, state of art for Z + 3, 4 jets, as well as many other high-multiplicity processes, was based on **Leading Order QCD**
→ **normalization uncertain**



Now available at **Next to Leading Order**, greatly reducing theoretical uncertainties





NLO Anatomy

- Two basic ingredients to any NLO QCD calculation:

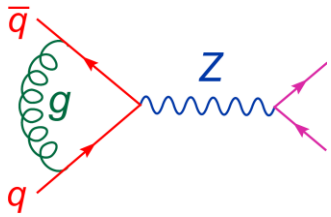
1. Virtual corrections from one-loop scattering amplitudes
2. Real-emission corrections from tree-level processes with one additional parton

- Each has infrared divergences, which cancel in the sum (for IR safe observables). Usually treated separately.
- Flexible methods required to handle experimental cuts, jet definitions, many requested kinematical distributions.

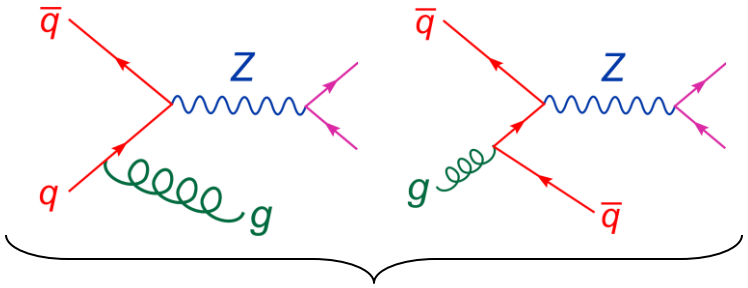
See however Belitsky talk

$\hat{\sigma}^{(1)}$

NLO



1 loop

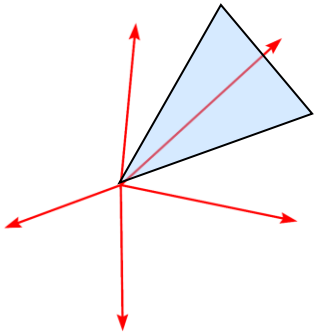


tree + 1 parton

Subtraction methods for NLO real-emission

$$\begin{aligned}
\sigma_n^{\mathsf{NLO}} &= \int d\sigma_n^{\mathsf{NLO}} = \int_n d\sigma^V + \int_{n+1;\epsilon} d\sigma^R \\
&= \int_n d\sigma^V + \int_{n+1;\epsilon} d\sigma^A + \int_{n+1;\epsilon=0} [d\sigma^R - d\sigma^A] \\
&= \int_n [d\sigma^V + \int_1 d\sigma^A]_{\epsilon=0} + \int_{n+1;\epsilon=0} [d\sigma^R - d\sigma^A]
\end{aligned}$$

- Subtraction term $d\sigma^A$ should match $d\sigma^R$ pointwise on $(n+1)$ phase space
- Factorization of $d\sigma^A$ needed to allow integral to be split, combined with $d\sigma^V$



Dipole subtraction

Catani, Seymour, hep-ph/9602227, hep-ph/9605323

$$d\sigma^A = \sum_{\text{dipoles}} d\sigma^B \otimes dV_{\text{dipole}}$$

sum over colors, convolution over momentum fractions

$$\int_{n+1} d\sigma^A = \sum_{\text{dipoles}} \int_n d\sigma^B \otimes \int_1 dV^{\text{dipole}}$$

$$= \int_n d\sigma^B \otimes I$$

For hadrons in initial state, also convolute over initial-state splitting

Poles in ε cancel universal IR poles in

$$d\sigma^V = d\sigma^B \otimes I^{(1)}$$

Also FKS method:
Frixione, Kunszt, Signer (1995)

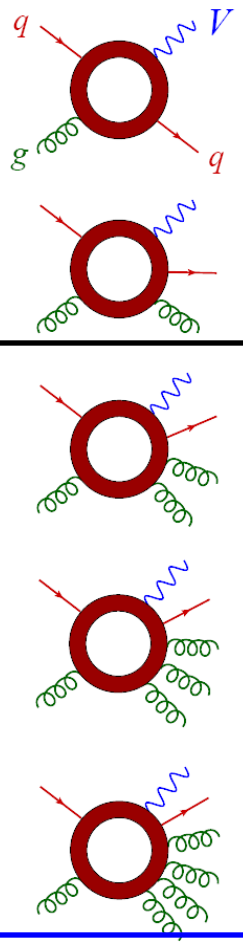
- Efficient implementations of Catani-Seymour and FKS methods for high-multiplicity tree processes [AMEGIC, COMIX, MadFKS, MadDipole,…]

→ **one-loop amplitudes as main NLO bottleneck** (until recently)

The one-loop QCD challenge

***pp* → *W* + *n* jets** (amplitudes with most gluons)

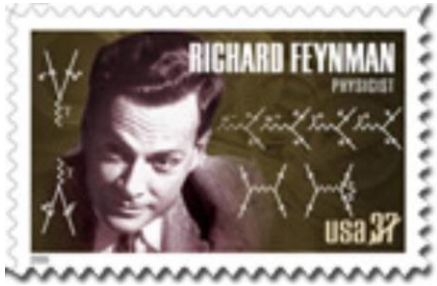
# of jets	# 1-loop Feynman diagrams
1	11
2	110
3	1,253
4	16,648
5	256,265

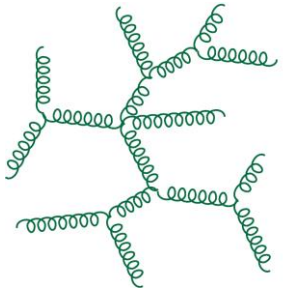


Current limit with Feynman diagrams

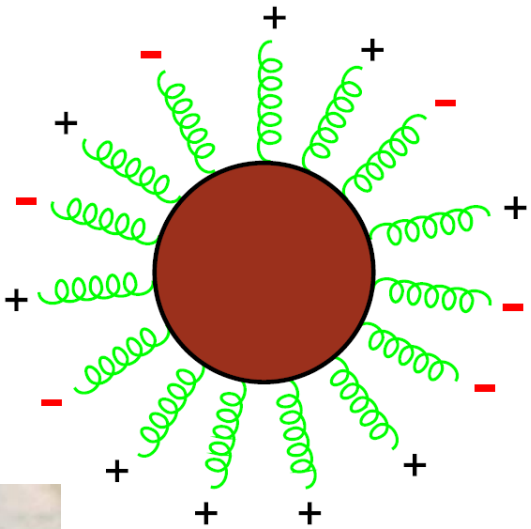
Current limit with on-shell methods

Feynman diagrams vs. On-shell methods: Granularity vs. Fluidity





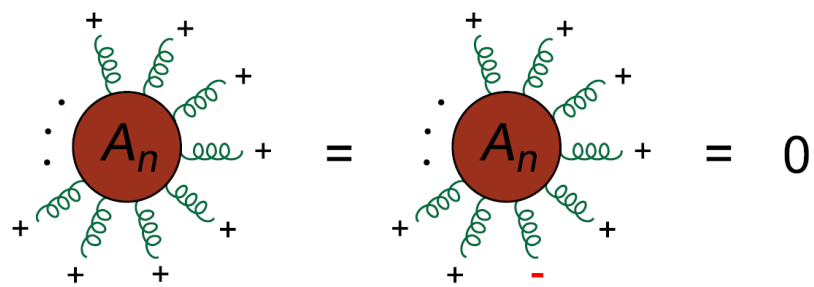
+ …



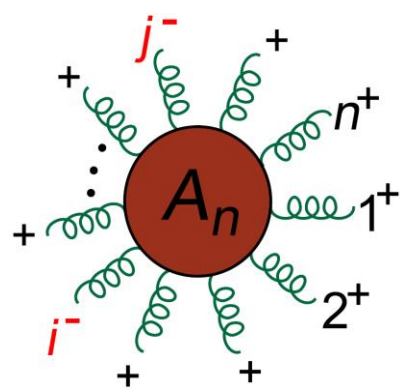
Helicity Formalism Exposes Tree-Level Simplicity in QCD

Many tree-level helicity amplitudes either vanish or are very short

right-handed $h = +1$ left-handed $h = -1$



Analyticity makes it possible to recycle this simplicity into loop amplitudes



$$= \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$$

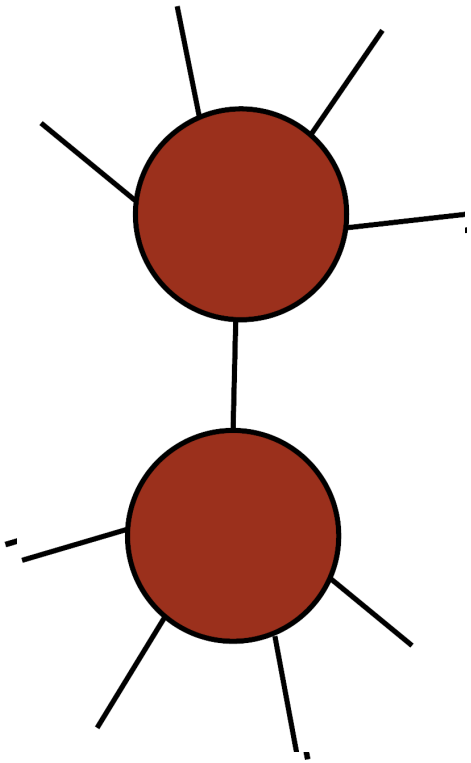
Parke-Taylor formula (1986)

Recycling “Plastic” Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

Picture leads directly to BCFW (on-shell) recursion relations

Britto, Cachazo, Feng, Witten, hep-th/0501052

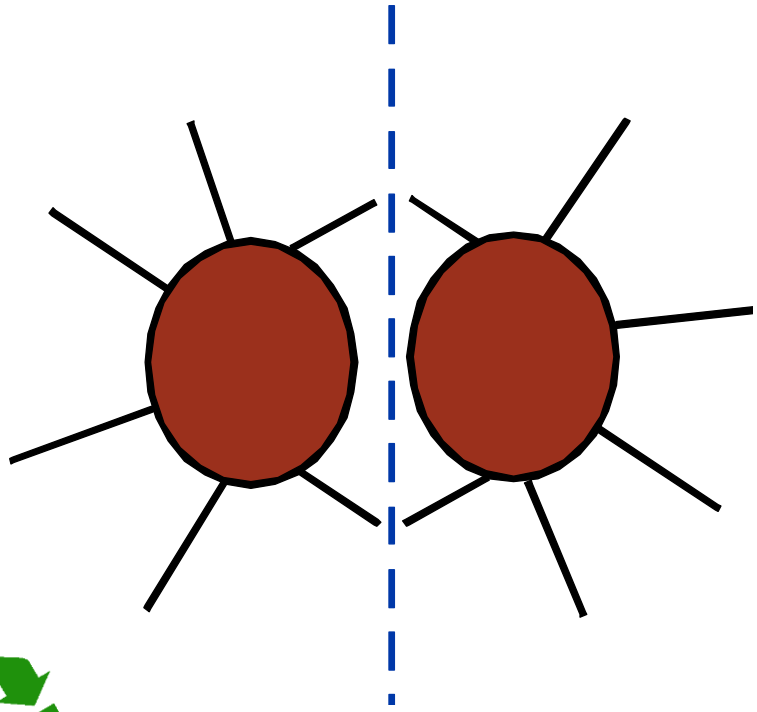


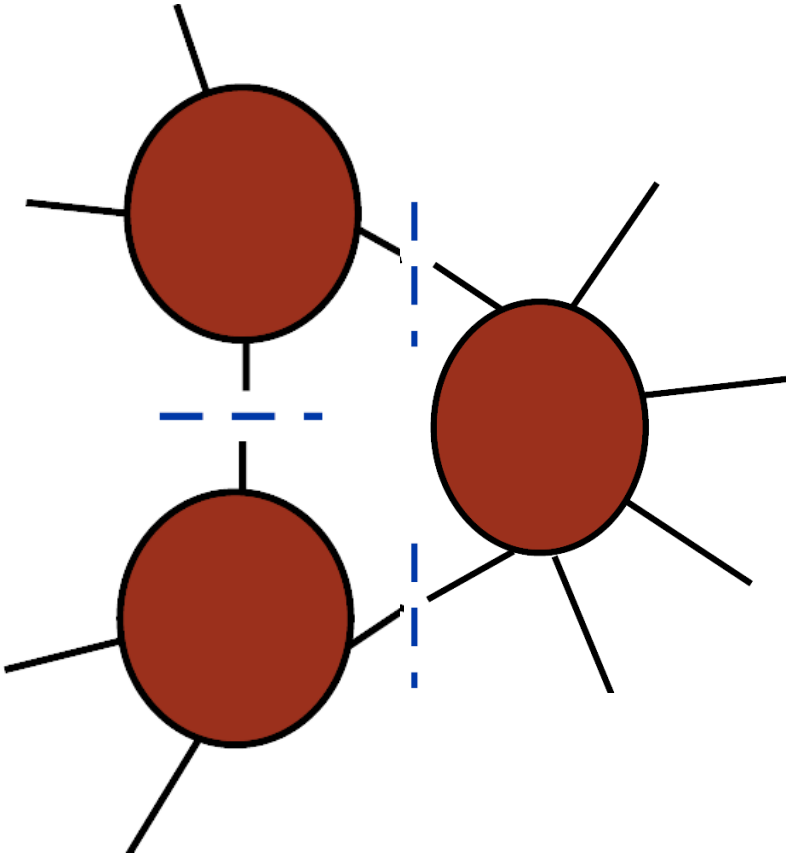
Trees recycled into trees

Branch cut information → Generalized Unitarity (One-loop Plasticity)

Ordinary unitarity:
put 2 particles on shell

Generalized unitarity:
put 3 or 4 particles on shell





Trees recycled into loops!

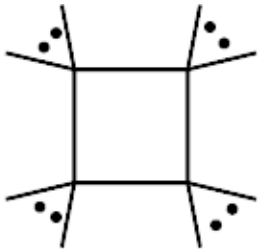
One-Loop Amplitude Decomposition

Bern, LD, Dunbar, Kosower (1994)

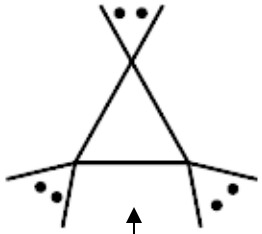
Missing from the old, nonperturbative analytic *S*-matrix

coefficients can be determined from products of trees using (generalized) unitarity

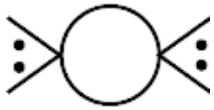
$$A^{1\text{-loop}} = \sum_i d_i$$



$$+ \sum_i c_i$$



$$+ \sum_i b_i$$

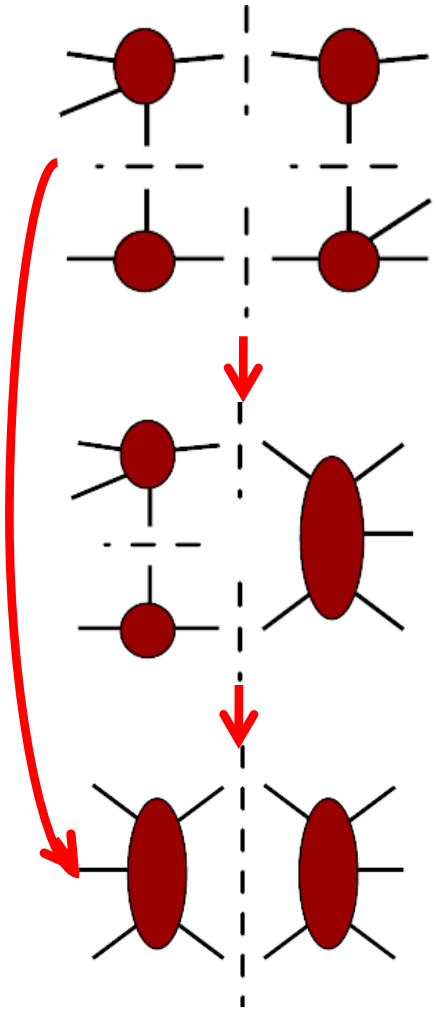


$$+ R + \mathcal{O}(\epsilon)$$

Known functions (integrals), same for all amplitudes

rational part; from D-dimensional trees, or recursively

Full amplitude determined hierarchically



Each box coefficient comes uniquely from 1 "quadruple cut"

Britto, Cachazo, Feng, hep-th/0412103

Ossola, Papadopolous, Pittau, hep-ph/0609007; Mastrolia, hep-th/0611091; Forde, 0704.1835; Ellis, Giele, Kunszt, 0708.2398; Berger et al., 0803.4180;…

Each triangle coefficient from 1 triple cut, but "contaminated" by boxes

Each bubble coefficient from 1 double cut, removing contamination by boxes and triangles

Rational part depends on all of above

Many Automated On-Shell One Loop Programs

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…
+ **Sherpa** → NLO *W,Z* + 3,4,5 jets pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO *WWW, WWZ*, ... Binoth+OPP, 0804.0350
NLO $ttb\bar{b}$, $tt\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009
MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621
HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO *W* + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI (→ GoSAM): Mastrolia, Ossola, Reiter, Tramontano, 1006.0710

NGluon: Badger, Biedermann, Uwer, Yundin, 1011.2900, 1209.0098, 1309.6585

OpenLoops: Cascioli, Maierhofer, Pozzorinit, 1111.5206, 1312.0546

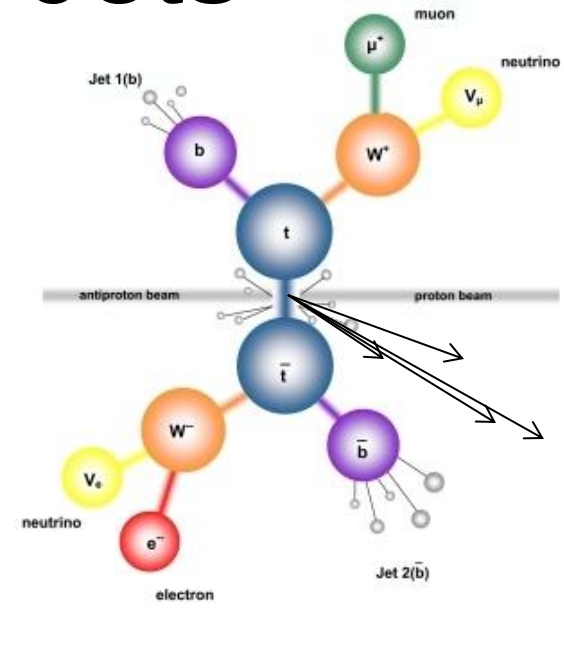
As a result…

Dramatic increase recently in rate of NLO QCD predictions for new processes!

One indicator of NLO progress

pp → W + 0 jet	1978	Altarelli, Ellis, Martinelli
pp → W + 1 jet	1989	Arnold, Ellis, Reno
pp → W + 2 jets	2002	Campbell, Ellis
pp → W + 3 jets	2009	BH+Sherpa Ellis, Melnikov, Zanderighi
pp → W + 4 jets	2010	BH+Sherpa
pp → W + 5 jets	2013	BH+Sherpa

Top Quark Pairs + Jets



- Like (W,Z) + jets, very important bkgd
- Cross sections large

– no electroweak couplings

- Jets boost $t\,\bar{t}$ system, increase MET,

provide jets to pass various signal cuts.

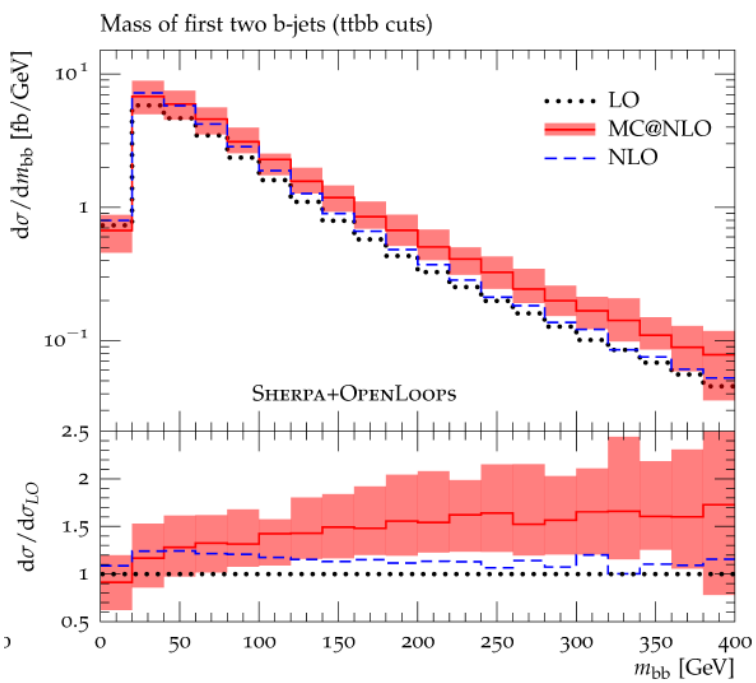
- State of art:
- **NLO *tt* + 1 jet:** Dittmaier, Uwer, Weinzierl, hep-ph/0703120,…
- **+ top decays:** Melnikov, Schulze, 1004.3284
- **+ NLO parton shower:** Kardos, Papadopoulos, Trócsányi, 1101.2672
- **NLO *tt* + *bb*:** Bredenstein, Denner, Dittmaier, Pozzorini, 0905.0110, 1001.4006; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723
- **+ NLO parton shower:** Cascioli, Maierhofer, …, 1309.5912
- **NLO *tt* + 2 jets:** Bevilacqua, Czakon, Papadopoulos, Worek, 1002.4009

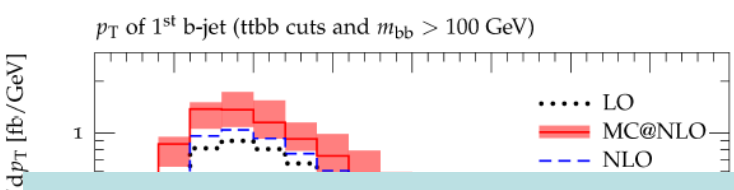
NLO $pp \to t\bar{t}\,b\bar{b}$ at LHC

Background to $t\,\bar{t}$ + *Higgs*, $H \to b\bar{b}$ at LHC (for λ_t)

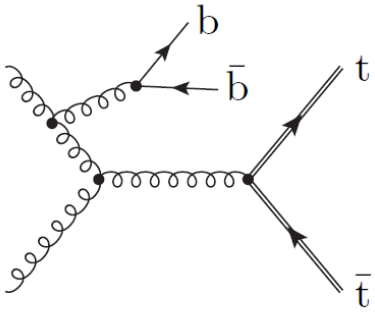
First done using Feynman diagrams — Bredenstein et al., 0807.1248, 0905.0110

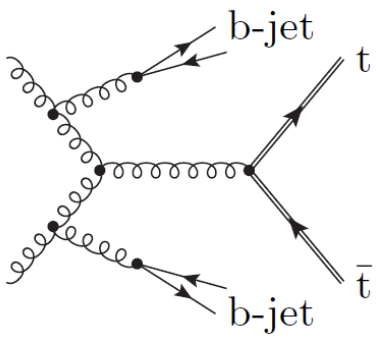
Recomputed using unitarity/OPP (**CutTools and OpenLoops) and interfaced to parton shower** — Bevilacqua et al., 0907.4723; Cascioli et al., 1309.5912



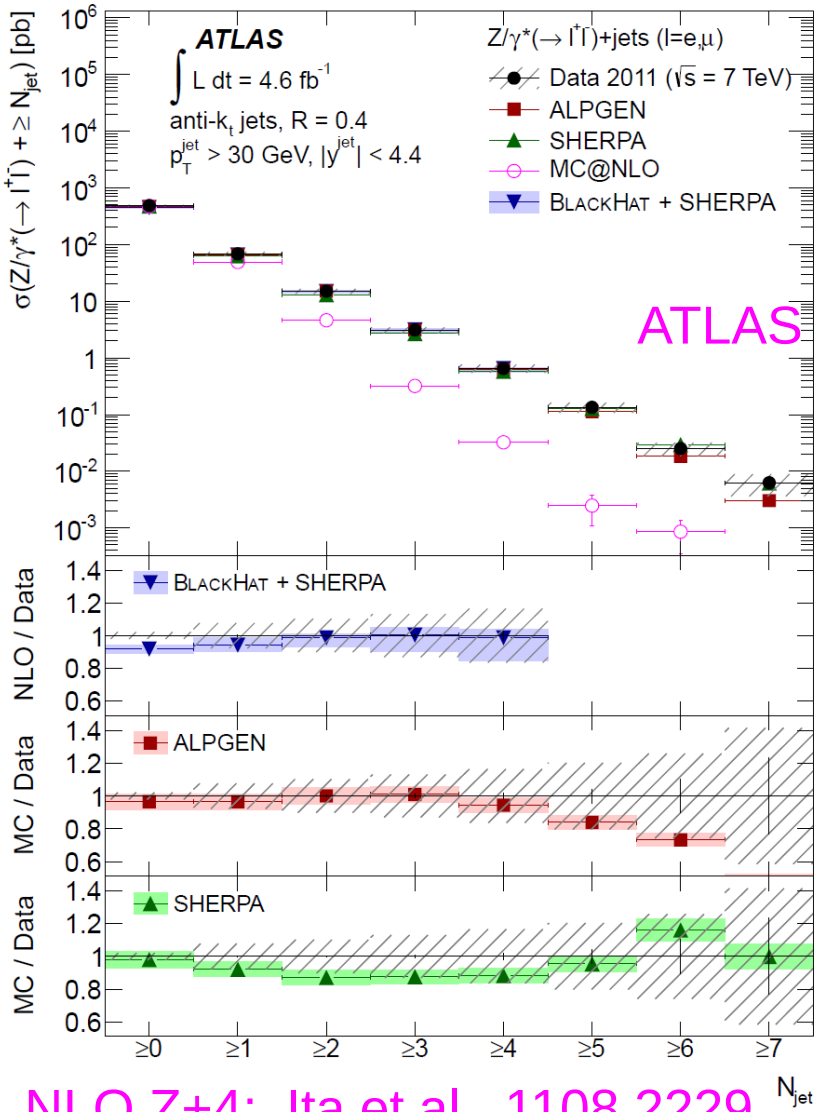


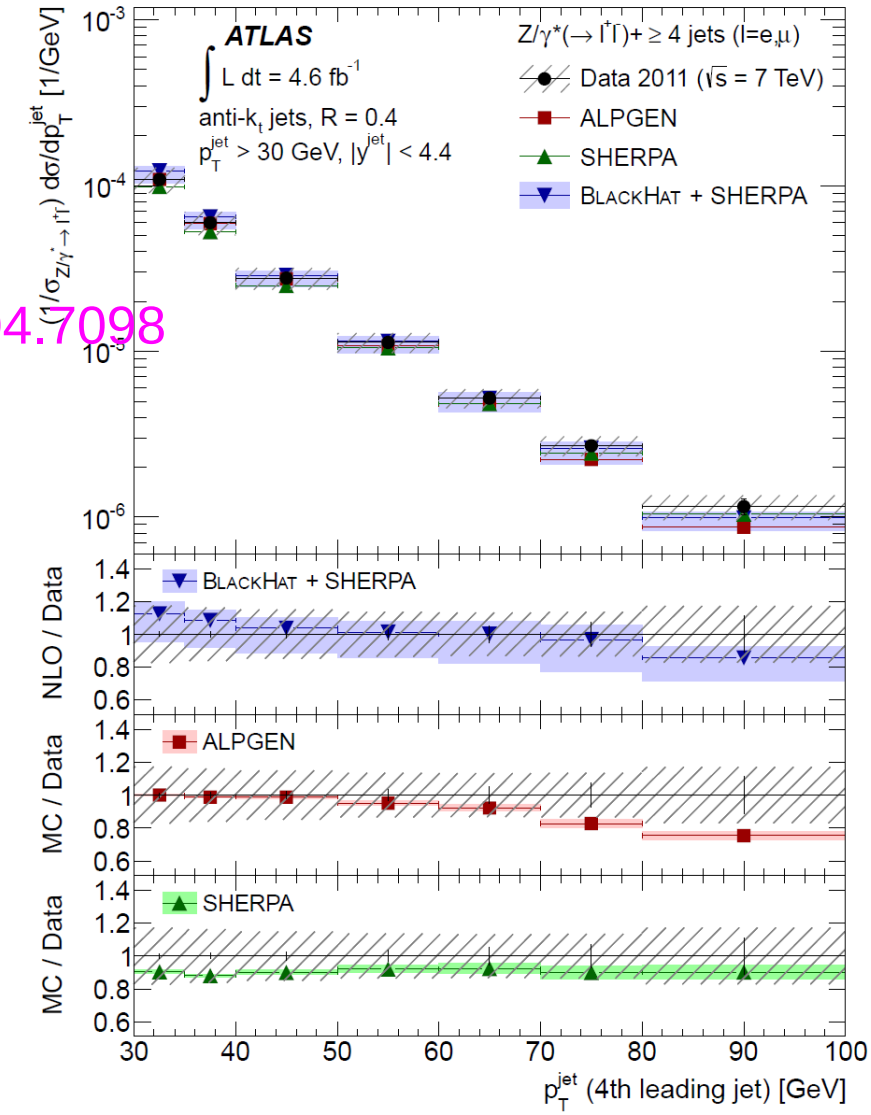
Large corrections from shower due to double g → bb splitting





NLO $pp \rightarrow Z + 1,2,3,4$ jets vs. ATLAS 2011 data

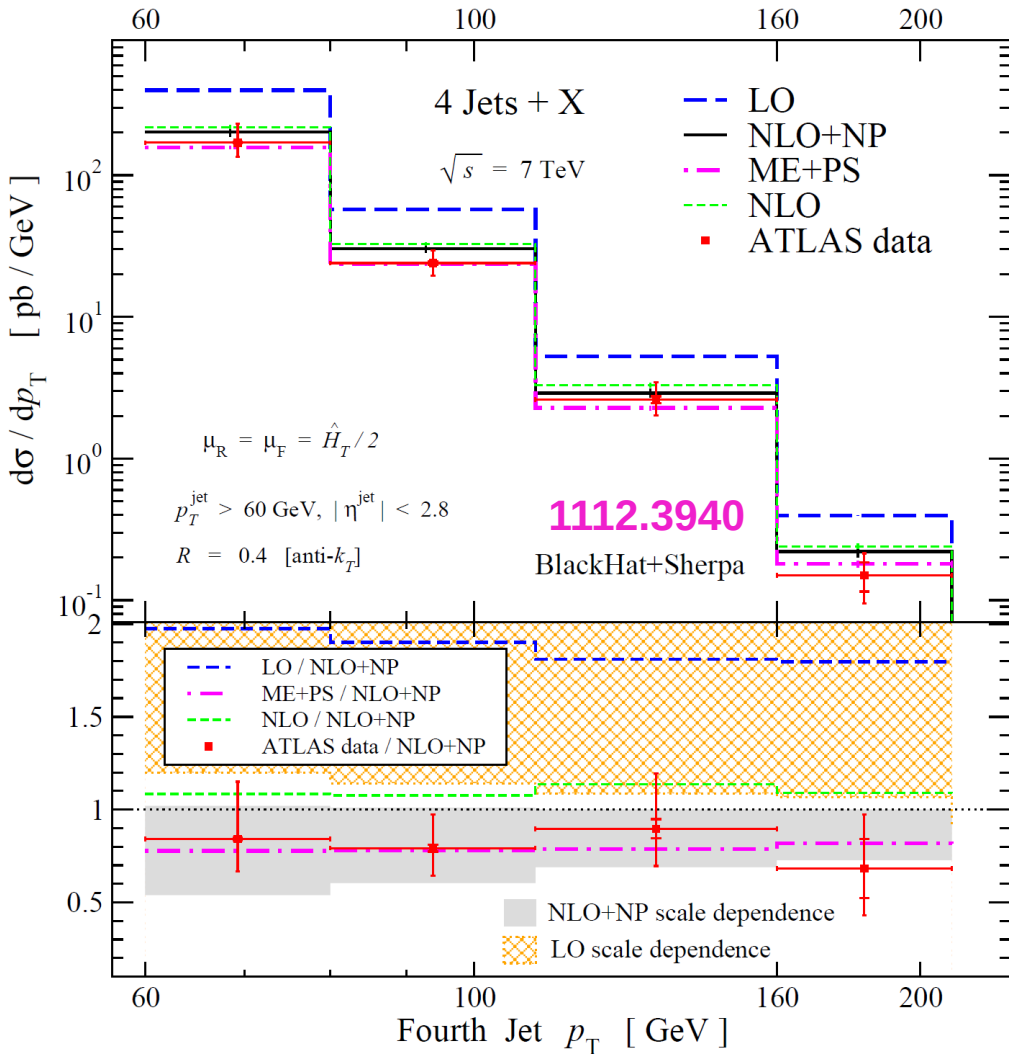




ATLAS 1304.7098

NLO Z+4: Ita et al., 1108.2229

Pure QCD: $pp \rightarrow 4$ jets vs. ATLAS data



4 jet events might hide pair production of 2 colored particles, each decaying to a pair of jets

Detailed study of multi-jet QCD dynamics may help understand other channels

NNLO

- Needed for high precision in benchmark processes
- Until recently, limited by "real-emission bottleneck" to 2 → 1 processes:

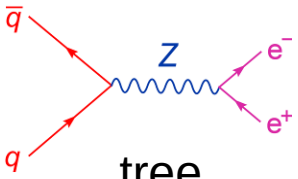
  W, Z, Higgs
- Now that is rapidly being overcome, for 2 → 2 processes such as

  $t\bar{t}$, dijets, H + 1 jet, VV

NNLO Anatomy

Example of Z production at hadron colliders

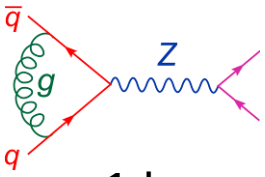
LO $\hat{\sigma}^{(0)}$



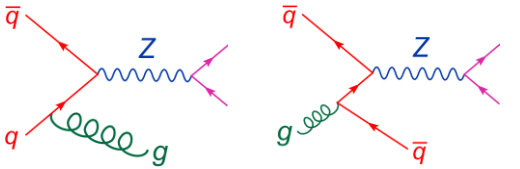
tree

convolute with pdfs
apply cuts

NLO $\hat{\sigma}^{(1)}$



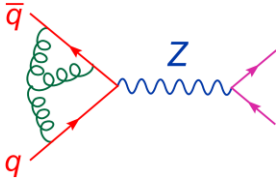
1 loop



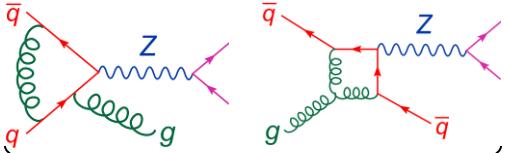
tree + 1 parton

dim. reg. $D = 4 - 2\epsilon$
first, cancel infrared divergences ($1/\epsilon^2$) between virtual & real

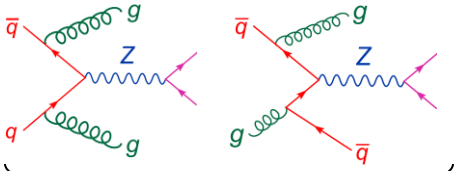
NNLO $\hat{\sigma}^{(2)}$



2 loops



1 loop + 1 parton



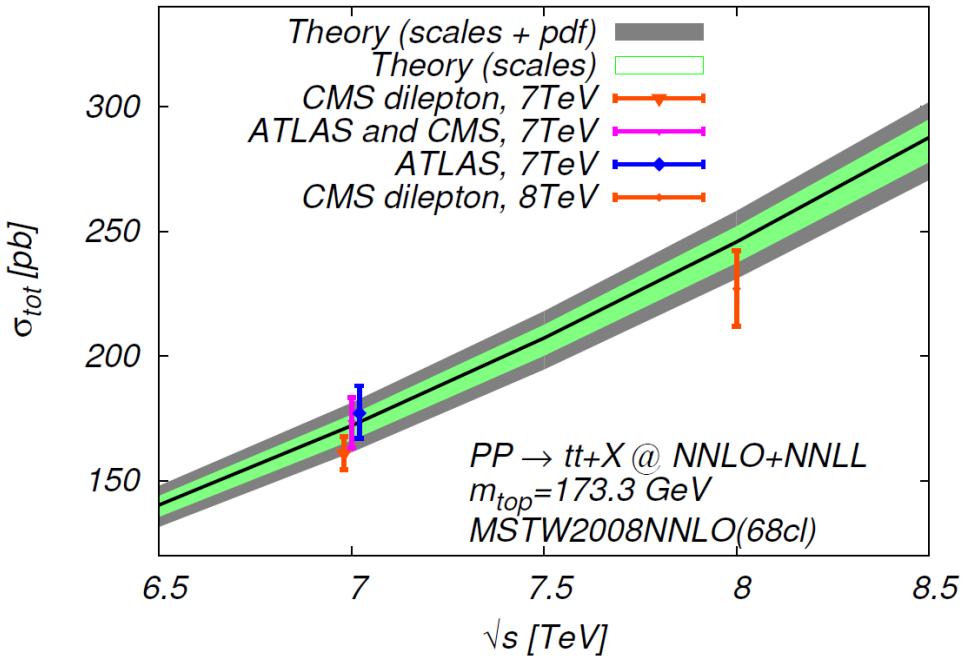
tree + 2 partons

intricate ($1/\epsilon^4$) IR cancellations

NNLO $p\overset{(-)}{p} \to t\bar{t}$

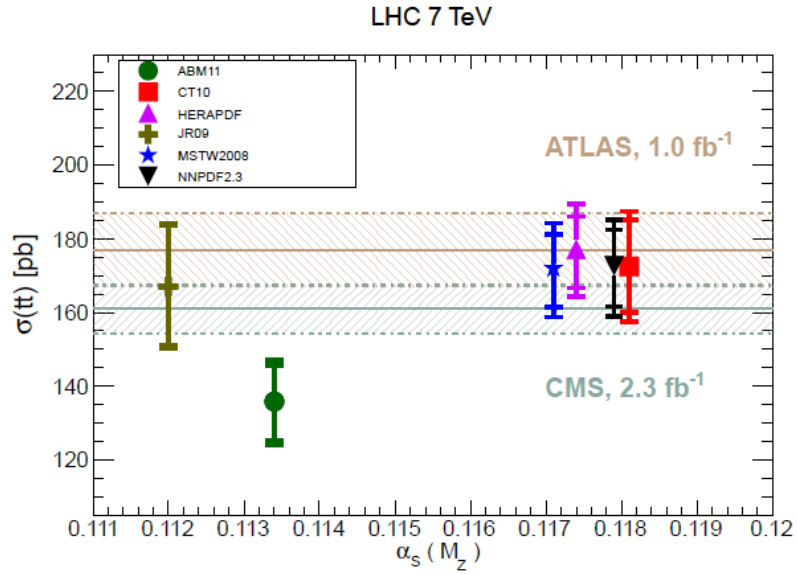
Bärnreuther, Czakon, Fielder, Mitov, 1204.5201, 1210.6832, 1303.6254, …

- 2-loop virtual computed numerically
- Total cross section only; looking forward to distributions, such as forward-backward asymmetry at Tevatron



Czakon, Fielder, Mitov, 1303.6254

- Constrains gluon pdf:



Czakon, Mangano, Mitov, Rojo, 1303.7215

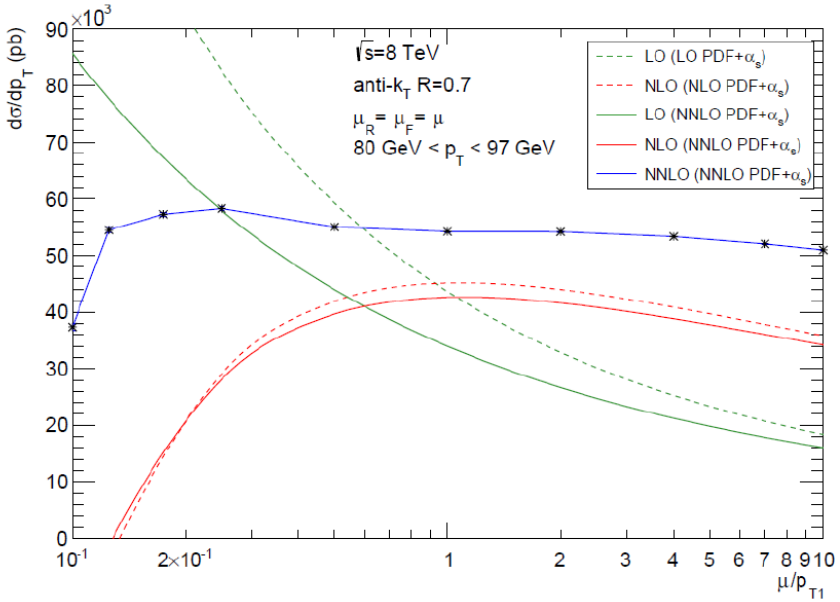
NNLO di-jet production

Currie, Gehrmann, Gehrmann-de Ridder, Glover, Heinrich, Pires, Wells, 1112.3613, 1301.4693, 1301.7310, 1310.3993, …

- 2-loop amplitudes (and 1-loop amplitudes) known analytically (2000-2003)

Anastasiou, Glover, Tejeda-Yeomans; Bern, LD, DeFreitas

- Antenna subtraction formalism for intricate double-real contributions



✓ Completed:

- ✓ $gg \to gg$ leading colour
- ✓ $gg \to gg$ sub-leading colour
- ✓ $q\bar{q} \to gg$ leading colour

✓ Other processes in progress

NNLO Higgs + 1 jet

Boughezal, Caola, Melnikov, Petriello, Schulze, 1302.6216

- Amplitudes also known analytically
  Gehrmann, Jaquier, Glover, Koukoutsakis, 1112.3554
- Generic subtraction method,

related to that used

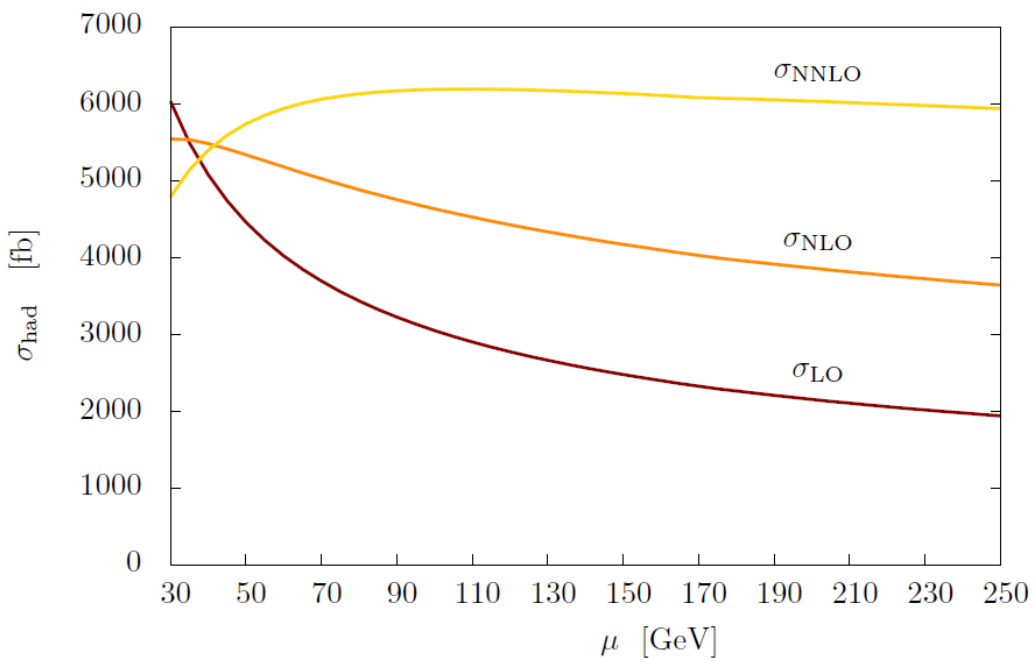
for $pp \rightarrow tt$

Czakon, 1005.0274;

Boughezal et al., 1111.7041

- $gg \rightarrow H + jet$

completed so far →



NNLO $pp \to VV \quad (V = W,Z,\gamma)$

- Uses q_T subtraction method for processes: $pp \to X[\text{color-singlet objects}]$
- Divide computation into:

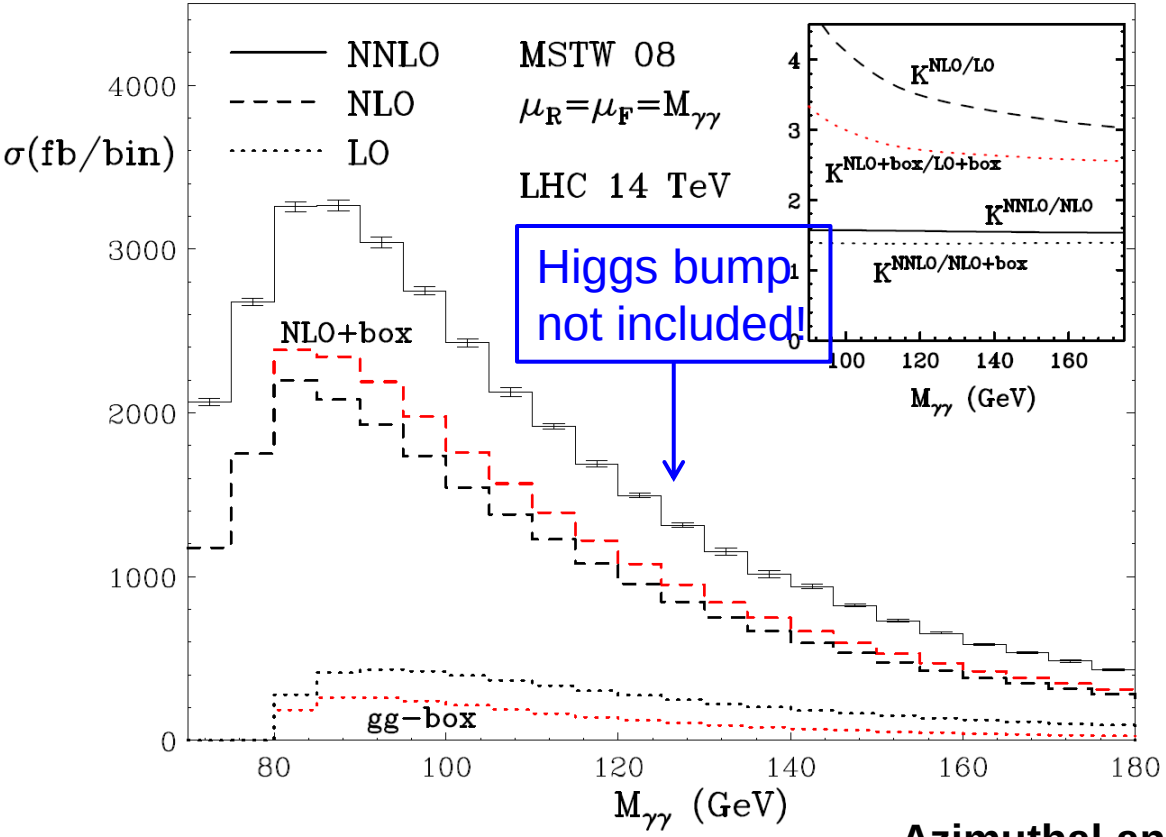
✓ $q_T(X) > 0$: use NLO results for $X + \text{jet}$

✓ $q_T(X) = 0$: use universal $q_T \to 0$ limit, related to soft gluon resummation of q_T distribution Catani, Grazzini, hep-ph/0703012 [Higgs]; Catani, Cieri, Ferrera, De Florian, Grazzini, 0903.2120 [W,Z]; 1311.1654

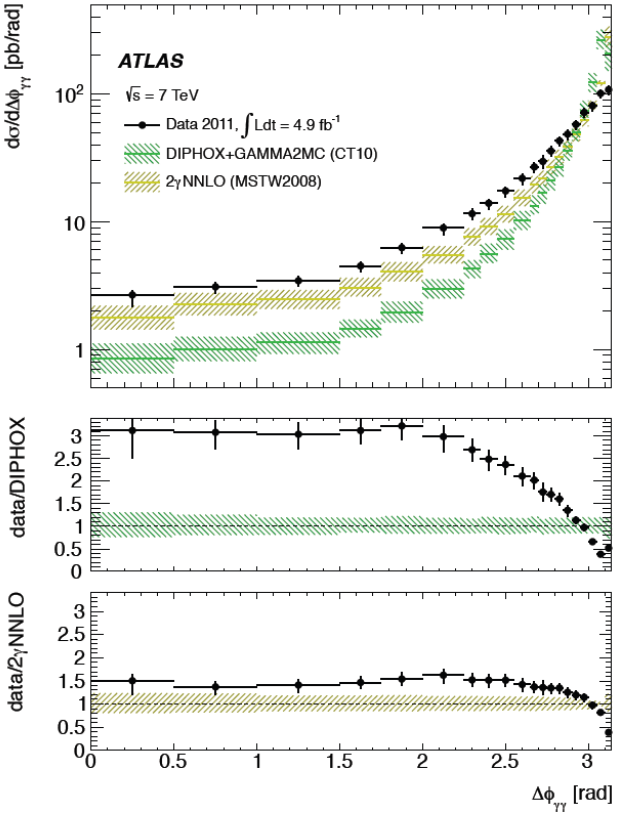
NNLO $pp \rightarrow \gamma\gamma$

Catani, Cieri, Ferrera, De Florian, Grazzini, 1110.1375

2-loop amplitudes: Anastasiou, Glover, Tejeda-Yeomans, hep-ph/0201274



Higgs bump not included!



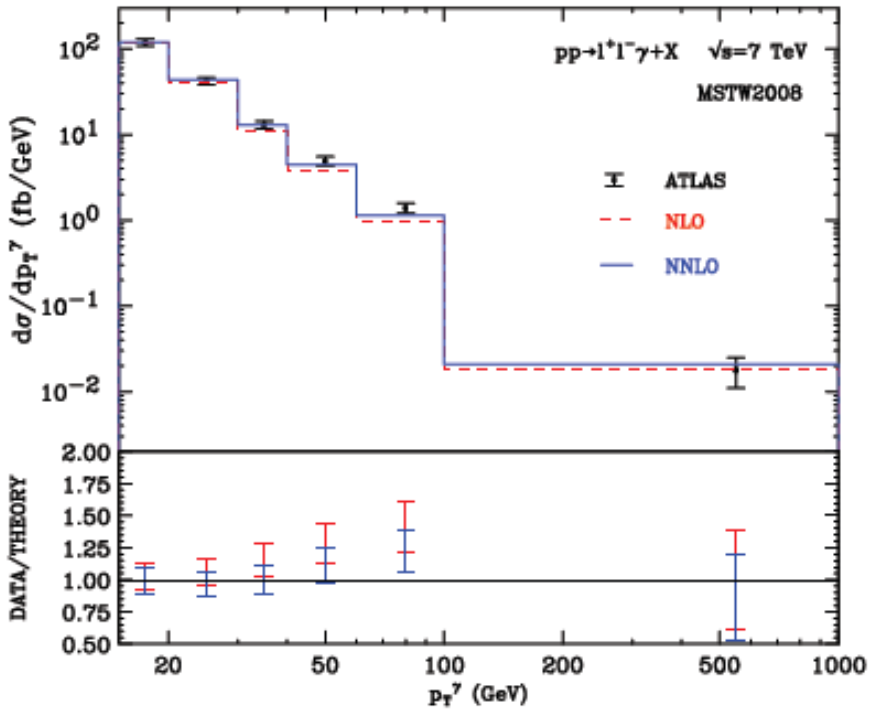
Azimuthal angle separation → huge corrections

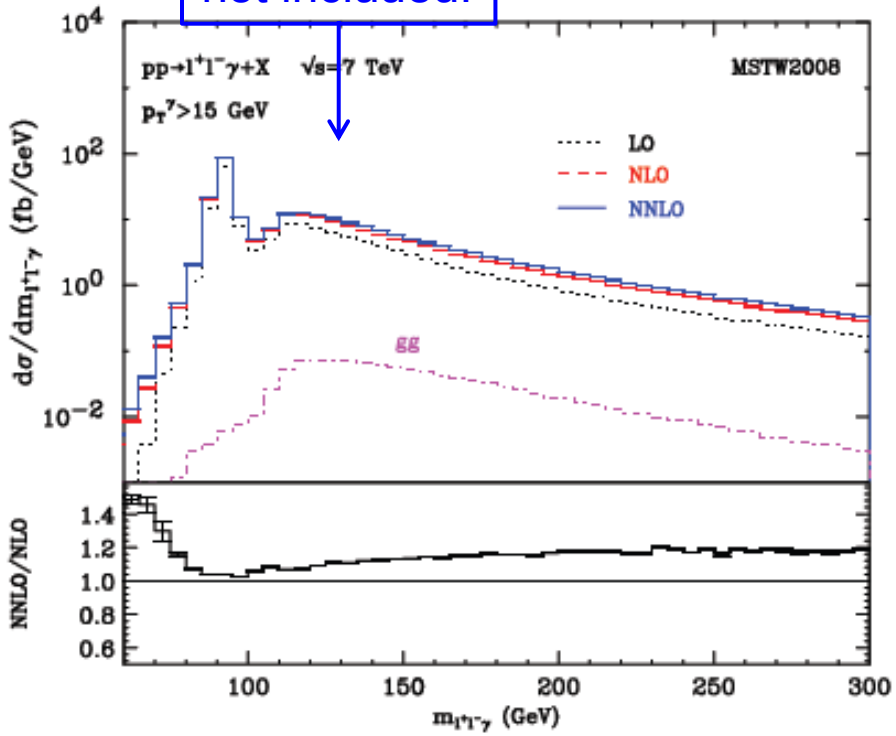
NNLO $pp \to Z\gamma$ (also $W\gamma$)

Grazzini, Kallweit, Rathlev, Torre, 1309.7000

2-loop amplitudes: Gehrmann, Tancredi, 1112.1531

Higgs bump not included!



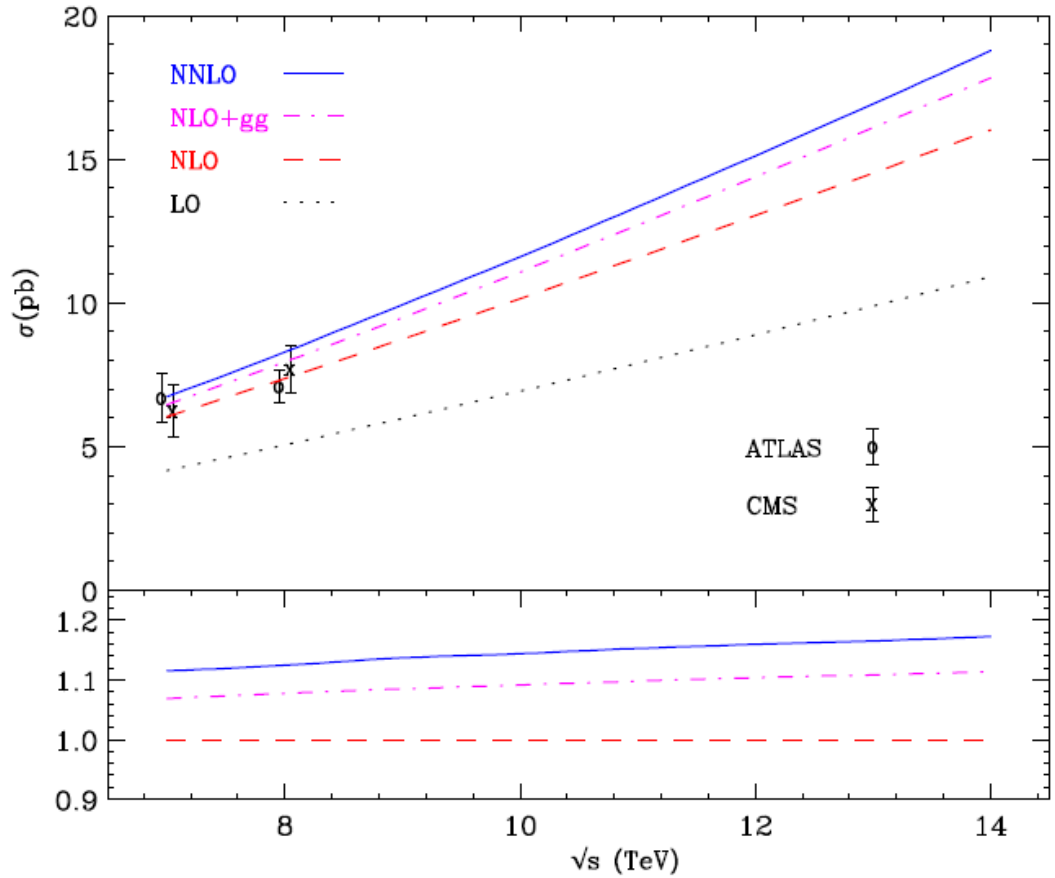


NNLO $pp \to ZZ$

Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Tancredi, Weihs, Rathlev, 1405.2219

2-loop master integrals for amplitudes:

Gehrmann, von Manteuffel, Tancredi, Weihs, 1404.4853



$\sqrt{s}$ (TeV)	σ_{LO} (pb)	σ_{NLO} (pb)	σ_{NNLO} (pb)
7	$4.167^{+0.7\%}_{-1.6\%}$	$6.044^{+2.8\%}_{-2.2\%}$	$6.735^{+2.9\%}_{-2.3\%}$
8	$5.060^{+1.6\%}_{-2.7\%}$	$7.369^{+2.8\%}_{-2.3\%}$	$8.284^{+3.0\%}_{-2.3\%}$

Conclusions

- In past 5 years, frontier for computing LHC processes at NLO in α_s has moved from 2→3 to 2→4,5, and even 6, thanks to great improvements in computing 1-loop amplitudes.
- Good agreement with many ATLAS+CMS measurements.
- Work ongoing to interface these results to Monte Carlos.
- NNLO frontier now moving from 2→1 to 2→2 processes, thanks to better subtraction methods and some new 2→2 2-loop amplitudes.
- NNLO *VV* results improve agreement with LHC data.
- Next NNLO bottleneck may be 2→3 2-loop amplitudes. Much progress being made along the 1-loop lines, but still a ways to go.

Good news:

Trees can also be recycled into multi-loops!

Extra Slides

Fixed order vs. Monte Carlo

- Previous plots NLO but fixed-order, few partons: no model of long-distance effects included; cannot pass through a detector simulation
- Methods available for matching NLO parton-level results to parton showers, with NLO accuracy:
  - **MC@NLO** Frixione, Webber (2002) ; …; SHERPA implementation
  - **POWHEG** Nason (2004); Frixione, Nason, Oleari (2007)
- Now implemented for increasingly complex final states, e.g. W + 1,2,3 jets Höche et al, 1201.5882