2 jet production in DIS at ~~NNLL~~+O(α_s)

N^{3}LL

Daekyoung Kang (MIT)

In collaboration with

Iain Stewart (MIT) and **Chris Lee** (LANL)

May 13, 2014

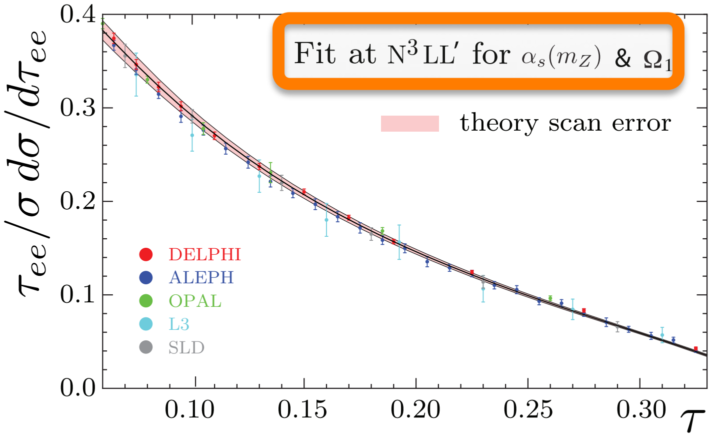
QCD Evolution Workshop

Jet event shape: Thrust

$$\tau_{ee} = 1 - \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{p}_i \cdot \vec{n}|$$ Farhi

- Up to $O(\alpha_s^3)$+N^{3}LL — Becher and Schwartz; Abbate, Fickinger, Hoang, Mateu, Stewart

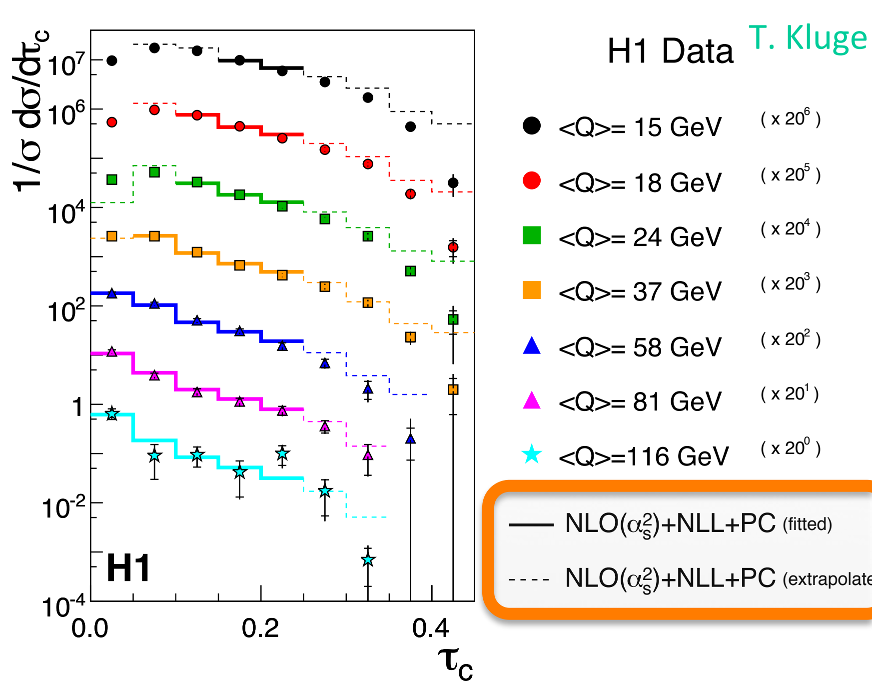
$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$



$$\tau_{\rm DIS} = 1 - \frac{1}{E_J} \sum_{i\in\mathcal{H}_J} |\vec{p}_i \cdot \vec{n}|$$

- one hemisphere
- Up to $O(\alpha_s^2)$+NLL — Antonelli, Dasgupta, Salam

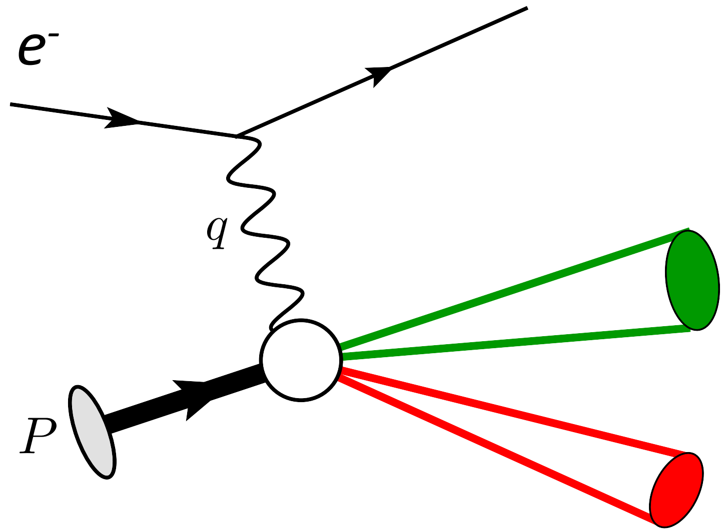
$$\alpha_s(m_Z) = 0.1198 \pm 0.0013(\text{exp.}) {}^{+0.0056}_{-0.0043}(\text{th.})$$

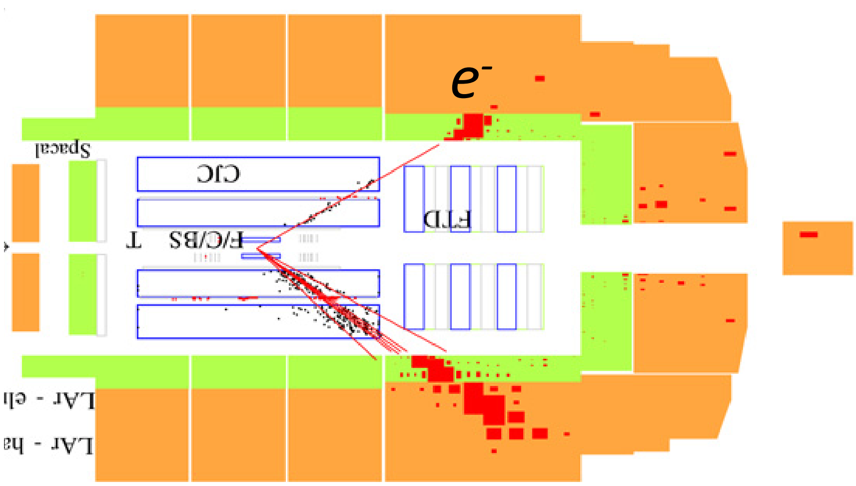


- *Higher precision in DIS?* NNLL or higher ?

Outline

- ***1**-jettiness* in **3** ways in DIS
- NNLL + O(α_s) for one way
- N^{3}LL results for two ways
- Summary





H1 Event from *www-h1.desy.de*

Event shape: 1-jettiness

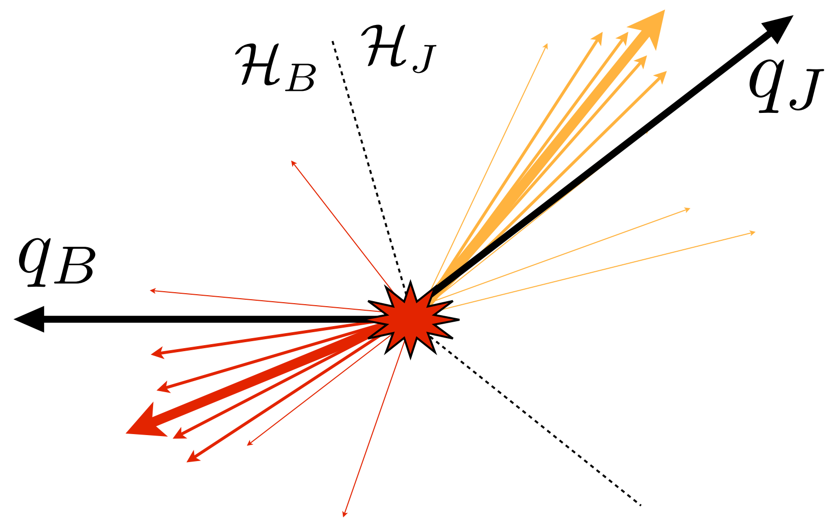
- **N-jettiness**
  - Generalization of thrust
  - N-jet limit: $\tau_N \to 0$

$$\tau_N = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_1 \cdot p_i, \dots, q_N \cdot p_i\}$$

Stewart, Tackmann, Waalewijn

- **1-jettiness:** 1 jet + 1 ISR
  - q_B, q_J are axes to project particle mom.
  - Considering 3 ways to define q_J
  - min. groups particles into 2 regions

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$



Why 1-jettiness?

DIS thrust: Non-Global Log beyond NLL

Dasgupta, Salam

Unknown how to resum NGL

1-jettiness: No NGL, N^{n}LL (n>1) accessible

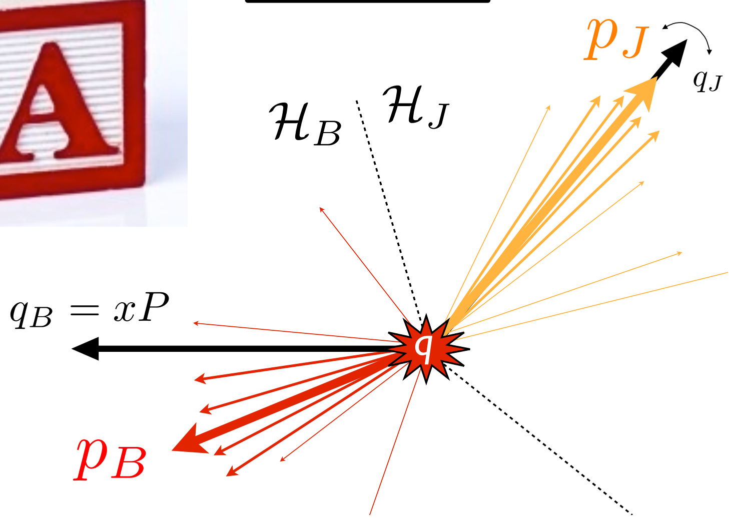
derive factorization thm. by using SCET

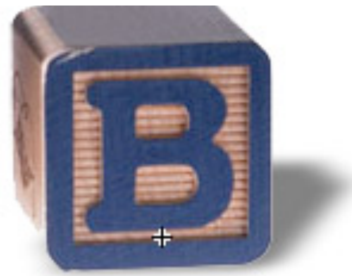
accuracy systematically improved with higher order ME's

1-jettiness in 3 ways

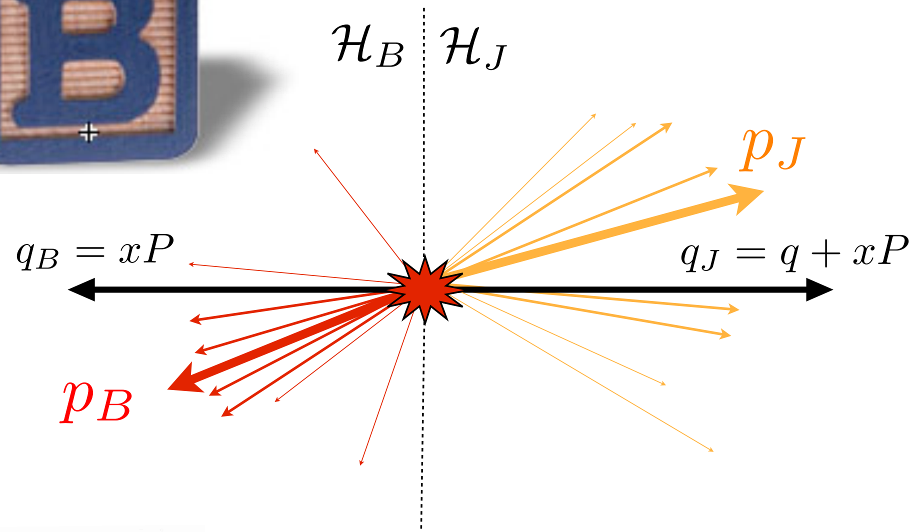
DK, Lee, Stewart 2013

CM frame



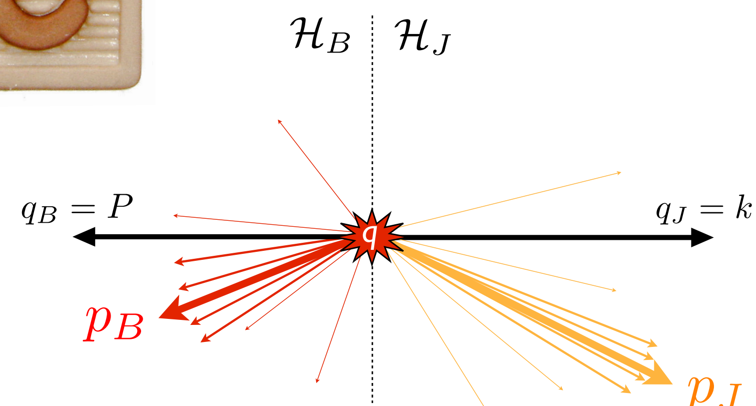


Breit frame



$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

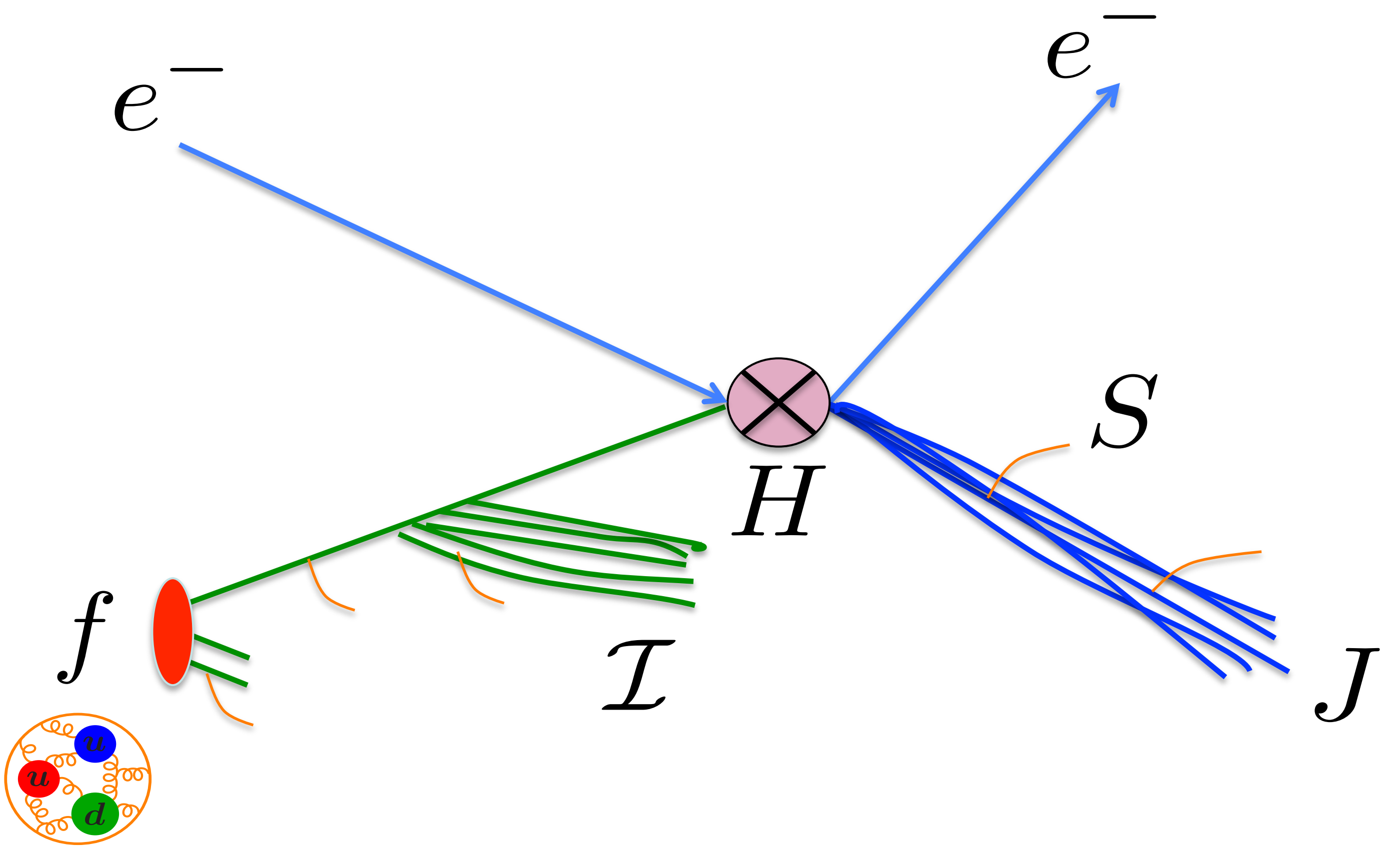
CM frame



Kang, Mantry, Qiu PRD2012, 2013

same axes as  but different weighting for Jet and Beam regions

Factorization theorems



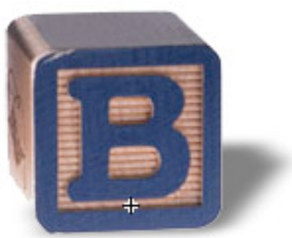
$$\sigma \sim H \times B \otimes J \otimes S$$

$$B = f \otimes \mathcal{I}$$

Factorization theorems

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx\,dQ^2\,d\tau_1^a} = H_q(\mu)\int dt_B\,dt_J\,dk_s\,\delta\left(\tau_1^a - \frac{t_B}{Q^2} - \frac{t_J}{Q^2} - \frac{k_s}{Q}\right)$$

$$\times B_q\left(t_B, x, \mu\right)\;J_q\left(t_J, \mu\right)\;S\left(k_s, \mu\right) + \left(q \leftrightarrow \bar{q}\right)$$



$$\frac{1}{\sigma_0}\frac{d\sigma}{dx\,dQ^2\,d\tau_1^b} = H_q(\mu)\int dt_B\,dt_J\,dk_s\,\delta\left(\tau_1^a - \frac{t_B}{Q^2} - \frac{t_J}{Q^2} - \frac{k_s}{Q}\right)$$

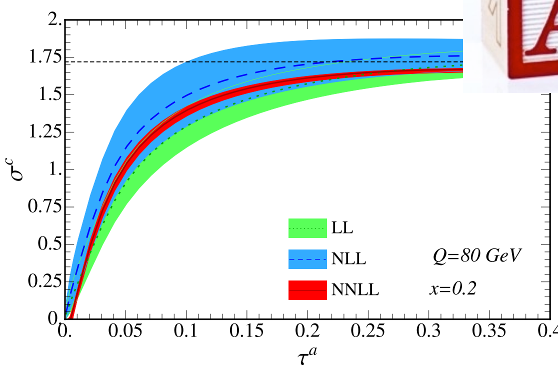
$$\times \int d^2\vec{p}_\perp\,B_q\left(t_B, x, \vec{p}_\perp^{\,2}, \mu\right)\;J_q\left(t_J - \vec{p}_\perp^{\,2}, \mu\right)\;S\left(k_s, \mu\right) + \left(q \leftrightarrow \bar{q}\right)$$

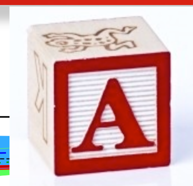
Transverse momentum dependent Beam function

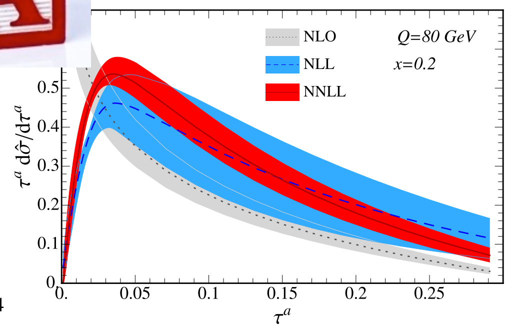
$$\frac{1}{\sigma_0}\frac{d\sigma}{dx\,dQ^2\,d\tau_1^c} = H_q(\mu)\int dt_B\,dt_J\,dk_s\,\delta\left(\tau_1^a - \frac{t_B}{Q^2} - \frac{t_J}{xQ^2} - \frac{k_s}{\sqrt{x}Q}\right)$$

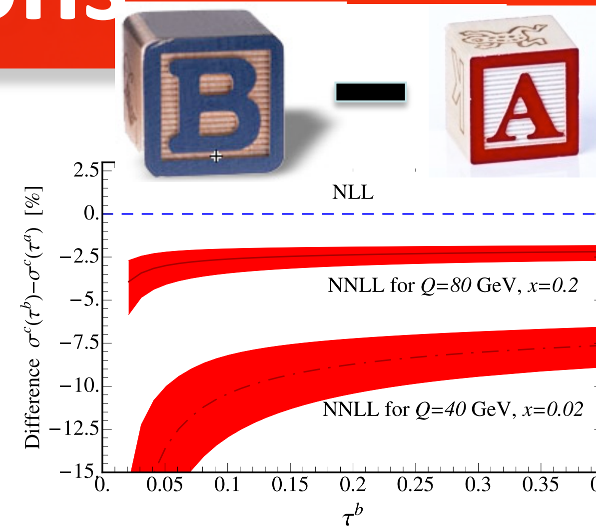
$$\times \int d^2\vec{p}_\perp\,B_q\left(t_B, x, \vec{p}_\perp^{\,2}, \mu\right)\;J_q\left(t_J - (\vec{q}_\perp + \vec{p}_\perp)^2, \mu\right)\;S\left(k_s, \mu\right) + \left(q \leftrightarrow \bar{q}\right)$$

NNLL predictions

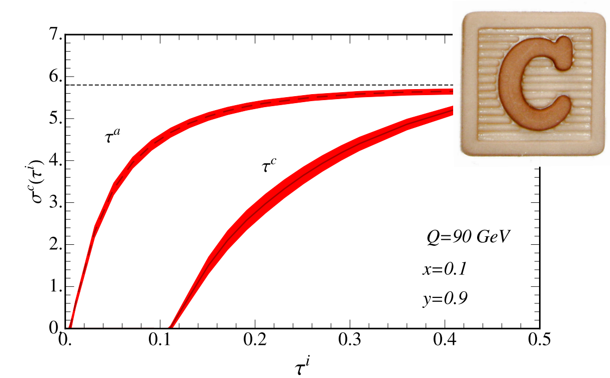








DK, Lee, Stewart 2013

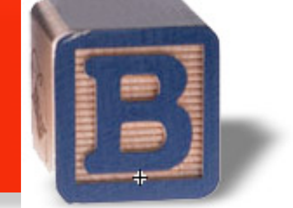


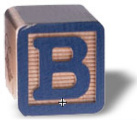
- One order higher than DIS thrust resummation (NLL)
- Higher precision?

$$d\tilde{\sigma} = \exp\left[L\sum_{k=1}^{\infty}(\alpha_s L)^k + \sum_{k=1}^{\infty}(\alpha_s L)^k + \alpha_s\sum_{k=0}^{\infty}(\alpha_s L)^k + \cdots\right] + \mathrm{NS}(\alpha_s)$$

singular part: LL, NLL, NNLL, N^{3}LL,... nonsingular part: $O(\alpha_s)$, $O(\alpha_s^2)$,...

Nonsingular part at $O(\alpha_s)$



- 

 amenable to analytic calculation (this talk)

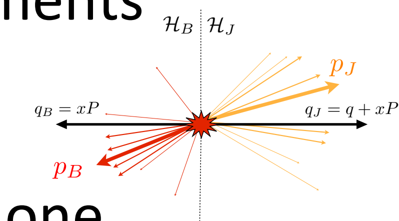
  DK, Lee, Stewart 2014

  - 

 requires jet algorithm and is done numerically

    Kang, Liu, Mantry 1312.0301

- Jet region has been measured in H1 and ZEUS experiments
  - difficult to measure the beam region



- 

 can be obtained from measuring jet region alone, while 

 requires measuring two regions.

$$\tau_1^b \overset{\text{Breit}}{=} \frac{1}{Q}\sum_{i\in X} \min\{n_z\cdot p_i, \bar{n}_z\cdot p_i\}$$
$$= \frac{1}{Q}\left[\sum_{i\in\mathcal{H}_J^b}(E_i - p_{z\,i}) + \sum_{i\in\mathcal{H}_B^b}(E_i + p_{z\,i})\right]$$
$$= \frac{1}{Q}\left[\sum_{i\in X}(E_i + p_{z\,i}) - 2\sum_{i\in\mathcal{H}_J^b} p_{z\,i}\right],$$

$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q}\sum_{i\in\mathcal{H}_J^b} p_{z\,i}$$

Antonelli, Dasgupta, Salam JHEP 2000

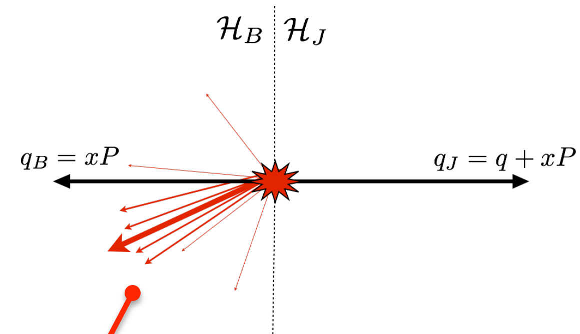
$$\tau_1^c \overset{\text{CM}}{=} \frac{1}{xy\sqrt{s}}\sum_{i\in X}\min\{n_z\cdot p_i, \bar{n}_z\cdot p_i\}$$
$$= \frac{1}{xy\sqrt{s}}\left[\sum_{i\in X}(E_i + p_{z\,i}) - 2\sum_{i\in\mathcal{H}_J^c} p_{z\,i}\right]$$

$$\tau_1^c \overset{\text{CM}}{=} \frac{1}{x}\left(1 - \frac{2}{y\sqrt{s}}\sum_{i\in\mathcal{H}_J^c} p_{z\,i}\right)$$

Nonsingular part at $O(\alpha_s)$

DK, Lee, Stewart

$$\frac{d\sigma}{dx\,dQ^2\,d\tau_1} = \frac{4\pi\alpha^2}{Q^4}\left[(1+(1-y)^2)F_1 + \frac{1-y}{x}F_L\right]$$



Nonsingular part of F1

$$B_q = Q_f^2\frac{\alpha_s C_F}{2\pi}\Bigg[N_1(\tau,x) + N_0(\tau,x) + \int_x^{\frac{1}{1+\tau}}\frac{dz}{z}f_q\Big(\frac{x}{z}\Big)R^q(\tau,z)$$
$$+ (1+\tau)\,f_q(x(1+\tau))\Delta_2^q(\tau) + \delta(\tau-1)\int_x^{1/2}\frac{dz}{z}f_q\Big(\frac{x}{z}\Big)\,\Delta_1^q(z)\Bigg]$$

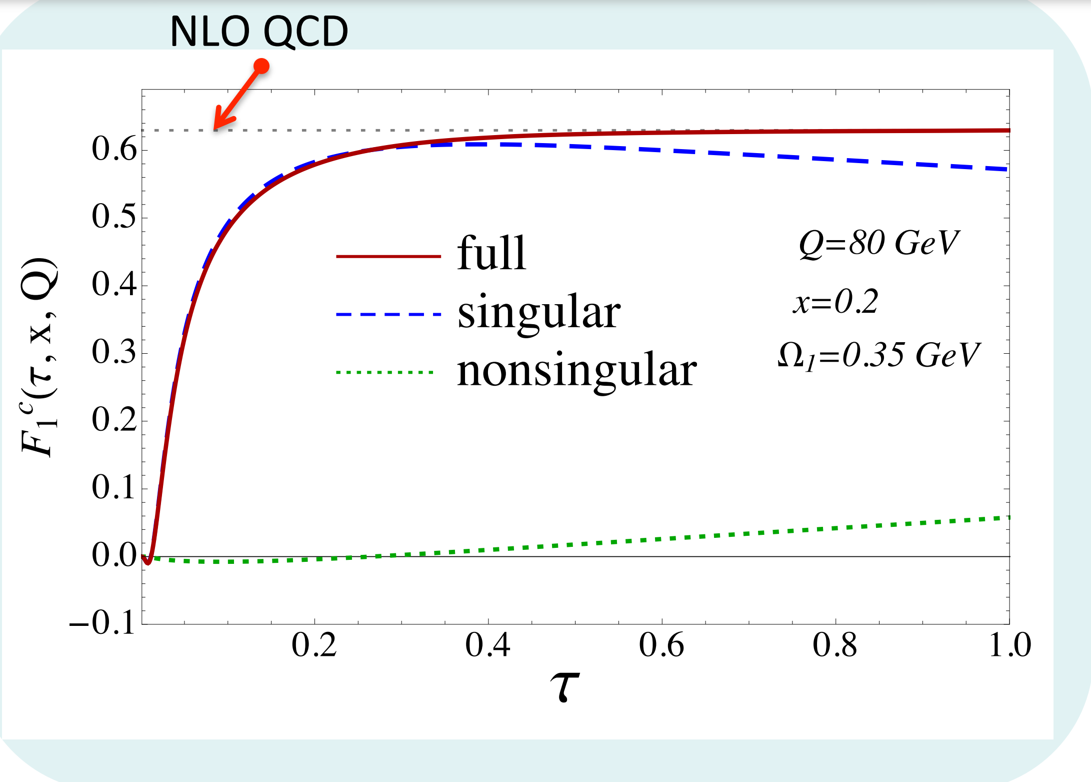
$$N_1(\tau,x) = -4\frac{\ln\tau}{\tau}\Big[(1+\tau/2)f_q(x(1+\tau)) - f_q(x)\Big]$$

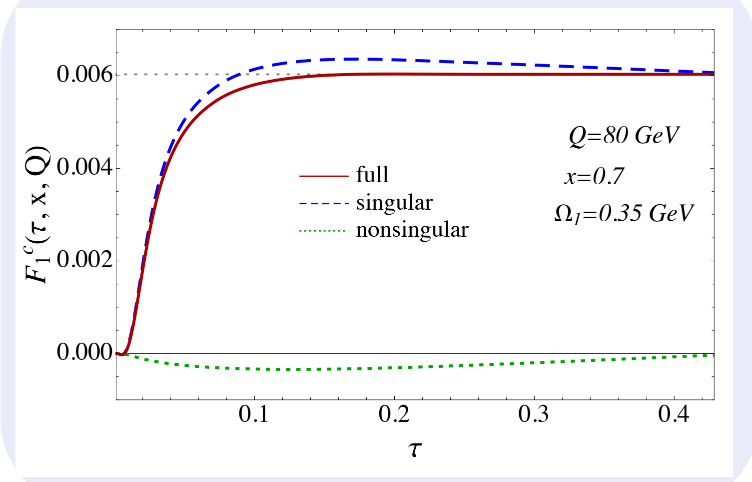
enhanced at small x

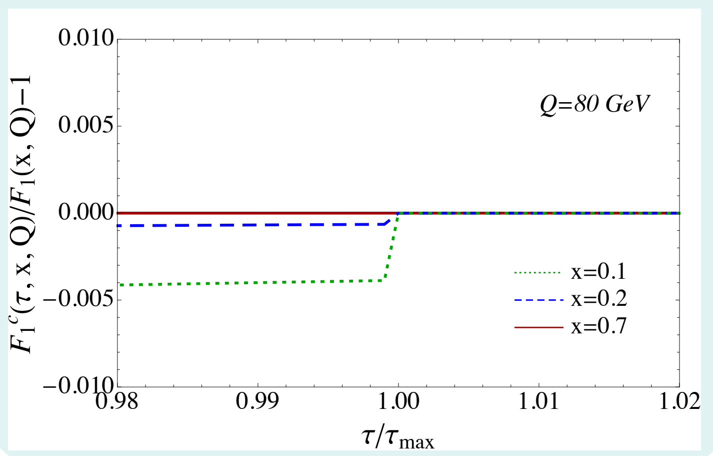
singular term cancels

$$\Delta_1^q(x) = \frac{(1-2x)(1-4x)}{2(1-x)} + \frac{1+x^2}{1-x}\ln\Big(\frac{1-x}{x}\Big)$$

Cumulant at NNLL+ $O(\alpha_s)$



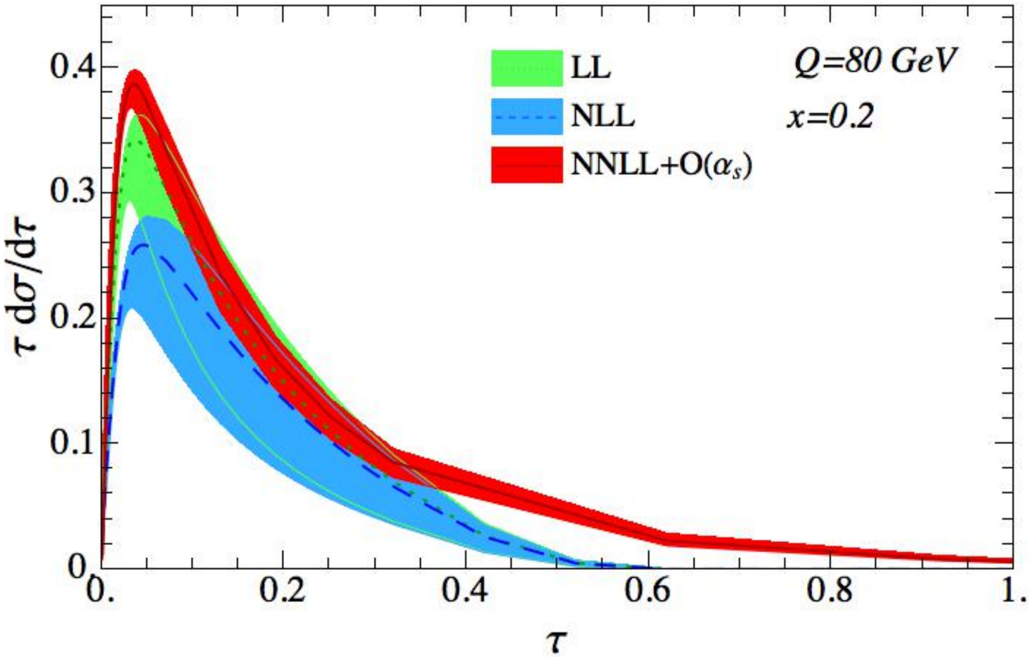


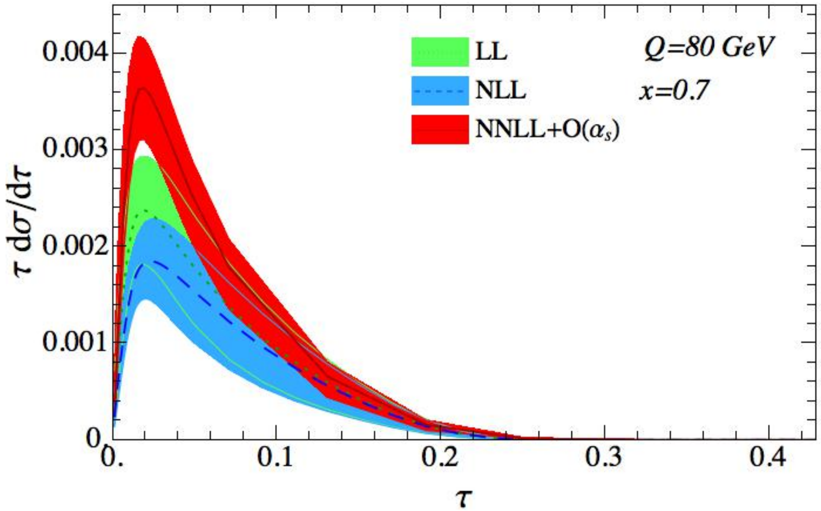


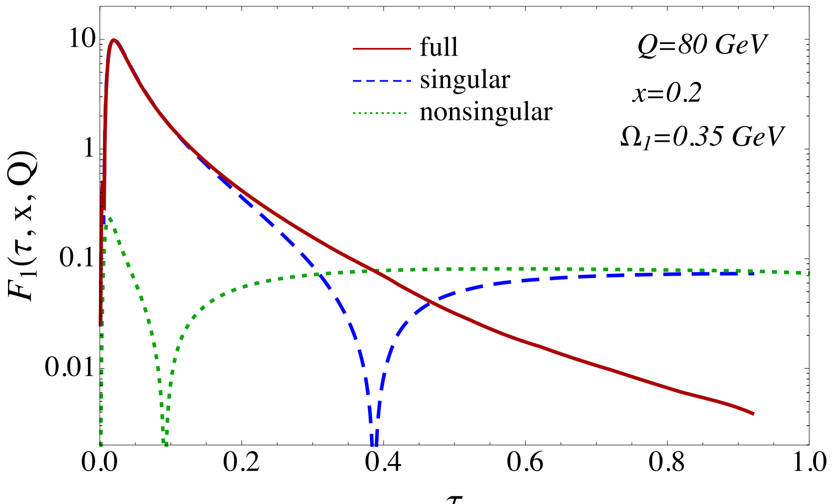
- The singular undershoots for small x and overshoots for larger x
- The nonsingular is sensitive to x.
- Small discontinuities at τ_{max}

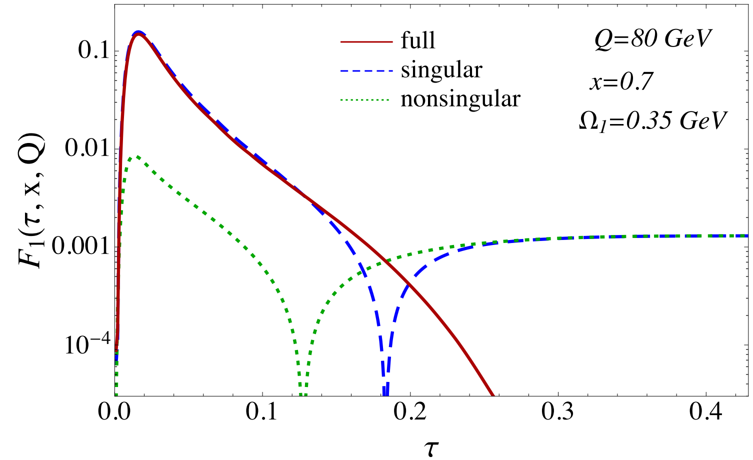
Distribution at NNLL+ $O(\alpha_s)$

DK, Lee, Stewart









Toward N^{3}LL

	$\Gamma[\alpha_s]$	$\gamma[\alpha_s]$	$\beta[\alpha_s]$	$\{H, J, B, S\}[\alpha_s]$
LL	α_s	1	α_s	1
NLL	α_s^2	α_s	α_s^2	1
NNLL	α_s^3	α_s^2	α_s^3	α_s
N^{3}LL	α_s^4	α_s^3	α_s^4	α_s^2

Pade approx.

$$\Gamma_3^q = (1 \pm 2)\frac{(\Gamma_2^q)^2}{\Gamma_1^q}$$

0.2 % in e$^+$e$^-$ thrust

trans. mom. *B* up to 1 loop

B function up to 2 loops

Gaunt, Stahlhofen, Tackmann 1401.5478

$S_{ee} = S_{ep} = S_{pp}$ up to 2 loops

Lee and Zhang

Kelley, Schabinger, Schwartz, Zhu

Soft function at 2 loop

Lee and Zhang

- Wilson lines are different.

e⁺e⁻: $\langle 0|\bar{T}\left[\tilde{Y}_{\bar{n}}^{\dagger}\tilde{Y}_{n}\right]\ \delta(\cdots)\,T\left[\tilde{Y}_{n}^{\dagger}\tilde{Y}_{\bar{n}}\right]|0\rangle$

ep: $\langle 0|\bar{T}\left[Y_{\bar{n}}^{\dagger}\tilde{Y}_{n}\right]\ \delta(\cdots)\,T\left[\tilde{Y}_{n}^{\dagger}Y_{\bar{n}}\right]|0\rangle$

pp: $\langle 0|\bar{T}\left[Y_{\bar{n}}^{\dagger}Y_{n}\right]\ \delta(\cdots)\,T\left[Y_{n}^{\dagger}Y_{\bar{n}}\right]|0\rangle$

incoming and outgoing lines give different sign in the Eikonal propagator

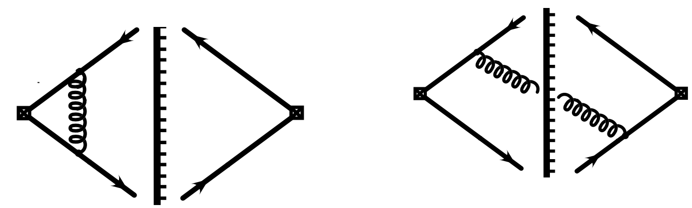
$$\frac{i}{n\cdot k\pm i\epsilon}$$

The sign could matter in the loop integral.

- Well known at $O(\alpha_s)$:

virtual is scaleless and zero.

no loop in the real.



- at $O(\alpha_s{}^2)$:

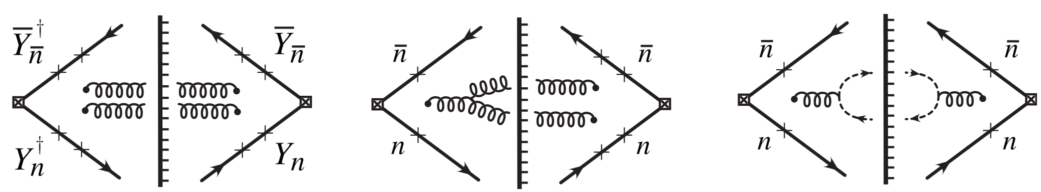
virtual are scaleless and zero.

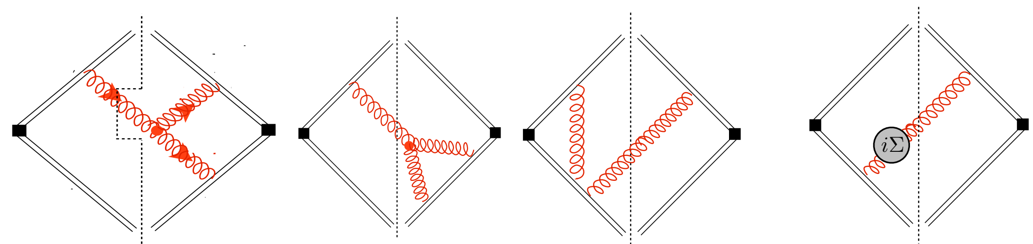
2 gluon cut has no loop.

1 gluon cut needs to be checked.

Nontrivial only for triple gluon vertex

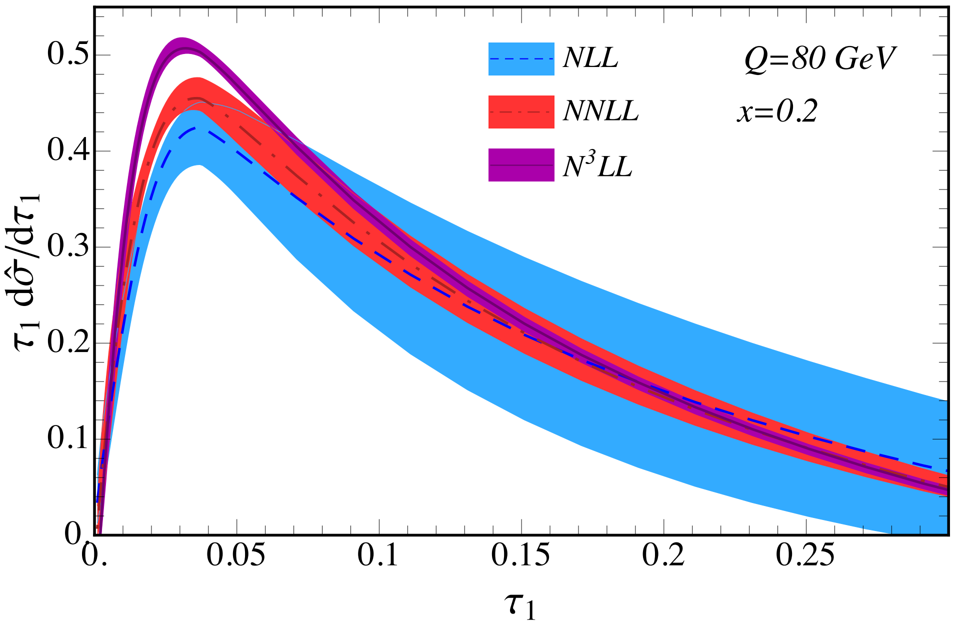
Same for e⁺e⁻, ep, pp!

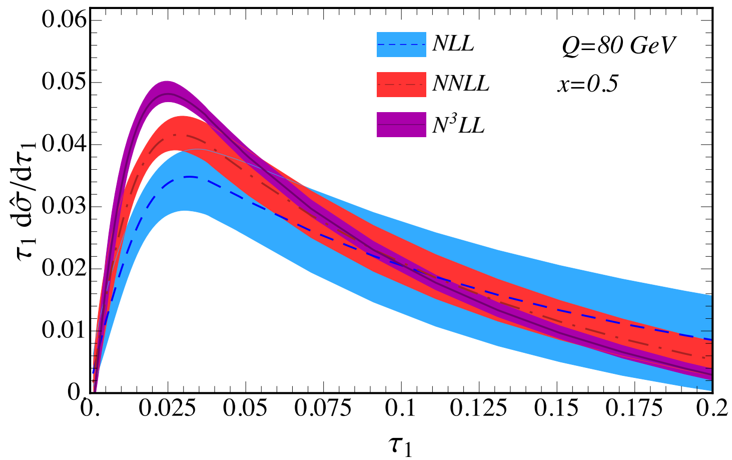


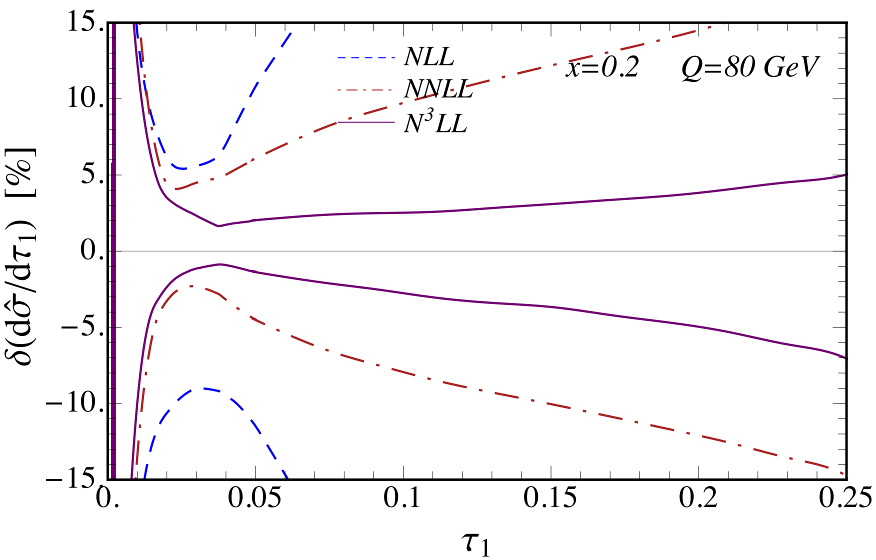


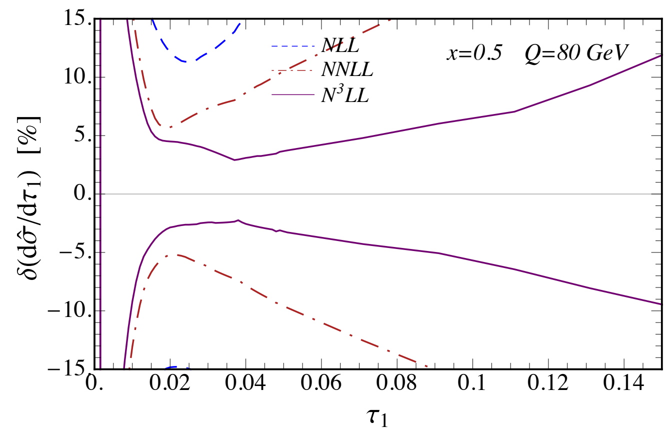
N^3LL results

DK, Lee, Stewart



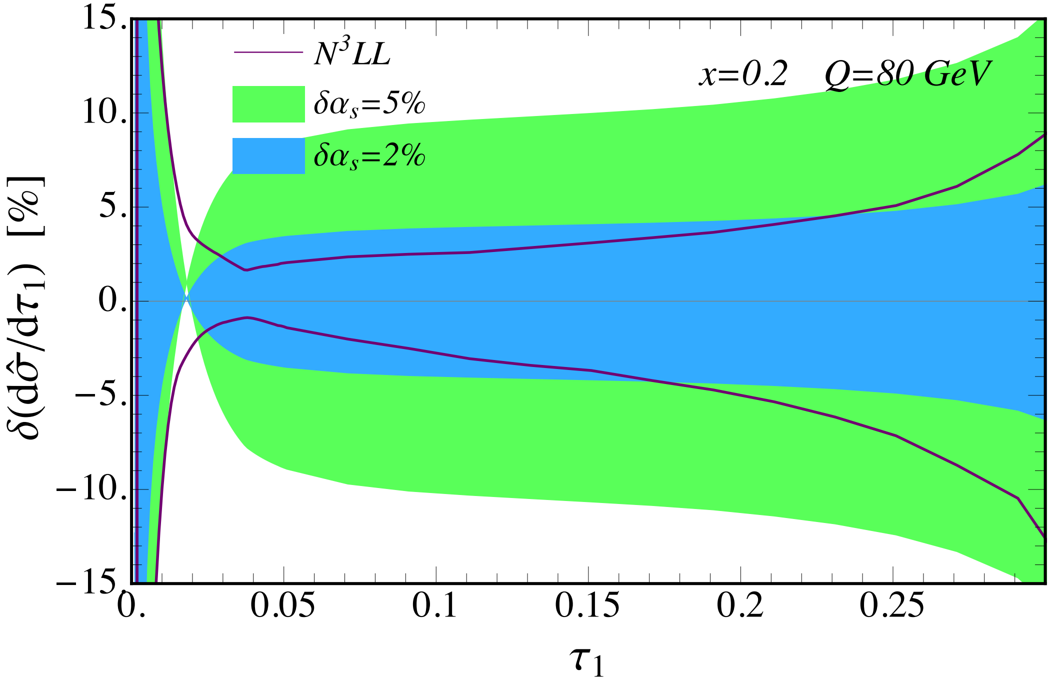


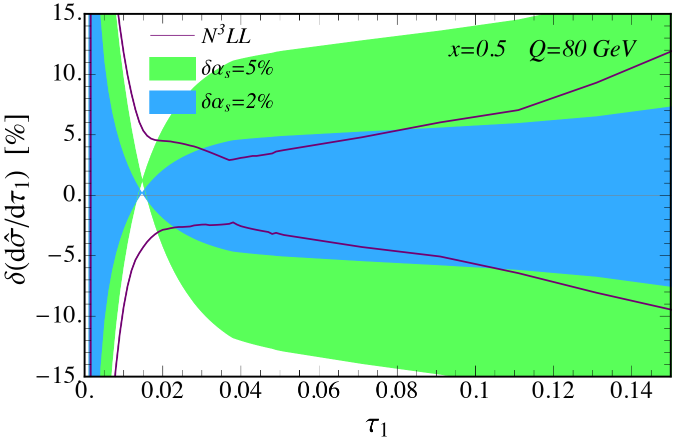




Sensitivity to α_s variation

DK, Lee, Stewart



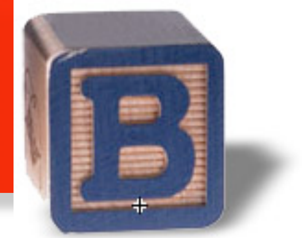


- *H1 analysis (2006)* $\alpha_s(m_Z) = 0.1198 \pm 0.0013(\mathrm{exp.}) {}^{+0.0056}_{-0.0043}(\mathrm{th.})$
- Better δσ at N³LL => *$\delta\alpha_s$=2% at x =0.2 ~0.5*
- $\delta\alpha_s$=2% by MSTW PDF uncertainty

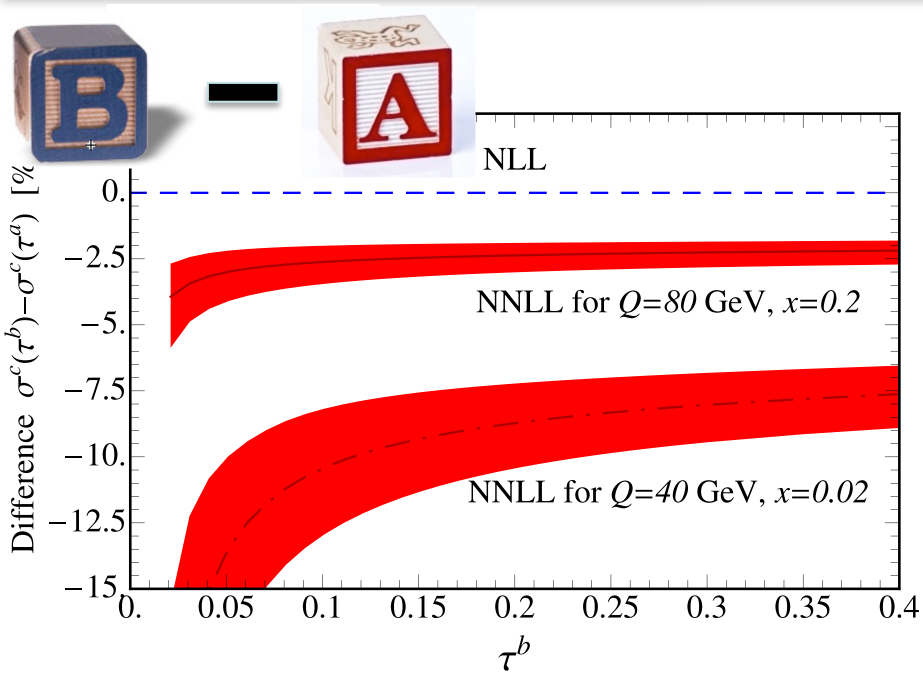
4% theory uncertainty

($\delta\alpha_s$= *0.1 %* or less by 4-loop cusp anomalous dim.)

Approximate $N^3LL+O(\alpha_s)$



DK, Lee, Stewart



$$\sigma_b = \sigma_a + \Delta\sigma_b$$

Difference from beam functions:

a few terms in fixed order part

The same RG evolution

- difference (< perturbative uncertainty) can be ignored.

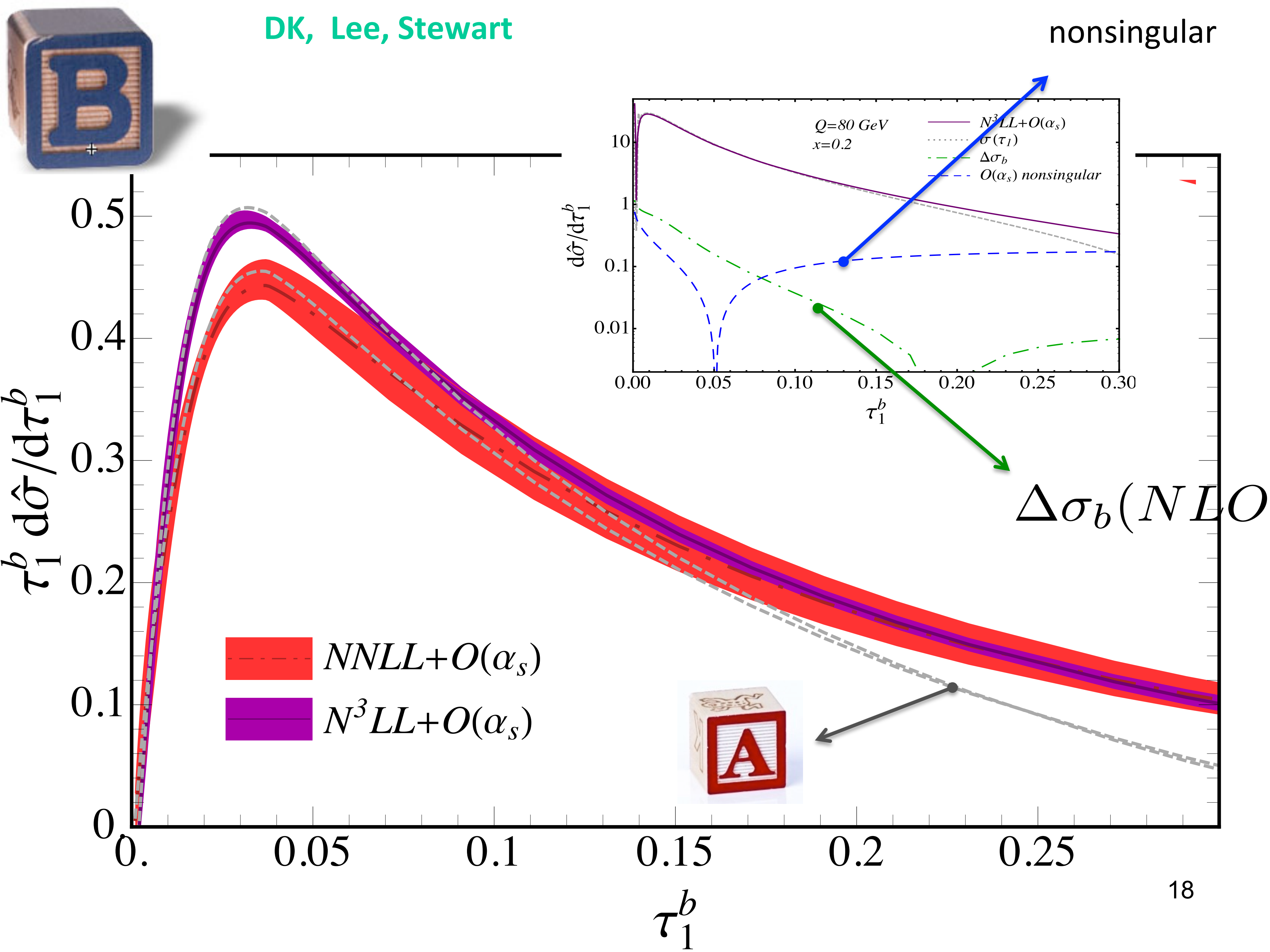
assuming $\Delta\sigma_b$ (NNLO) ≈ α_s $\Delta\sigma_b$(NLO) ≈1% < $\delta\sigma$ at N^3LL ≈ 2~5%

$$\sigma_b(N^3LL) \approx \sigma_a(N^3LL) + \Delta\sigma_b(NLO)$$

- *For*  *we also have $O(\alpha_s)$ nonsingular part.*

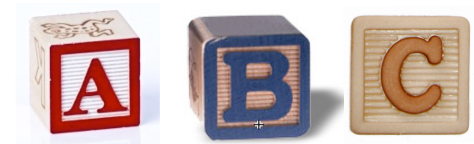
DK, Lee, Stewart

nonsingular



Summary

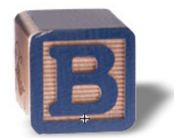
- Factorization thms for 1-jettiness



$$\sigma \sim H \times B \otimes J \otimes S$$

$$B = f \otimes \mathcal{I}$$

- N^3LL predictions for 

- Approx. $N^3LL+O(\alpha_s)$ predictions for 

- Accuracy $\delta\alpha_s$= 2% at x =0.2~0.5

  better than $\delta\alpha_s$ = 4% theory uncertainty in H1 analysis

  comparable to MSTW PDF uncertainty

Backup

NNLL+ $O(\alpha_s)$

Kang, Liu, Mantry

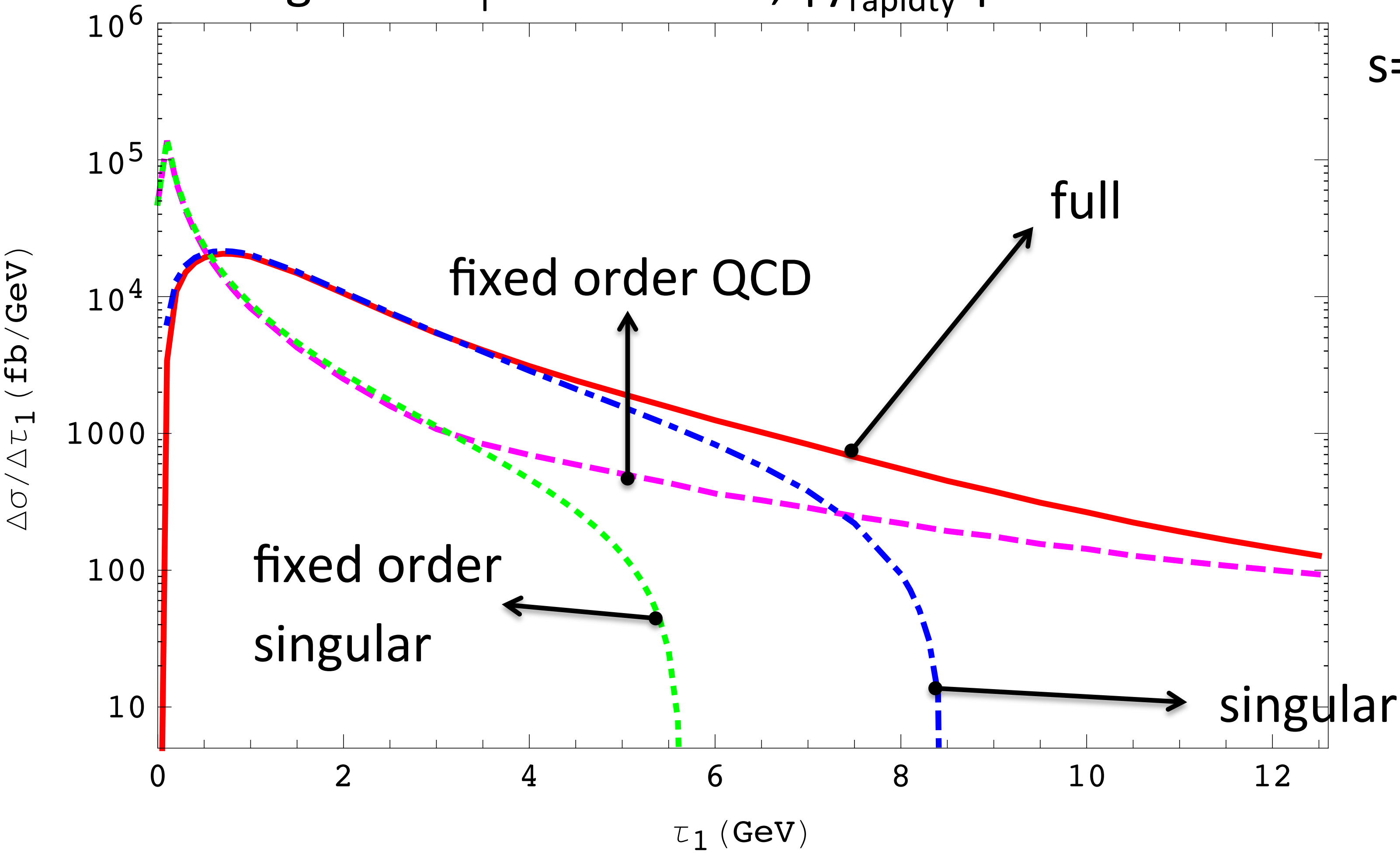
1312.0301

$$\tau_1 = \sum_k \min\left\{\frac{2q_A\cdot p_k}{Q_a}, \frac{2q_J\cdot p_k}{Q_J}\right\}$$

$$Q_a = x_A A Q_e$$
$$Q_J = 2K_{J_T}\cosh y_K$$

integrated P_T=20~30 GeV, $|y_{rapidty}|$<2.5

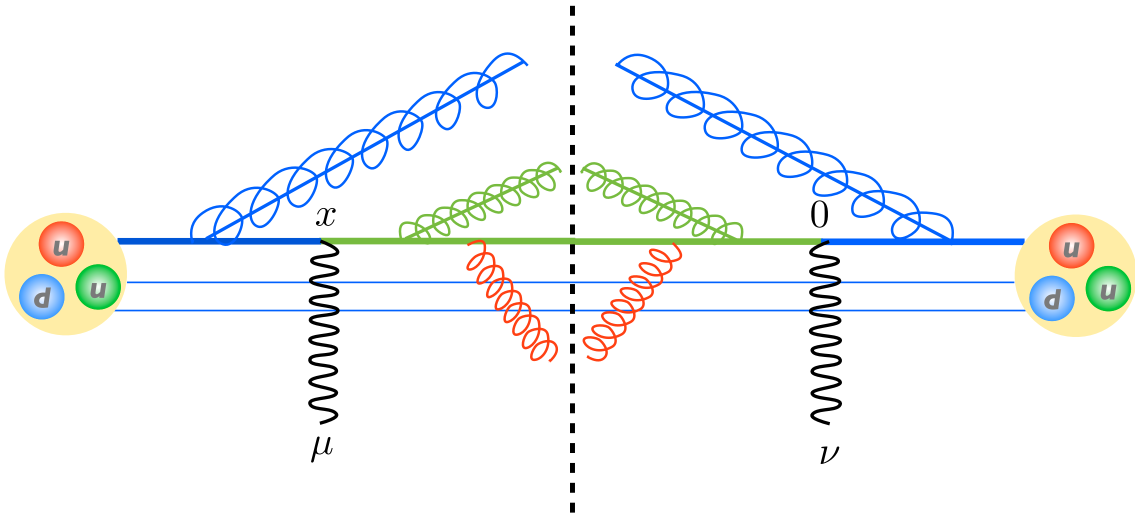
s=(90 GeV)[2]

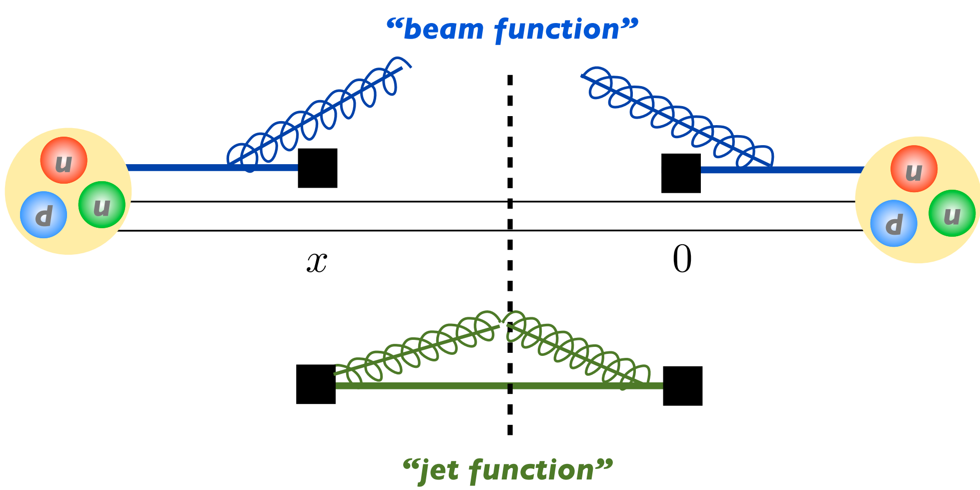


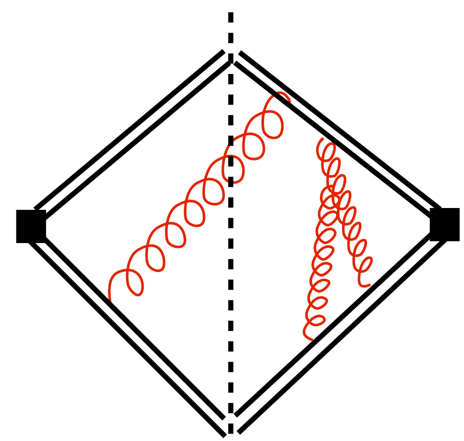
Beam, Jet, Soft functions

from Chris Lee's talk

in SCET 2013







"soft function"

Nonpertubative Effect

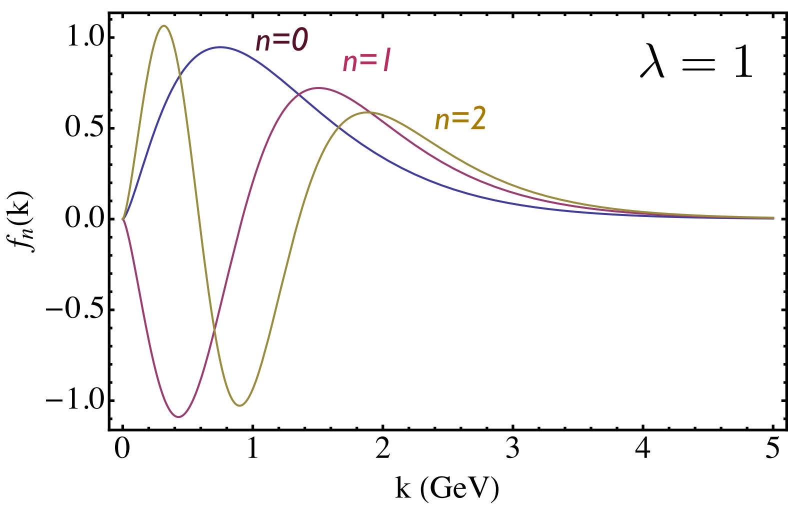
- Estimating nonperturbative part of soft function
- For $\tau \gg \Lambda_{QCD}/Q$
  OPE gives power correction with $\mathcal{O}(\Lambda_{QCD}/\tau Q)$ suppression

$$\sigma(\tau) = \sigma_{\rm pert}(\tau) - \boxed{\frac{2\Omega}{Q}}\frac{d\sigma_{\rm pert}(\tau)}{d\tau} \approx \sigma_{\rm pert}(\tau - 2\Omega/Q)$$

  - $\Omega \sim \Lambda_{QCD}$: nonpertubative matrix element
- For $\tau \geq \Lambda_{QCD}/Q$
  significant nonpertubative effect
  convolving shape function
  consistent with power correction

$$\sigma(\tau) = \int dk \sigma_{\rm pert}(\tau - k/Q)\, F(k)$$

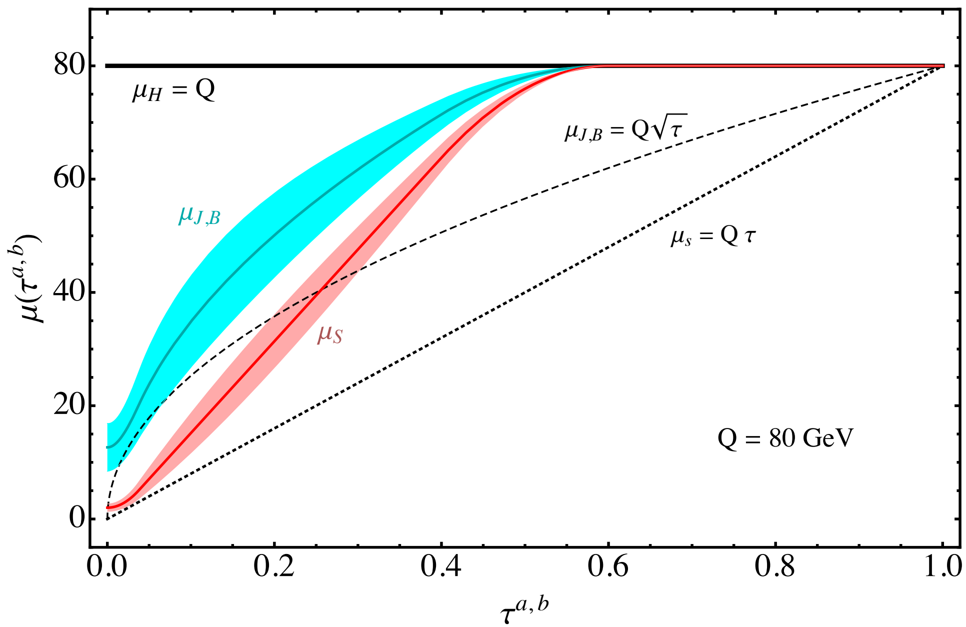
$$\to \sigma_{\rm pert}(\tau) - \boxed{\left(\int dk\, \frac{k}{Q} F(k)\right)} \frac{d\sigma_{\rm pert}(\tau)}{d\tau}$$

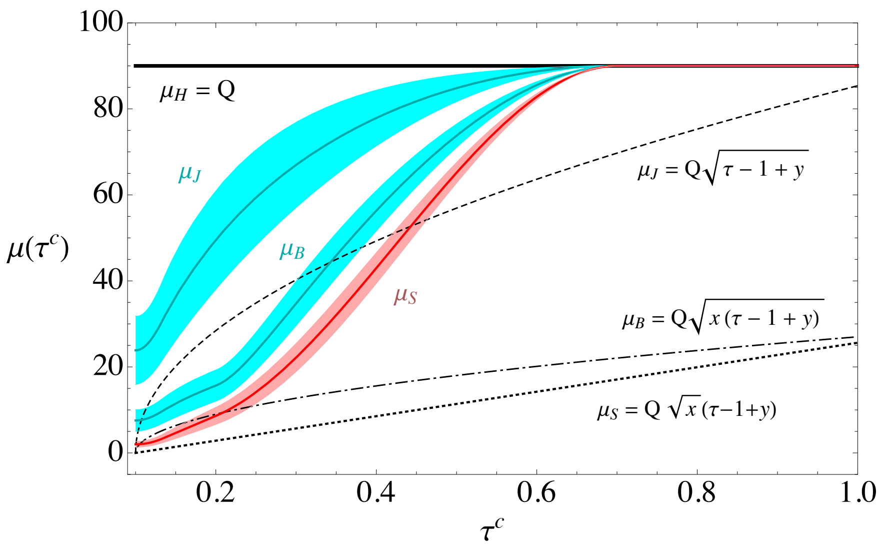


$$F(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{N} c_n f_n\left(\frac{k}{\lambda}\right)\right]^2$$

Ligeti, Tackmann, Stewart

Choice of scales





- For $\Lambda_{QCD} \ll \tau \ll 1$

  $\mu_H = Q \quad \mu_{B,J} = \sqrt{\tau} Q$

  $\mu_S = \tau Q$

- For $\tau \sim \Lambda_{QCD}/Q$

  significant nonperturbative effect

  soft scale freezing at $\mu_S \sim \Lambda_{QCD}$

  $\mu_{B,J} \sim \sqrt{\Lambda_{QCD} Q}$

- For $\tau \sim 1$

  no hierarchy in scales

  no large logs

  $\mu_H \sim \mu_{B,J} \sim \mu_S \sim Q$

Resummation and RGE

- Fourier transformation

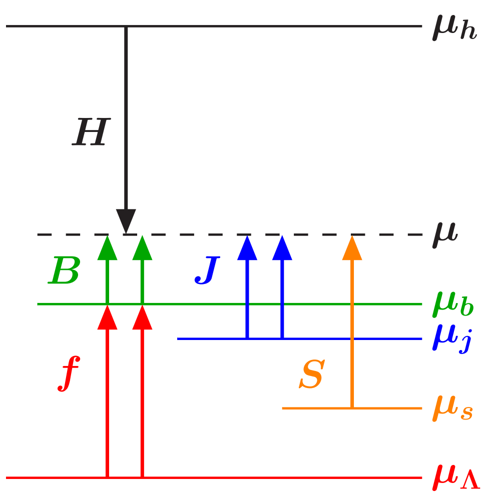
y : conjugate variable of τ_1

$$\frac{d\tilde{\sigma}}{dy} = \int d\tau_1 \, e^{-iy\tau_1} \frac{d\sigma}{d\tau_1} = H(\mu) \, \widetilde{B}_q(y,x,\mu) \, \widetilde{J}_q(y,\mu) \, \widetilde{S}(y,\mu)$$

$$\ln \frac{d\tilde{\sigma}}{dy} = \underbrace{L \sum_{k=1}^{\infty} (\alpha_s L)^k}_{\text{LL}} + \underbrace{\sum_{k=1}^{\infty} (\alpha_s L)^k}_{\text{NLL}} + \underbrace{\alpha_s \sum_{k=0}^{\infty} (\alpha_s L)^k}_{\text{NNLL}} + \cdots$$

$$L = \log(iy)$$

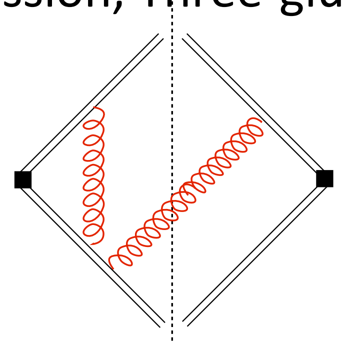
- Resumming large logs
  - No large logs in each function at its natural scale μ_i
  - *RG evolution* from μ_i to common scale μ



soft function at 2 loop

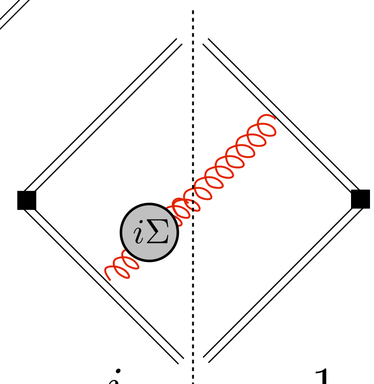
- At two loops, there are 0, 1, or 2 real gluons in the cut soft function diagrams
  - All purely virtual diagrams are scaleless and **zero** in dim. reg.
  - All diagrams with two real gluons independent of $\pm i\epsilon$ prescriptions.
  - Only need to consider diagrams with **one** real gluon:
- Three topologies: Independent emission, Three-gluon vertex, Bubble

  - Independent emission: 1 cut gluon diagrams contain a scaleless virtual loop



etc. = 0

  - Bubble Diagrams just contribute to charge renormalization, 1 gluon cut independent of eikonal $\pm i\epsilon$ prescriptions.



See 1105.3676

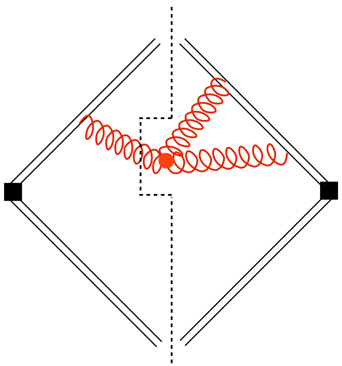
$$\sim g^2 n\cdot\bar{n} \int \frac{d^D p}{(2\pi)^D} 2\pi\delta(p^2)\theta(p^0)\widehat{\mathcal{M}}(p) \frac{-i}{p^2 + i\Sigma(p^2)} \frac{1}{n\cdot p \pm i\epsilon} \frac{1}{\bar{n}\cdot p \pm i\epsilon}$$

Measurement on cut gluon

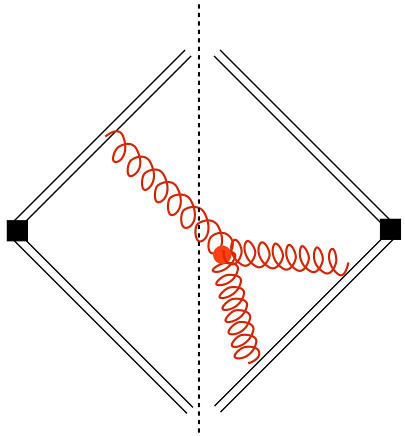
Signs irrelevant

soft function at 2 loop

- Three-gluon vertex



etc. $\propto n^2, \bar{n}^2 = 0$

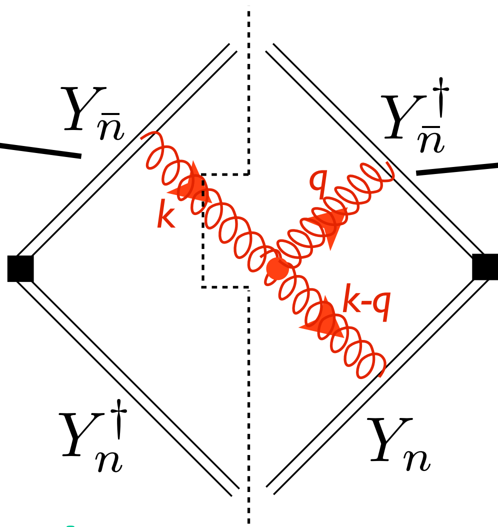


etc., turns out scaleless in DR = 0

1105.3676

- Only nontrivial diagrams to check:

$$\frac{1}{\bar{n}\cdot k \pm i\epsilon}$$



$$\frac{1}{\bar{n}\cdot q \pm i\epsilon}$$

$\pm i\epsilon$ in these eikonal propagators change from ep to ee

can choose to perform q loop integral by doing $n\cdot q$ integral by contours first, leaves real integrals, making $\pm i\epsilon$ sign changes irrelevant.

Lee and Zhang

missing particles in forward region

$$\eta = -\ln(\tan\theta/2)$$

- Proton remnants and particles moving very forward region out of detector coverage: $0 < \theta < \theta_{\rm cut}\,,\ \eta > \eta_{\rm cut}$
  - H1 : $\theta_{\rm cut} = 4^\circ (0.7^\circ)$ and $\eta_{\rm cut} = 3.4(5.1)$ for main cal. (PLUG cal.)
  - ZEUS: $\theta_{\rm cut} = 2.2^\circ$ and $\eta_{\rm cut} = 4.0$ for FCAL

$$\Delta\eta = \ln\frac{E_p^{\rm lab}}{E_p^{\rm CM}} = \ln\frac{920}{157} = 1.8$$

- Boost to CM frame: $\eta^{\rm CM} = \eta - \Delta\eta$
  - H1: $\eta_{\rm cut}^{\rm CM} = 1.6(3.3)$, $\boxed{e^{-\eta_{\rm cut}^{\rm CM}} = 0.2(0.04)}$
  - ZEUS: $\eta_{\rm cut}^{\rm CM} = 2.2$, $\boxed{e^{-\eta_{\rm cut}^{\rm CM}} = 0.1}$

Suppression factor!

- Maximum missing measurement: $\tau_{\rm miss} = \dfrac{2q_B\cdot p_{\rm miss}}{Q^2} = \dfrac{m_T}{Q_B}e^{-\eta}$
  - $m_T^{\rm max} = E_p^{\rm lab}\sin\theta_{\rm cut}$

    about 64(11) GeV for H1 and 32 GeV for ZEUS

$$Q_B = \sqrt{y/x}Q,\ xQ$$