
THE EVOLUTION OF SOFT-COLLINEAR EFFECTIVE THEORY

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FLAVORS OF SCET

- $SCET_I$: Hadronic Jets, measured with Thrust, jet masses, etc.
- $SCET_{II}$: (Recoil-sensitive) Jet Broadening, k_T -dependent distributions
- $SCET_G$: SCET with Glaubers (relevant for heavy-ion collisions)



EFFECTIVE FIELD THEORIES

- Simplify “full theory” by integrating out heavy or UV degrees of freedom. Replace with “effective theory” of the accessible, low-energy degrees of freedom.
- e.g. effective EW theory (integrate out W, Z bosons), heavy quark effective theory (integrate out momentum modes $\sim m_Q$)
- EFT must reproduce same IR, low-energy behavior of full theory. Mismatch in UV encoded in Wilson or “matching” coefficients.
- EFT may possess additional symmetries or computational simplifications beyond full theory (e.g. heavy quark spin-flavor symmetry in HQET, soft-collinear decoupling in SCET.)
- EFT is an approximation to full theory to order-by-order in a power counting expansion in a small parameter λ



FLASHBACK: HEAVY QUARK EFFECTIVE THEORY

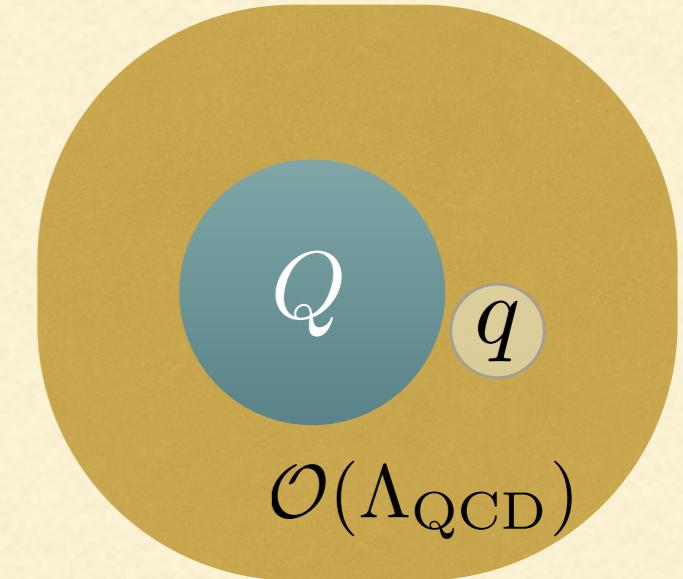
see Isgur, Wise (90s); Manohar, Wise (2000)

- Expansion of QCD in $1/m_Q$, HQET fields describes small momentum fluctuations around heavy quark at rest:

$$p_Q = m_Q v + k$$

$$k \sim \Lambda_{\text{QCD}}$$

$$v = (1, \vec{0})$$



- Factor out large momentum phases, integrate out large momentum fluctuations:

$$\psi(x) = e^{im_Q v \cdot x} \psi_v(x)$$

$$\mathcal{L}_Q = (\bar{h}_v \quad \bar{H}_v) \begin{pmatrix} iD & iD \\ iD & iD - 2m_Q \end{pmatrix} \begin{pmatrix} h_v \\ H_v \end{pmatrix}$$

$$h_v(x) \equiv \frac{1+\not{v}}{2} \psi_v(x) \quad H_v(x) \equiv \frac{1-\not{v}}{2} \psi_v(x)$$

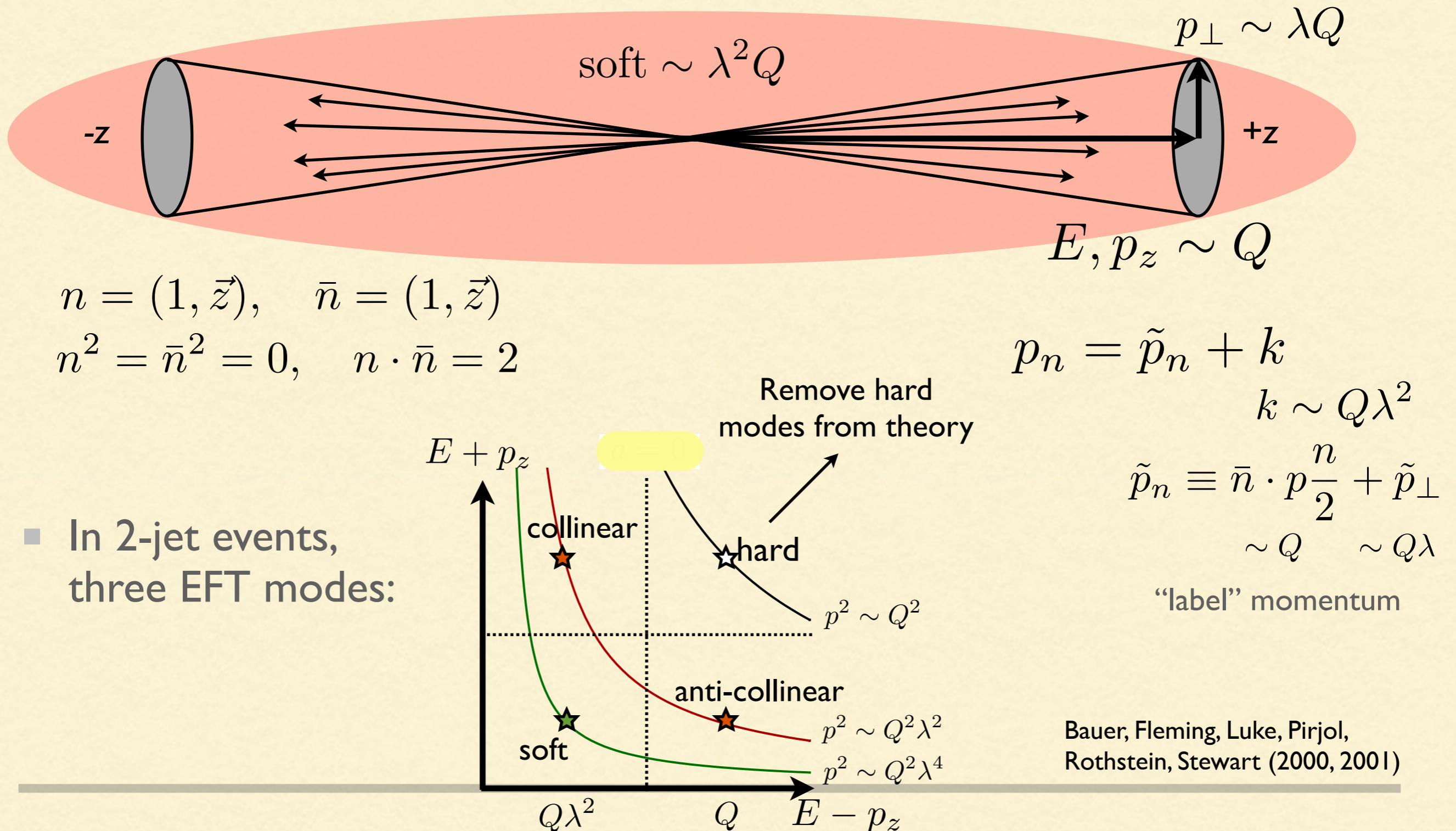
integrate out H_v

- Simpler Lagrangian at leading power for HQET field, enhanced **spin-flavor symmetry**, systematic power expansion

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v(x)(iv \cdot D)h_v(x) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right)$$

SOFT-COLLINEAR EFFECTIVE THEORY: MODES

- Similarly to HQET, SCET modes describe small fluctuations around *lightlike* momentum trajectories:



SCET LAGRANGIAN

Bauer, Fleming, Pirjol, Stewart
(2000, 2001)

- Factor out large momentum phase and project out “heavy” and “light” components of fermion fields:

$$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \psi_{n,p}(x)$$

$$\xi_{n,p} = \frac{\not{n}\not{\bar{n}}}{4} \psi_{n,p}$$

massless
effective mass Q ,
integrate out

- Similarly for gluon fields. Resulting leading-power Lagrangian:

$$\mathcal{L}_{\text{SCET}} = \sum_n (\mathcal{L}_{qn} + \mathcal{L}_{gn}) + \mathcal{L}_s$$

$$\mathcal{L}_{qn} = \bar{\xi}_n(x) \left[in \cdot D + i \not{D}_\perp^c W_n(x) \frac{1}{i\bar{n} \cdot \mathcal{P}} W_n^\dagger(x) i \not{D}_\perp^c \right] \frac{\not{n}}{2} \xi_n(x)$$

$$\mathcal{L}_{gn} = \frac{1}{2g^2} \text{Tr} [i\mathcal{D}^\mu + gA_n, i\mathcal{D}^\nu + gA_n^\nu]^2$$

$$\mathcal{D}^\mu = \bar{n} \cdot \mathcal{P} \frac{n^\mu}{2} + \mathcal{P}_\perp^\mu + in \cdot D \frac{\bar{n}^\mu}{2}$$

$$\mathcal{L}_s = \mathcal{L}_{\text{QCD}}[q_s, A_s]$$

\mathcal{P}^μ “label momentum operator”

$$\xi_n(x) = \sum_{\tilde{p}} e^{i\tilde{p}\cdot x} \xi_{n,\tilde{p}}(x) \quad \text{similarly for collinear gluons}$$

SCET FEYNMAN RULES

■ Collinear quarks:

$$\begin{array}{c} \tilde{p} + k \\ \hline \text{---} \rightarrow \text{---} \end{array} = i \frac{\not{\epsilon}}{2} \frac{\bar{n} \cdot \tilde{p}}{n \cdot k \bar{n} \cdot \tilde{p} + \tilde{p}_\perp^2 + i\epsilon}$$
$$\begin{array}{c} \text{---} \rightarrow \text{---} \\ \text{---} \rightarrow \text{---} \quad \text{---} \end{array} = igT^A n_\mu \frac{\not{\epsilon}}{2}$$
$$\begin{array}{c} \text{---} \rightarrow \text{---} \\ \text{---} \rightarrow \text{---} \quad \text{---} \end{array} = igT^A \left(n_\mu + \frac{\gamma_\mu^\perp \tilde{p}_\perp}{\bar{n} \cdot \tilde{p}} + \frac{\tilde{p}'_\perp \gamma_\mu^\perp}{\bar{n} \cdot \tilde{p}'} - \frac{\tilde{p}'_\perp \tilde{p}_\perp}{\bar{n} \cdot \tilde{p} \bar{n} \cdot \tilde{p}'} \bar{n}_\mu \right) \frac{\not{\epsilon}}{2}$$

■ Similarly for collinear gluons.

SCET OPERATOR MATCHING

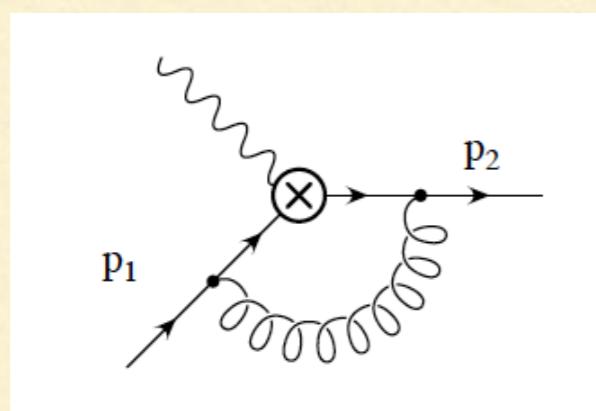
- Besides Lagrangian, also need to match QCD currents onto SCET operators.

- e.g. for e^+e^- to 2 jets, DIS, or Drell-Yan:

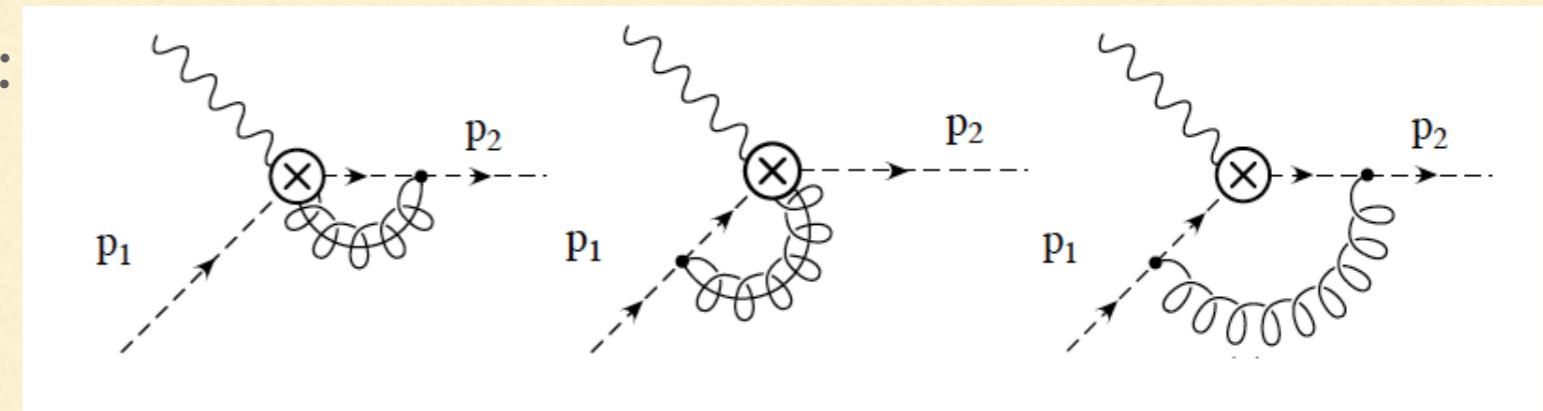
$$\bar{\psi} \gamma^\mu \psi \longrightarrow C_2(\mu) [\bar{\xi}_{n_1} W_{n_1}]_{\tilde{p}_1} \gamma^\mu [W_{n_2}^\dagger \xi_{n_2}]_{p_2}$$

- One-loop matching, difference of QCD and SCET loop graphs. IR divergences match, leaving over UV-dependent Wilson coefficient:

QCD:



SCET:



$$C_2(\mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi} \left[-\ln^2 \frac{\mu^2}{-\tilde{p}_1 \cdot \tilde{p}_2} - 3 \ln \frac{\mu^2}{-\tilde{p}_1 \cdot \tilde{p}_2} - 8 + \frac{\pi^2}{6} \right]$$

Manohar (2003);
Bauer, CL, Manohar,
Wise (2003)

SOFT-COLLINEAR DECOUPLING

Bauer, Pirjol, Stewart (2001)

- At leading power, soft-collinear interactions are eikonal:

$$\bar{\xi}_n(in \cdot D_s)\xi_n = \bar{\xi}_n(in \cdot \partial + gn \cdot A_s)\xi_n$$

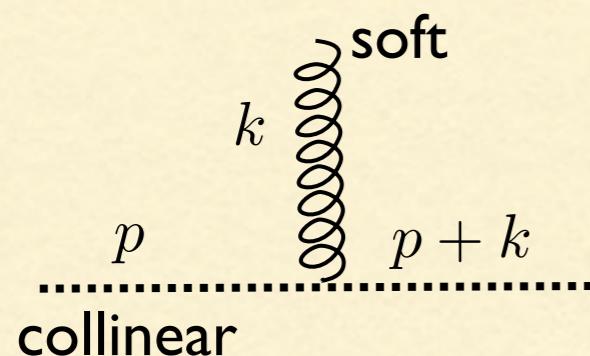
- Soft-collinear interactions resummed into Wilson lines:

$$Y_n(x) = P \exp \left[ig \int_{-\infty}^0 ds n \cdot A_s(ns + x) \right]$$

- Perform field redefinition:

$$\xi_n(x) = Y_n(x)\xi_n^{(0)}(x)$$

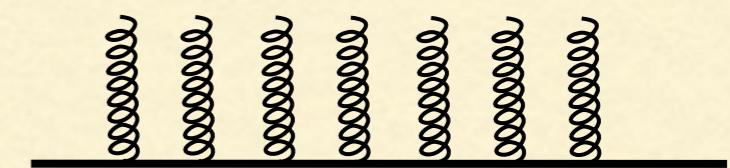
$$\bar{\xi}_n(in \cdot D_s)\xi_n \longrightarrow \bar{\xi}_n^{(0)}(in \cdot \partial)\xi_n^{(0)}$$



soft gluons “see”
only direction
and color of jet

$$\sim \cancel{p} \frac{ig\gamma^\mu t^a}{(p+k)^2} \cancel{p}$$

keep leading order part of diagram

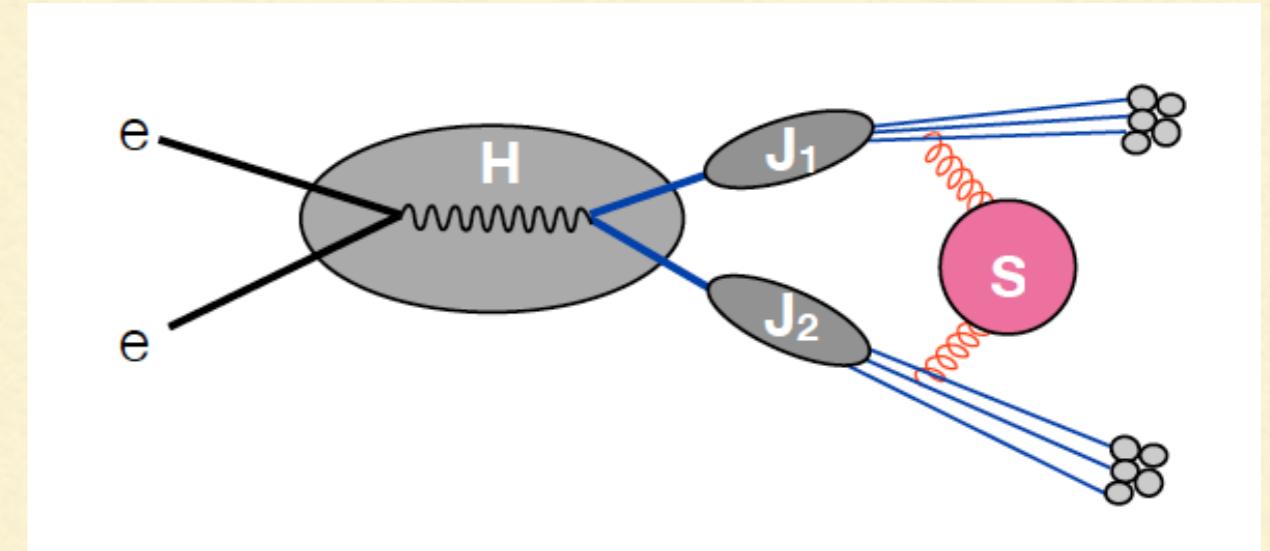


$$\frac{n^\mu t^a}{n \cdot k} \cancel{\not{n}}$$

FACTORIZATION FOR EVENT SHAPES

Collins, Soper, Sterman

- Two-jet event shapes in e^+e^- ,
e.g. thrust:

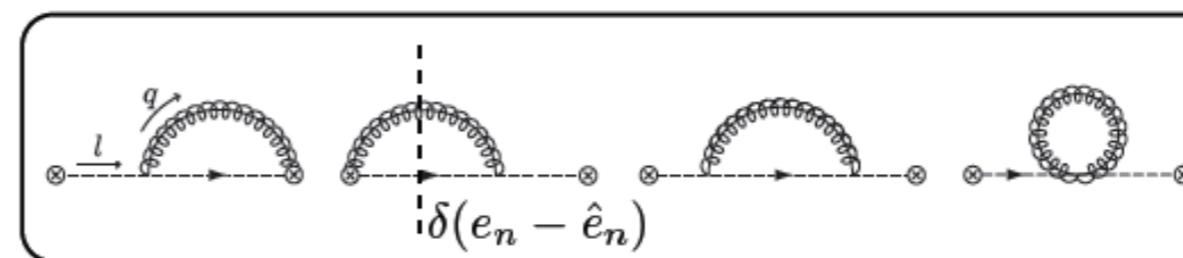


$$\frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = H_2(Q^2, \mu) \int dt_n dt_{\bar{n}} dk_S \delta\left(\tau - \frac{t_n + t_{\bar{n}}}{Q^2} - \frac{k_S}{Q}\right) J_n(t_n, \mu) J_{\bar{n}}(t_{\bar{n}}, \mu) S_2(k_S, \mu)$$

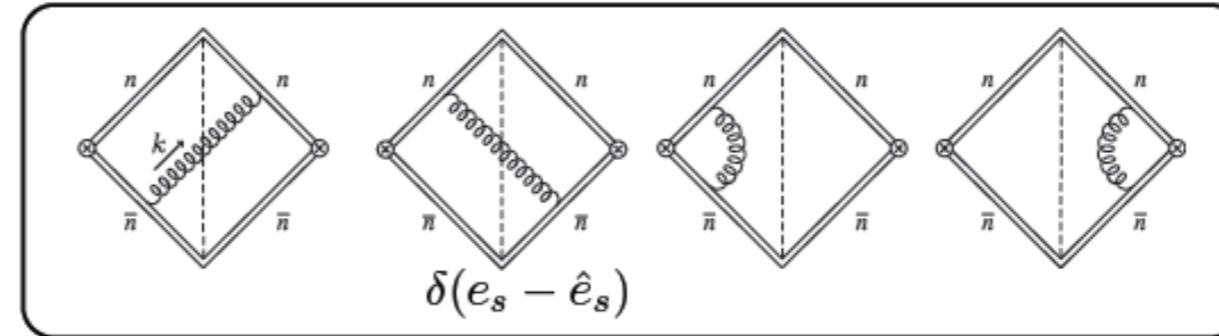
Fleming, Hoang, Mantry, Stewart (2007)

Bauer, Fleming, CL, Sterman (2008)

- Jet Function



- Soft Function



RESUMMATION OF LARGE LOGARITHMS

- Computing in fixed-order perturbation theory, we encounter large logs in the two-jet kinematic region:

$$\sigma(\tau) = \int_0^\tau d\tau_1 \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = 1 + \frac{\alpha_s}{4\pi} \left(F_{12} \ln^2 \tau - F_{11} \ln \tau + F_{10} \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left(F_{24} \ln^4 \tau + F_{23} \ln^3 \tau + F_{22} \ln^2 \tau + F_{21} \ln \tau + F_{20} \right) + \dots$$

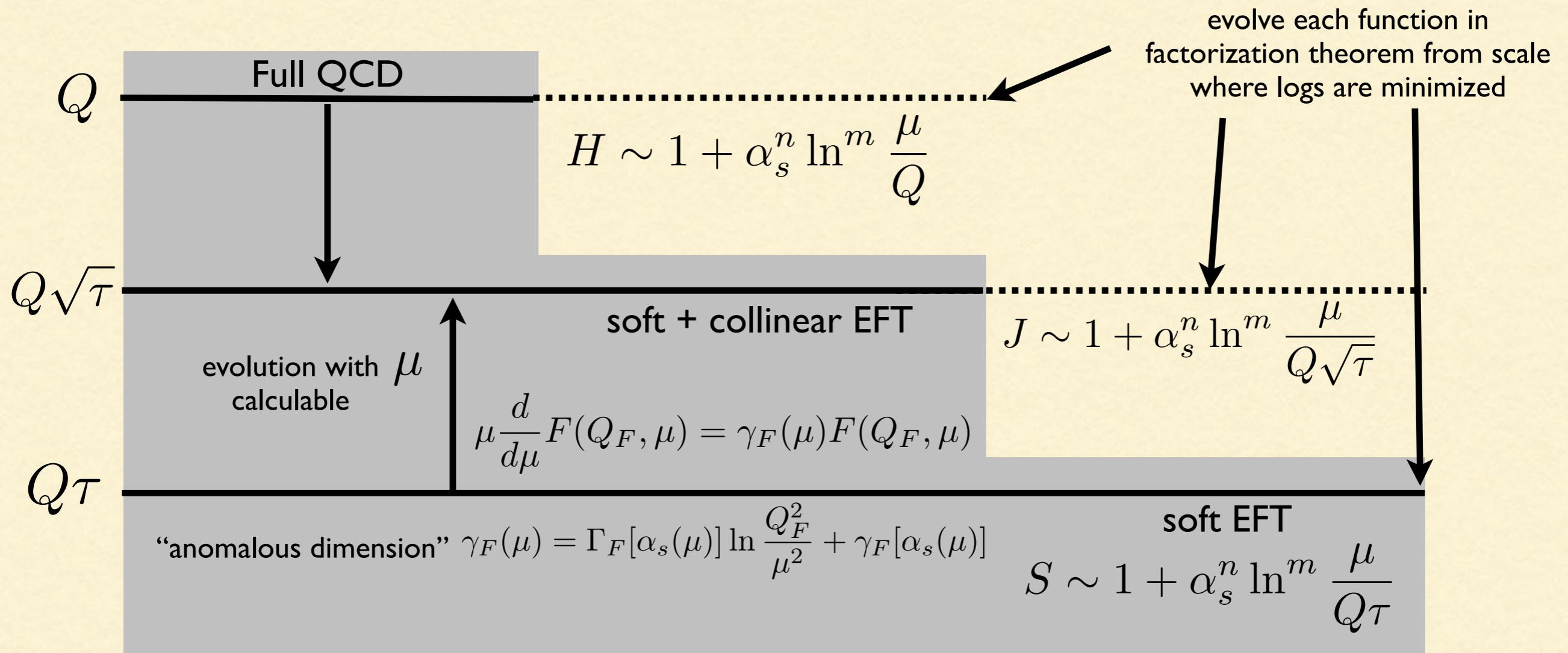
- Need to reorganize perturbative series to regain convergence:

$$\ln \sigma(\tau) \sim \alpha_s (\ln^2 \tau + \ln \tau) + \alpha_s^2 (\ln^3 \tau + \ln^2 \tau + \ln \tau) + \alpha_s^3 (\ln^4 \tau + \ln^3 \tau + \ln^2 \tau + \ln \tau) + \dots$$

Leading Log (LL)
 Next-to-Leading Log (NLL)
 NNLL

RESUMMATION FROM RG EVOLUTION

- SCET RG gives equations for evolution of hard, jet, and soft functions in factorization theorem with energy scale μ .



- Solutions of evolution equations contain logs resummed to all orders in α_s

SCET VS. “DIRECT” QCD RESUMMATION

- Resummed thrust distribution derived in “traditional” or “direct” QCD:

$$\sigma(\tau) = \mathcal{N}(Q) \exp \left[\sum_{n=2}^{\infty} \frac{1}{n!} \bar{E}^{(n)} \partial_{\bar{E}'}^n \right] \frac{e^{\bar{E}(\ln 1/\tau)}}{\Gamma(1 - \bar{E}'(\ln 1/\tau))}$$

CTTW (1993)
Berger, Kucs, Sterman (2003)

$$E^{(n)} = \frac{d^n E}{d[\ln(1/\tau)]^n}$$

$$\begin{aligned} E(\ln \nu) &= 2 \int_{Q(e^{\gamma_E} \nu)^{-1/2}}^Q \frac{d\mu'}{\mu'} \left\{ A[\alpha_s] \ln \left(\frac{\mu'}{Q} \right)^2 + B_J[\alpha_s] \right\} \\ &\quad - 2 \int_{Q(e^{\gamma_E} \nu)^{-1}}^{Q(e^{\gamma_E} \nu)^{-1/2}} \frac{d\mu'}{\mu'} \left\{ A[\alpha_s] \ln \left(\frac{\mu'}{Q(e^{\gamma_E} \nu)} \right)^2 + B_S[\alpha_s] \right\} \end{aligned}$$

- Form derived from solutions of RGEs in SCET:

$$\begin{aligned} \sigma(\tau) &= e^{K_H + 2K_J + K_S} \left(\frac{\mu_H}{Q} \right)^{\omega_H} \left(\frac{\mu_J}{Q\tau^{1/2}} \right)^{\omega_J} \left(\frac{\mu_S}{Q\tau} \right)^{\omega_S} H_2(Q^2, \mu_H) \\ &\quad \times \tilde{J} \left(\partial_\Omega + \ln \frac{\mu^2}{Q^2 \tau}, \mu_J \right)^2 \tilde{S} \left(\partial_\Omega + \ln \frac{\mu_S}{Q\tau}, \mu_S \right) \frac{e^{\gamma_E \Omega}}{\Gamma(1 - \Omega)} \end{aligned}$$

cf. Becher, Neubert (2006)

$$K_F(\mu, \mu_F) = \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'} \left\{ \Gamma_F[\alpha_s(\mu')] \ln \frac{\mu'}{\mu_F} + \gamma_F[\alpha_s(\mu')] \right\} \quad \Omega = 2\omega_J + \omega_S$$

$$\omega_F(\mu, \mu_F) = \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'} \Gamma_F[\alpha_s(\mu')]$$

SCET VS. “DIRECT” QCD RESUMMATION

Almeida, Ellis, CL, Sterman, Sung, Walsh, 1401.4460

- Showed that two forms are exactly equal with the dictionary:

$$A[\alpha_s] = \Gamma_{\text{cusp}}[\alpha_s] \quad B_F[\alpha_s] = \gamma_F[\alpha_s] - \frac{d \ln \tilde{F}(0, \mu_F)}{d \ln \mu_F}$$

$$\mathcal{N}(Q) = H(Q) \tilde{J}(0, Q)^2 \tilde{S}(0, Q)$$

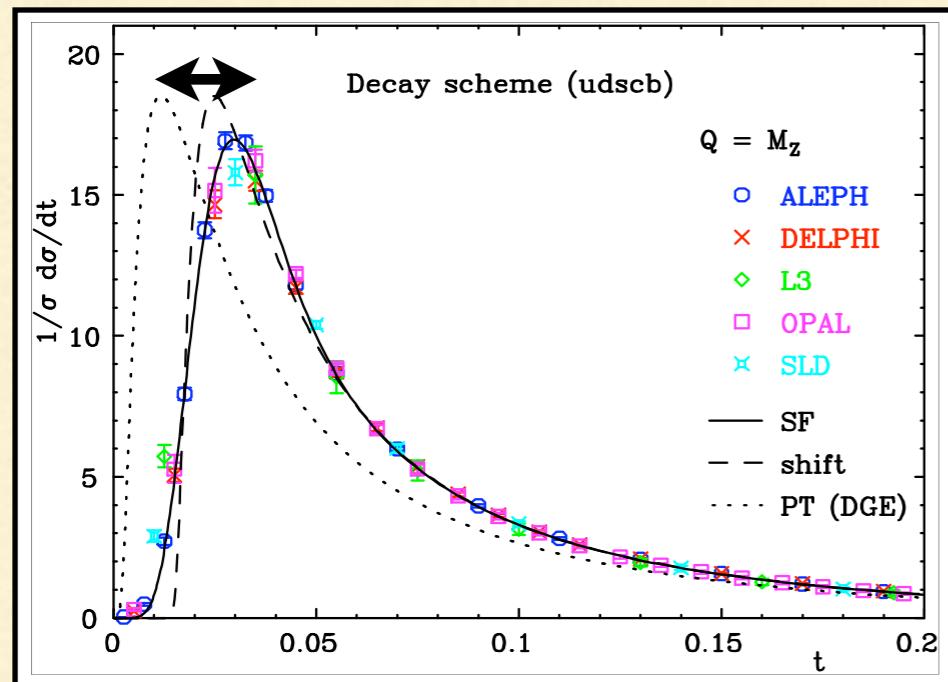
$$E = 2E_J + E_S \quad E_F(\mu, \mu_F) = K_F(\mu, \mu_F) - \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'} \frac{d \ln \tilde{F}(0, \mu')}{d \ln \mu'}$$

- Leads to unified formula, generalizing dQCD to variable jet/soft scales, and SCET to more explicitly exponentiated form:

$$\begin{aligned} \sigma(\tau) &= \sigma_0 H(\mu_H) \left(\frac{\mu_H}{Q} \right)^{\omega_H} e^{K_H} \tilde{J}(0, \mu)^2 \tilde{S}(0, \mu) e^{2E_J(\mu, \mu_J) + E_S(\mu, \mu_S)} \\ &\times \exp \left[\sum_{n=2}^{\infty} \frac{1}{n!} \left(2E_J^{(n)}(\mu_J) \partial_{2E'_J}^n + E_S^{(n)}(\mu_S) \partial_{E'_S}^n \right) \right] \\ &\times \left(\frac{Q(e^{-\gamma_E} \tau)^{1/2}}{\mu} \right)^{2E'_J(\mu, \mu_J)} \left(\frac{Q\tau}{\mu e^{\gamma_E}} \right)^{E'_S(\mu, \mu_S)} \frac{1}{\Gamma(1 + 2E'_J + E'_S)} \end{aligned}$$

NONPERTUBATIVE HADRONIZATION CORRECTIONS

- Soft power corrections shift mean values of event shapes



$$\langle e \rangle = \langle e \rangle_{\text{PT}} + c_e \frac{\Omega_1}{Q}$$

c_e observable dependent,
calculable coefficient

Ω_1 universal
nonperturbative
parameter

conjecture from single
soft gluon emission:
Dokshitzer, Webber
(1995, 1997)

proof to all orders in
soft gluon emission

CL, Sterman (2006, 2007)

- SCET: First rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

$$\Omega_1 = \frac{1}{N_C} \text{Tr} \langle 0 | \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger \mathcal{E}_T(\eta) Y_n \bar{Y}_{\bar{n}} | 0 \rangle$$

generalization
to massive hadrons

Mateu, Thaler, Stewart (2012)

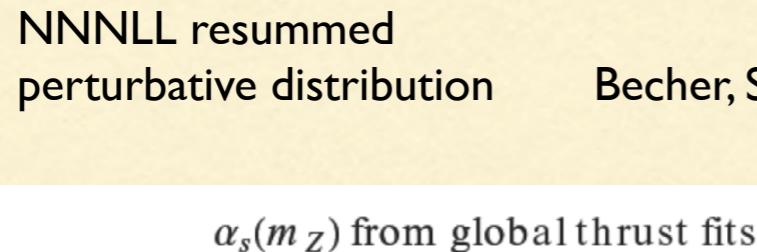


soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

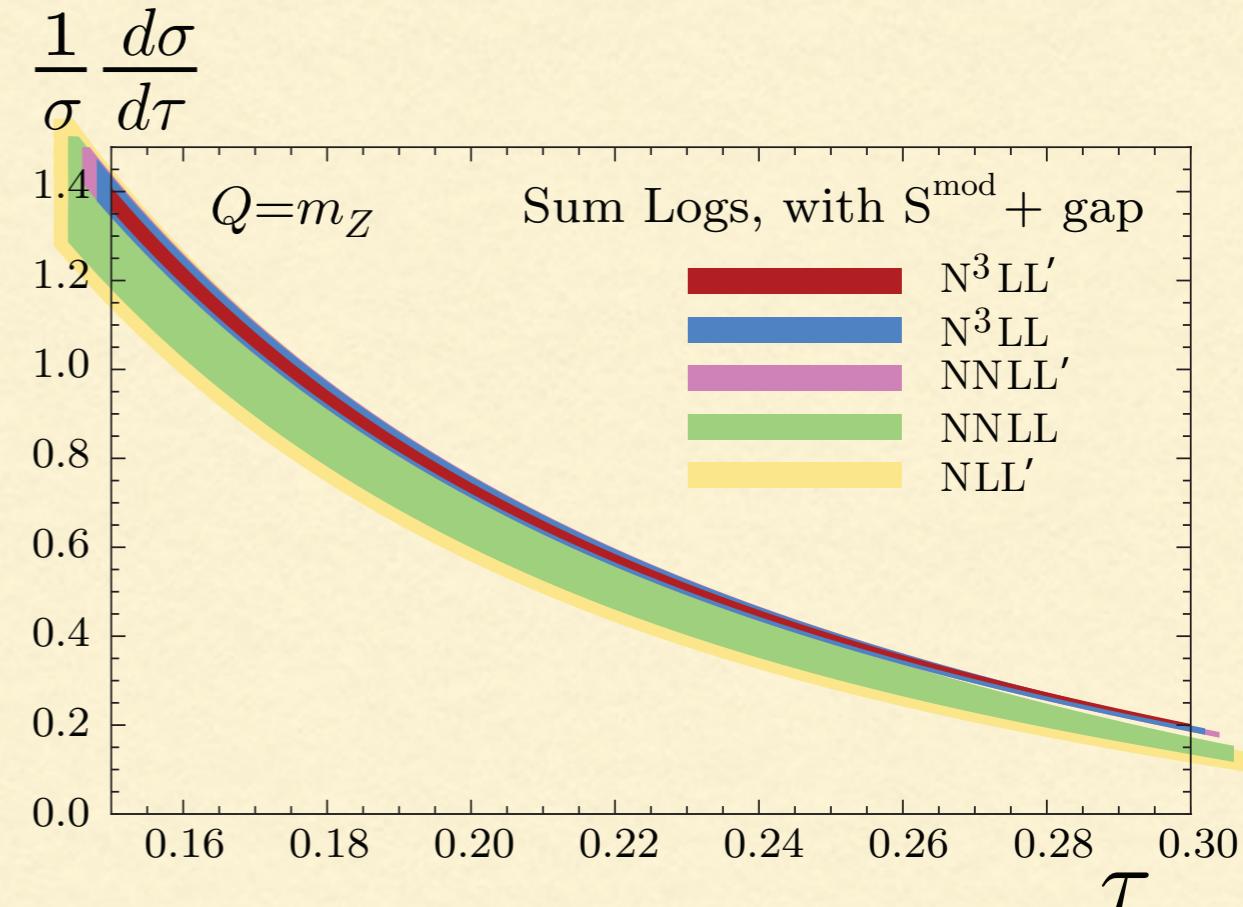
HIGH PRECISION STRONG COUPLING EXTRACTION

NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang,
Mateu, Stewart (2010)



Abbate, Fickinger, Hoang, Mateu, Stewart (2010)



GenEvA Collaboration (2012) fit to LEP data gave consistent value (improved Monte Carlo with NLL and higher-order theory input)

Generically, better perturbative calculations + rigorous treatment of nonperturbative corrections gives smaller α_s

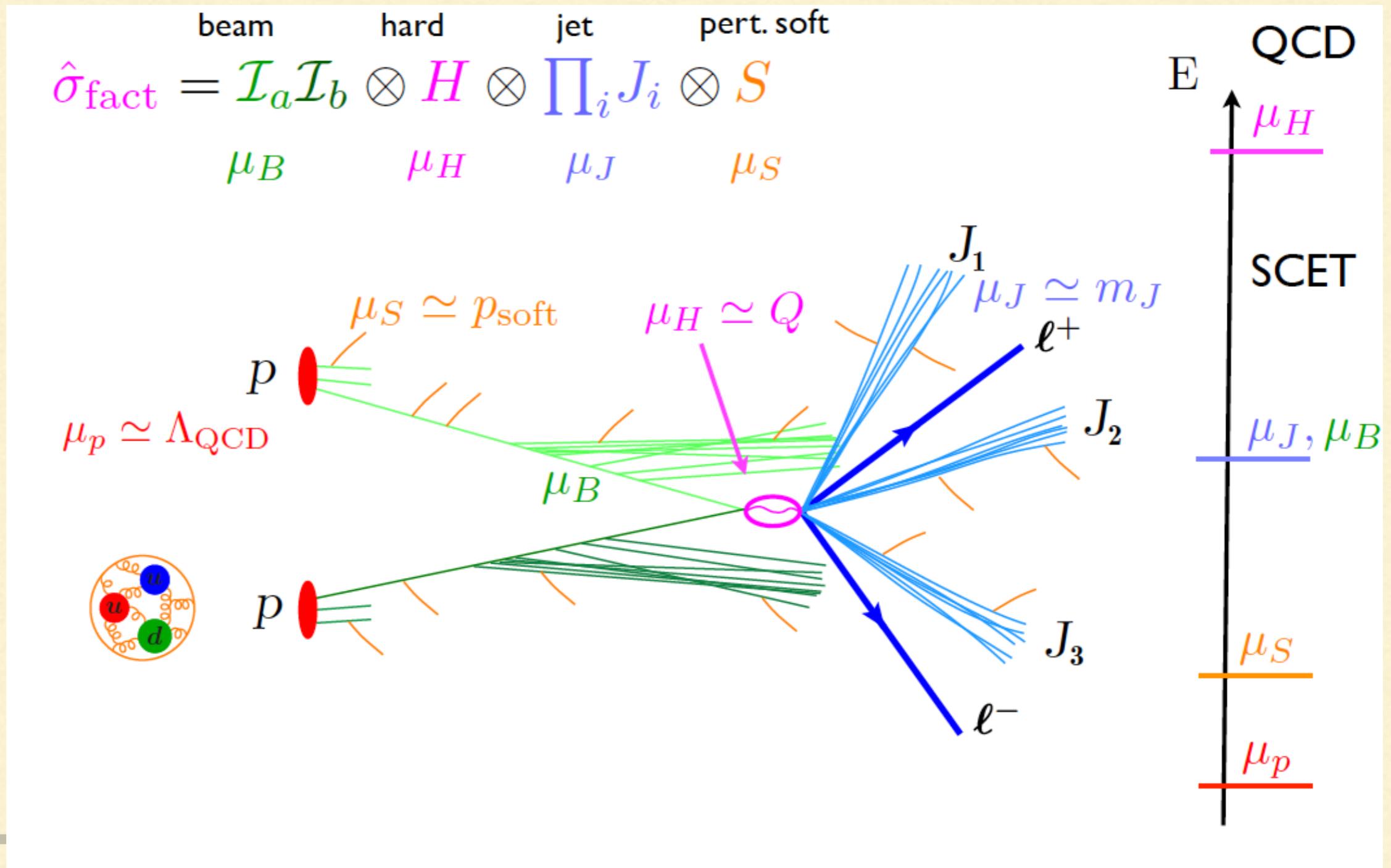
PROTON COLLISIONS: BEAM FUNCTIONS AND PDFS

Nonperturbative factorization: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F_{\text{had}}$

Perturbative factorization:

$$\hat{\sigma}_{\text{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

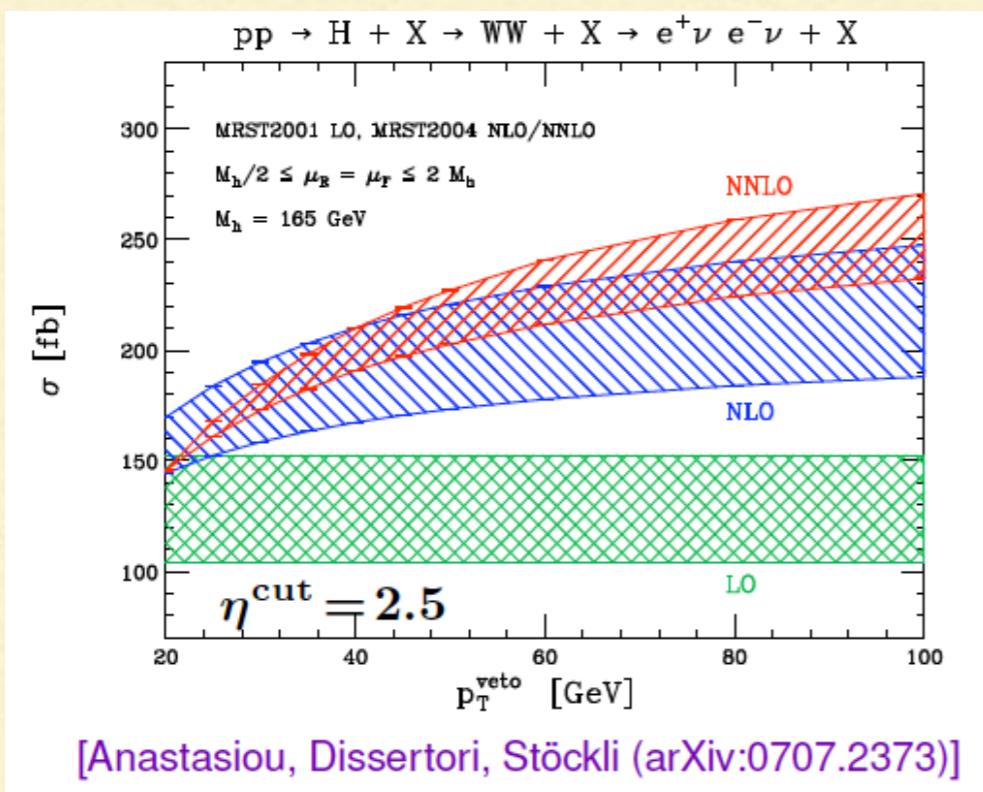
beam	hard	jet	pert. soft
μ_B	μ_H	μ_J	μ_S



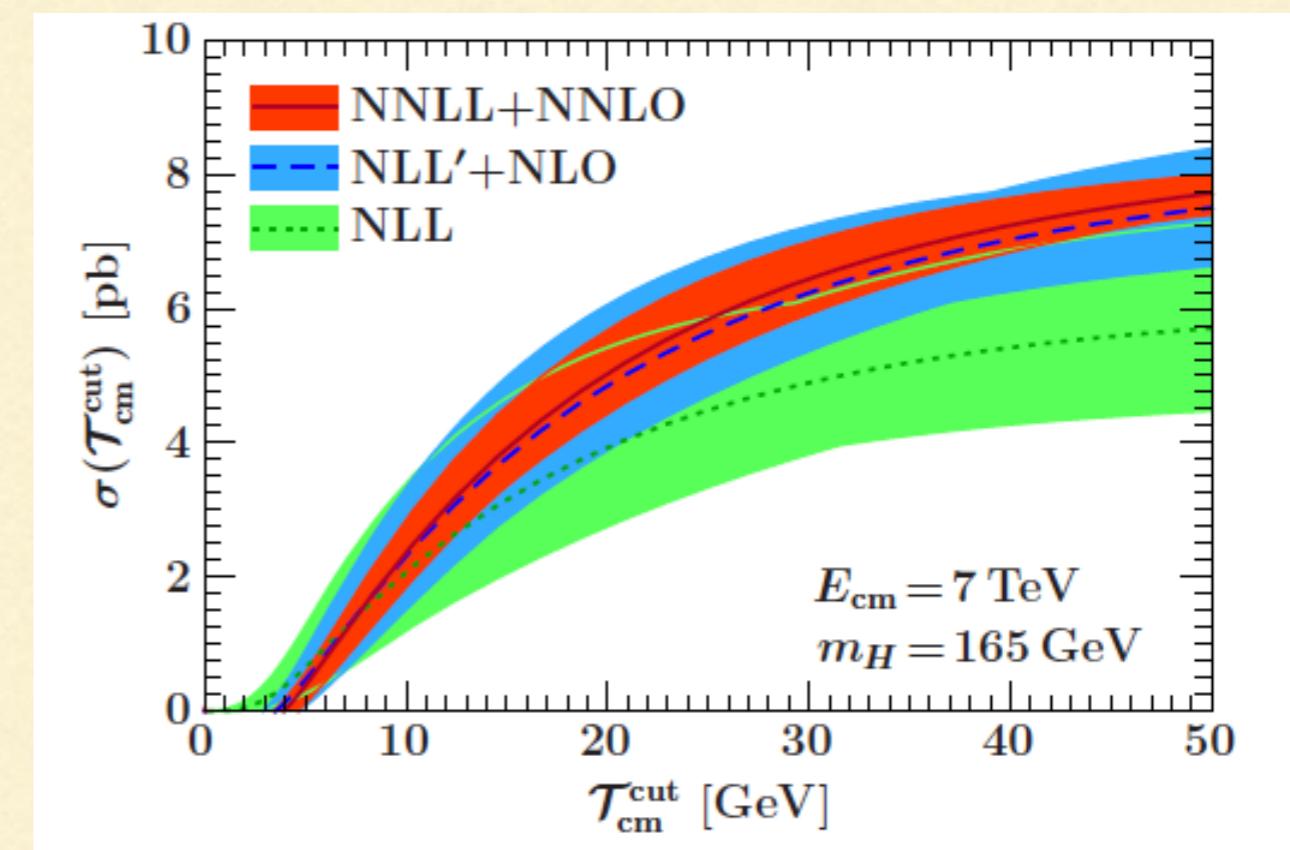
EVENT SHAPES IN pp COLLISIONS

- “Beam thrust” in pp collisions as Higgs jet veto (like DY):

Fixed-order:



Resummed in SCET:



$$\mathcal{T}_{\text{cm}} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

Berger, Marcantonini, Stewart, Tackmann, Waalewijn (2010)

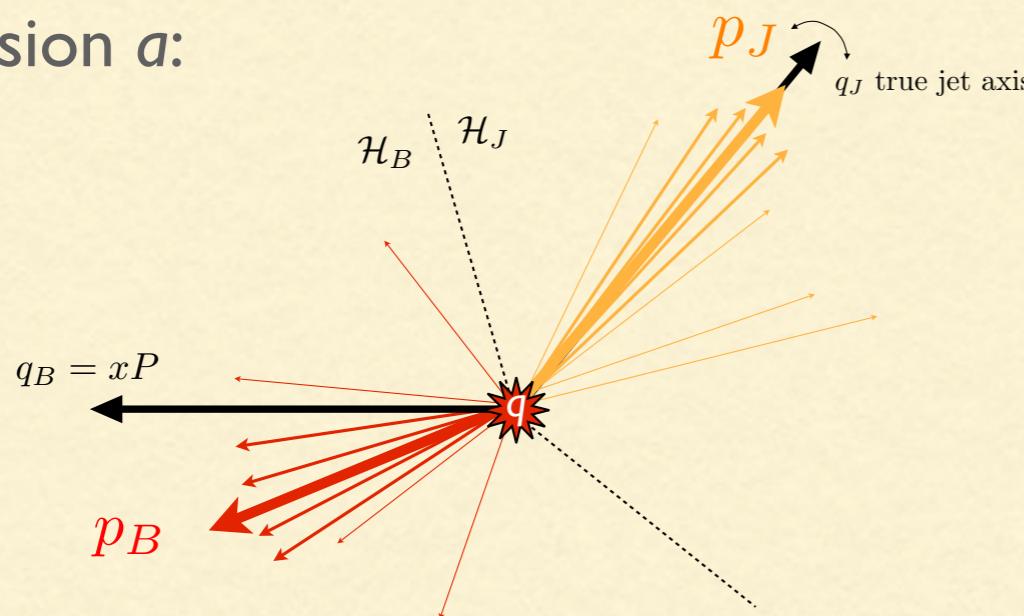
EVENT SHAPES IN DIS:

D. Kang, CL, I. Stewart (2013)
also Z. Kang, Liu, Mantry, Qiu
(2012, 2013)

- “I-jettiness” in DIS measures final states with beam radiation + one additional jet

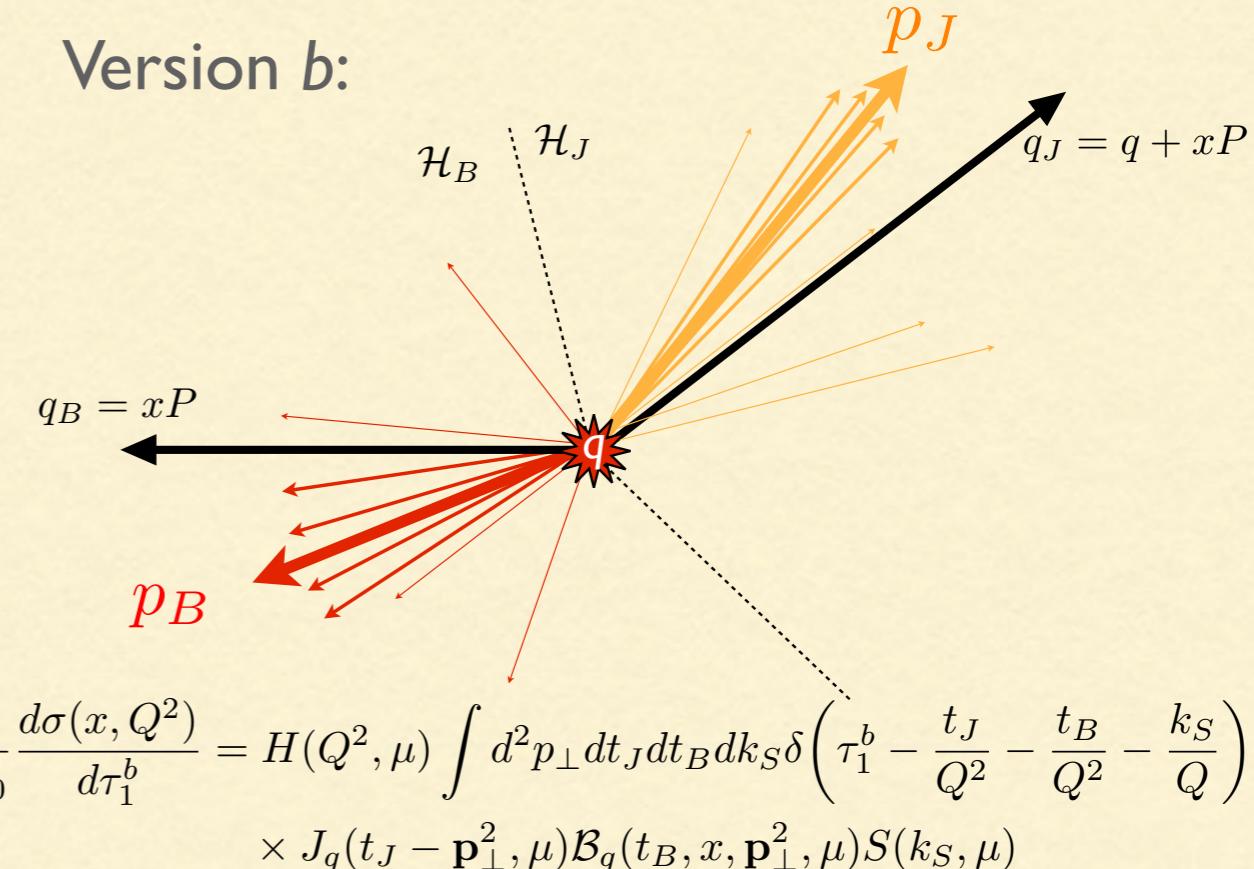
$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Version *a*:



$$\begin{aligned} \frac{1}{\sigma_0} \frac{d\sigma(x, Q^2)}{d\tau_1^a} &= H(Q^2, \mu) \int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \\ &\times J_q(t_J, \mu) B_q(t_B, x, \mu) S(k_S, \mu) \end{aligned}$$

Version *b*:

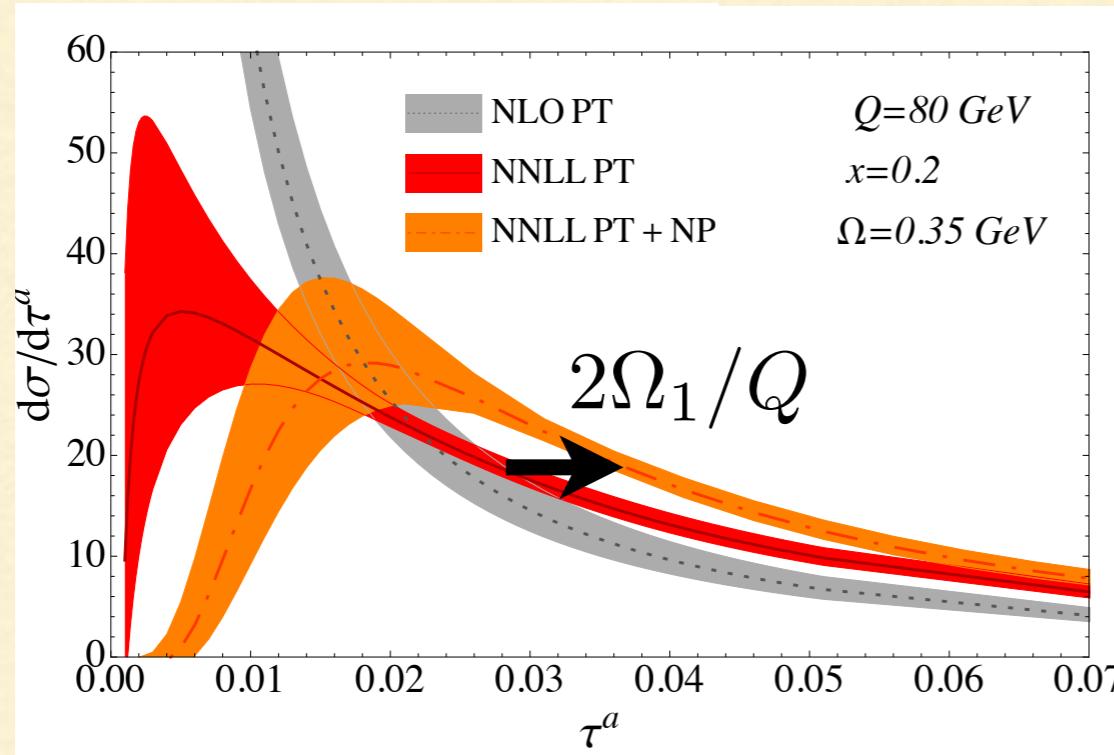


- Choice of axes can induce sensitivity to transverse momentum dependence of (perturbative) initial-state radiation.
- Resummation possible to NNLL or (for version *a*) NNNLL

D. Kang, CL, I. Stewart (2014)

POWER CORRECTIONS IN pp AND DIS

- Universal nonperturbative shift in 3 versions of DIS I-jettiness:

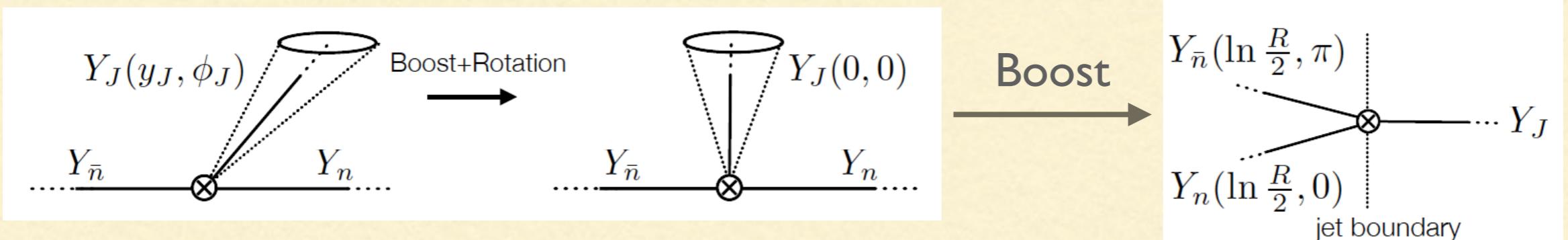


Using factorization theorems and boost invariance properties of soft Wilson lines, can prove that:

$$\Omega_1^a = \Omega_1^b = \Omega_1^c$$

D. Kang, CL, I. Stewart (2013)

- Surprising relation also to leading NP correction to jet mass in pp to 1 jet



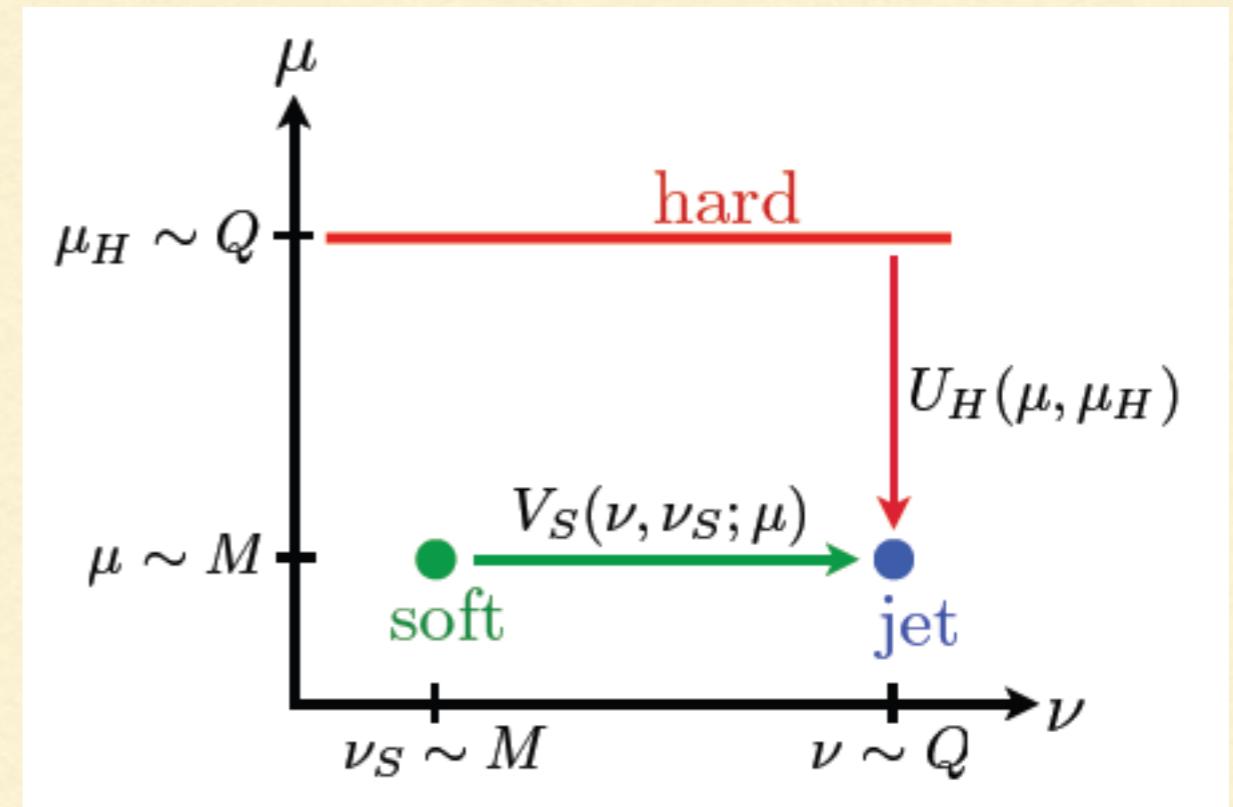
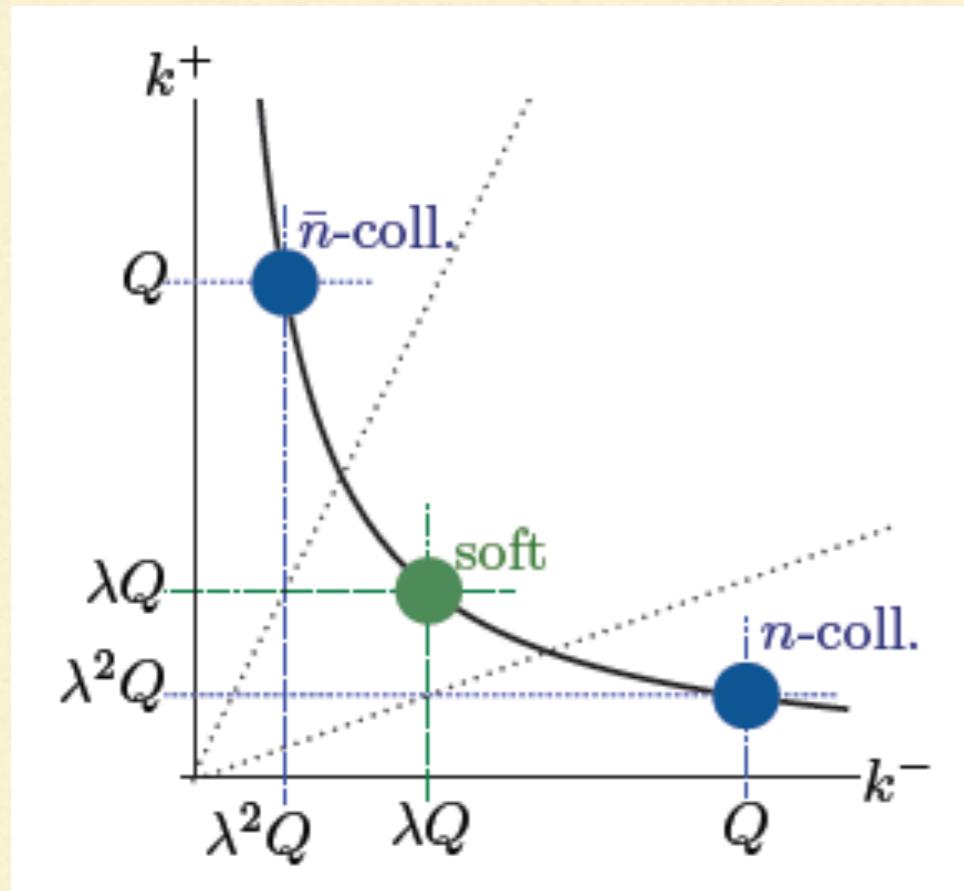
Stewart, Tackmann,
Waalewijn (2014)

For $R \ll 1$, the beam Wilson lines fuse and $\Omega = \frac{R}{2} \Omega_0 + \dots$

The universal Ω_0^q can be extracted from DIS event shapes

SCET_{II}: RAPIDITY LOGS AND RENORMALIZATION

- For more exclusive or for TMD distributions, collinear and soft modes have same virtuality, distinguished only by rapidity



Chiu, Jain, Neill, Rothstein (2012)

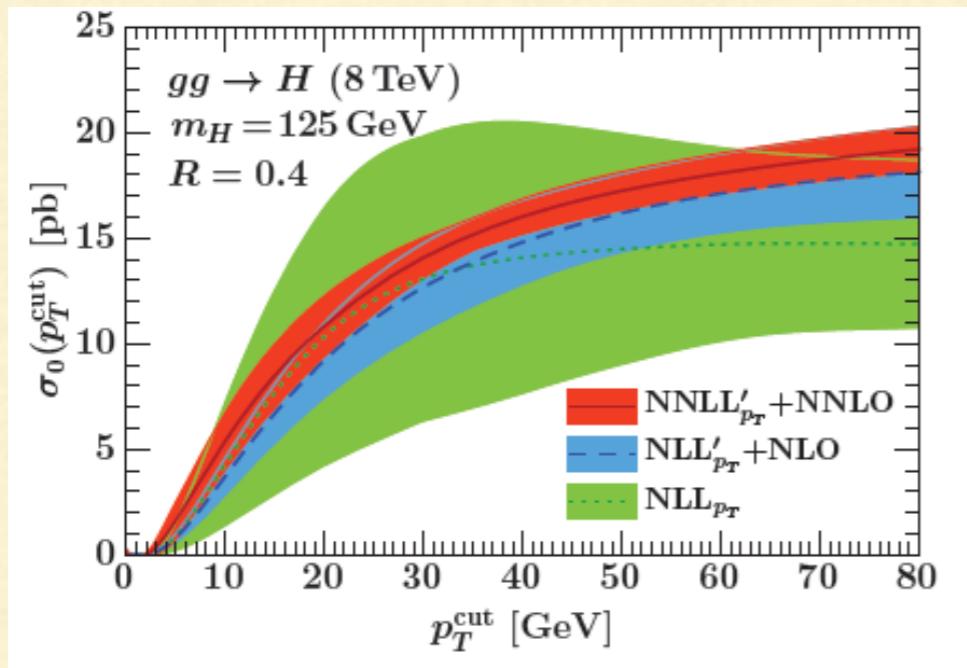
- Need additional regulator for rapidity divergences, evolution both in virtuality and in rapidity scales.

Chiu, Jain, Neill, Rothstein (2012)

Echevarria, Idilbi, Scimemi

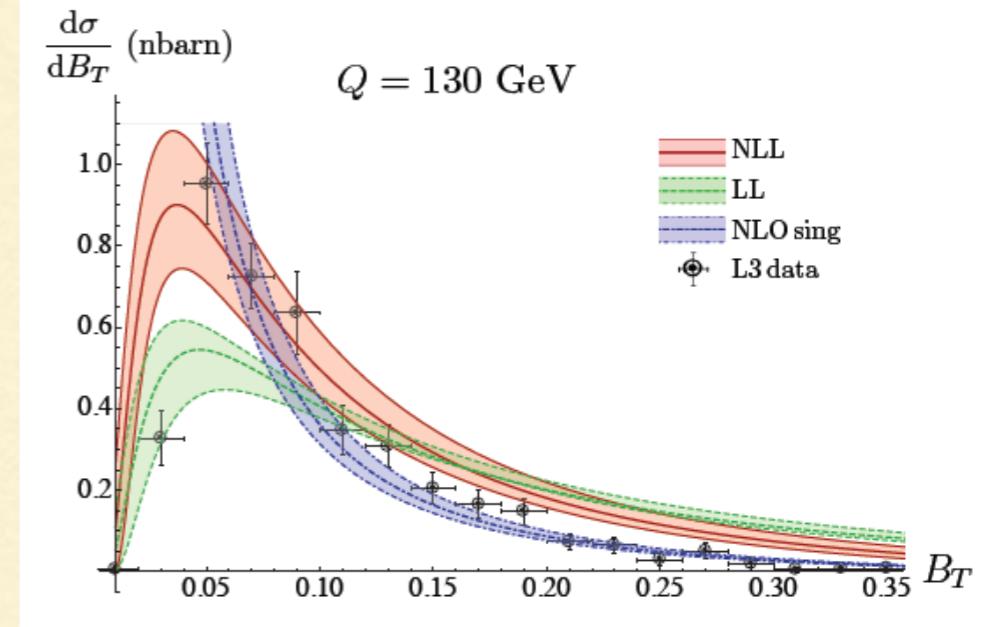
SCET_{II}: EFT FOR TMD DISTRIBUTIONS

- p_T veto in pp to Higgs + 0-jet



Stewart, Tackmann, Walsh, Zuberi (2013)

- Jet broadening

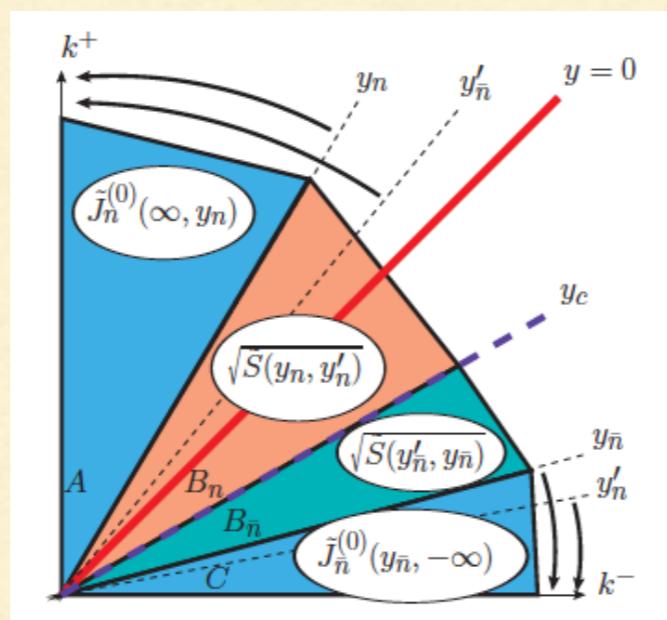


Chiu, Jain, Neill, Rothstein (2012)

See also: “Recoil-free” observables,

Larkoski, Neill, Thaler (2014)

- TMDPDFs



Echevarria, Idilbi, Scimemi (2012)

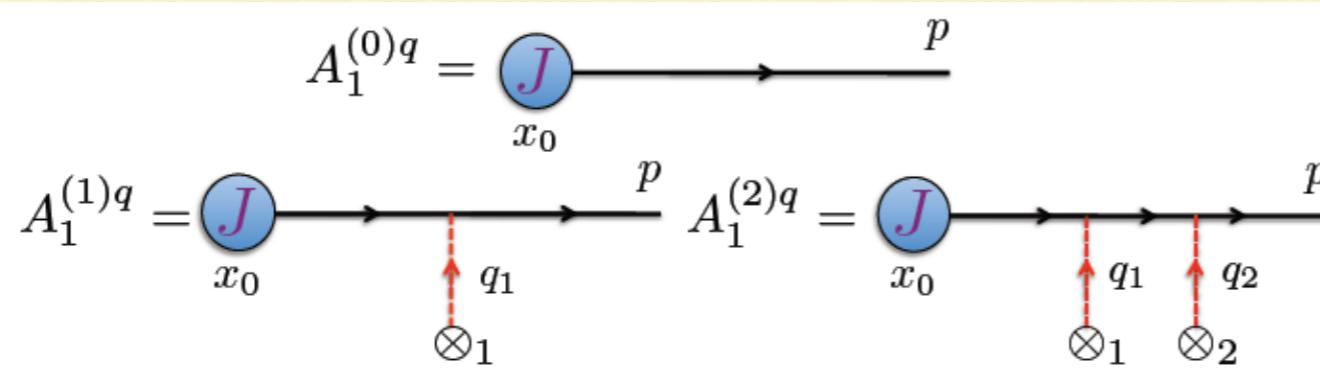
SCET_G: WITH GLAUBERS

- Add to SCET Glauber modes exchanged between jets and scattering centers in a dense medium:

Idilbi, Majumder

D'Eramo, Liu, Rajagopal

Ovanesyan, Vitev



$$p \sim Q(1, \lambda^2, \lambda)$$

$$q_i \sim Q(\lambda^2, \lambda^2, \lambda)$$

Feynman rules in covariant gauge:

Ovanesyan, Vitev

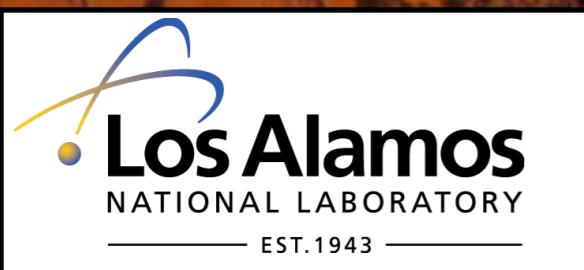
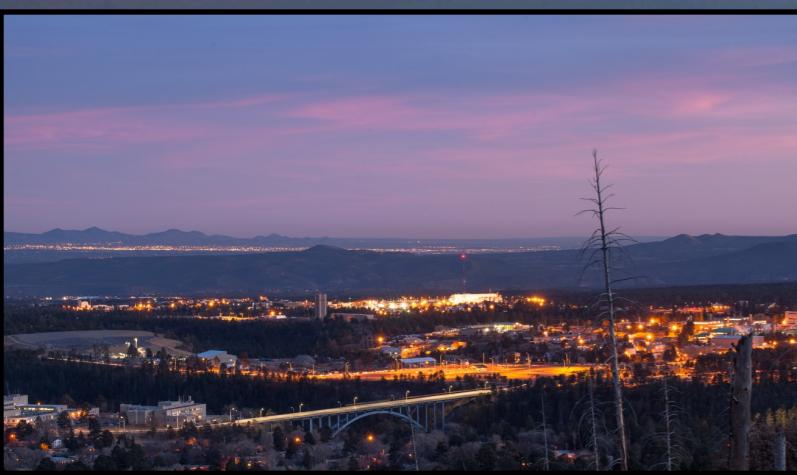
The diagram shows two Feynman rules. The top rule, labeled R_ξ , shows a horizontal line with arrows pointing right, labeled p and p' , with a vertical red dashed line with an arrow pointing up from it, labeled q_1 . To its right is the equation $= i v(q_{1\perp}) (b_1)_R (b_1)_{T_i} \frac{\bar{n}}{2}$. The bottom rule shows a horizontal line with arrows pointing right, labeled p and p' , with a vertical red dashed line with an arrow pointing up from it, labeled q_1 . The line has labels μ, a and ν, b at its ends. To its right is the equation $= v(q_{1\perp}) f^{abc_1} (c_1)_{T_i} \left[g^{\mu\nu} \bar{n} \cdot p + \bar{n}^\mu q_{1\perp}^\nu - \bar{n}^\nu q_{1\perp}^\mu - \frac{1-\frac{1}{\xi}}{2} (\bar{n}^\nu p^\mu + \bar{n}^\mu p^\nu) \right]$.

SCET, GLAUBERS AND REGGE BEHAVIOR

- Much recent activity in understanding Glauber interactions and Regge behavior from SCET:
- S. Fleming: Glauber exchange in SCET and BFKL equation, I404.5672
- J. Donoghue, B. Kamal El-Menoufi, G. Ovanesyan: Regge behavior in EFT, I405.1731
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