New results from the quantum statistical approach to parton distributions

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Outline

- Basic procedure to construct the statistical polarized parton distributions
- Essential features from unpolarized and polarized Deep Inelastic Scattering data
- Special predictions for the flavor structure of the sea
- Helicity asymmetries for $W^{\pm}$ production
- Transverse momentum dependence (TMD) extention:
  Transverse energy sum rule. Gaussian shape with no x, k_T factorization
  Melosh-Wigner effects mainly in low x, Q^2 region
- Conclusions

Collaboration with Claude Bourrely and Franco Buccella

- A Statistical Approach for Polarized Parton Distributions
  Euro. Phys. J. C23, 487 (2002)
- Recent Tests for the Statistical Parton Distributions
  Mod. Phys. Letters A18, 771 (2003)
- The Statistical Parton Distributions: status and prospects
  Euro. Phys. J. C41,327 (2005)
- The extension to the transverse momemtum of the statistical parton distributions
  Mod. Phys. Letters A21, 143 (2006)
- Strangeness asymmetry of the nucleon in the statistical parton model
  Phys. Lett. B648, 39 (2007)
- How is transversity related to helicity for quarks and antiquarks in a proton?
  Mod. Phys. Letters A24, 1889 (2009)
- Semiinclusive DIS cross sections and spin asymmetries in the quantum statistical parton distributions approach, Phys. Rev. D83, 074008 (2011)
- The transverse momentum dependent statistical parton distributions revisited
  Int. Journal of Mod. Phys. A28, 1350026 (2013)
- $W^{\pm}$ bosons production in the quantum statistical parton distributions approach

Our motivation and goals

- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features

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- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features
- Will parametrize our PDF in terms of a rather small number of physical parameters, at variance with standard polynomial type parametrizations
- Will be able to construct simultaneously unpolarized and polarized PDF: A UNIQUE CASE ON THE MARKET!
- Will be able to describe physical observables both in DIS and hadronic collisions
- Will make some very specific challenging predictions, from the behavior of unpolarized and polarized PDF, either in the sea quark region or in the valence region
- Will also consider the case of the elusive polarized gluon distribution

Basic procedure

Use a simple description of the PDF, at input scale Q_0^2, proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, *plus* sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and *minus* sign for gluons, corresponds to a Bose-Einstein distribution. X_{0p} is a constant which plays the role of the *thermodynamical potential* of the parton *p* and $\bar{x}$ is the *universal temperature*, which is the same for all partons.

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From the chiral structure of QCD, we have two important properties, allowing to RELATE quark and antiquark distributions and to RESTRICT the gluon distribution:
- Potential of a quark q^h of helicity *h* is opposite to the potential of the corresponding antiquark $\bar{q}^{-h}$ of helicity *-h*, $X_{0q}^h = -X_{0\bar{q}}^{-h}$.
- Potential of the gluon G is zero, $X_{0G} = 0$.

The polarized PDF $q^{\pm}(x, Q_0^2)$ at initial scale Q_0^2

For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{A X_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ ,$$

consequently for antiquarks of helicity $h = \mp$

$$x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A} (X_{0q}^h)^{-1} x^{\bar{b}}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ .$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for $\bar{q}$).
Extra term is absent in Δq and q_v also in $u - d$ or $\bar{u} - \bar{d}$.
The additional factors X_{0q}^h and $(X_{0q}^h)^{-1}$ are coming from TMD (see below)

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The additional factors X_{0q}^h and $(X_{0q}^h)^{-1}$ are coming from TMD (see below)

For strange quarks and antiquarks, s and $\bar{s}$, use the same procedure which leads to $xs(x, Q_0^2) \neq x\bar{s}(x, Q_0^2)$ and $x\Delta s(x, Q_0^2) \neq x\Delta\bar{s}(x, Q_0^2)$ (Phys. Lett. B648, 39 (2007)).

For gluons we use a Bose-Einstein expression given by $xG(x, Q_0^2) = \frac{A_G x^b G}{\exp(x/\bar{x}) - 1}$, with a vanishing potential and the same temperature $\bar{x}$. For the polarized gluon distribution $x\Delta G(x, Q_0^2)$ we take a similar expression at initial scale (positive for all x)

Essential features from the DIS data

From well established features of u and d extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have $X_{0u}^{+} + X_{0u}^{-} > X_{0d}^{+} + X_{0d}^{-}$
- $\Delta u(x) > 0$, therefore $X_{0u}^{+} > X_{0u}^{-}$
- $\Delta d(x) < 0$, therefore $X_{0d}^{-} > X_{0d}^{+}$.

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- $\Delta u(x) > 0$, therefore $X^+_{0u} > X^-_{0u}$
- $\Delta d(x) < 0$, therefore $X^-_{0d} > X^+_{0d}$.

So we expect X^+_{0u} to be the largest potential and X^+_{0d} the smallest one. In fact, from our fit we have obtained the following ordering

$$X^+_{0u} > X^-_{0d} \sim X^-_{0u} > X^+_{0d} \, .$$

This ordering has important consequences for $\bar{u}$ and $\bar{d}$, namely

Essential features from DIS data

- $\bar{d}(x) > \bar{u}(x)$, flavor symmetry breaking expected from Pauli exclusion principle. This was already confirmed by the violation of the Gottfried sum rule (NMC).
- $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, a PREDICTION from 2002, in agreement with polarized DIS (see below) and has been more precisely checked at RHIC-BNL from $W^{\pm}$ production, already in active running phase (see below).

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- Note that since $u^{-}(x) \sim d^{-}(x)$, it follows that $\bar{u}^{+}(x) \sim \bar{d}^{+}(x)$, so we have

$$\Delta\bar{u}(x) - \Delta\bar{d}(x) \sim \bar{d}(x) - \bar{u}(x) \ ,$$

  i.e. the flavor symmetry breaking is almost the same for unpolarized and polarized distributions ($\Delta\bar{u}$ and $\Delta\bar{d}$ contribute to about 10% to the Bjorken sum rule).

Very few free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on $F_2^p(x,Q^2), F_2^n(x,Q^2), xF_3^{\nu N}(x,Q^2)$ and $g_1^{p,d,n}(x,Q^2)$, in correspondance with ten free parameters for the light quark sector with some physical significance:

* the four potentials $X_{0u}^+,\ X_{0u}^-,\ X_{0d}^-,\ X_{0d}^+,$
* the universal temperature $\bar{x}$,
* and $b,\ \bar{b},\ \tilde{b},\ b_G,\ \tilde{A}.$

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* the universal temperature $\bar{x}$,
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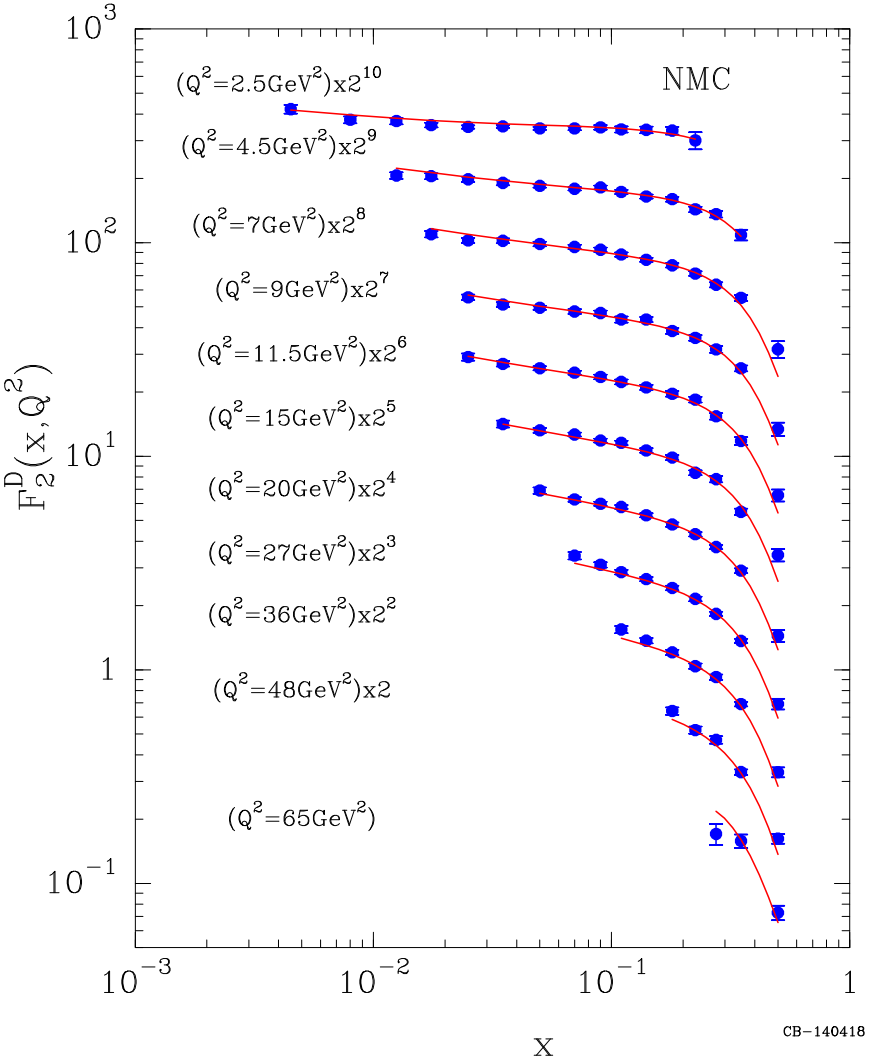
We also have three additional parameters, $A,\ \bar{A},\ A_G$, which are fixed by 3 normalization conditions .

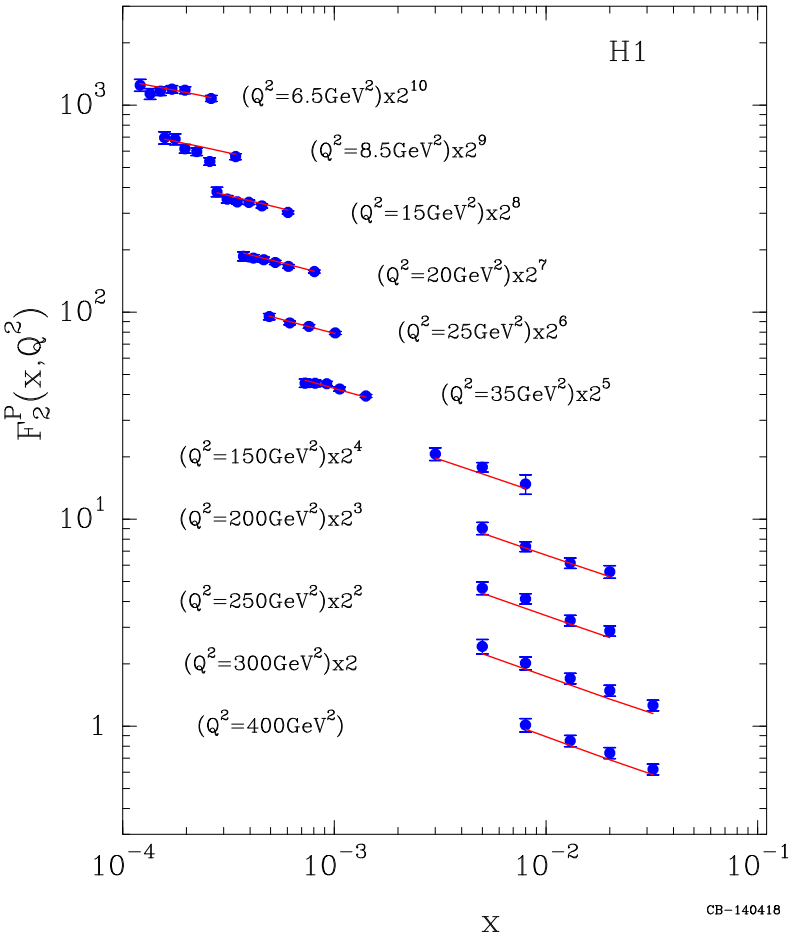
$$u - \bar{u} = 2, \quad d - \bar{d} = 1$$

and the momentum sum rule.

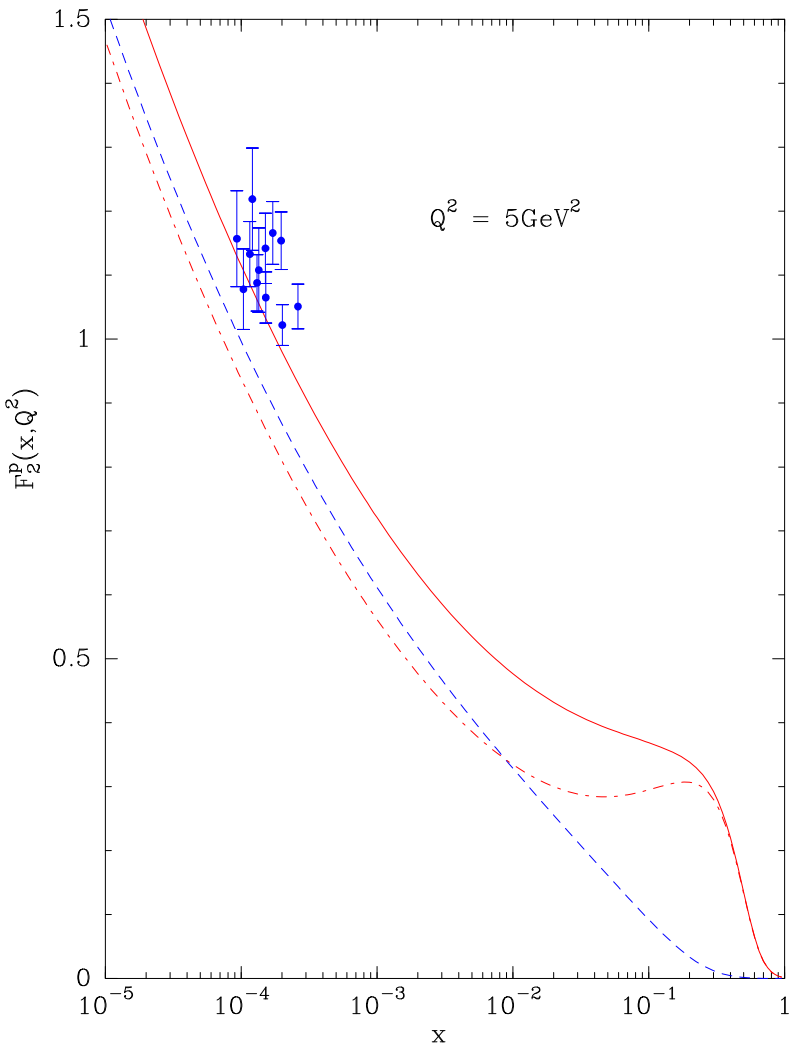
There are several additional parameters to describe the strange quark-antiquark sector and for the gluon polarization. We use the constraint $s - \bar{s} = 0$.
We note that potentials become smaller for heaviest quarks and since $X_{0s}^- > X_{0s}^+$, we will have $\Delta s < 0$ like for d-quarks.

Some data on $F_2^D(x,Q^2), F_2^p(x,Q^2)$



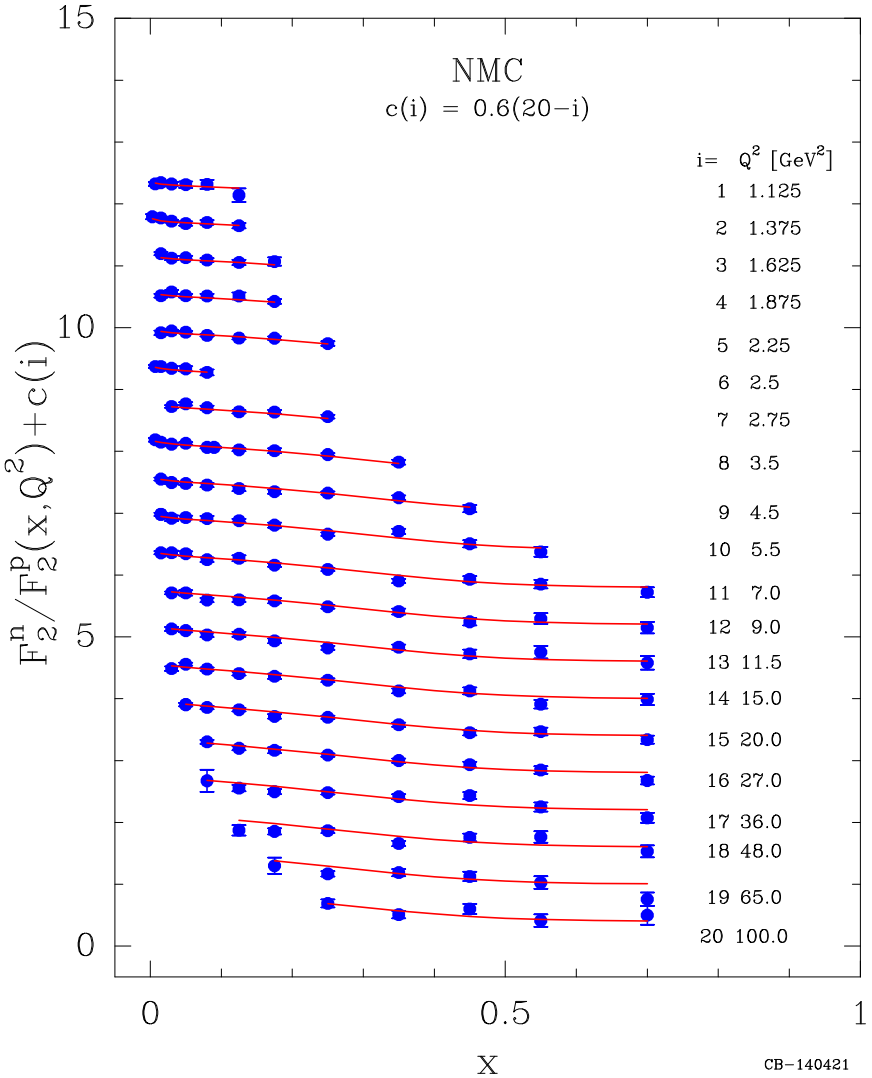


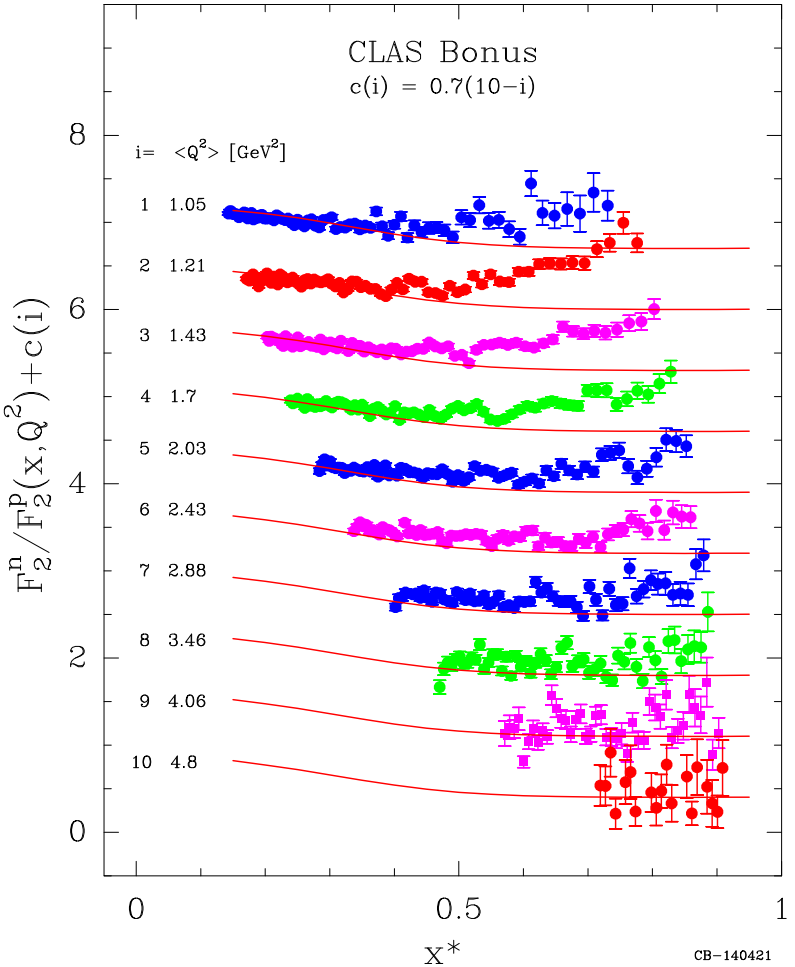
Effect of the evolution on the diffractive term



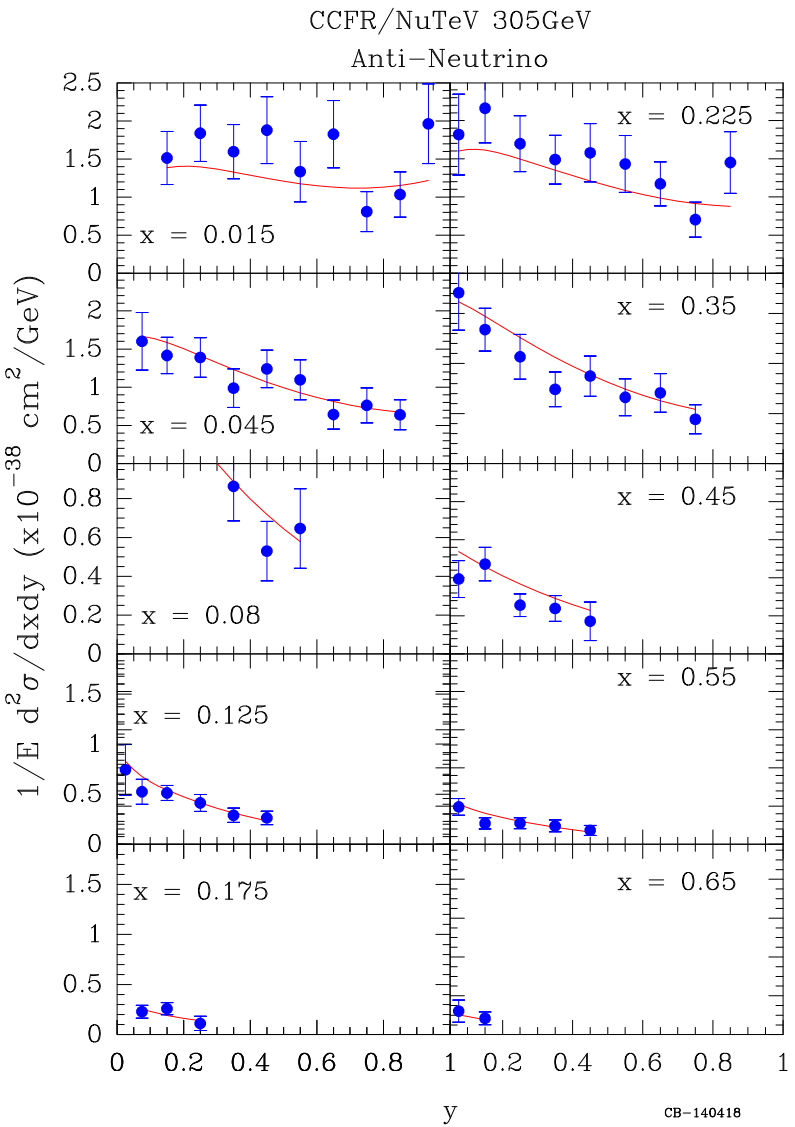
Both terms become comparable for small x, as soon as Q^2 takes off

Some data on $F_2^n(x,Q^2)/F_2^p(x,Q^2)$

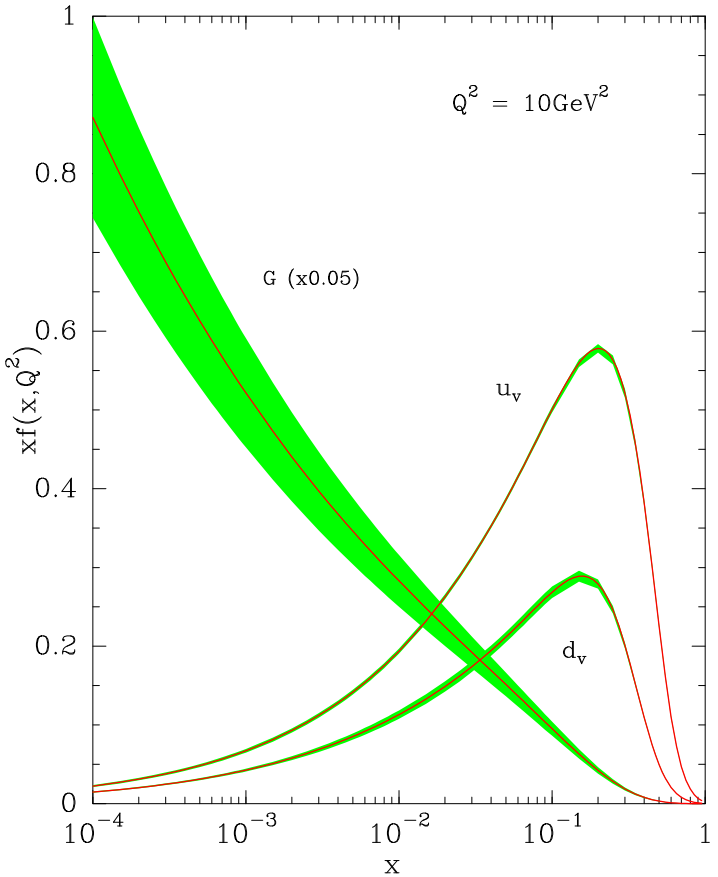


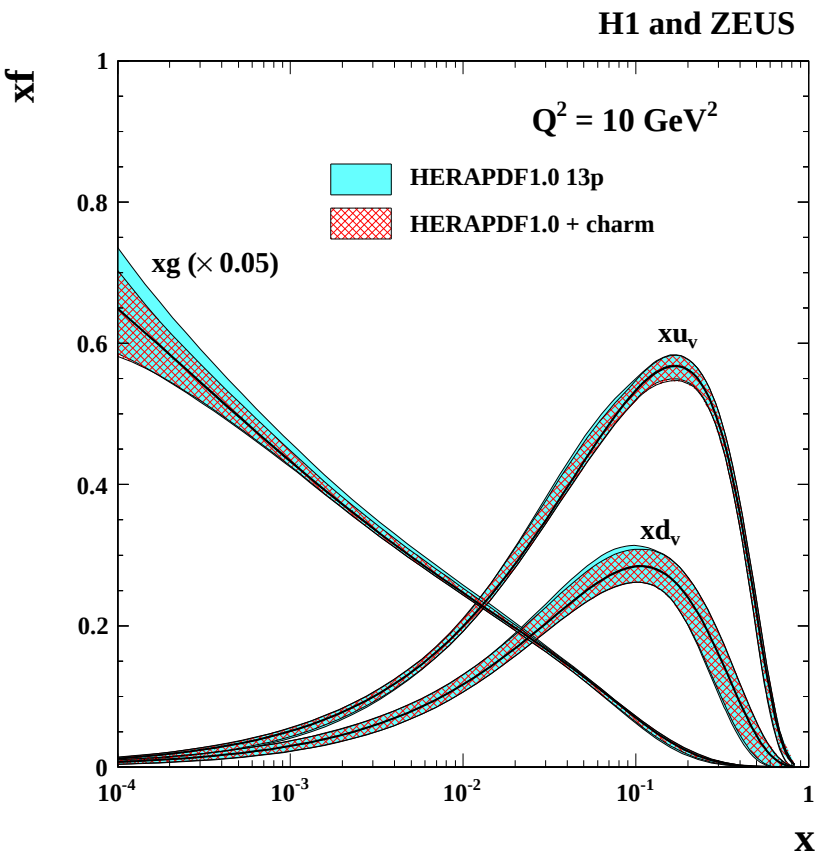


Some data on anti-neutrino



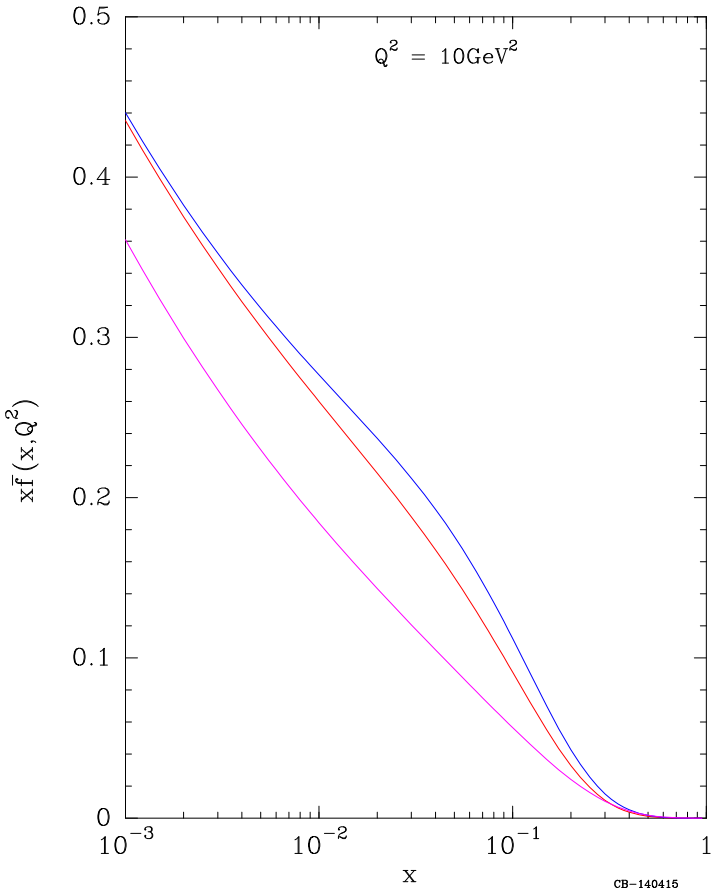
A global view of the unpolarized parton distributions

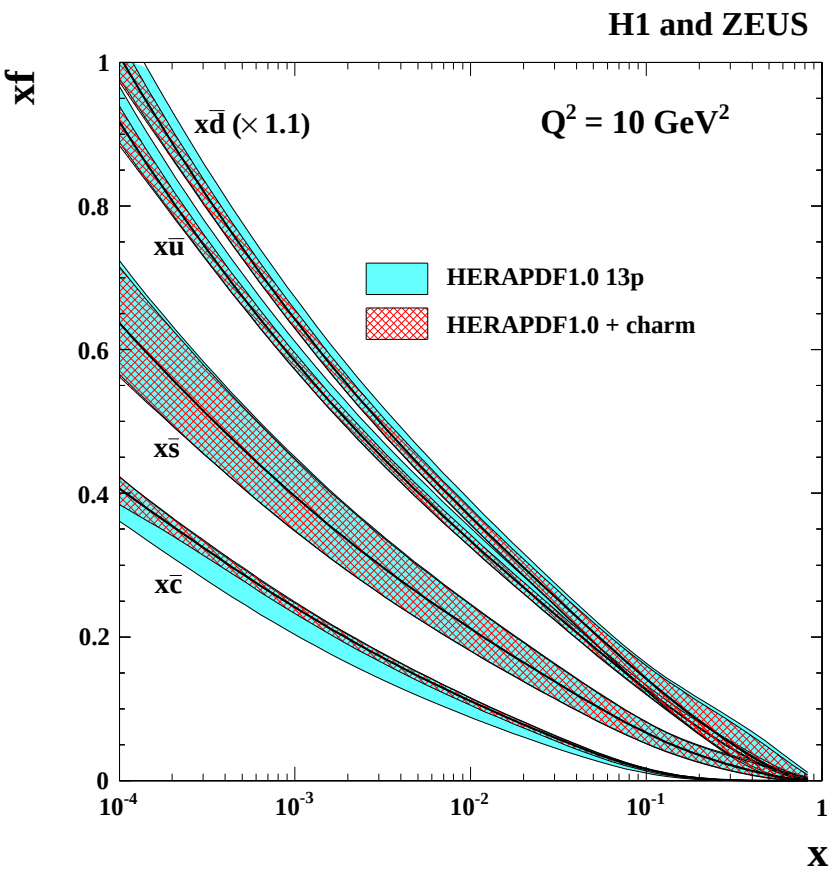




u_v, d_v agree well, but G grows a bit faster at low x

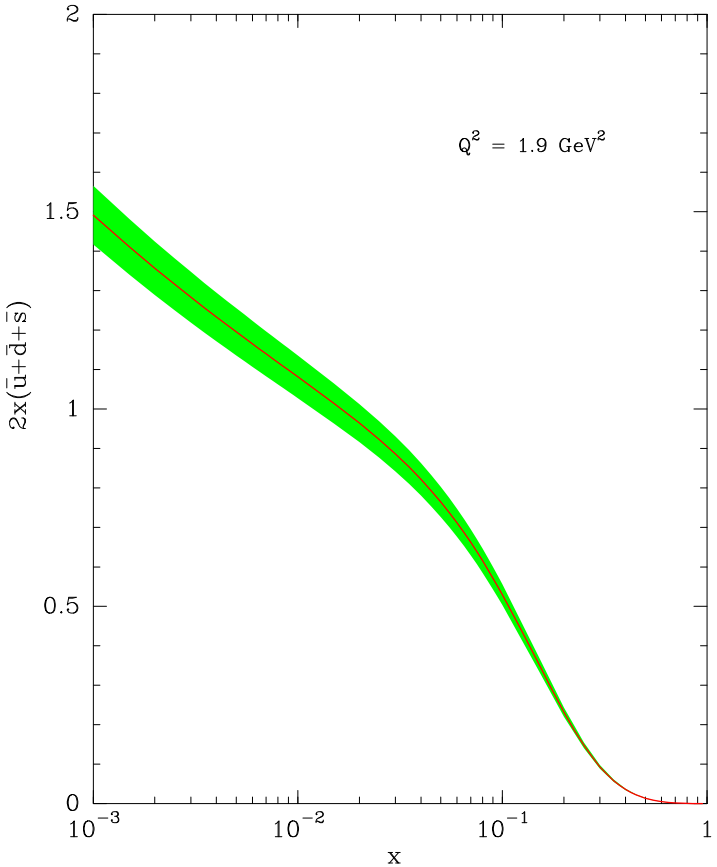
A global view of the unpolarized sea parton distributions

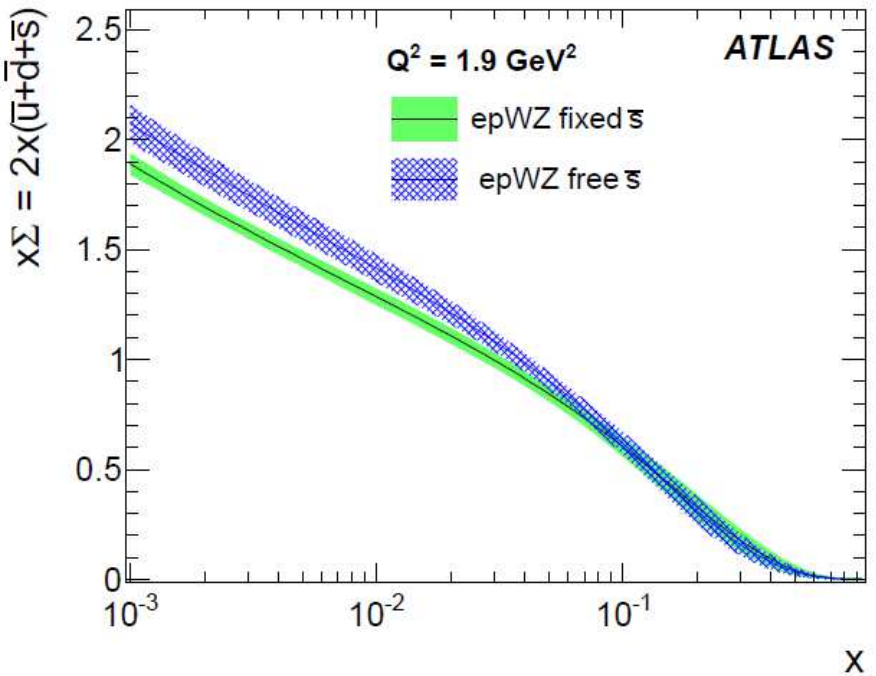




$\bar{s}$ agrees well but $\bar{u}, \bar{d}$ grow slower and we expect $\bar{d} > \bar{u}$ for all x

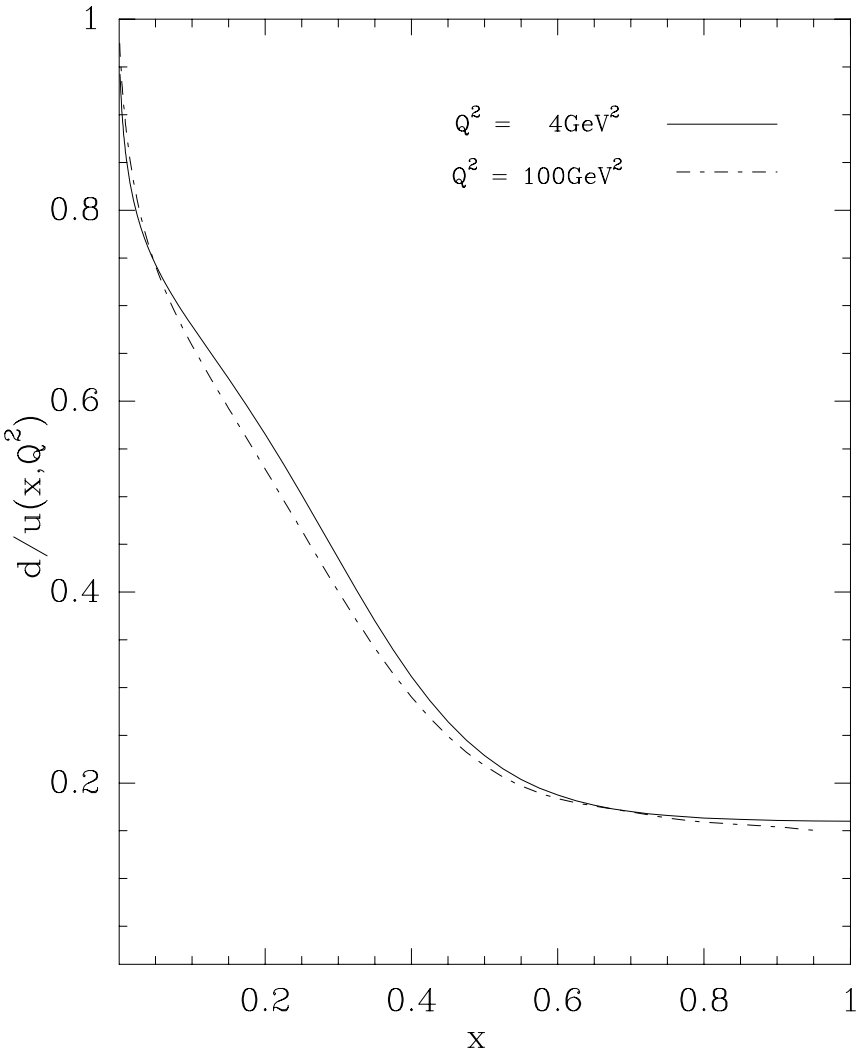
Unpolarized sea parton distributions from ATLAS + HERA



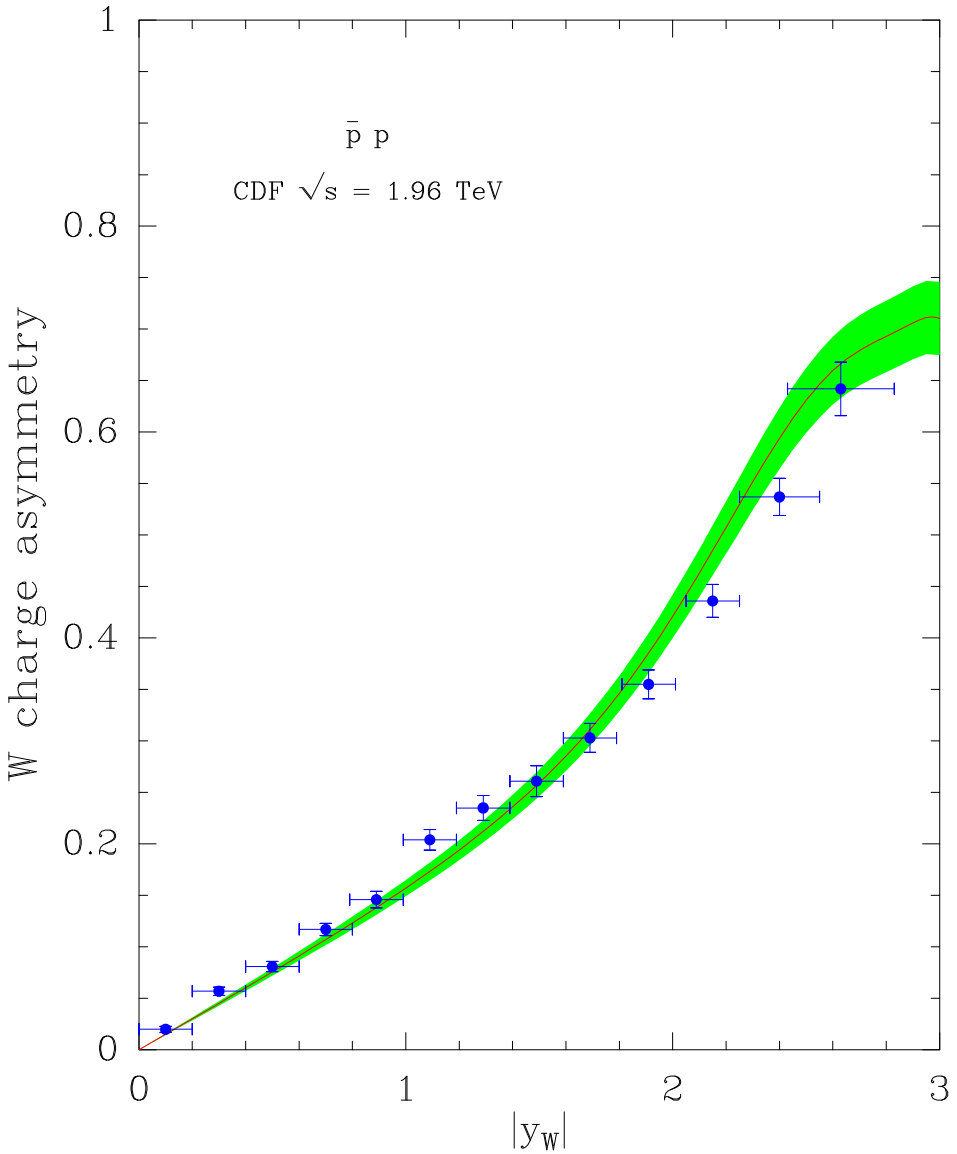


It grows slower than the data

The predicted d/u ratio versus x

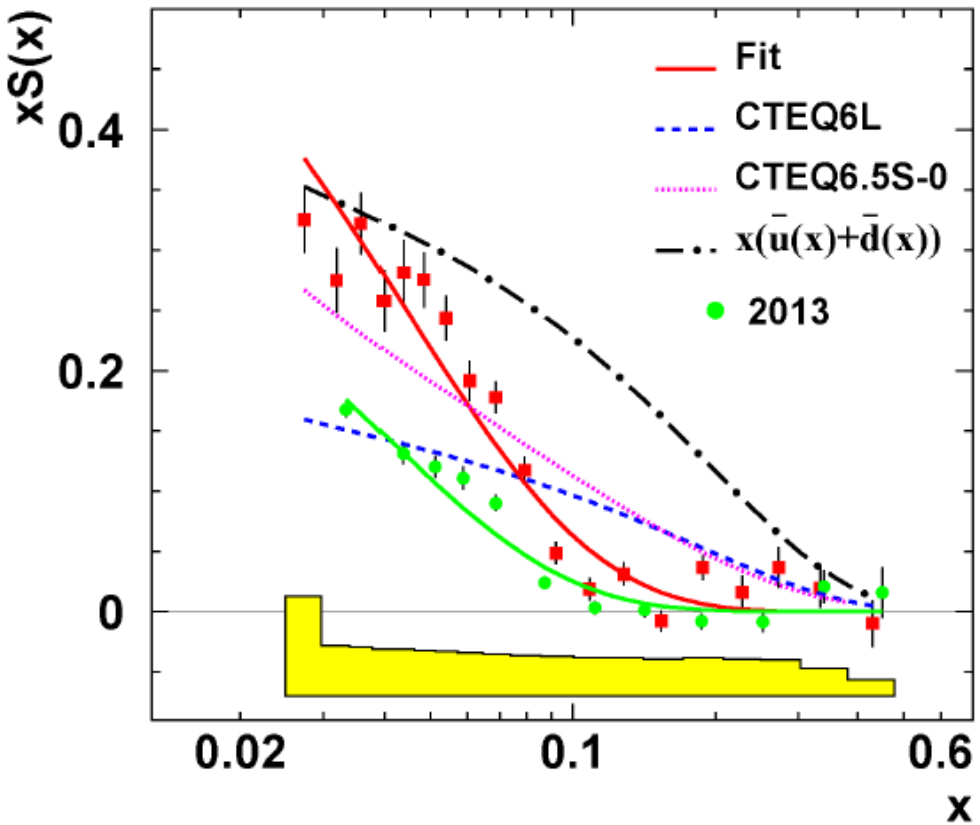


The predicted charge asymmetry from BBS, PLB 726, 296 (2013)



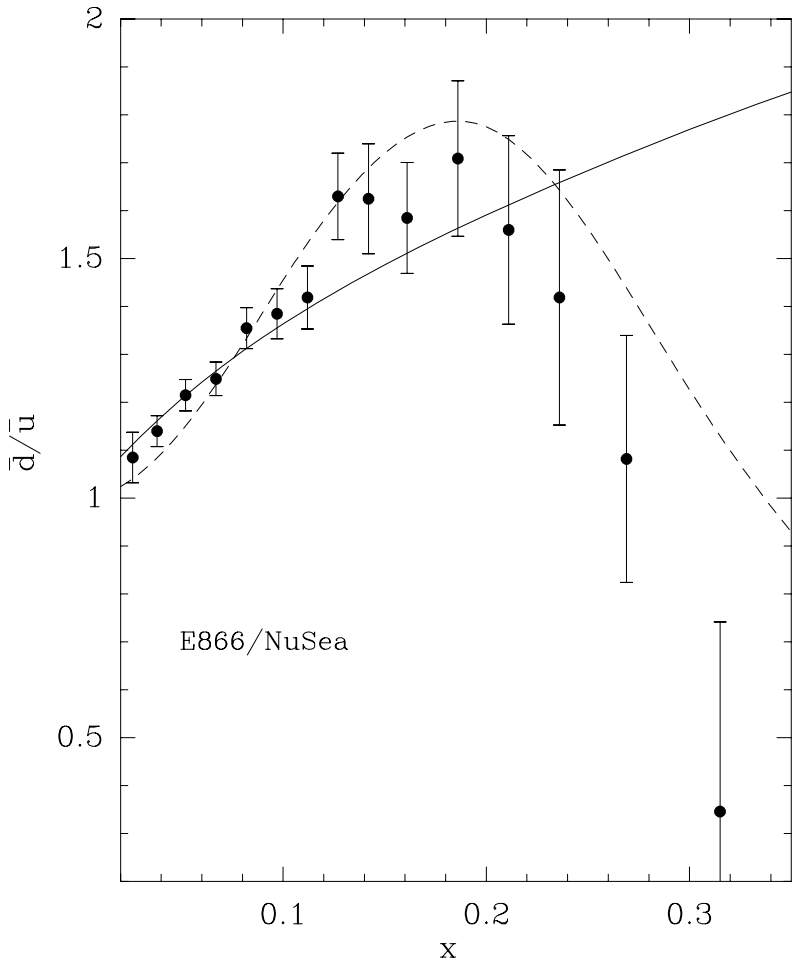
The $s(x, Q^2) + \bar{s}(x, Q^2)$ from HERMES (2013)

Comparison PLB666 with HERMES (2013)



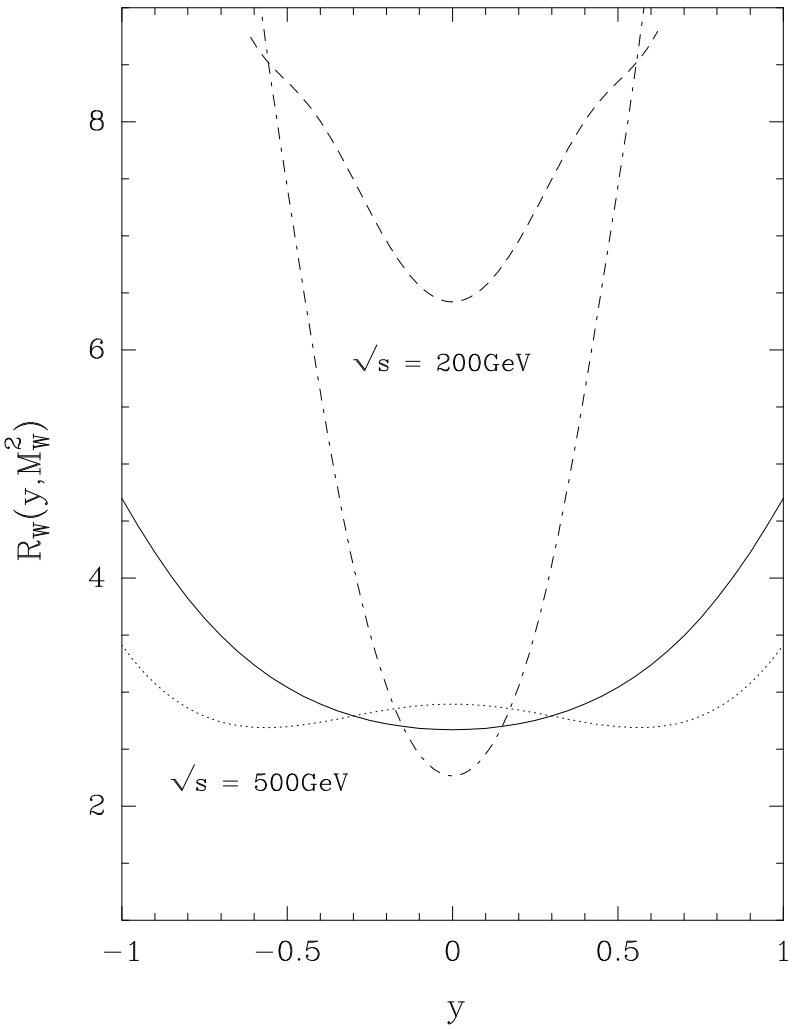
s-quark distributions mainly in the low x region. Not compatible with HERA data

Important issue: $\bar{d}/\bar{u}$ at large x and high Q^2



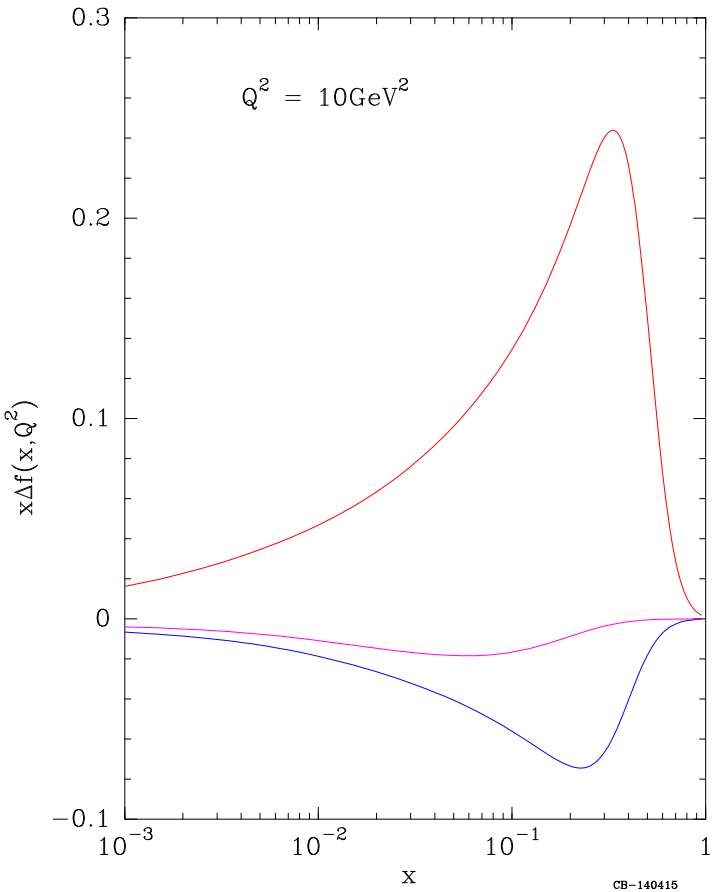
We look forward to the results of E906

Important issue: $\bar{d}/\bar{u}$ at large x and high Q^2

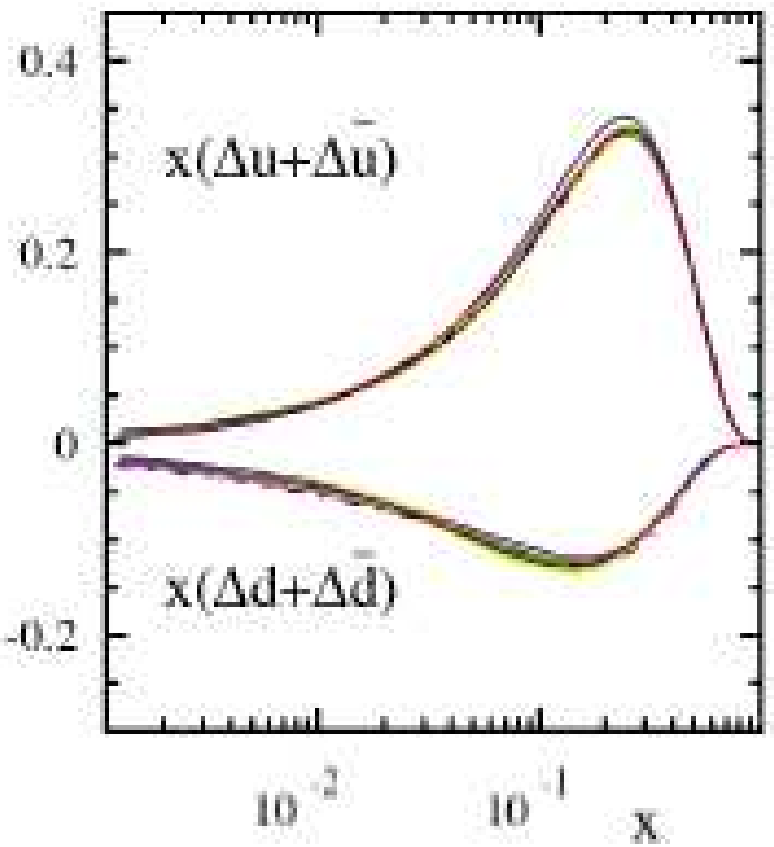


Ratio of $W^{\pm}$ cross sections: Another possible way to access it

A global view of the polarized parton distributions

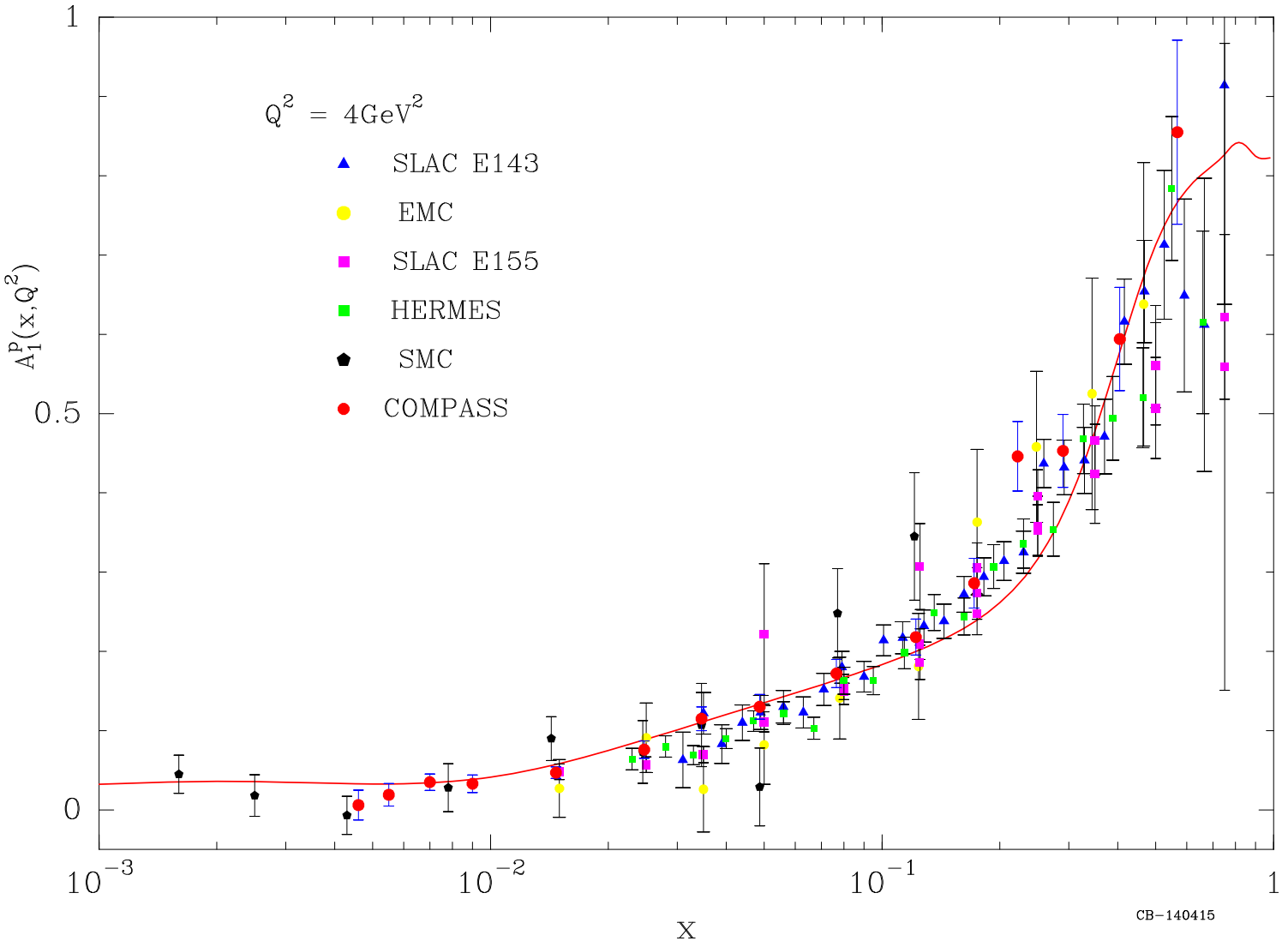


BBS



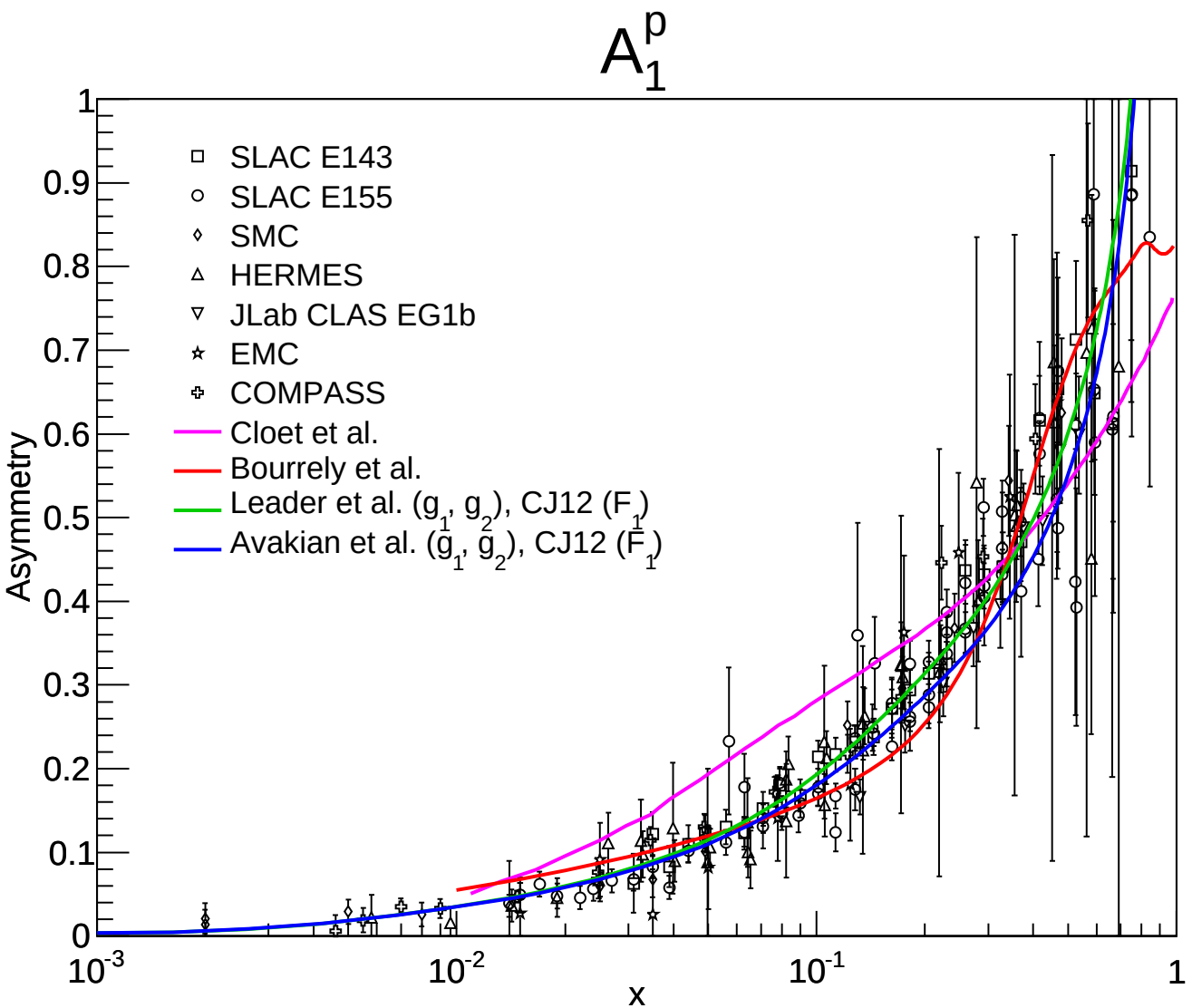
DSSV

A compilation of data on $A_1^p(x, Q^2)$

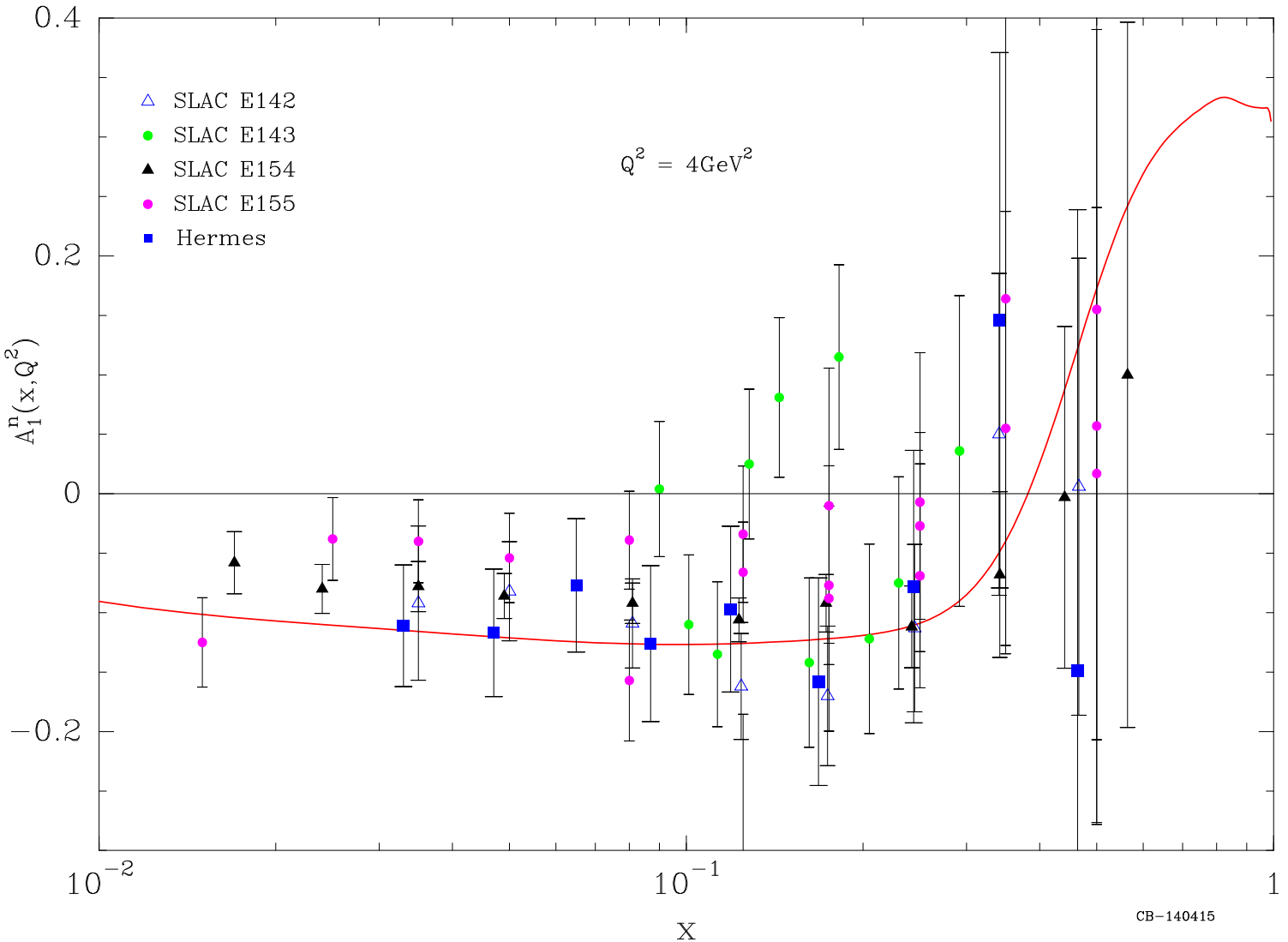


A compilation of data on $A_1^p(x, Q^2)$.

Comparison with models (thanks to D. Flay)



A compilation of data on $A_1^n(x, Q^2)$



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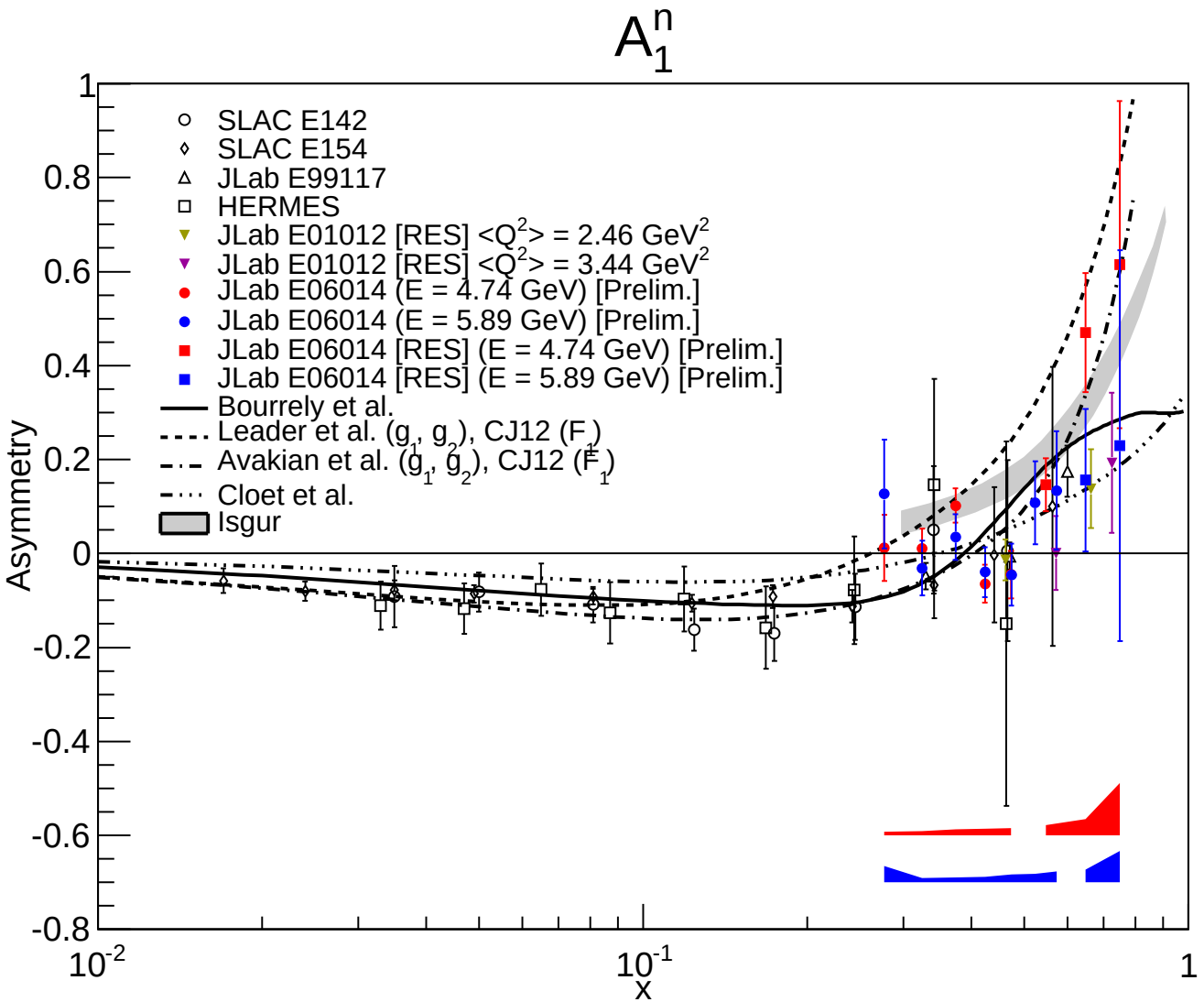
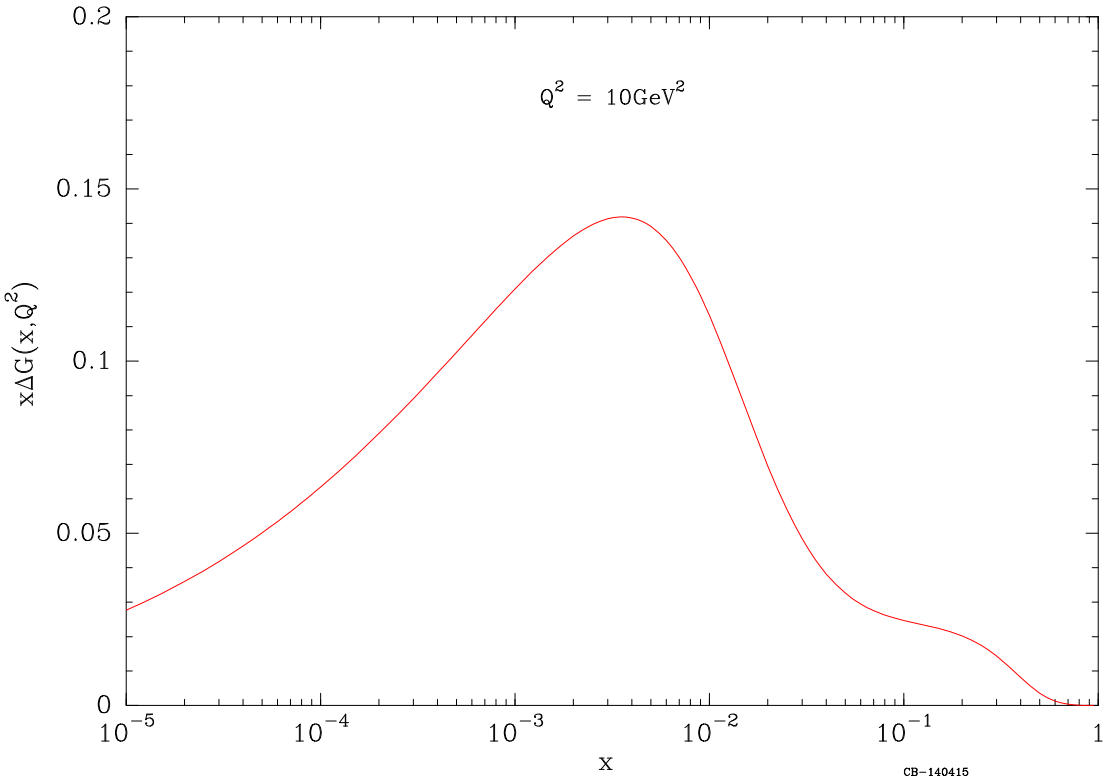
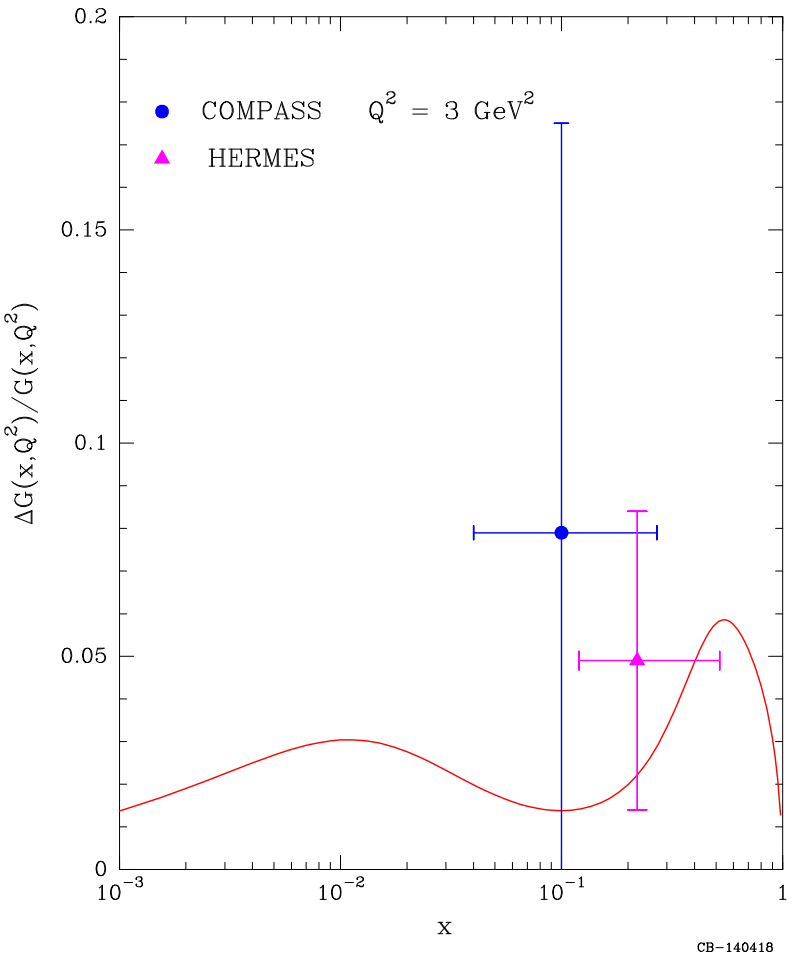


The elusive gluon polarization $x\Delta G(x, Q^2)$



It seems to be concentrated in the low x-region. PHENIX and STAR plan to access it.

The ratio $\Delta G(x, Q^2)/G(x, Q^2)$



Comparison with recent COMPASS data

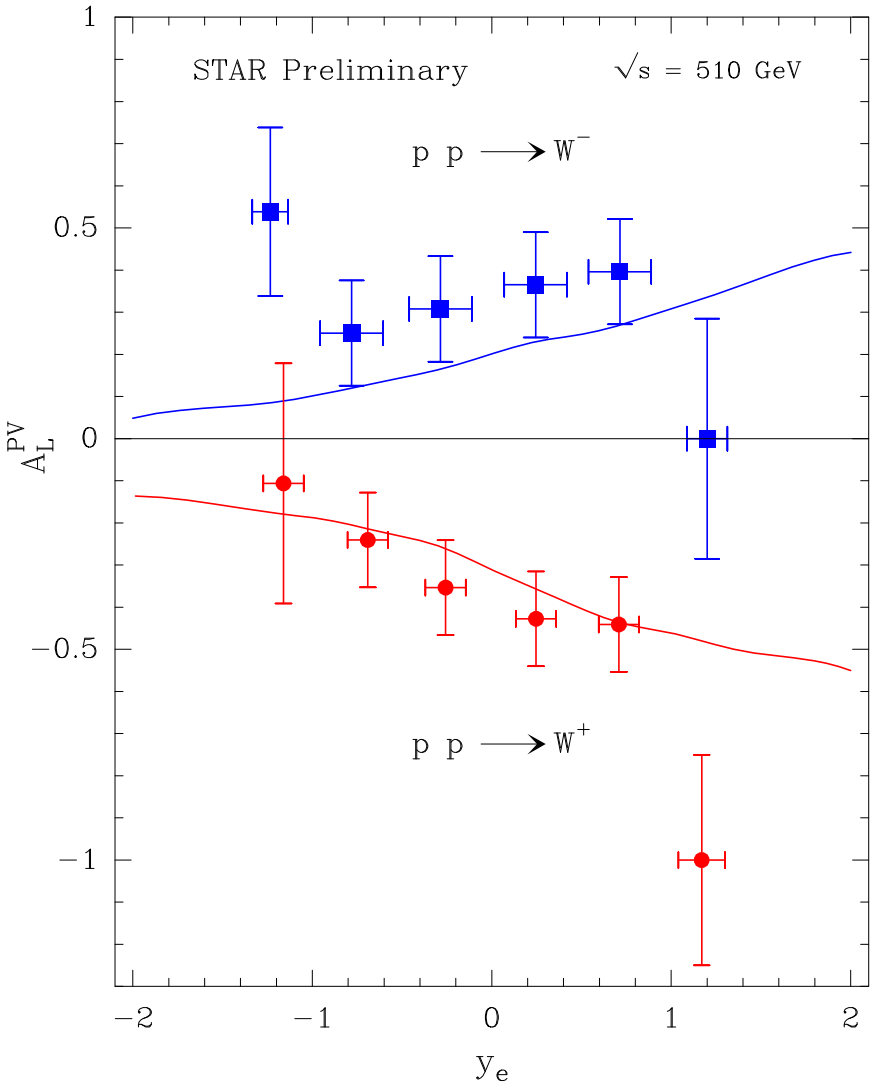
Helicity asymmetry in $W^{\pm}$ production at BNL-RHIC

Consider the processes $\overrightarrow{p}p \to W^{\pm} + X \to e^{\pm} + X$, where the arrow denotes a longitudinally polarized proton and the outgoing $e^{\pm}$ have been produced by the leptonic decay of the $W^{\pm}$ boson. The helicity asymmetry is defined as $A_L = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}$.
Here σ_h denotes the cross section where the initial proton has helicity h.
For W^- production, the numerator of the asymmetry is found to be proportional to

$$\Delta\bar{u}(x_1, M_W^2)d(x_2, M_W^2)(1 - \cos\theta)^2 - \Delta d(x_1, M_W^2)\bar{u}(x_2, M_W^2)(1 + \cos\theta)^2 \ ,$$

where θ is the polar angle of the electron in the $c.m.s.$, with $\theta = 0$ in the forward direction of the polarized parton. The denominator of the asymmetry has a similar form, with a plus sign between the two terms of the above expression. For W^+ production, the asymmetry is obtained by interchanging the quark flavors ($u \leftrightarrow d$).
We first show below the results of the calculations of the helicity asymmetries, versus the charged-lepton pseudo-rapidity and for a clear interpretation some explanations are required. At high negative η_e, one has $x_2 >> x_1$ and $\theta >> \pi/2$, so the first term above dominates and the asymmetry generated by the W^- production is driven by $\Delta\bar{u}(x_1)/\bar{u}(x_1)$, for medium values of x_1. Similarly for high positive η_e, the second term dominates and now the asymmetry is driven by $-\Delta d(x_1)/d(x_1)$, for large values of x_1. So we have a clear separation between these two contributions.

Helicity asymmetry in $W^{\pm}$ production at BNL-RHIC (BBS, PLB 726, 296 (2013))



Comparison with preliminary STAR data

Transverse momentum dependence (TMD) of the PDF

How to introduce the TMD of the PDF ?

There are several possibilities

- Assume factorization and simple Gaussian behavior for the PDF

$$q(x, k_T) = q(x) \frac{1}{\pi \mu_0^2} \exp[-k_T^2/\mu_0^2] \ ,$$

and also for the fragmentation function

$$D(z, q_T) = D(z) \frac{1}{\pi \mu_D^2} \exp[-q_T^2/\mu_D^2] \ .$$

A naive assumption which has no theoretical justification

- No factorization: Covariant approach, derivative method

- No factorization: The statistical distributions for quarks and antiquarks

(TMD) in the statistical approach

The parton distributions $p_i(x, k_T^2)$ of momentum k_T, must obey the momentum sum rule

$$\sum_i \int_0^1 dx \int x p_i(x, k_T^2) dk_T^2 = 1 \ ,$$

and also the transverse energy sum rule

$$\sum_i \int_0^1 dx \int p_i(x, k_T^2) \frac{k_T^2}{x} dk_T^2 = M^2 \ .$$

From the general method of statistical thermodynamics we are led to put $p_i(x, k_T^2)$ in correspondance with the following expression

$$\exp(\frac{-x}{\bar{x}} + \frac{-k_T^2}{x\mu^2}) \ ,$$

where μ^2 is a parameter interpreted as the transverse temperature.

So we have now the main ingredients for the extension to the TMD of the statistical PDF.

We obtain in a natural way the Gaussian shape with NO x, k_T factorization

(TMD) in the statistical approach

The quantum statistics distributions for quarks and antiquarks read in this case

$$xq^h(x,k_T^2) = \frac{F(x)}{\exp(x - X_{0q}^h)/\bar{x} + 1}\frac{1}{\exp(k_T^2/x\mu^2 - Y_{0q}^h) + 1} \ ,$$

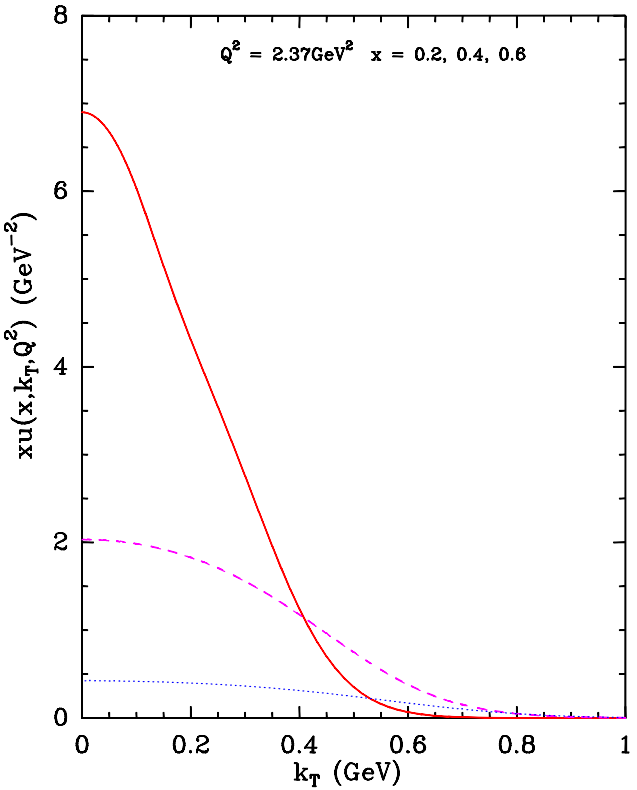
$$x\bar{q}^h(x,k_T^2) = \frac{\bar{F}(x)}{\exp(x + X_{0q}^{-h})/\bar{x} + 1}\frac{1}{\exp(k_T^2/x\mu^2 + Y_{0q}^{-h}) + 1},$$

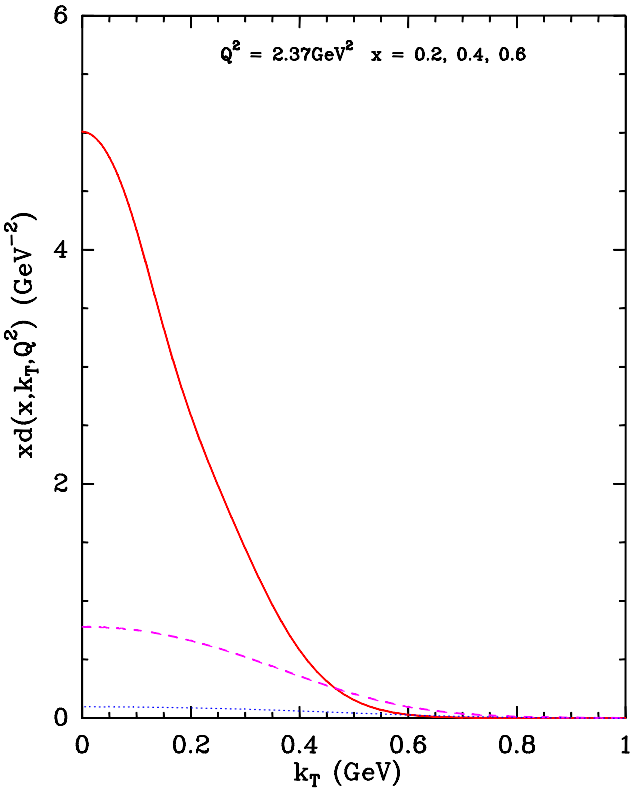
where

$$F(x) = \frac{Ax^{b-1}X_{0q}^h}{\ln(1+\exp Y_{0q}^h)\mu^2} = \frac{Ax^{b-1}}{k\mu^2} \ ,$$

because Y_{0q}^h are the thermodynamical potentials chosen such that $\ln(1+\exp Y_{0q}^h) = kX_{0q}^h$, in order to recover the factors X_{0q}^h, introduced earlier. Similarly for $\bar{q}$ we have $\bar{F}(x) = \bar{A}x^{2b-1}/k\mu^2$. This determination of the 4 potentials Y_{0q}^h can be achieved with the choice $k = 3.05$. Finally μ^2 will be determined by the transverse energy sum rule and one finds $\mu^2 = 0.198$GeV^2.

The statistical distributions u and d vs k_T





Melosh-Wigner effects

So far in all our quark or antiquark TMD distributions, the label "'h'" stands for the helicity along the longitudinal momentum and not along the direction of the momentum, as normally defined for a genuine helicity. The basic effect of a transverse momentum $k_T \neq 0$ is the Melosh-Wigner rotation, which mixes the components $q^{\pm}$ in the following way

$$q^{+MW} = \cos^2\theta\, q^+ + \sin^2\theta\, q^- \quad \text{and} \quad q^{-MW} = \cos^2\theta\, q^- + \sin^2\theta\, q^+ \ ,$$

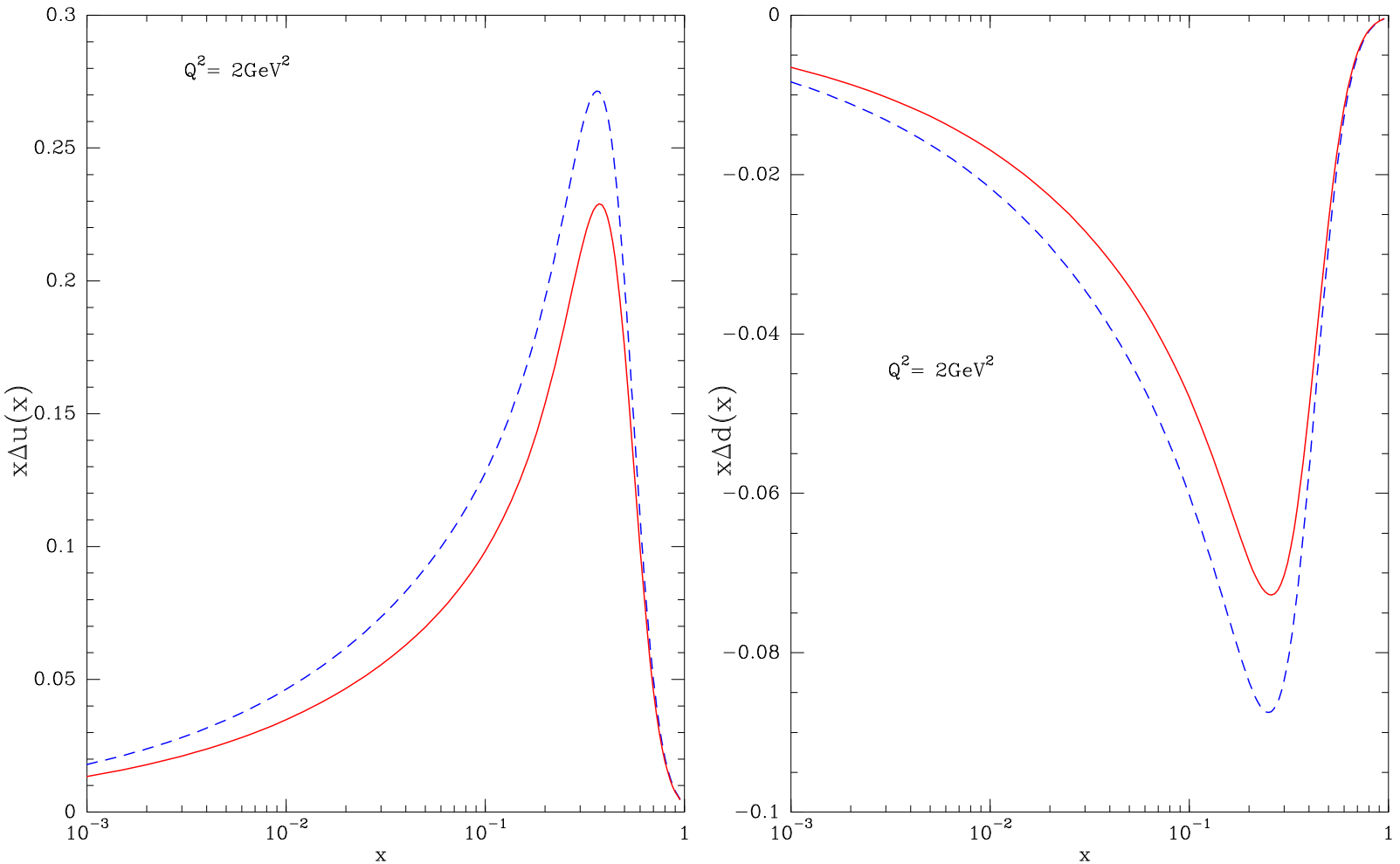
where, for massless partons, $\theta = \arctan\left(\frac{k_T}{p_0+p_z}\right)$, with $p_0 = \sqrt{k_T^2 + p_z^2}$.

It vanishes when either $k_T = 0$ or p_z goes to infinity.

Consequently $q = q^+ + q^-$ remains unchanged since $q^{MW} = q$,

whereas we have $\Delta q^{MW} = (\cos^2\theta - \sin^2\theta)\Delta q$.

Predicted quark helicity distributions



The effect is relevant for small Q^2 and mainly in the low x region

Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon.
- All unpolarized and polarized distributions depend upon a small number of free parameters, with some physical meaning.
- New tests against experiments in particular, for unpolarized and polarized sea distributions, are very satisfactory. s-quark distributions remain a problem
- Gluon helicity distribution is concentrated in the low x-region. Need to be confirmed
- Good predictive power but some special features remain to be verified, specially in the high x-region.
- Extension to TMD has been achieved and must be checked more accurately together with Melosh-Wigner effects in the low x-region