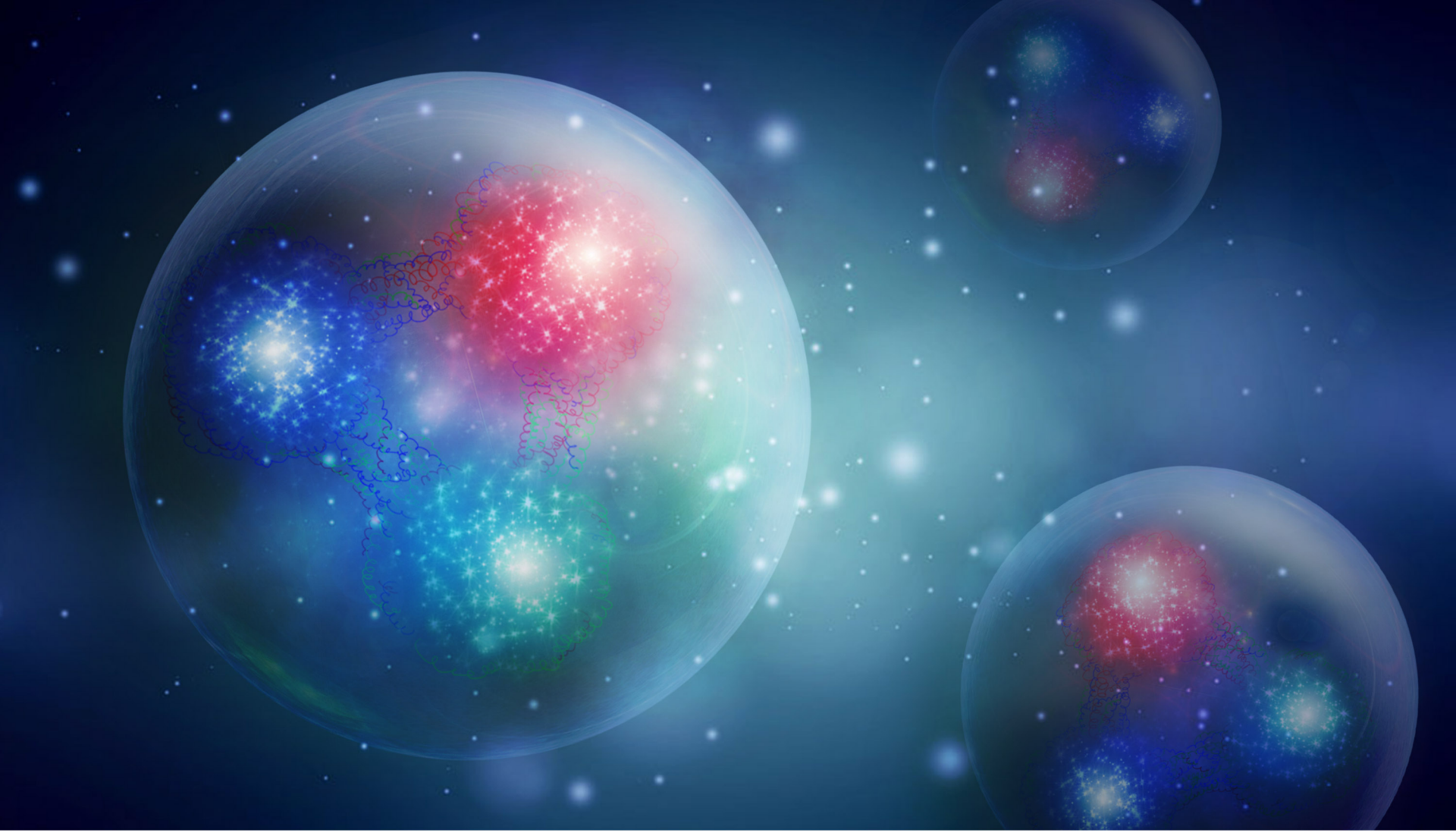
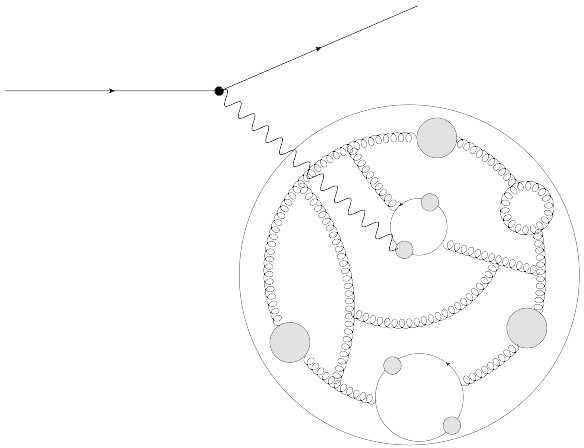


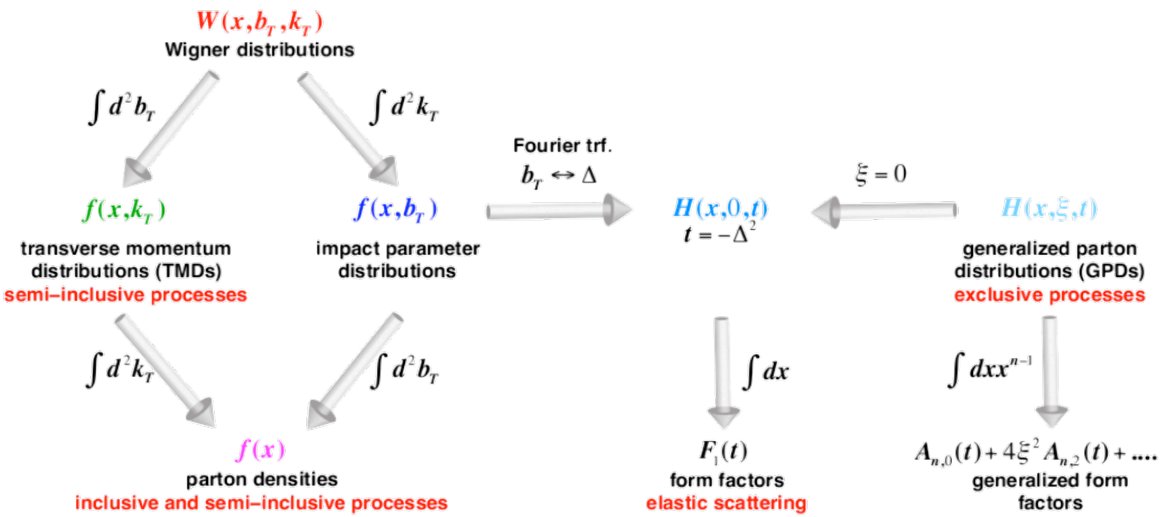
Rapidity factorization and TMDs

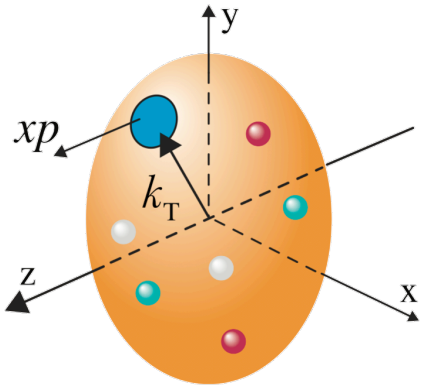
Andrey Tarasov

QCD Evolution, May 15, 2014

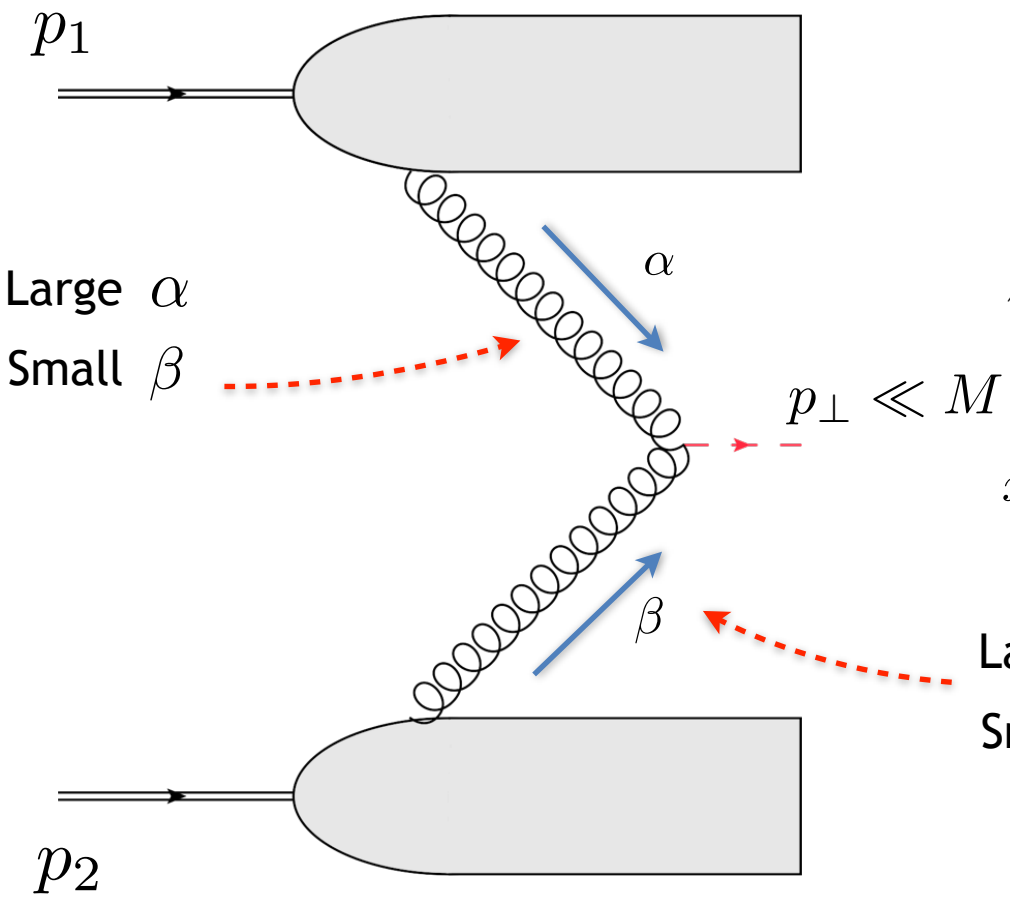
3D Picture of Hadrons







SCALAR PARTICLE PRODUCTION



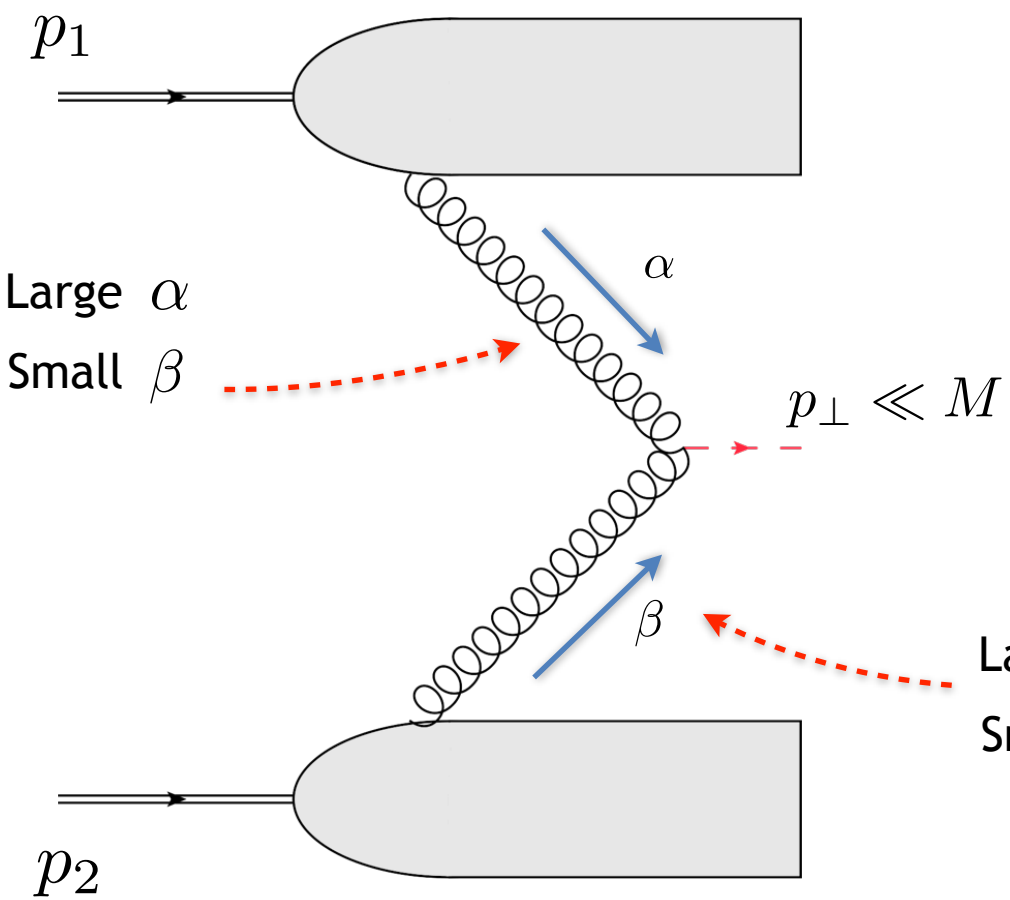
$$p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

$$\alpha = p_+/\sqrt{s} \qquad \beta = p_-/\sqrt{s}$$

$$z^\mu = \frac{2}{s} z_* p_1^\mu + \frac{2}{s} z_\bullet p_2^\mu + z_\perp^\mu$$

$$x_* = x_+\sqrt{\frac{s}{2}} \qquad x_\bullet = x_-\sqrt{\frac{s}{2}}$$

SCALAR PARTICLE PRODUCTION

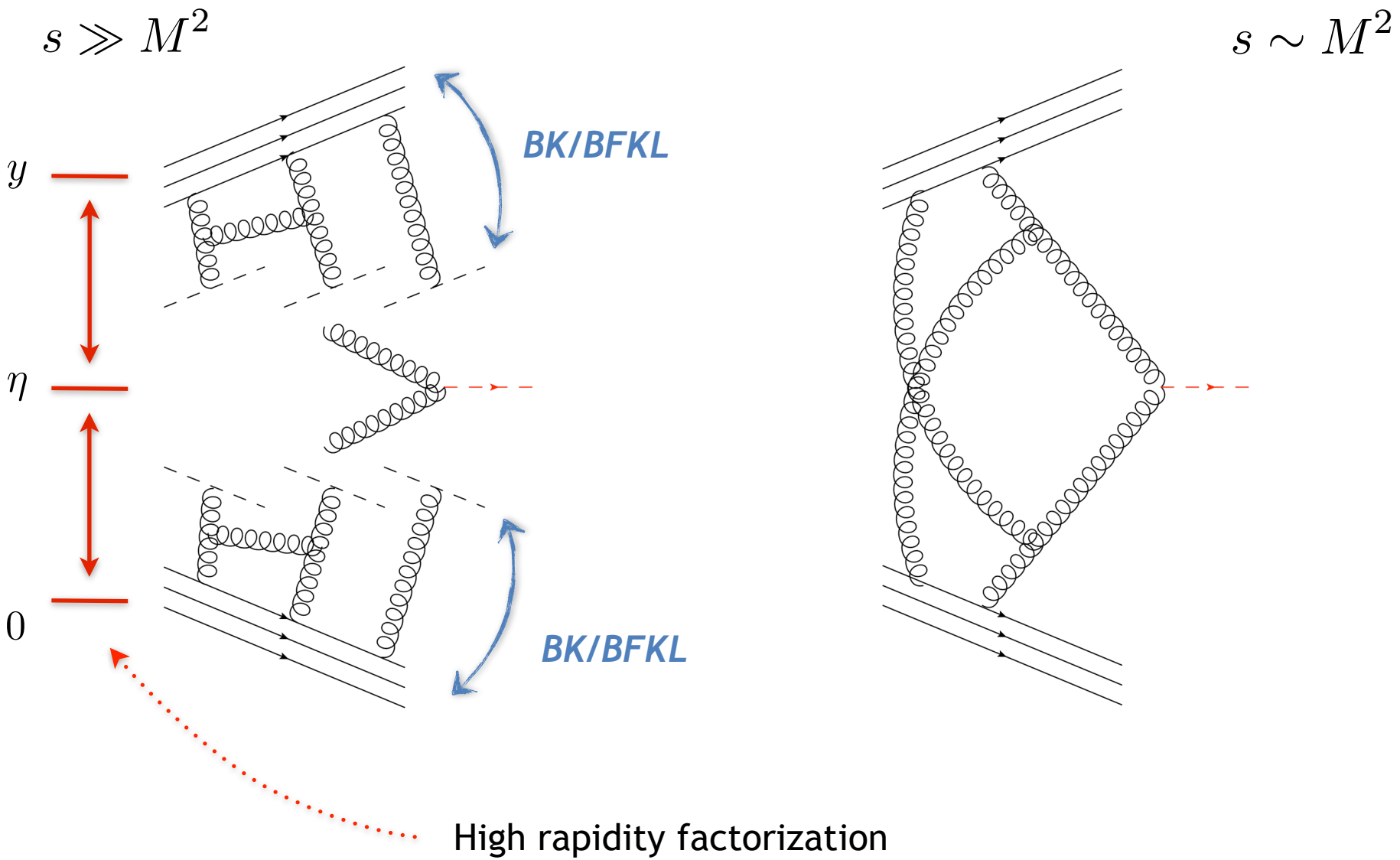


Two regimes

$s \sim M^2$

$s \gg M^2$

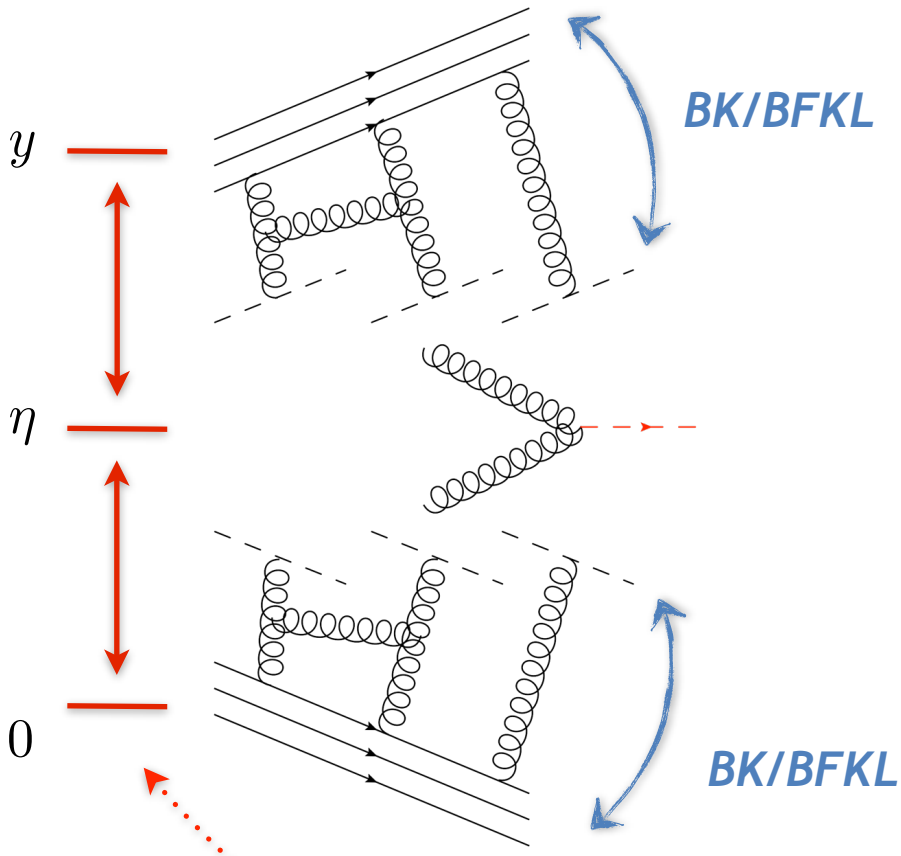
TWO REGIMES

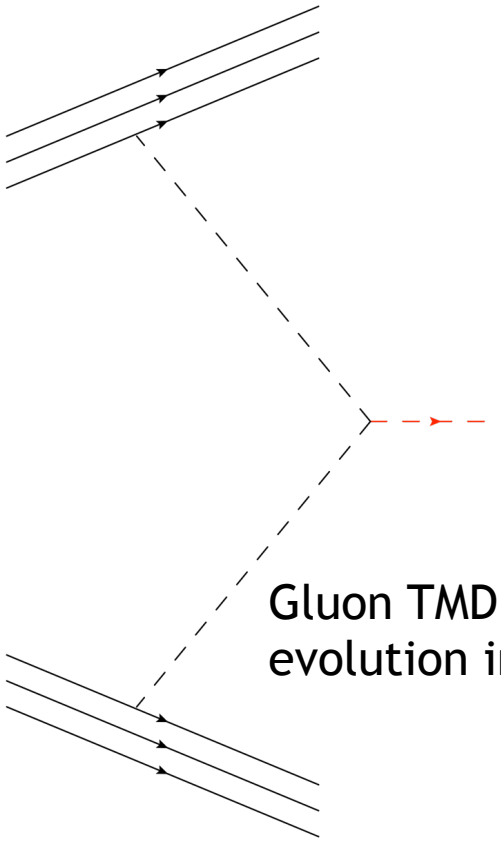

$s \gg M^2$
$s \sim M^2$
y
η
0
BK/BFKL
BK/BFKL
High rapidity factorization

TWO FORMALISMS

$s \gg M^2$

$s \sim M^2$

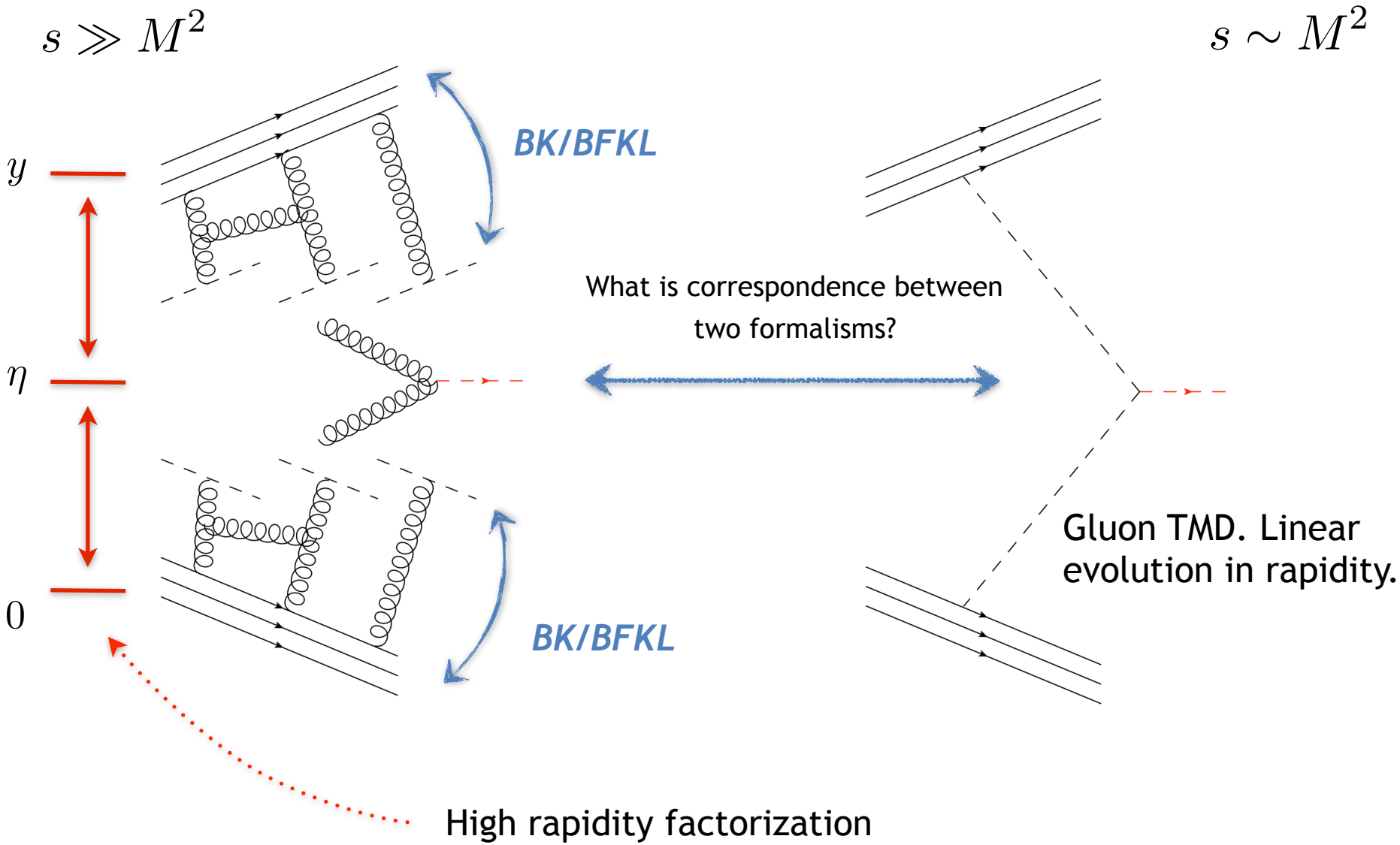




Gluon TMD. Linear evolution in rapidity.

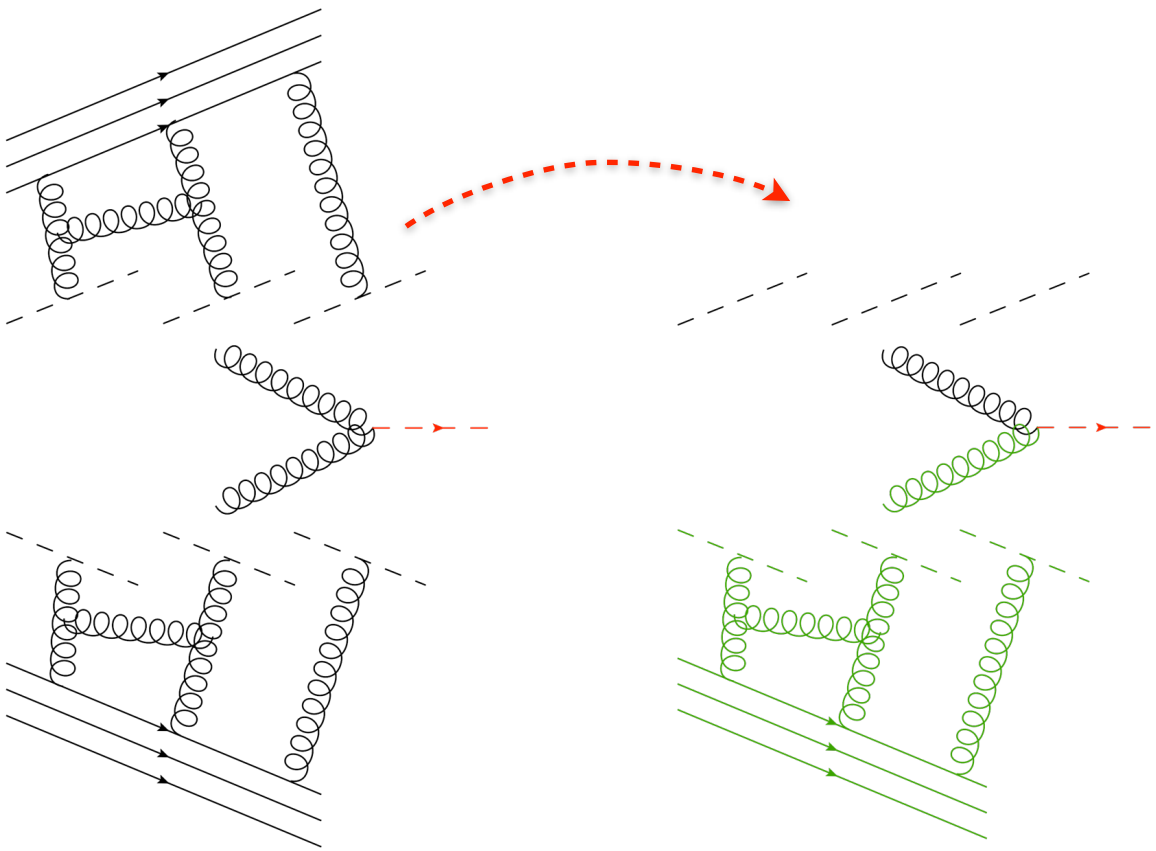
High rapidity factorization

TWO FORMALISMS



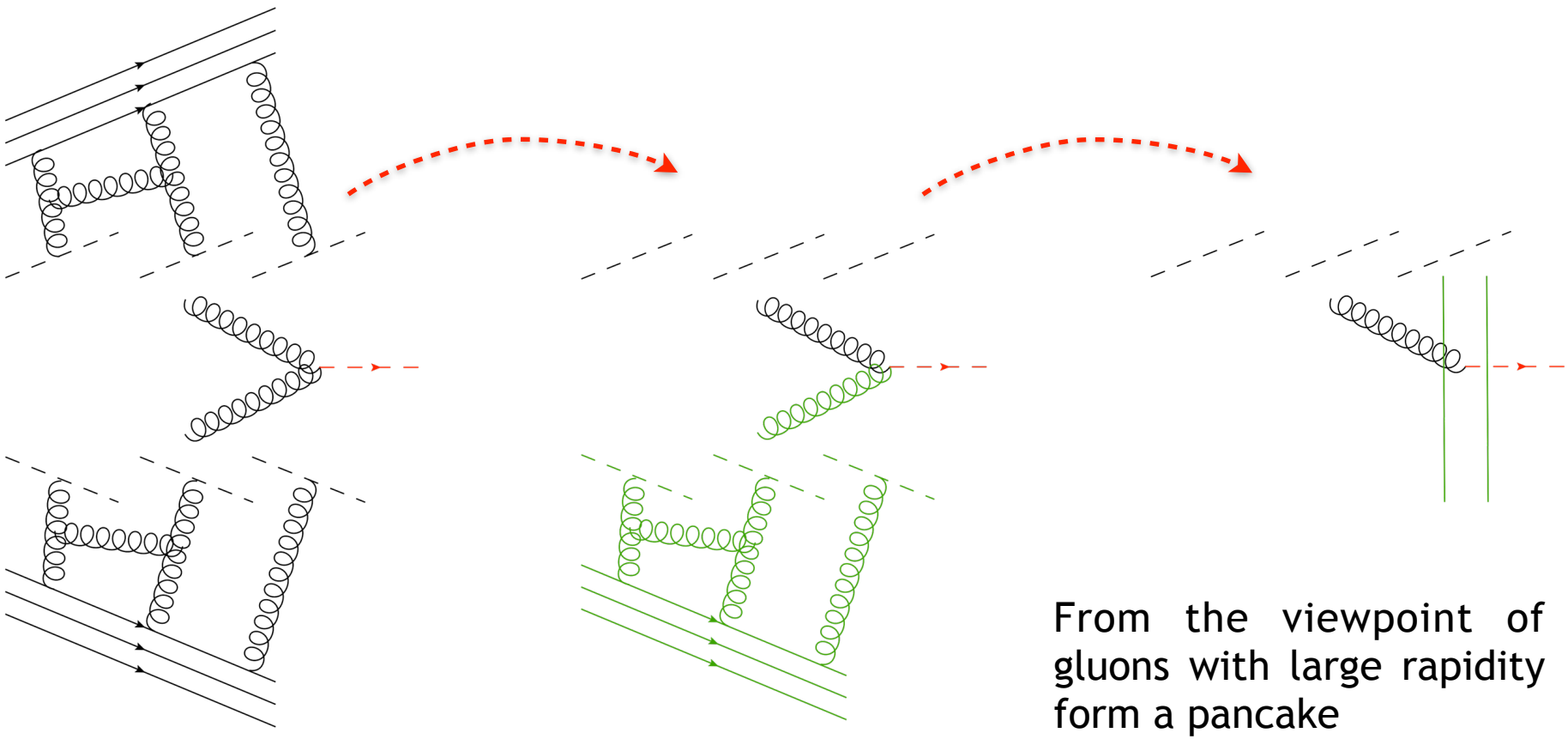
HIGH-RAPIDITY FACTORIZATION

$s \gg M^2$



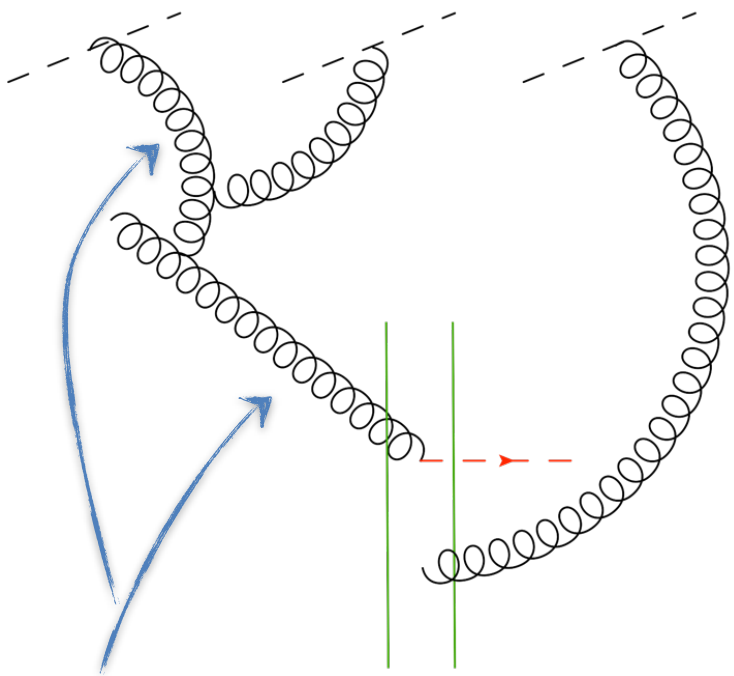
HIGH-RAPIDITY FACTORIZATION

$s \gg M^2$



From the viewpoint of gluons with large rapidity form a pancake

SCALAR PARTICLE PRODUCTION INSIDE A SHOCK-WAVE



All this gluons are inside the shock-wave

Background field

$$C_\mu = B_\mu + A_\mu$$

$$\alpha > a; \quad \alpha < a$$

QCD in background:

$$\mathcal{L} = \frac{1}{2} B^a_\mu (\mathcal{D}^2_{ab} g^{\mu\nu} - 2ig\mathcal{F}^{\mu\nu}_{ab}) B^b_\nu + \ldots$$

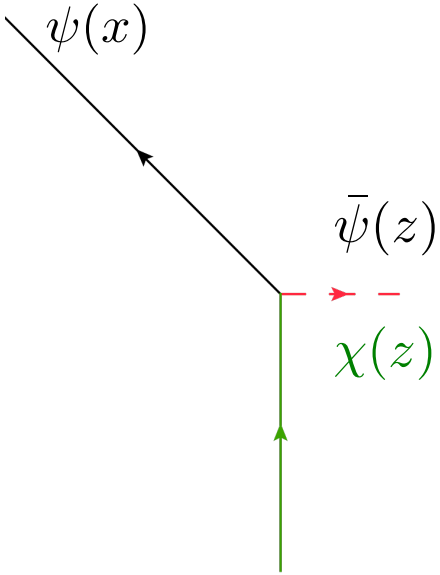
An external field is quite specific:

$$A_\bullet(z_*, z_\perp); \quad A_i = A_* = 0$$

$$\chi(z_*, z_\perp)$$

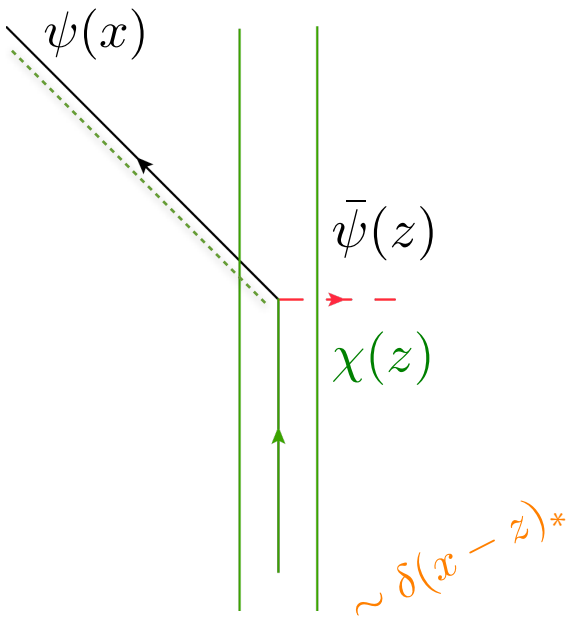
$$\not{p}_2 \chi = \gamma_* \chi = 0$$

DIAGRAM CALCULATION



$$-ig_h \int d^4z \langle \psi(x)\bar{\psi}(z)\chi(z)\phi(z)\rangle$$

DIAGRAM CALCULATION



$$-ig_h \int d^4z \langle \psi(x)\bar{\psi}(z)\chi(z)\phi(z)\rangle$$

$$g_h \int d^4z (x|\not{P}\frac{1}{\not{P}^2+i\epsilon}|z)\chi(z)\phi(z)$$

$\sim \not{p}_2$

$$(x|\not{P}\frac{1}{\not{P}^2+i\epsilon}|z) = (x|\frac{1}{(\alpha\pm i\epsilon)s}|z)\not{p}_2 + (x|(\alpha\not{p}_1+\frac{p_\perp^2}{s\alpha}\not{p}_2+\not{p}_\perp)\frac{\theta(-\alpha)}{p^2+i\epsilon}|z)[x_*,z_*] + \frac{ig}{4}(x|(\alpha\not{p}_1+\not{p}_\perp)\frac{\theta(-\alpha)}{\alpha(p^2+i\epsilon)}|z)[x_*,\sigma G,z_*]$$

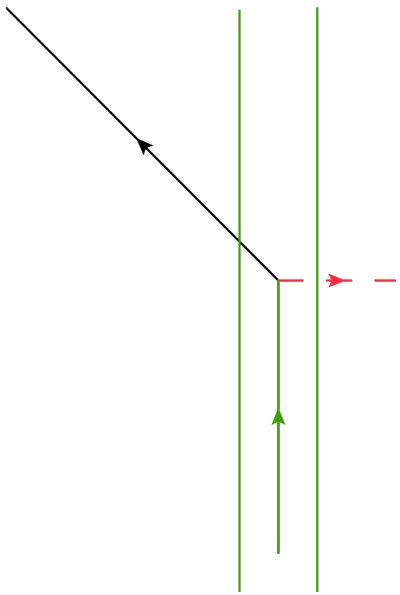
At this level we easily get factorization

$$\not{p}_2\chi = \gamma_*\chi = 0$$

$$g_h \int d^4z (x|\frac{\theta(-\alpha)}{\hat{p}+i\epsilon}|z)[-\infty,z_*]_{z_\perp}\chi(z)\phi(z)$$

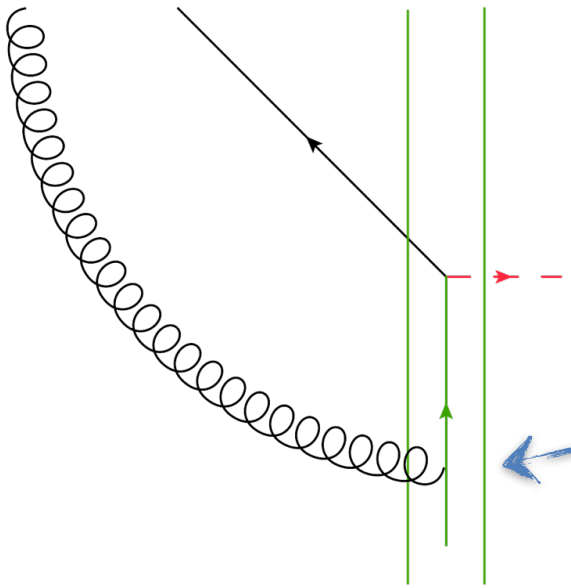
$$-ig_h \int d^4z \bar{\psi}(z)[-\infty,z_*]_{z_\perp}\chi(z)\phi(z)$$

COLOR ENTANGLEMENT



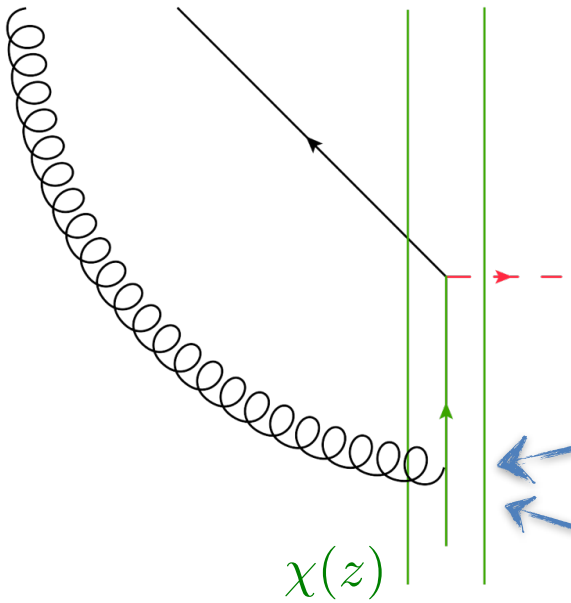
$$-ig_h \int d^4z \bar{\psi}(z)[-\infty, z_*]_{z_\perp} \chi(z)\phi(z)$$

COLOR ENTANGLEMENT



$$-ig_h \int d^4z \bar{\psi}(z)[-\infty, z_*]_{z_\perp} \chi(z)\phi(z)$$

COLOR ENTANGLEMENT



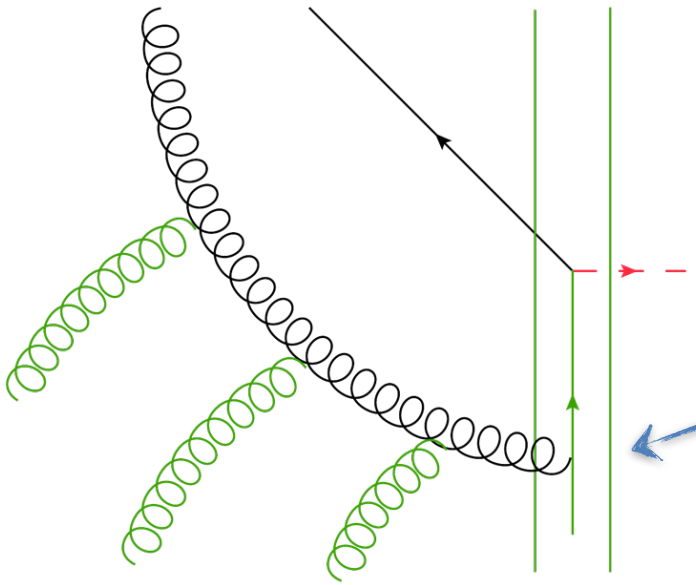
$$-ig_h \int d^4z\bar{\psi}(z)[-\infty, z_*]_{z_\perp}[z_\bullet, -\infty]_{z_\perp}\chi(z)\phi(z)$$

We get color entanglement!

$$[-\infty, z_*]t^a\chi(z)$$

To solve this problem one should take into account that the gluon is inside the shock-wave

COLOR ENTANGLEMENT



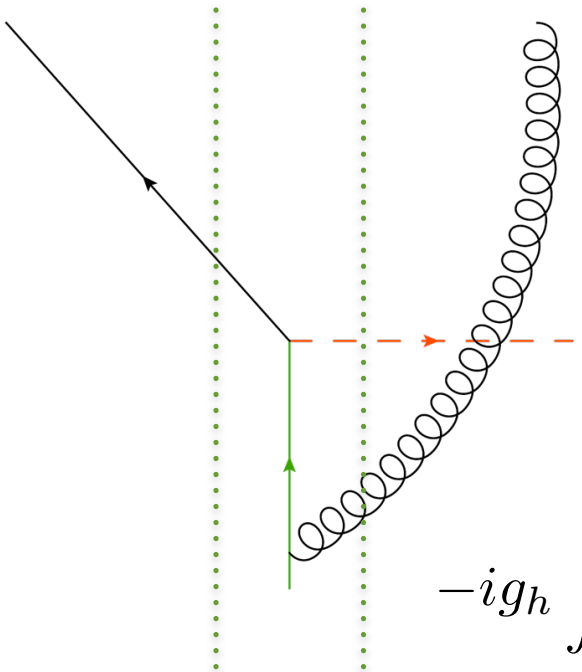
$$[-\infty, z_*]t^b[z_*, -\infty]^{ba}\chi(z) = [-\infty, z_*][z_*, -\infty]t^b[-\infty, z_*]\chi(z) = t^b[-\infty, z_*]\chi(z)$$

$$-ig_h \int d^4z\bar{\psi}(z)[-\infty, z_*]_{z_\perp}[z_\bullet, -\infty]_{z_\perp}\chi(z)\phi(z)$$

$$-ig_h \int d^4z\bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

To solve this problem one should take into account that the gluon is inside the shock-wave

COLOR ENTANGLEMENT



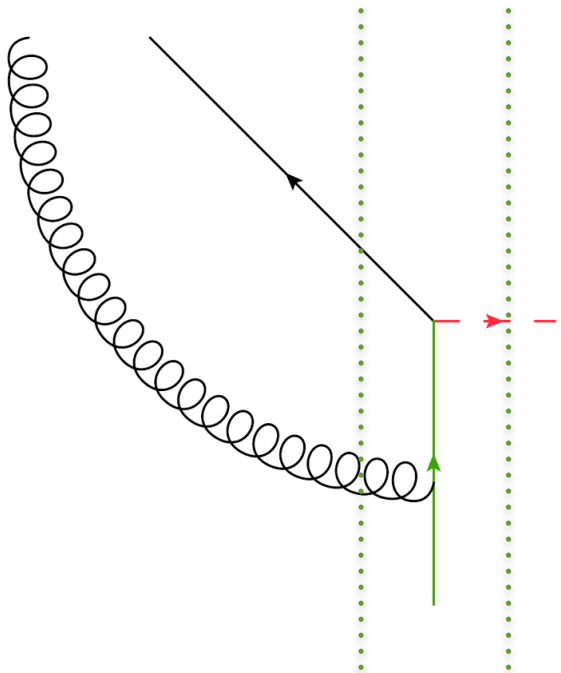
$$[-\infty, z_*]t^b[z_*, \infty]^{ba}\chi(z) =$$
$$[-\infty, z_*][z_*, \infty]t^b[\infty, z_*]\chi(z)$$
$$= [-\infty, \infty]t^b[\infty, z_*]\chi(z)$$

$$-ig_h \int d^4z\bar{\psi}(z)[-\infty, \infty]_{z_\perp}[z_\bullet, -\infty]_{z_\perp}[\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

Can't move this line to restore factorization

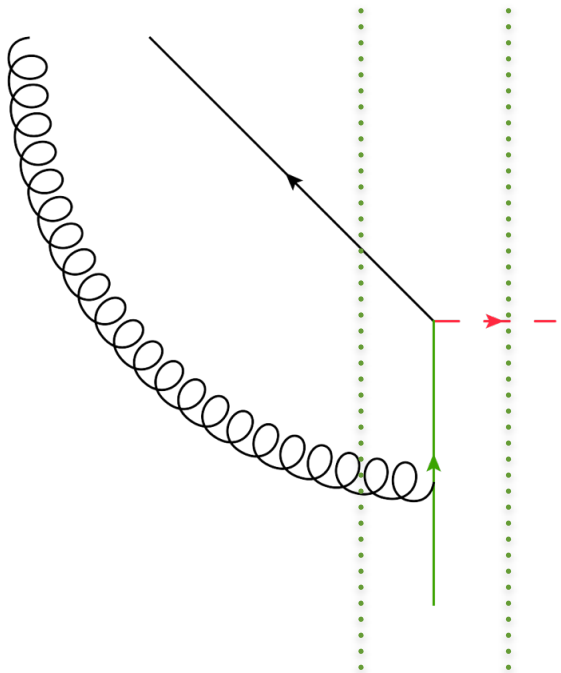
Solution will be presented on the future slides

COLOR ENTANGLEMENT

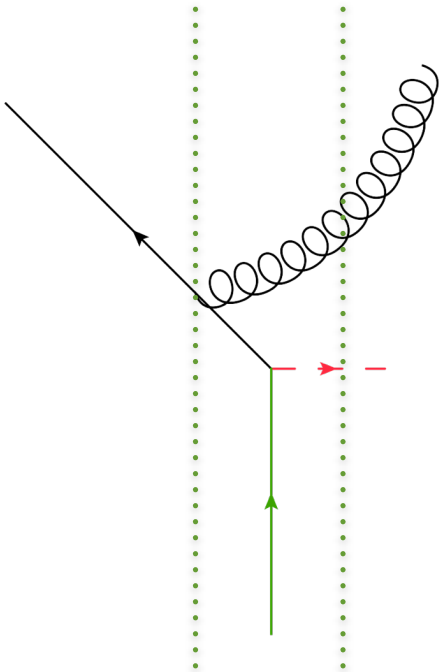


$$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

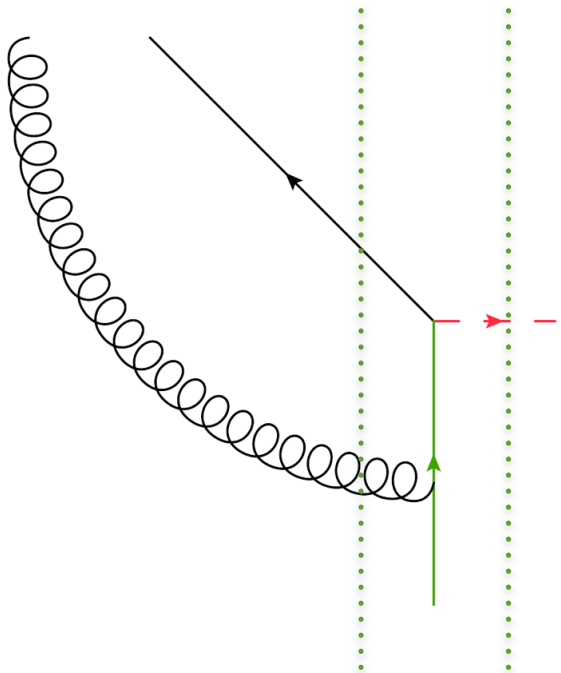
COLOR ENTANGLEMENT



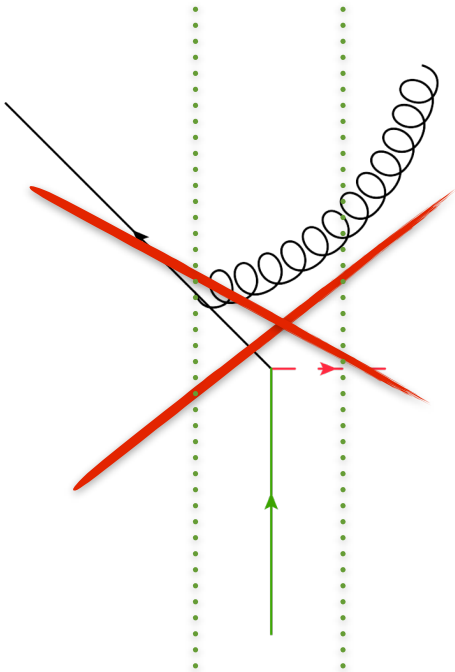
$$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$



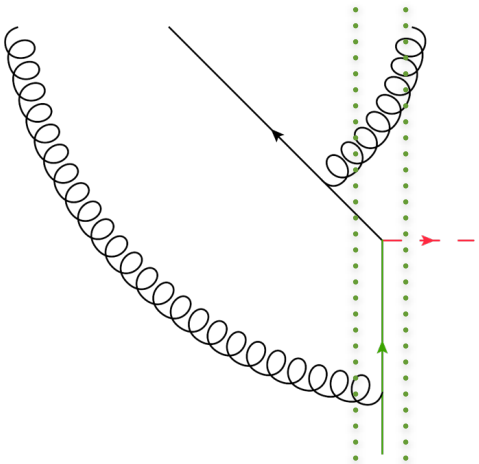
COLOR ENTANGLEMENT

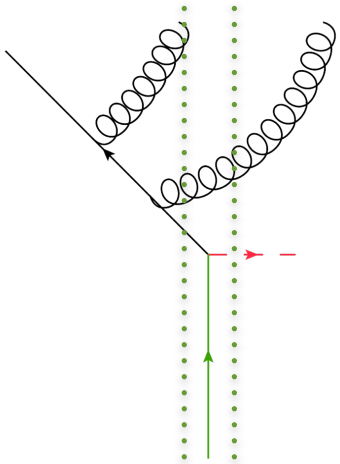


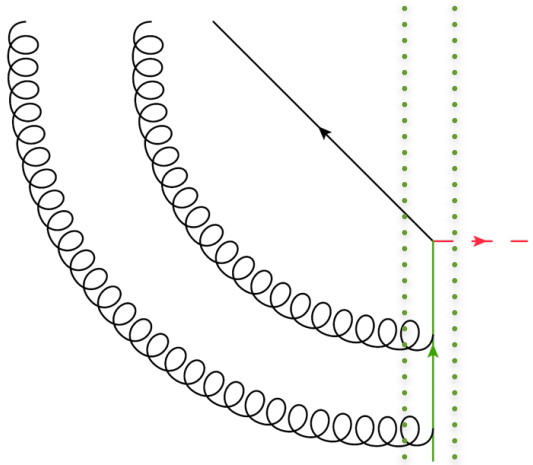
$$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

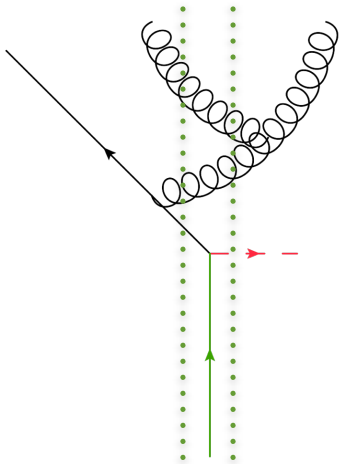


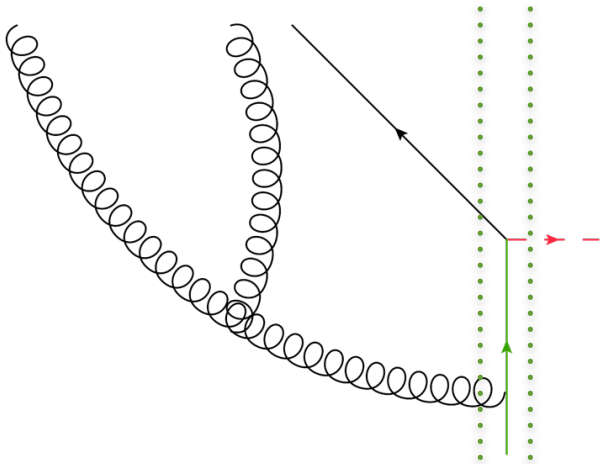
HIGHER ORDER CORRECTIONS





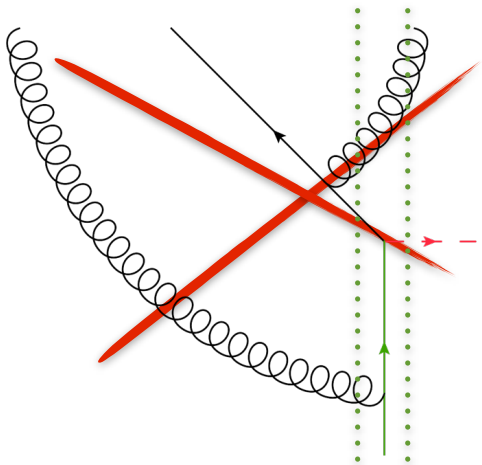


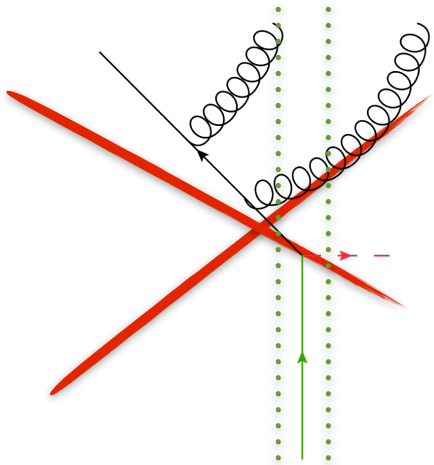


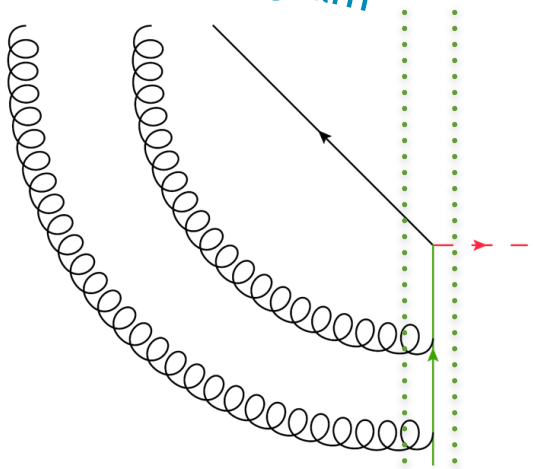


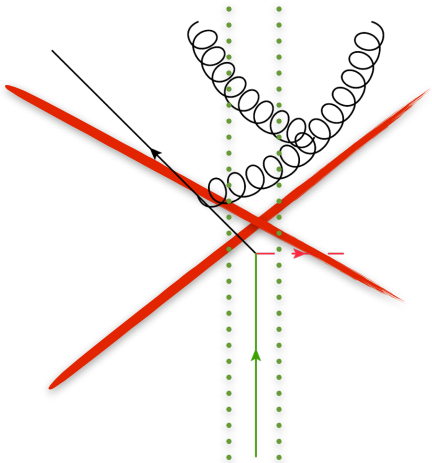
HIGHER ORDER CORRECTIONS

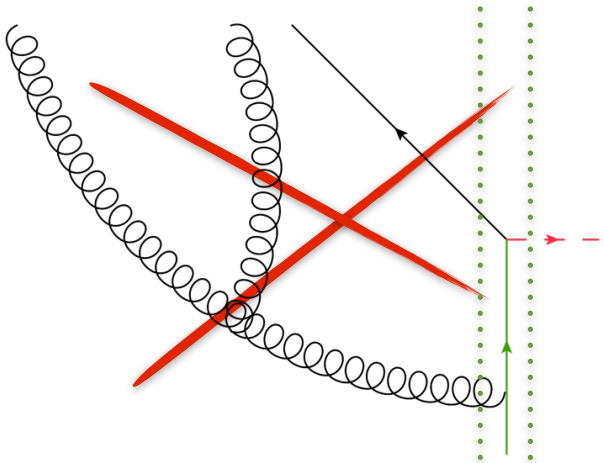
The only non-zero diagram



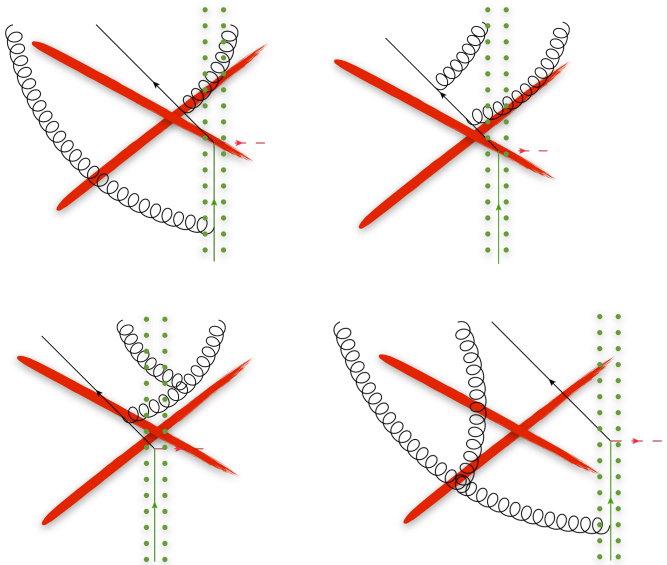




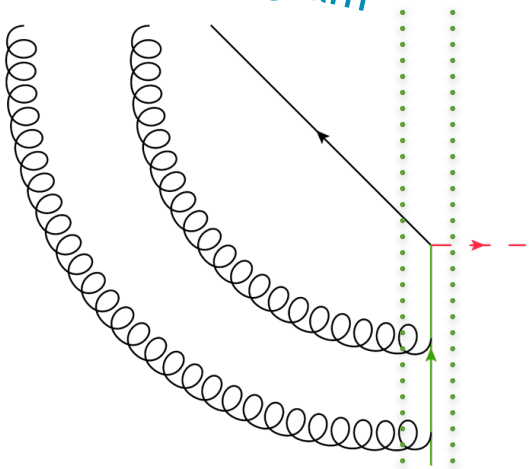




HIGHER ORDER CORRECTIONS



The only non-zero diagram



$$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

AXIAL AND FEYNMAN GAUGE

Feynman gauge: $$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

AXIAL AND FEYNMAN GAUGE

No such Wilson line in axial gauge

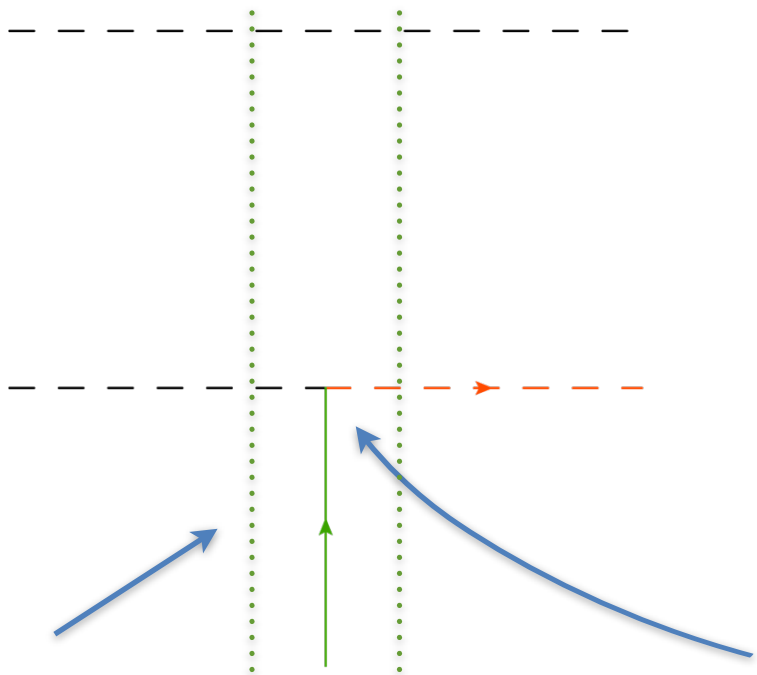
Feynman gauge: $$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

Axial gauge: $$-ig_h \int d^4z \bar{\psi}(z)[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$$

Axial and Feynman gauge

Feynman gauge: $-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$

Axial gauge: $-ig_h \int d^4z \bar{\psi}(z)[-\infty, z_*]_{z_\perp}\chi(z)\phi(z)$



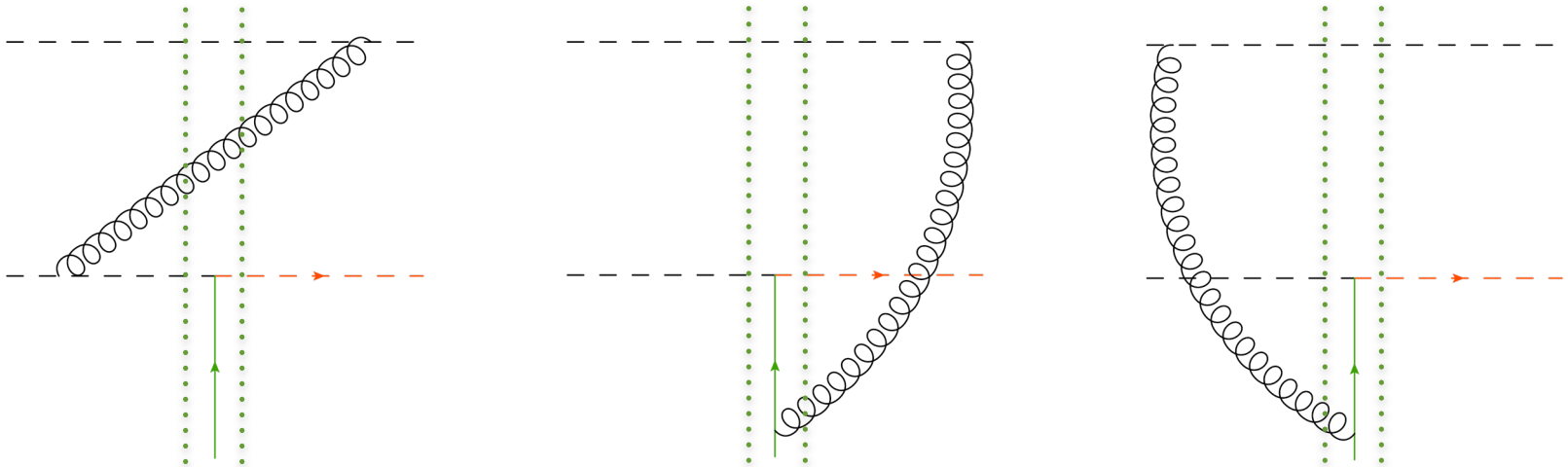
We calculate corrections to this diagram in both Feynman and axial gauge

$$\int d^4z[-\infty, z_*]\chi(z)\phi(z)$$

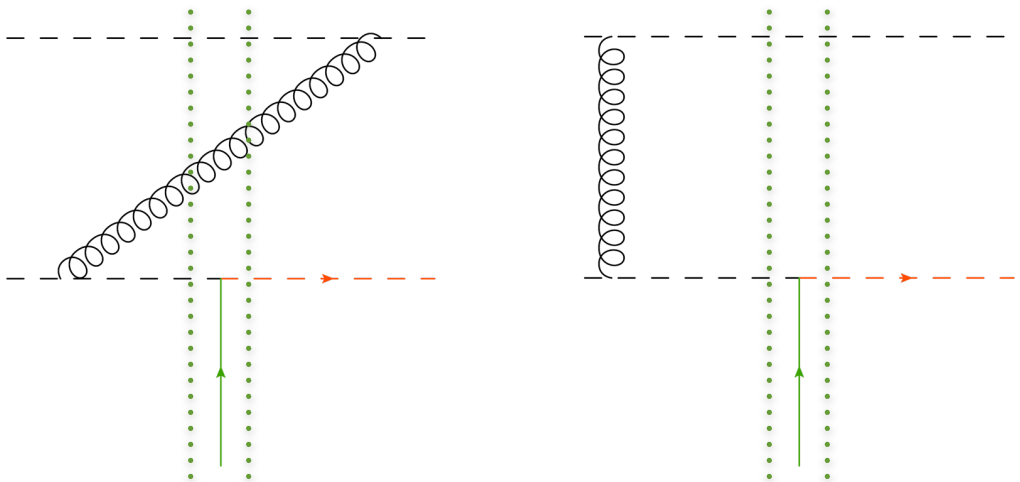
The shockwave

AXIAL AND FEYNMAN GAUGE

Feynman gauge:



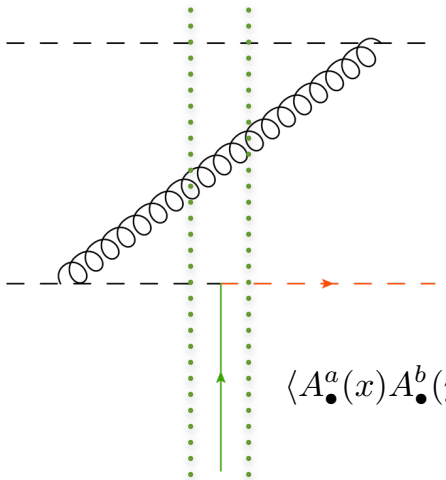
Axial gauge:



We calculated all these diagrams

Axial and Feynman gauge

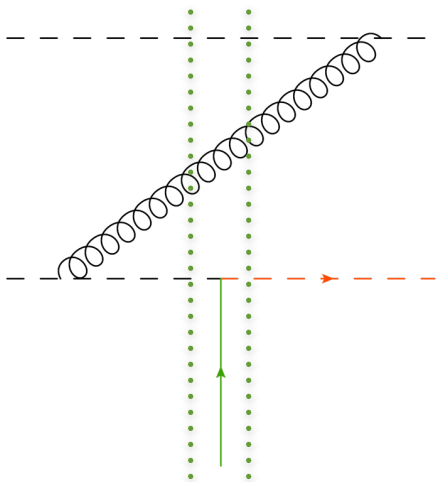
Feynman gauge:



$$\langle A^a_\bullet(x) A^b_\bullet(y)\rangle = \int_0^\infty \frac{d\alpha}{2\pi}\frac{e^{-i\alpha(x-y)_\bullet}}{\alpha}\Big\{-(x_\perp|2\frac{p_i}{p_\perp^2}U\frac{p_i}{p_\perp^2}|y_\perp)^{ab} + (x_\perp|U\frac{1}{p_\perp^2}|y_\perp)^{ab} + (x_\perp|\frac{1}{p_\perp^2}U|y_\perp)^{ab}\Big\}$$

1 2 3

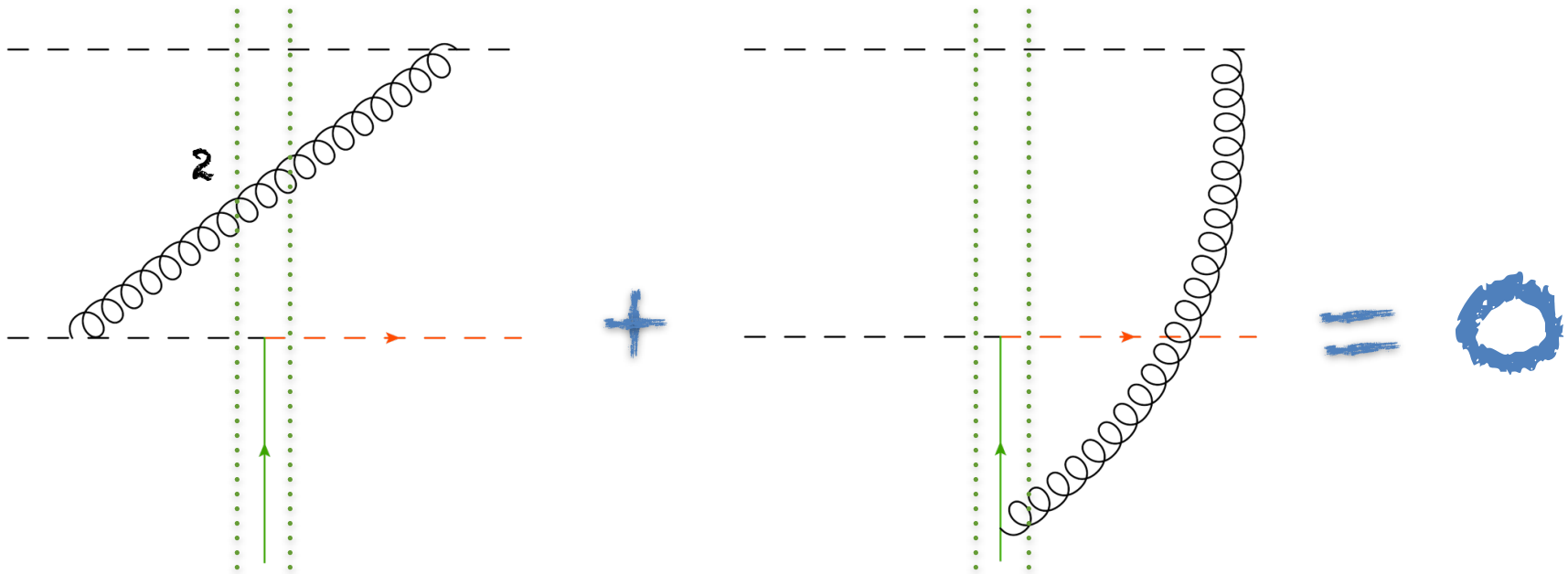
Axial gauge:



$$\langle A^a_\bullet(x) A^b_\bullet(y)\rangle = -\int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{\alpha}e^{-i\alpha(x-y)_\bullet} \times (x_\perp|2\frac{p_i}{p_\perp^2}U\frac{p_i}{p_\perp^2}|y_\perp)$$

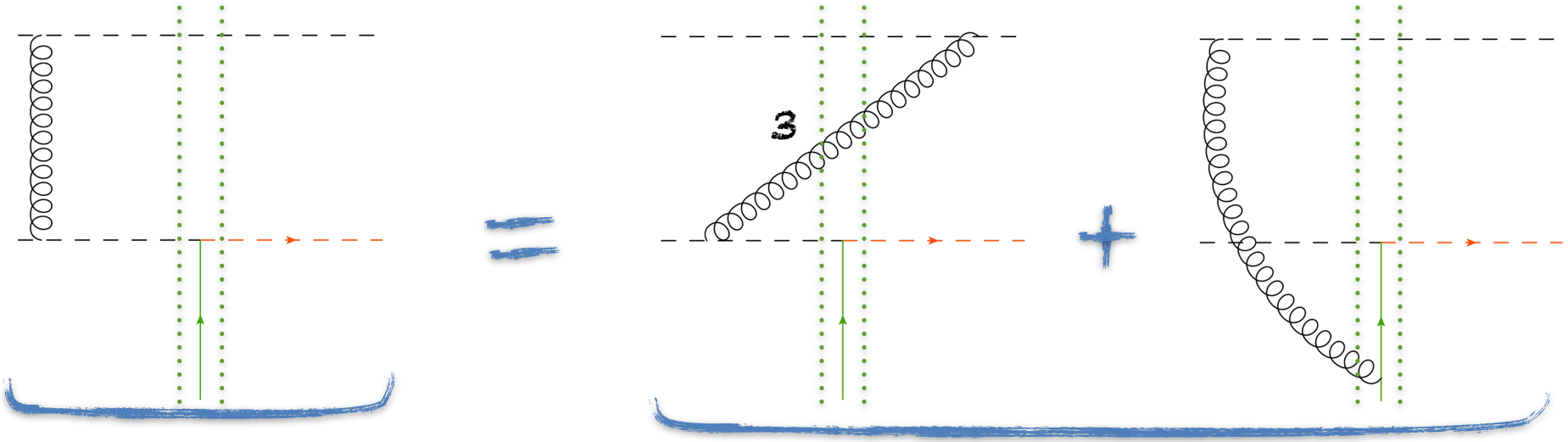
AXIAL AND FEYNMAN GAUGE

Feynman gauge:



Solution to color entanglement problem.

AXIAL AND FEYNMAN GAUGE



Axial gauge

Feynman gauge

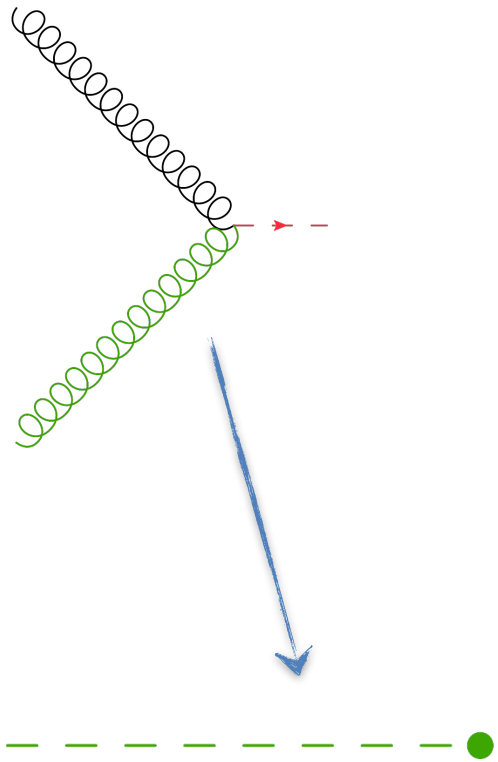
The result is the same in both gauges

$$-ig_h \int d^4z \bar{\psi}(z)[-\infty, z_*]_{z_\perp} \chi(z)\phi(z)$$

$$-ig_h \int d^4z \bar{\psi}(z)[z_\bullet, -\infty]_{z_\perp}[-\infty, z_*]_{z_\perp} \chi(z)\phi(z)$$

There is no direct correspondence between this two results

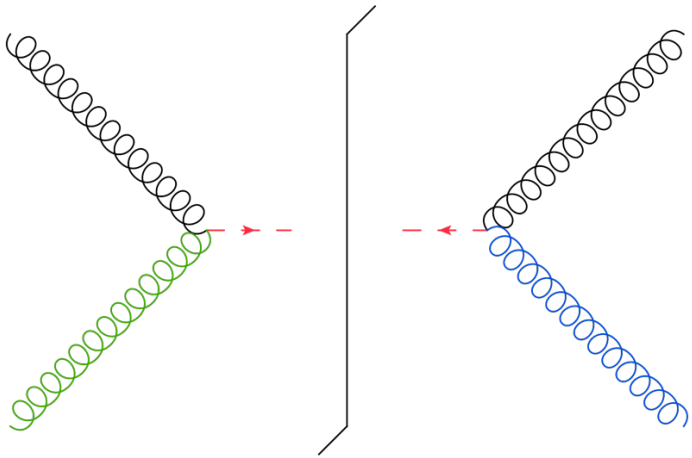
EVOLUTION EQUATIONS



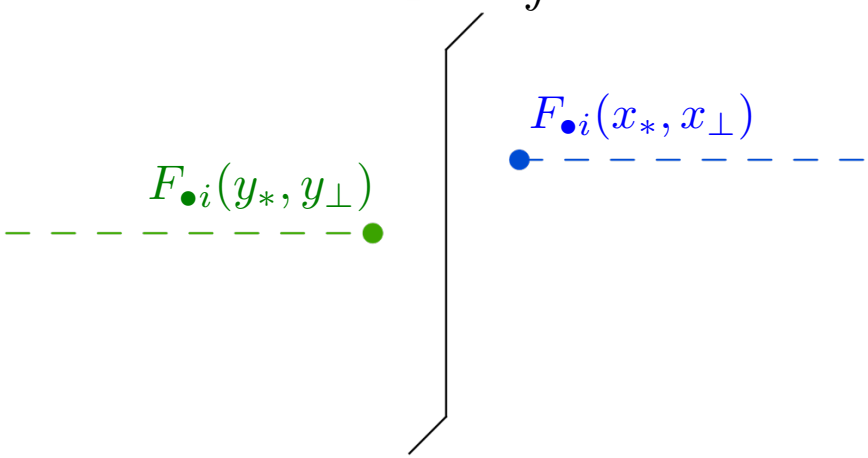
$$\frac{2i}{s}\int d^4y F^c_{*i}(y)[-\infty, y_*]^{cb} F^b_{\bullet i}(y)\Phi(y)$$

$$\frac{2i}{s}\int dy_* F^b_{\bullet i}(y_*, y_\perp)[y_*, -\infty]^{bc} e^{i\beta_h y_*}$$

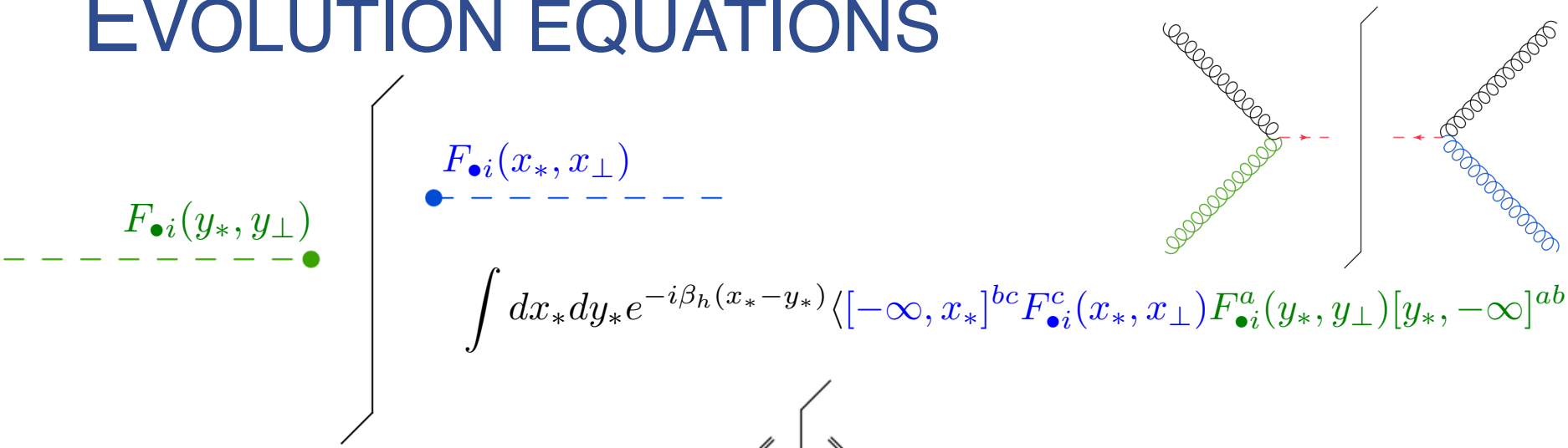
EVOLUTION EQUATIONS



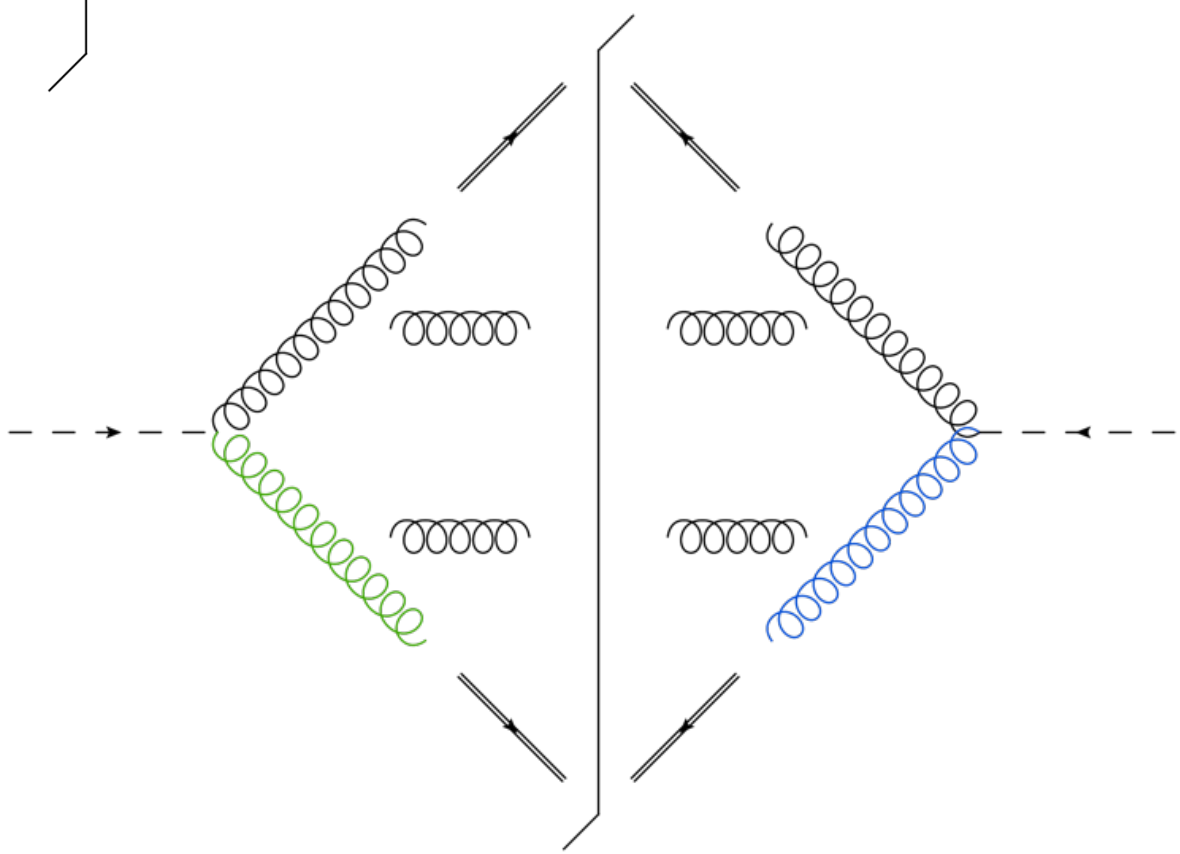
$$\int dx_* dy_* e^{-i\beta_h(x_*-y_*)} \langle [-\infty, x_*]^{bc} F^c_{\bullet i}(x_*, x_\perp) F^a_{\bullet i}(y_*, y_\perp) [y_*, -\infty]^{ab} \rangle$$



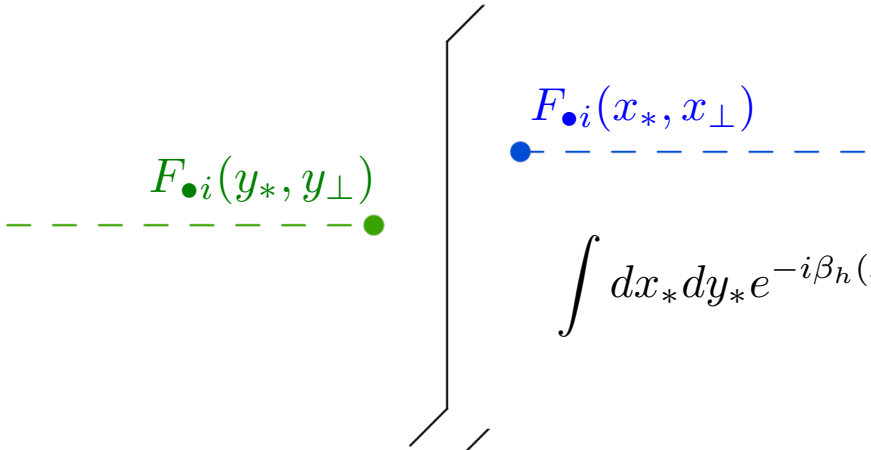
EVOLUTION EQUATIONS

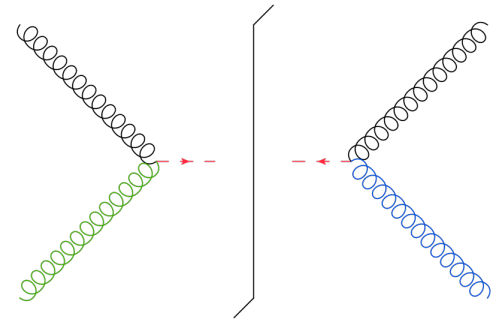


$$\int dx_* dy_* e^{-i\beta_h(x_*-y_*)} \langle [-\infty, x_*]^{bc} F^c_{\bullet i}(x_*, x_\perp) F^a_{\bullet i}(y_*, y_\perp) [y_*, -\infty]^{ab} \rangle$$

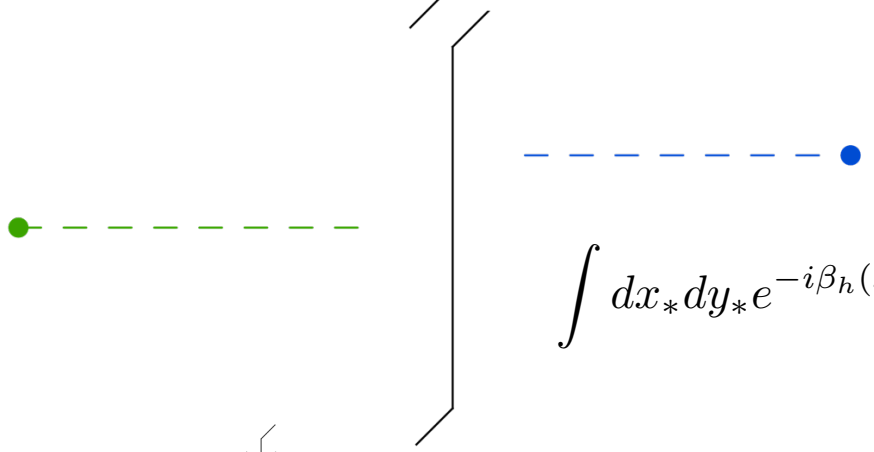


EVOLUTION EQUATIONS

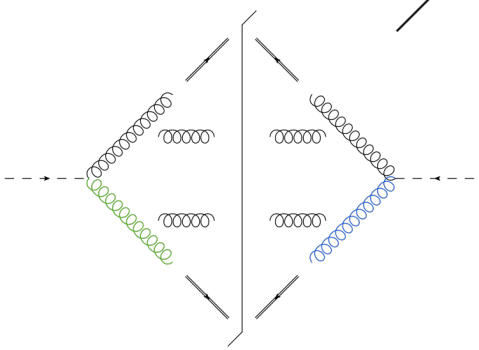




$$\int dx_* dy_* e^{-i\beta_h(x_*-y_*)} \langle [-\infty, x_*]^{bc} F^c_{\bullet i}(x_*, x_\perp) F^a_{\bullet i}(y_*, y_\perp) [y_*, -\infty]^{ab} \rangle$$

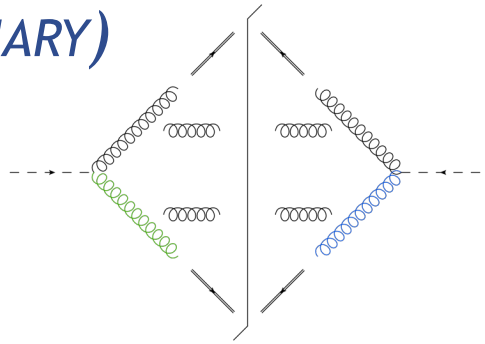


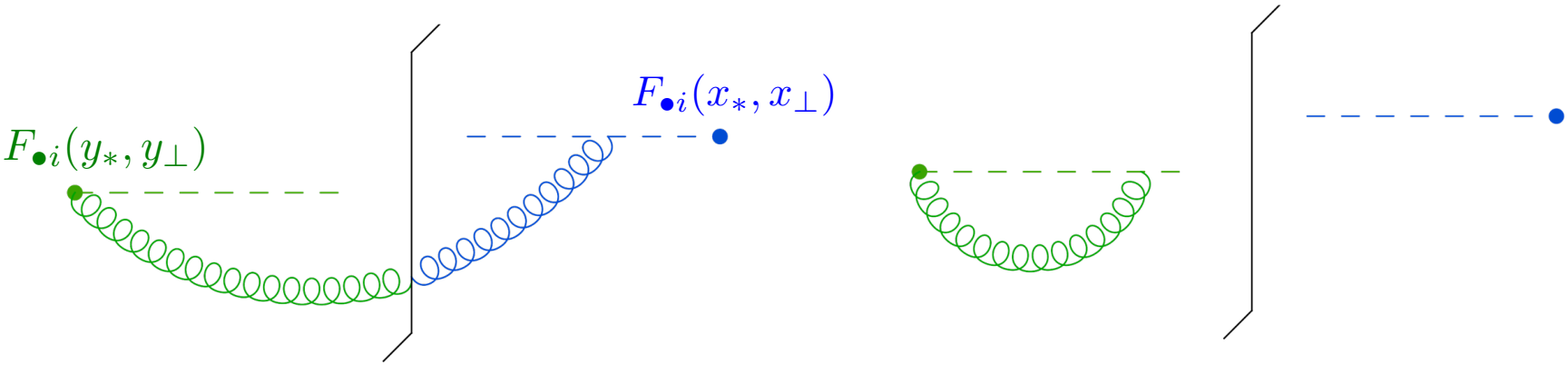
$$\int dx_* dy_* e^{-i\beta_h(x_*-y_*)} \langle F^c_{\bullet i}(x_*, x_\perp) [x_*, \infty]^{cb} [\infty, y_*]^{ba} F^a_{\bullet i}(y_*, y_\perp) \rangle$$



Direction of Wilson lines depends on whether we consider annihilation or particle production

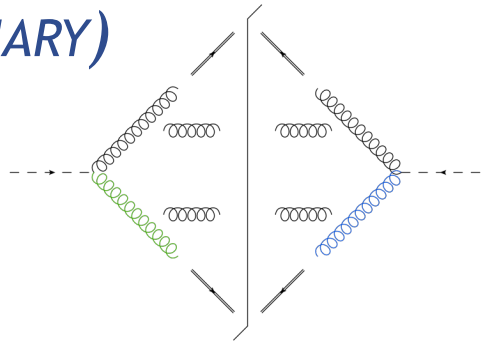
EVOLUTION EQUATIONS *(PRELIMINARY)*

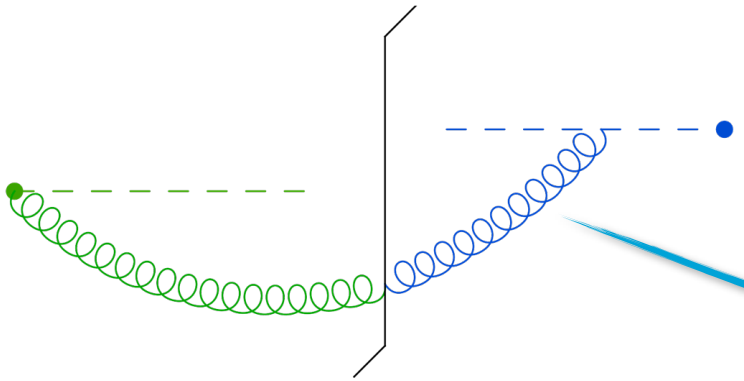




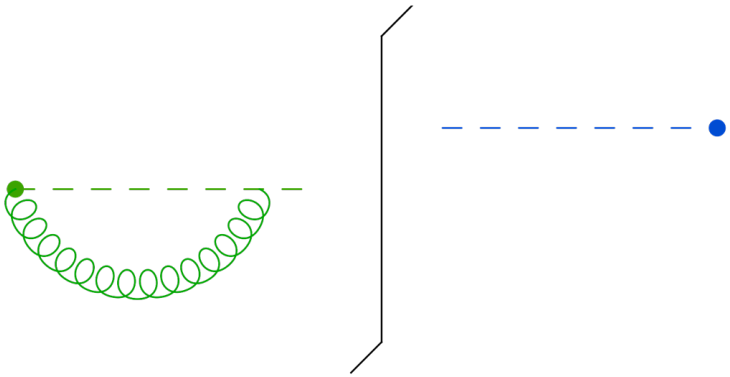
This diagrams are responsible for evolution

EVOLUTION EQUATIONS *(PRELIMINARY)*

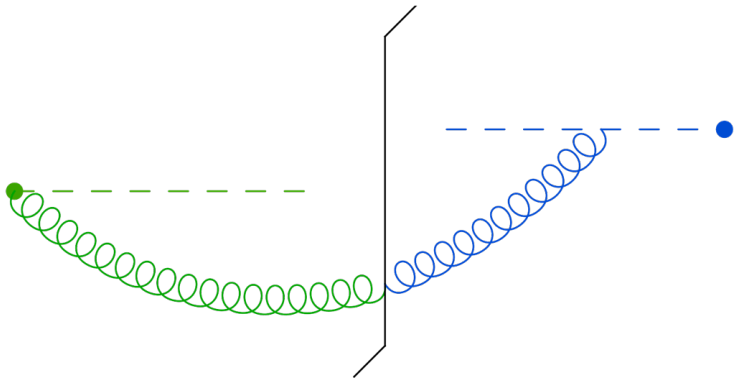


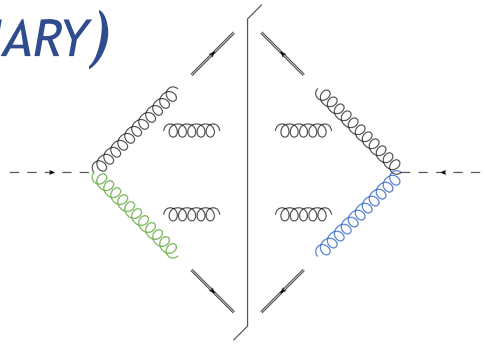


$$N_c \int dx_* dy_* e^{-i\beta_h(x_*-y_*)} F^a_{\bullet i}(x_*, x_\perp)[x_*, \infty]^{ac}[\infty, y_*]^{cb} F^b(y_*, y_\perp) \int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{\alpha}[(x_\perp|\frac{1}{p_\perp^2}|y_\perp) - (y_\perp|\frac{1}{p_\perp^2}|y_\perp)]$$



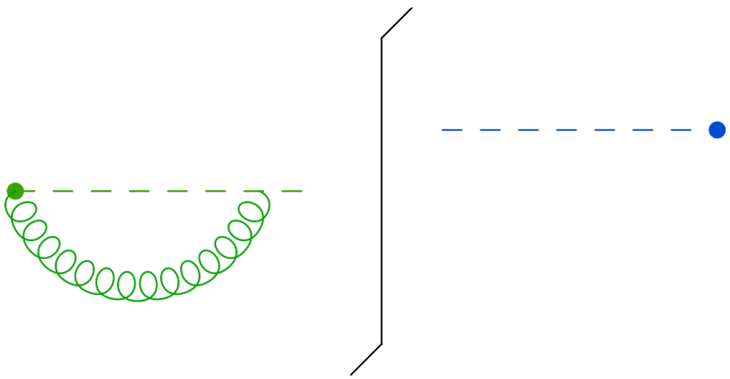
EVOLUTION EQUATIONS *(PRELIMINARY)*





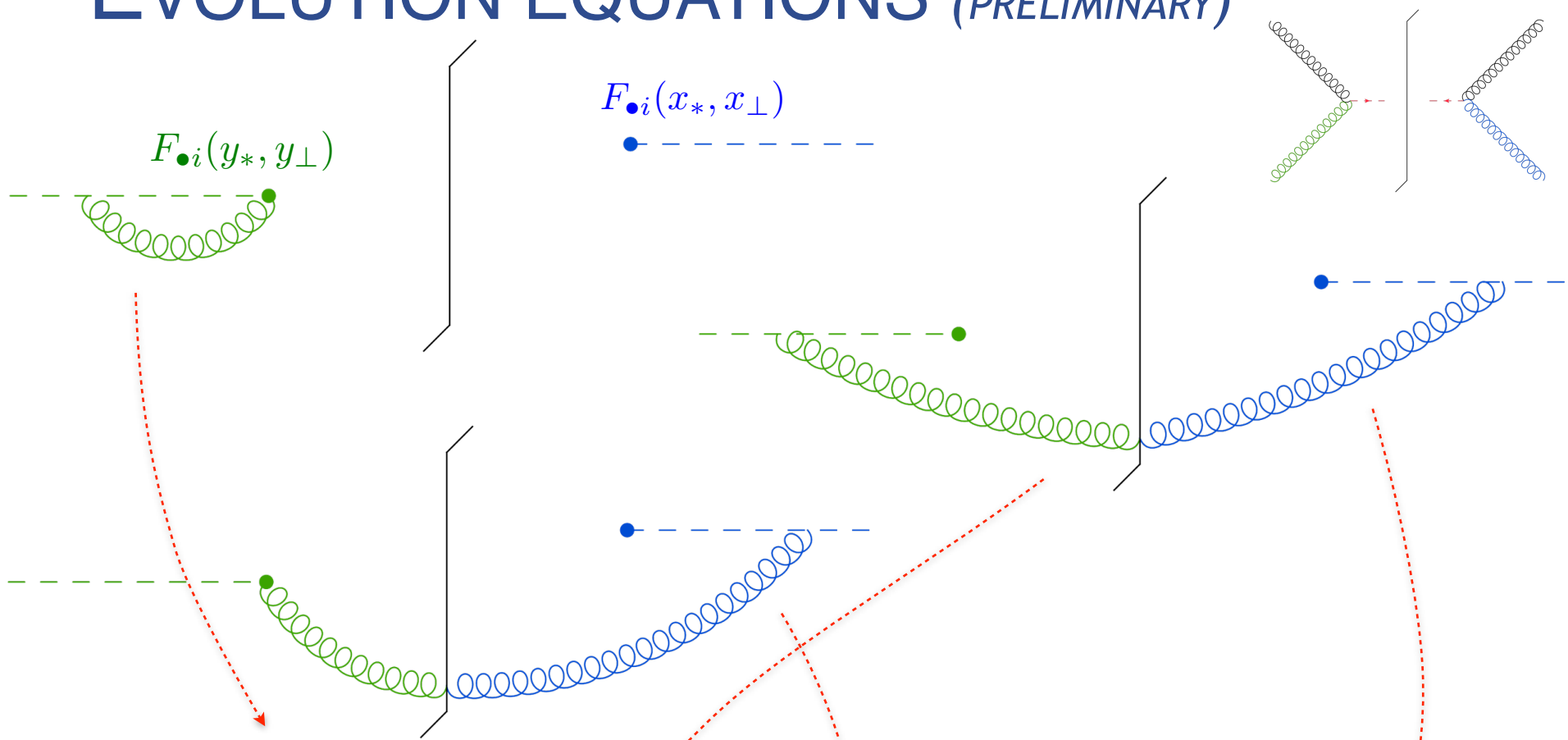
No infrared divergence

$$N_c \int dx_* dy_* e^{-i\beta_h(x_*-y_*)} F^a_{\bullet i}(x_*, x_\perp)[x_*, \infty]^{ac}[\infty, y_*]^{cb} F^b(y_*, y_\perp) \int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{\alpha}[(x_\perp|\frac{1}{p_\perp^2}|y_\perp) - (y_\perp|\frac{1}{p_\perp^2}|y_\perp)]$$



Ultraviolet divergence should be regularized

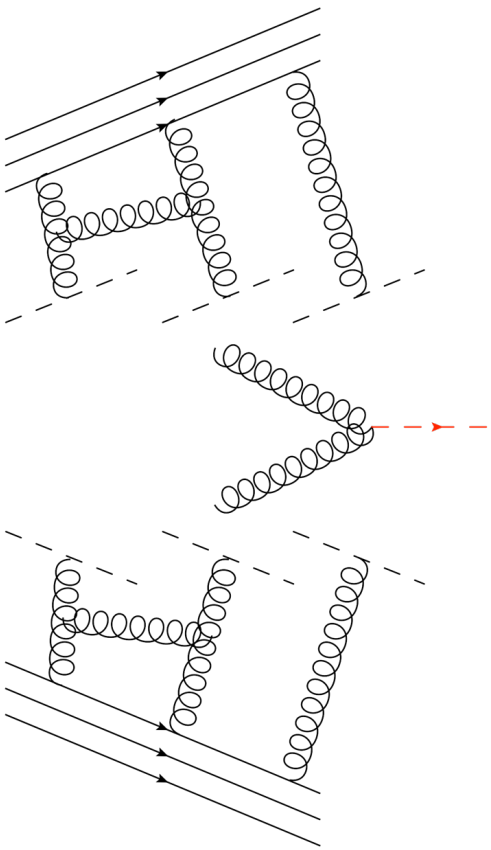
EVOLUTION EQUATIONS *(PRELIMINARY)*



$$-2N_c \mathcal{O}^m_{x_\perp} \mathcal{O}^m_{y_\perp} \int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{2\alpha}\int \frac{d^2p}{(2\pi)^2}\frac{1}{p_\perp^2} - 2T^a_{dk}T^b_{ld}\mathcal{O}^k_{x_\perp}(U^\dagger_{y_\perp}U_{y_\perp})^{ab}\mathcal{O}^l_{y_\perp}\int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{\alpha}(x_\perp|\frac{1}{p_\perp^2}|y_\perp)$$

$$+2T^a_{dk}T^b_{ld}\mathcal{O}^k_{x_\perp}(U^\dagger_{y_\perp}U_{y_\perp})^{ab}\mathcal{O}^l_{y_\perp}\int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{\alpha}(x_\perp|\frac{1}{p_\perp^2}|y_\perp) + 4T^a_{dk}T^b_{ld}\mathcal{O}^k_{x_\perp}\mathcal{O}^l_{y_\perp}\int_0^\infty \frac{d\alpha}{2\pi}\frac{1}{\alpha}(x_\perp|\frac{p_i}{p_\perp^2}(U^\dagger U)^{ab}\frac{p_i}{p_\perp^2}|y_\perp)$$

Infrared divergence still needs to be cancelled

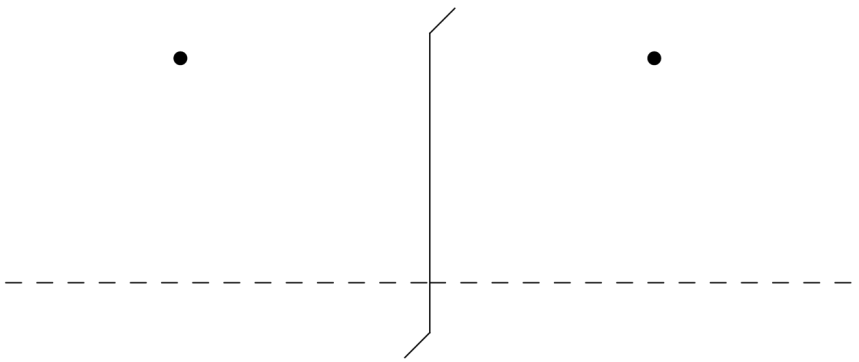


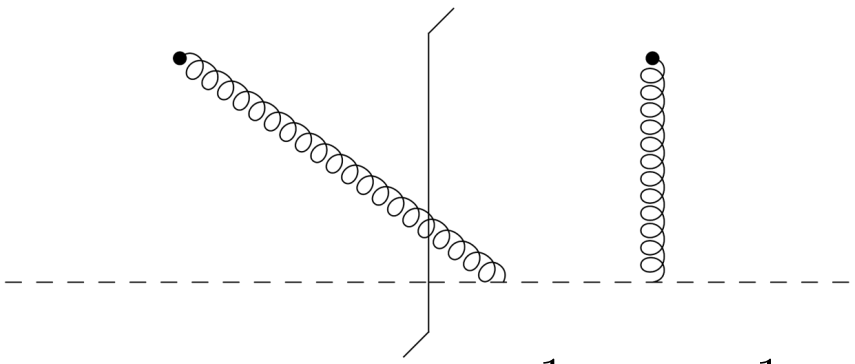
Thank you for your attention

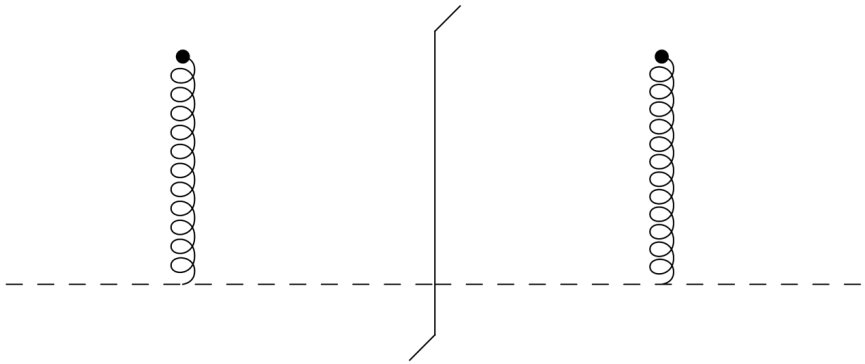
BACKUP SLIDES

Example in QED

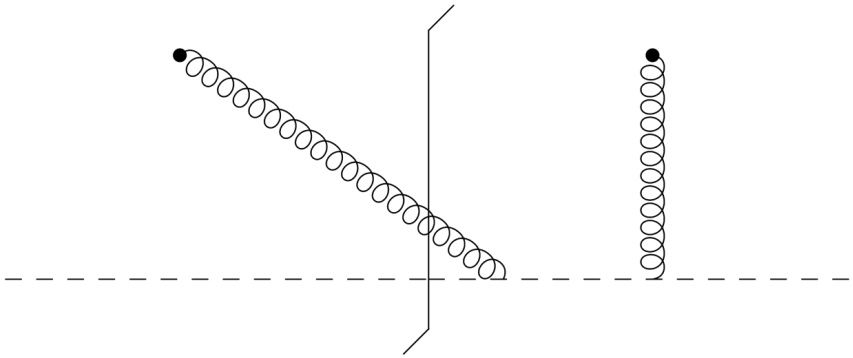
$$\sum_X \langle 0|F_{\mu\alpha}(x)[-\infty,\infty]_{z_\perp}|X\rangle\langle X|[\infty,-\infty]_{z_\perp}F_{\nu\alpha}(y)|0\rangle$$
$$= \int \mathcal{D}A e^{-iS} F_{\mu\alpha}(x)[-\infty,\infty] \int \mathcal{D}B e^{iS}[\infty,-\infty]F_{\mu\beta}(y)$$







$$\frac{1}{\beta - i\epsilon} - \frac{1}{\beta + i\epsilon} = 2\pi\delta(\beta)$$



$$\neq \sum_X \langle 0|F_{\mu\alpha}(x)|X\rangle\langle X|F_{\nu\alpha}(y)|0\rangle$$

EFFECTIVE ACTION

$$C_\mu = B_\mu + A_\mu$$

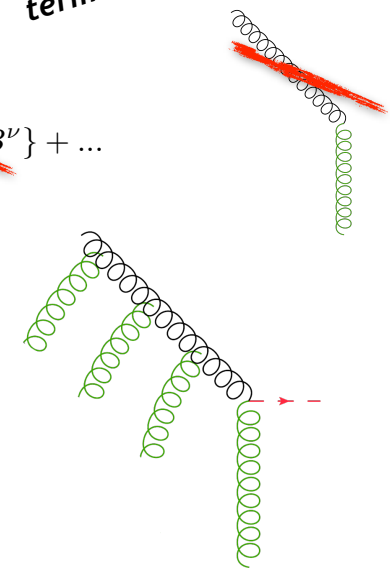
$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2$$

QCD

QCD in background field

$$\mathcal{L} = \frac{1}{2}B^a_\mu(\mathcal{D}^2_{ab}g^{\mu\nu} - 2ig\mathcal{F}^{\mu\nu}_{ab})B^b_\nu + 2Tr\{\cancel{\mathcal{D}^\mu F_{\mu\nu}B^\nu}\} + \ldots$$

with some surface terms



$$\int \mathcal{D}C \exp\Big\{iS(C) - \frac{i}{2}\int d^4z F^{\mu\nu}F_{\mu\nu}\Big\}$$

Subtract linear term to generate the same diagrams

This terms will generate an external field in QCD

Shock-wave background:

$$A_* = 0; \quad A_\bullet(z_*, z_\perp); \quad A_* = 0; \quad A_i = 0$$

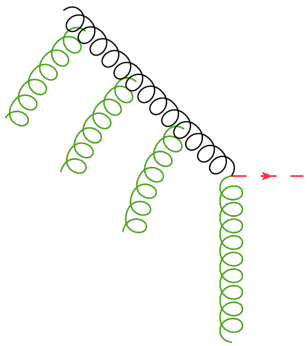
Source term for external field:

$$\exp\Big(-\frac{2i}{s}\int d^4z F_{i*}F_{i\bullet}\Big) = \exp\Big(\frac{2i}{s}\int dz_* d^2z_\perp [A_i(\infty p_2) - A_i(-\infty p_2)]F_{i\bullet}\Big)$$

EFFECTIVE ACTION

Shock-wave background:

$$A_* = 0; \quad A_\bullet(z_*, z_\perp); \quad A_* = 0; \quad A_i = 0$$



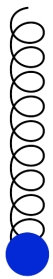
Source term for external field:

$$\exp\Big(-\frac{2i}{s}\int d^4z F_{i*}F_{i\bullet}\Big) = \exp\Big(\frac{2i}{s}\int dz_* d^2z_\perp [A_i(\infty p_2) - A_i(-\infty p_2)]F_{i\bullet}\Big)$$

We checked it at least in the first order

Generation of external field:

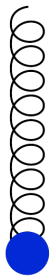
$$\int \mathcal{D}C\; C_\mu \exp\Big\{iS(C) + \frac{2i}{s}\int dz_* d^2z_\perp [A_i(\infty p_2) - A_i(-\infty p_2)]F_{i\bullet}\Big\} = A_\mu$$



EFFECTIVE ACTION

Shock-wave background:

$$A_* = 0; \quad A_\bullet(z_*, z_\perp); \quad A_* = 0; \quad A_i = 0$$



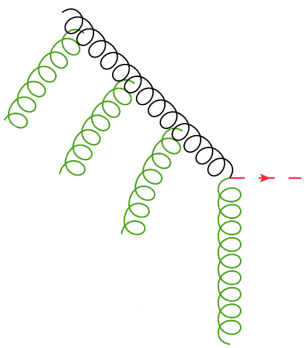
Source term for external field:

$$\exp\Big(-\frac{2i}{s}\int d^4z F_{i*}F_{i\bullet}\Big) = \exp\Big(\frac{2i}{s}\int dz_* d^2z_\perp [A_i(\infty p_2) - A_i(-\infty p_2)]F_{i\bullet}\Big)$$

We checked it at least in the first order

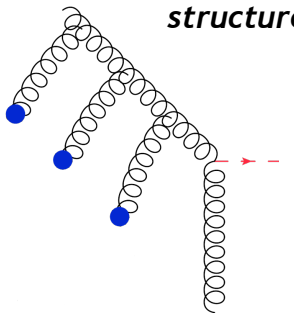
Generation of external field:

$$\int \mathcal{D}C\; C_\mu \exp\Big\{iS(C) + \frac{2i}{s}\int dz_* d^2z_\perp [A_i(\infty p_2) - A_i(-\infty p_2)]F_{i\bullet}\Big\} = A_\mu$$



The only Wilson line in axial gauge

We get very reach structure



$$\frac{2i}{s}\int d^4z F^c_{*i}(z)[-\infty, z_*]^{cb} F^b_{\bullet i}(z)\Phi(z)$$

EVOLUTION EQUATIONS *(PRELIMINARY)*