RIKEN BNL Research Center

Transverse single-spin asymmetries in proton-proton collisions within collinear factorization

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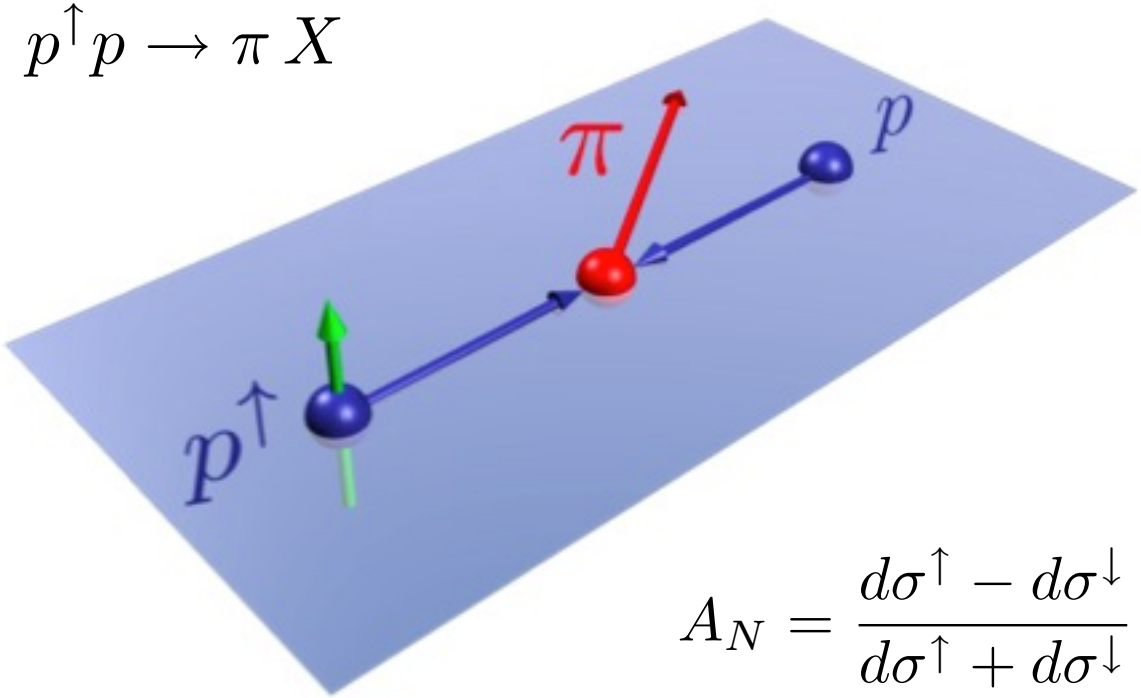
Outline

- Motivation
  - What are transverse single-spin asymmetries (TSSAs)?
  - Collinear twist-3 formalism
- A puzzle with TSSAs
  - "Sign mismatch" between the Qiu-Sterman function and the Sivers function
  - Insight from TSSAs in inclusive DIS
  - The role of twist-3 fragmentation in TSSAs
- Summary and outlook

Motivation

- TSSAs in proton-proton collisions

$$p^{\uparrow} p \to \pi X$$



$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

Data available from RHIC (BRAHMS, PHENIX, STAR) and FNAL (E704)

(Figure thanks to K. Kanazawa)

➢ Collinear twist-3 formalism

$$
\begin{aligned}
d\sigma = \; & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
+ \; & H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
+ \; & H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

Collinear twist-3 approach

(Efremov and Teryaev (1982, 1985); Qiu and Sterman (1992, 1999))

$P_{hT} >> \Lambda_{QCD}$

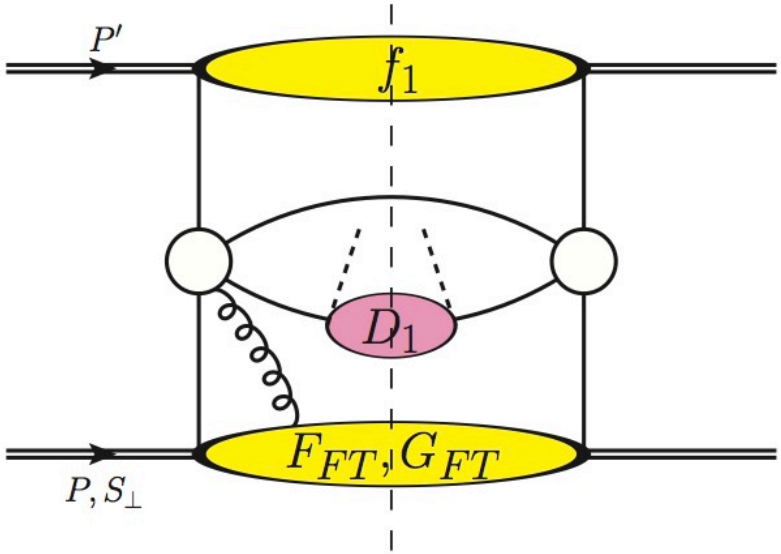
➢ Collinear twist-3 formalism

$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}$$

Collinear twist-3 approach

(Efremov and Teryaev (1982, 1985); Qiu and Sterman (1992, 1999))

$P_{hT} >> \Lambda_{QCD}$



- T-odd effect ➡ need to generate an imaginary part ➡ soft-gluon pole (SGP) or soft-fermion pole (SFP) ➡ internal particle goes on-shell
- One can also have SGPs with tri-gluon correlations

- SGP term (Qiu and Sterman (1999), Kouvaris, et al. (2006)):

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S+T/z}\phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right)\frac{1}{x}\left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

Qiu-Sterman function

- SFP term (Koike and Tomita (2009); Kanazawa and Koike (2011)):

$$E_h \frac{d^3\Delta\sigma^{\rm SFP}}{dP_h^3} = \frac{\alpha_s^2}{S}\frac{M_N\pi}{2}\epsilon^{pnP_hS_\perp}\int_{z_{min}}^1 \frac{dz}{z^3}\int_{x'_{min}}^1 \frac{dx'}{x'}\int \frac{dx}{x}\frac{1}{x'S+T/z}\delta\left(x - \frac{-x'U/z}{x'S+T/z}\right)$$
$$\times\Bigg[\sum_{a,b,c}\left(G_F^a(0,x)+\tilde{G}_F^a(0,x)\right)\{q^b(x')\left(D^c(z)\hat{\sigma}_{ab\to c}+D^{\bar{c}}(z)\hat{\sigma}_{ab\to\bar{c}}\right)$$
$$+q^{\bar{b}}(x')\left(D^c(z)\hat{\sigma}_{a\bar{b}\to c}+D^{\bar{c}}(z)\hat{\sigma}_{a\bar{b}\to\bar{c}}\right)\}$$
$$+\sum_{a,b}\left(G_F^a(0,x)+\tilde{G}_F^a(0,x)\right)\left(q^b(x')D^g(z)\hat{\sigma}_{ab\to g}+q^{\bar{b}}(x')D^g(z)\hat{\sigma}_{a\bar{b}\to g}\right)$$
$$+\sum_{a,c}\left(G_F^a(0,x)+\tilde{G}_F^a(0,x)\right)G(x')\left(D^c(z)\hat{\sigma}_{ag\to c}+D^{\bar{c}}(z)\hat{\sigma}_{ag\to\bar{c}}\right)$$
$$+\sum_{a}\left(G_F^a(0,x)+\tilde{G}_F^a(0,x)\right)G(x')D^g(z)\hat{\sigma}_{ag\to g}\Bigg]$$

$$T_F \sim G_F \sim F_{FT}$$
$$\tilde{T}_F \sim \tilde{G}_F \sim G_{FT}$$

- Tri-gluon correlators (Beppu, Kanazawa, Koike, Yoshida (2013)):

$$E_{P_h}\frac{d^3\Delta\sigma}{d^3P_h} = \frac{2\pi M_N\alpha_s^2}{S}\epsilon^{P_h pnS_\perp}\sum_{i,j}\int\frac{dx}{x}\int\frac{dx'}{x'}f_i(x')\int\frac{dz}{z^2}D_j(z)\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{z\hat{u}}$$

$$\times\left[\zeta_{ij}\left(\frac{d}{dx}O(x)-\frac{2O(x)}{x}\right)\hat{\sigma}^{(O)}_{gi\to j}+\left(\frac{d}{dx}N(x)-\frac{2N(x)}{x}\right)\hat{\sigma}^{(N)}_{gi\to j}\right]$$

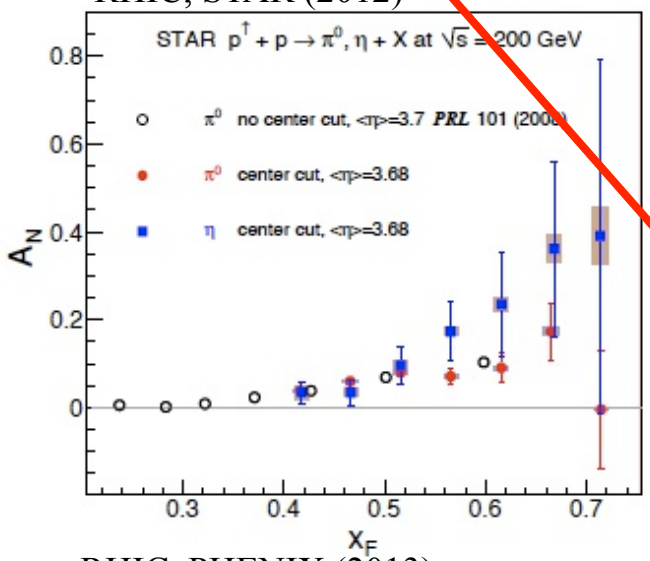
➡ For many years the SGP term involving the Qiu-Sterman function was thought to be the dominant contribution to TSSAs in $p^\uparrow p \to hX$

- A puzzle with TSSAs (the "sign mismatch" issue)

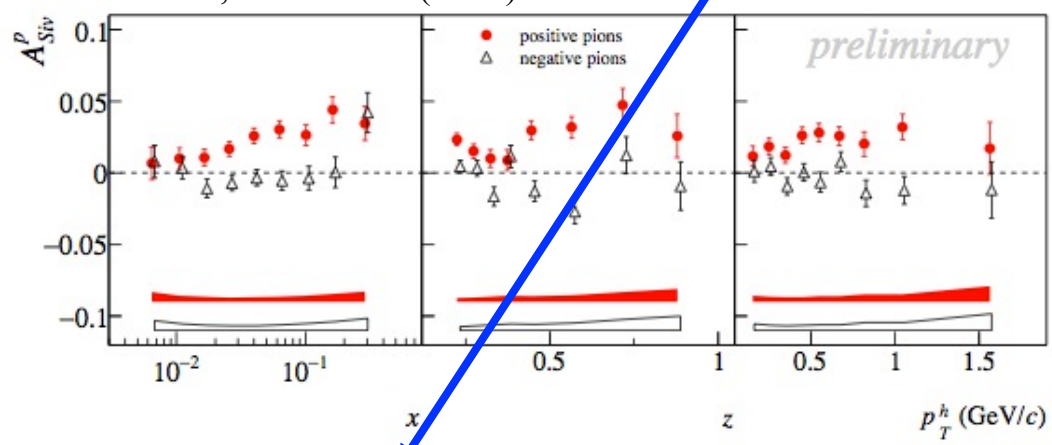
$$p^{\uparrow} p \to h\, X$$

$$\ell N^{\uparrow} \to \ell'\, h\, X$$

RHIC, STAR (2012)



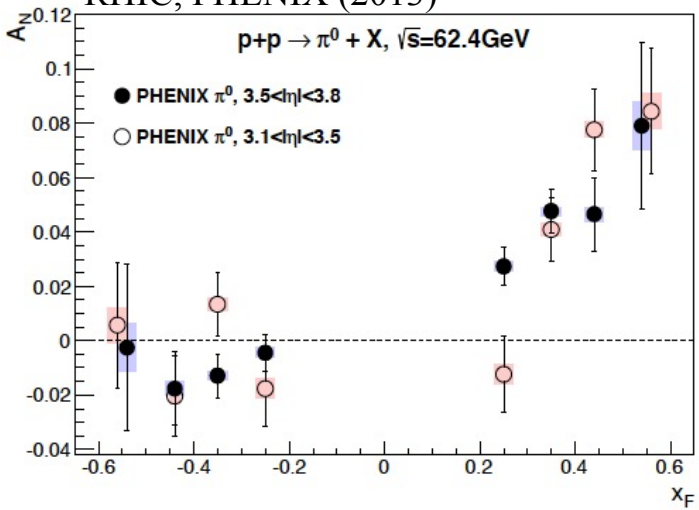
CERN, COMPASS (2013)



$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

$$F_{FT} \sim T_F$$

RHIC, PHENIX (2013)



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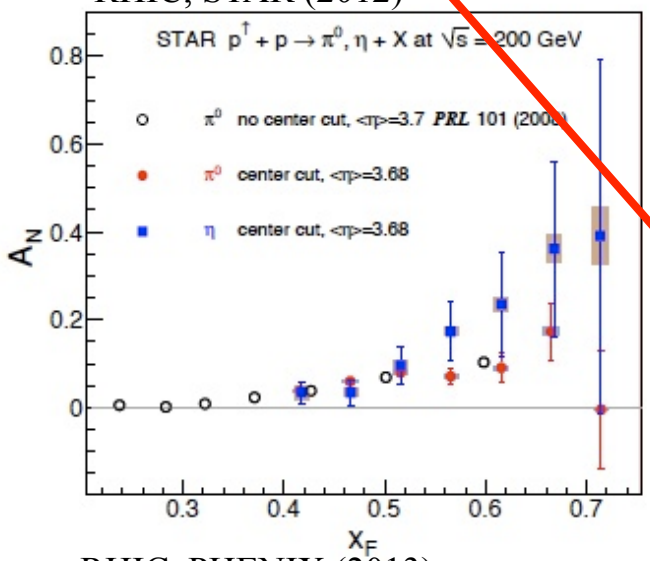

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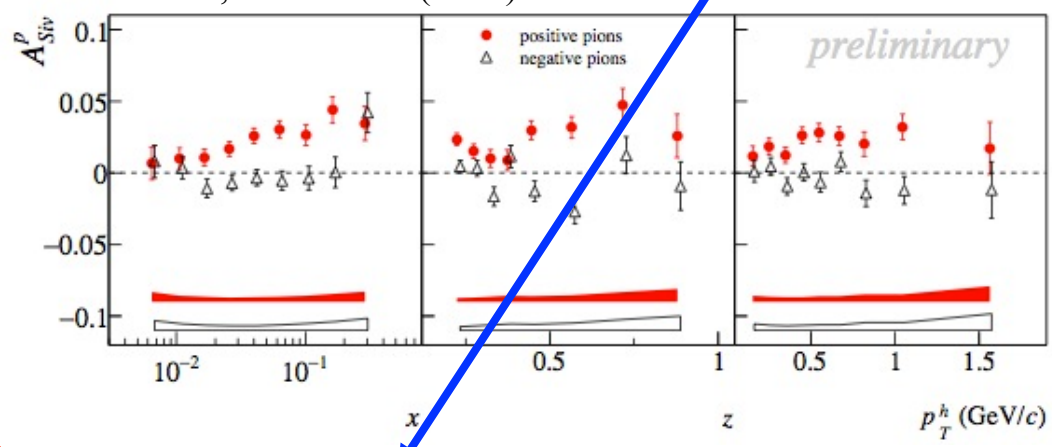

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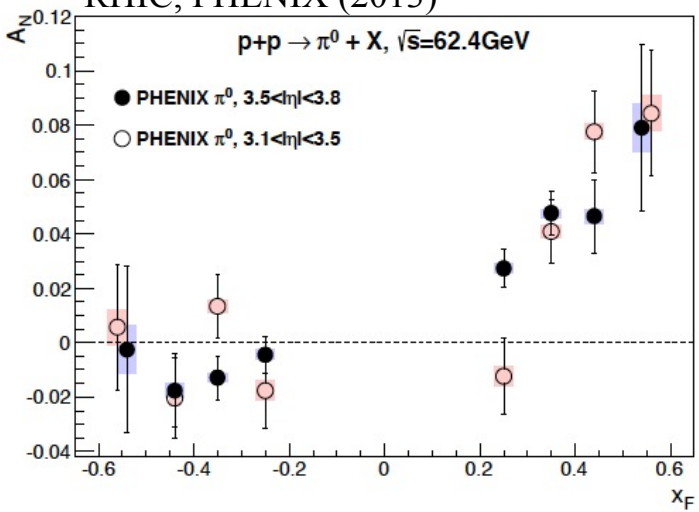
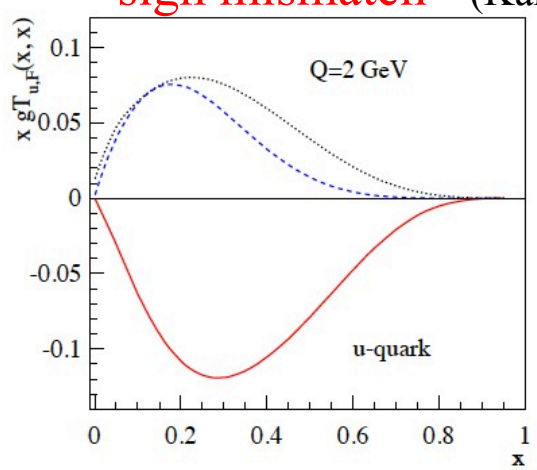
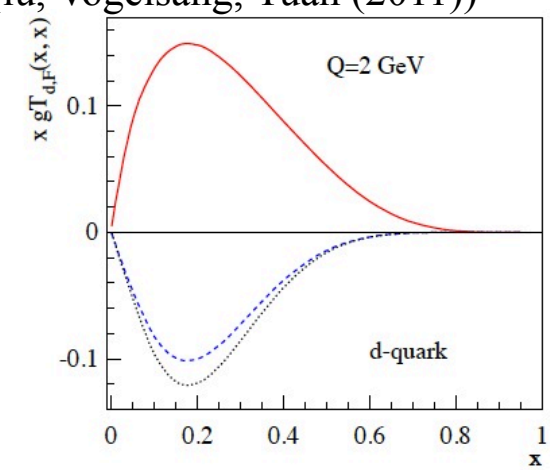


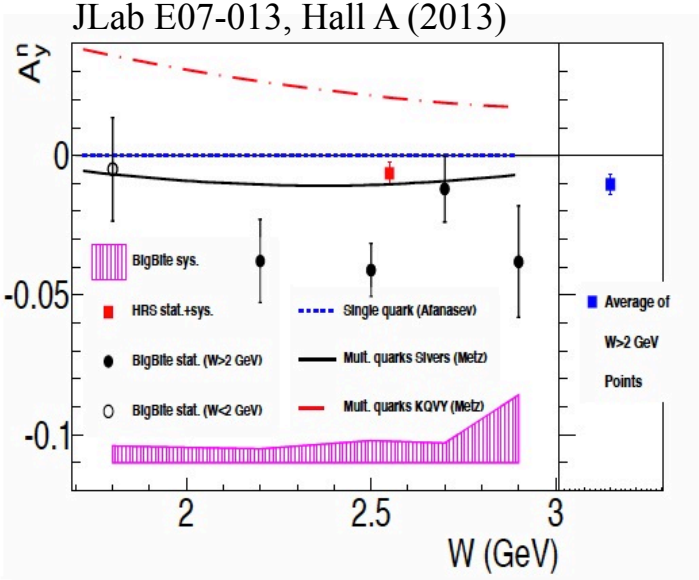
"sign mismatch" (Kang, Qiu, Vogelsang, Yuan (2011))





- TSSA in inclusive DIS (Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou, PRD **86** (2012))

JLab E07-013, Hall A (2013)



Neutron TSSA

Sivers input agrees reasonably well with the JLab data

➡ Node in k_T for the Sivers function can be ruled out/Also node in x is disfavored from proton data from HERMES (see also Kang and Prokudin (2012))

➡ FIRST INDICATION that the Sivers effect is intimately connected to the re-scattering of the active parton with the target remnants (PROCESS DEPENDENT)

KQVY input gives the <u>wrong sign</u> ➡ SGP contribution on the side of the transversely polarized incoming proton cannot be the main cause of the large TSSAs seen in pion production (i.e., $T_F(x,x)$ term)

$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$

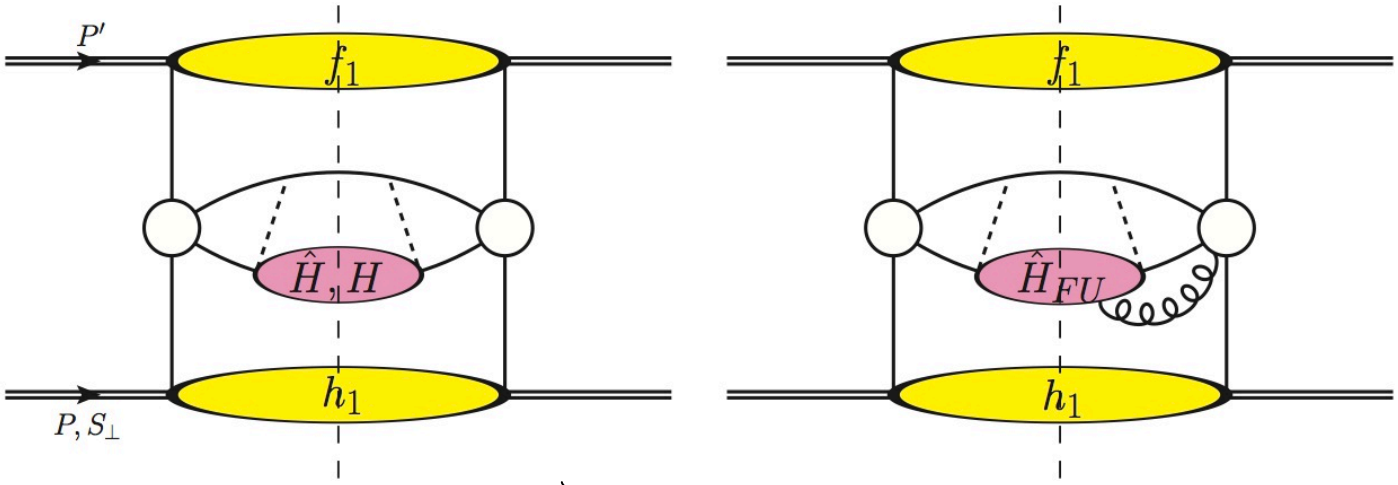
$$\boxed{+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}} \rightarrow$$

Negligible (Kanazawa and Koike (2000))

$$\boxed{+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}}$$

$$+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}$$

- Collinear twist-3 fragmentation term:



$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{2M_h^2} H_1^{\perp\, h/q}(z, z^2\vec{k}_\perp^{\,2})$$

Collins-type function

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1) = H^{h/q}(z) + 2z\hat{H}^{h/q}(z)$$

3-parton correlator

➡ There are 2 independent (unpolarized) collinear twist-3 FFs

Collinear twist-3 fragmentation structure is richer than that for the TMD formalism

- Calculation of twist-3 fragmentation term (Metz and DP, PLB **723** (2013))

$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\,S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'}\,\frac{1}{x'S+T/z}\,\frac{1}{-x\hat{u}-x'\hat{t}}$$
$$\times\,\frac{1}{x}\,h_1^a(x)\,f_1^b(x')\left\{\left(\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz}\right)S_{\hat{H}}^i + \frac{1}{z}\,H^{C/c}(z)\,S_H^i\right.$$
$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\,PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{C/c,\Im}(z,z_1)\,\frac{1}{\xi}\,S_{\hat{H}_{FU}}^i\right\}$$

➡ First time we have a complete pQCD result for this term in *pp* within the collinear twist-3 approach

➡ Also has been studied for TSSA in SIDIS (Kanazawa and Koike (2013))

➡ "Derivative term" has been calculated previously (Kang, Yuan, Zhou (2010))

➡ Derivative and non-derivative piece combine into a "compact" form as on the distribution side

➡ Must determine numerical significance of 3-parton fragmentation correlator

Unpolarized FF (DSS)

Unpolarized PDF (GRV98)

Distribution term (SGP)

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$

$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}} \right) \frac{1}{x} \left[T_{a,F}(x,x) - x \left(\frac{d}{dx} T_{a,F}(x,x) \right) \right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

Fragmentation term

$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S} \epsilon_{\perp\mu\nu} S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \frac{1}{-x\hat{u} - x'\hat{t}}$$

$$\times \frac{1}{x} h_1^a(x) f_1^b(x') \left\{ \left(\hat{H}^{C/c}(z) - z \frac{d\hat{H}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{z} H^{C/c}(z) S_H^i \right.$$

$$\left. + 2z^2 \int \frac{dz_1}{z_1^2} PV \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{C/c,\Im}(z,z_1) \frac{1}{\xi} S_{\hat{H}_{FU}}^i \right\}$$

Transversity PDF (Torino13)

Recall: $H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z,z_1)$

➢ The role of twist-3 fragmentation in TSSAs

(Kanazawa, Koike, Metz, DP, arXiv:1404.1033, submitted to PRD)

- Numerical study (Note: we only use $\sqrt{S} = 200$ GeV data ➔ higher P_T values)

Distribution term

➡ SGP: $\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$, Sivers function taken from Torino group (2009/2013)

$$F_{FT} \sim T_F$$

➡ SFP/Tri-gluon: neglect for now

Fragmentation term

➡ Transversity: taken from Torino group (2013), but allow β parameters to be free

➡ $\hat{H}^{h/q}(z)$: use Collins function extracted by the Torino group (2013)

$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{2M_h^2} H_1^{\perp\, h/q}(z, z^2\vec{k}_\perp^{\,2})$$

➡ $\hat{H}_{FU}^{h/q,\Im}(z, z_1)$ ➔ use the following ansatz:

$$\frac{\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\Im}(z,z_1)}{D^{\pi^+/(u,\bar{d})}(z)\, D^{\pi^+/(u,\bar{d})}(z/z_1)} = \frac{N_{\rm fav}}{2 I_{\rm fav} J_{\rm fav}}\, z^{\alpha_{\rm fav}} (z/z_1)^{\alpha'_{\rm fav}} (1-z)^{\beta_{\rm fav}} (1-z/z_1)^{\beta'_{\rm fav}}$$

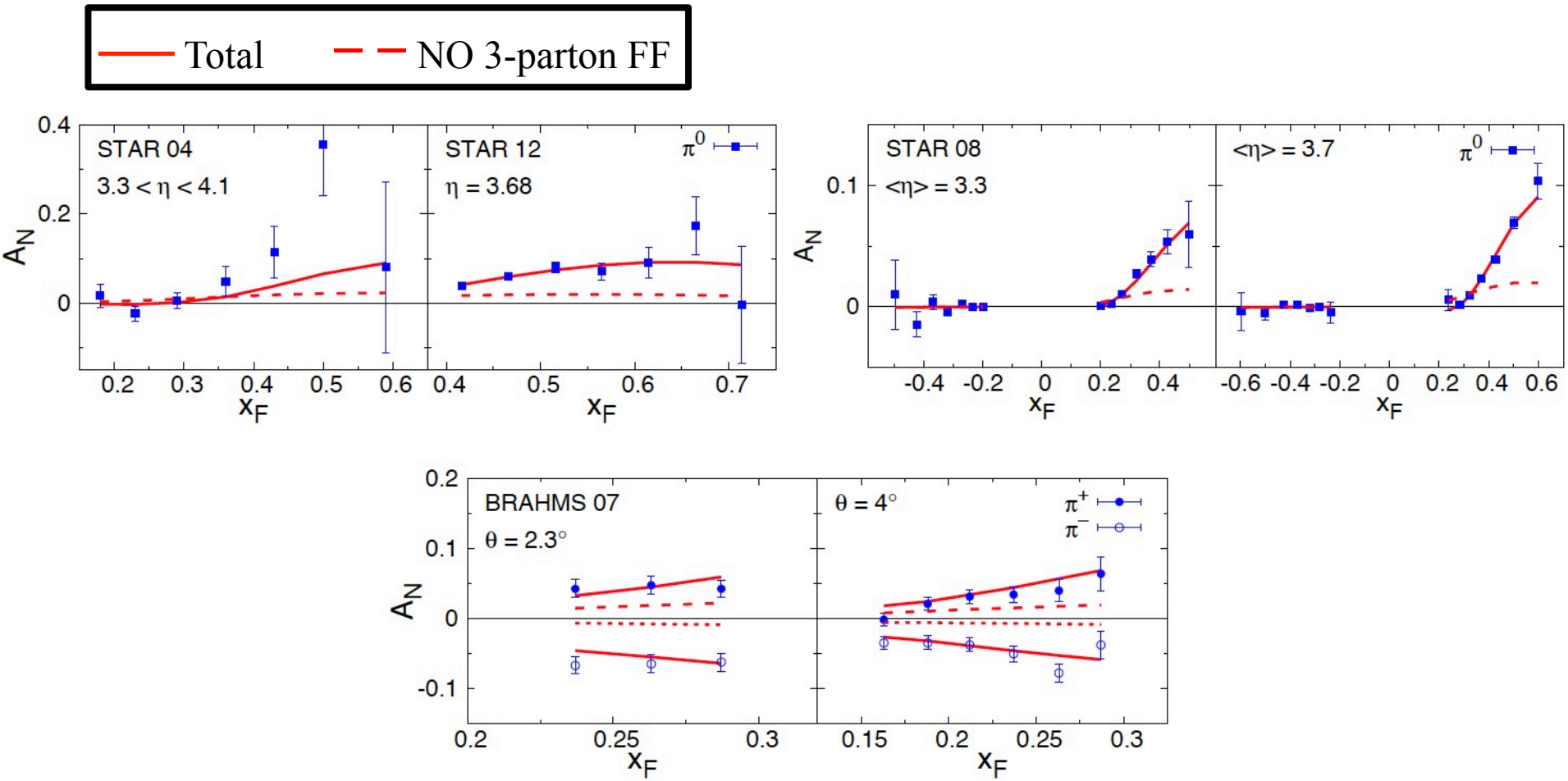
(similar for disfavored, π^- defined through c.c., π^0 defined as average of π^+ and π^-)

8 free parameters: N_{fav}, $\alpha_{fav} = \alpha'_{fav}$, β_{fav}, $\beta'_{fav} = \beta'_{dis}$

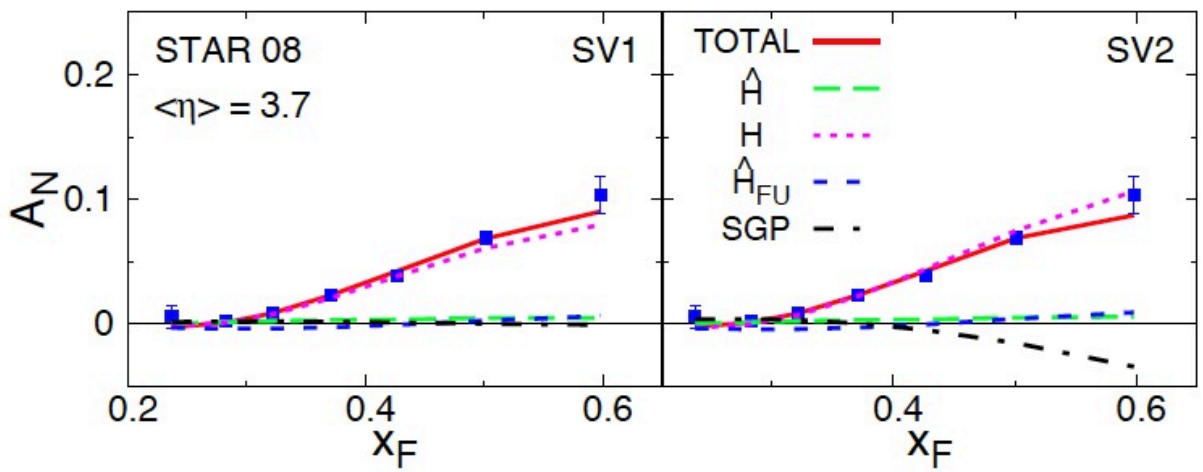
N_{dis}, $\alpha_{dis} = \alpha'_{dis}$, β_{dis}, $\beta_u^T = \beta_d^T$

$\chi^2/\text{d.o.f.} = 1.03$	
$N_{\text{fav}} = -0.0338$	$N_{\text{dis}} = 0.216$
$\alpha_{\text{fav}} = \alpha'_{\text{fav}} = -0.198$	$\beta_{\text{fav}} = 0.0$
$\beta'_{\text{fav}} = \beta'_{\text{dis}} = -0.180$	$\alpha_{\text{dis}} = \alpha'_{\text{dis}} = 3.99$
$\beta_{\text{dis}} = 3.34$	$\beta_u^T = \beta_d^T = 1.10$

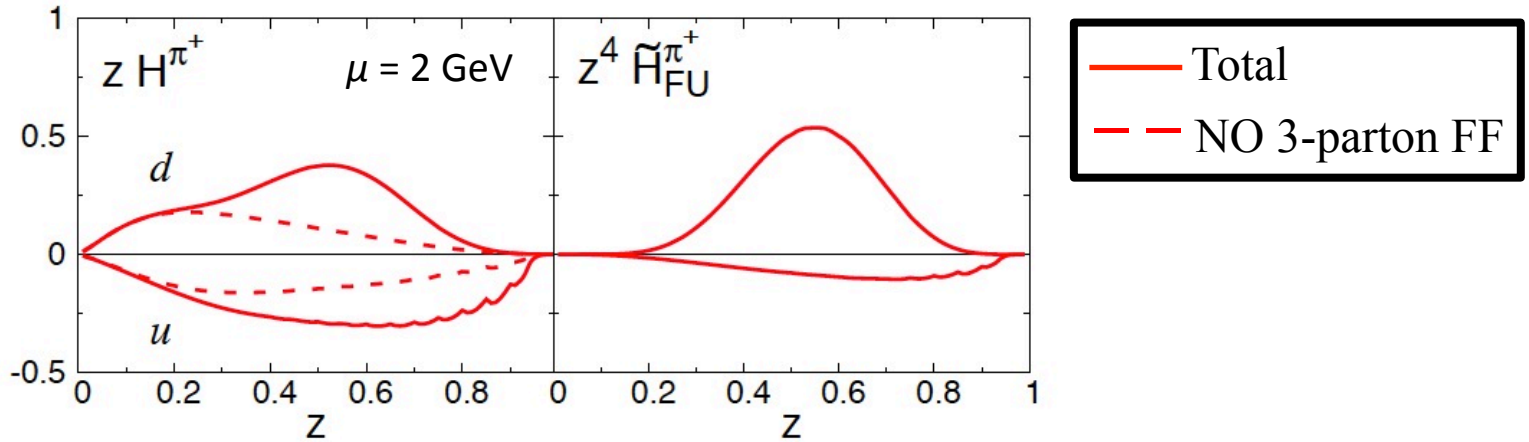
➡ Above parameters are from using 2009 Sivers function (SV1). Using 2013 Sivers function (SV2) gives similar values and $\chi^2/d.o.f. = 1.10$

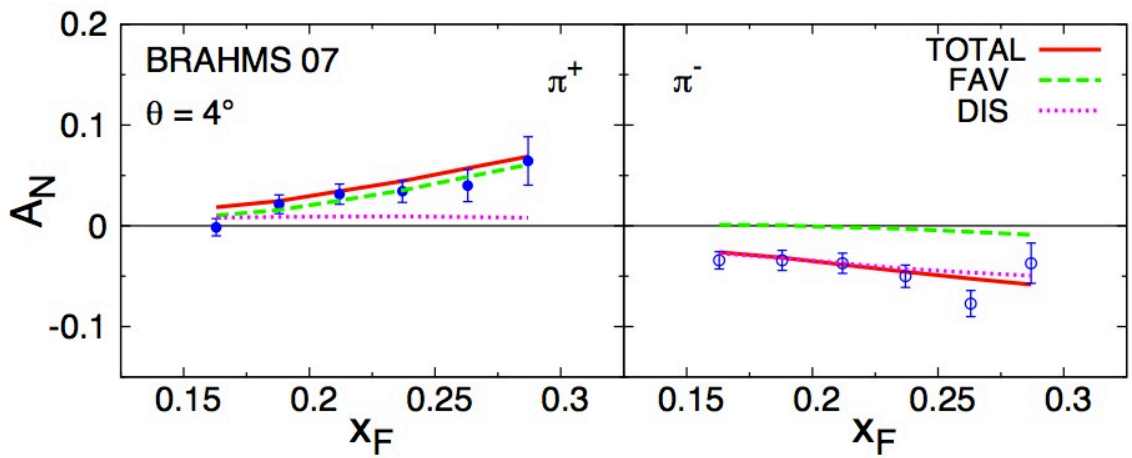


- Including the (total) fragmentation term leads to very good agreement with the RHIC data, especially with its characteristic rise towards large x_F
- Without the 3-parton FF, one has difficulty describing the RHIC data



- *H* term is dominant; Sivers-type, Collins-type, and $\hat{H}_{FU}$ terms are negligible
- SV1 – 2009 Sivers function from Torino group ➔ flavor-*independent* large-*x* behavior
- SV2 – 2013 Sivers function from Torino group ➔ flavor-*dependent* large-*x* behavior and slower decrease at large-*x* than SV1
  - Including 3-parton FF, one can accommodate such a Sivers function through the *H* term
  - Without the 3-parton FF, one would have serious issues handling such a (negative) SGP contribution to obtain a (large) positive A_N

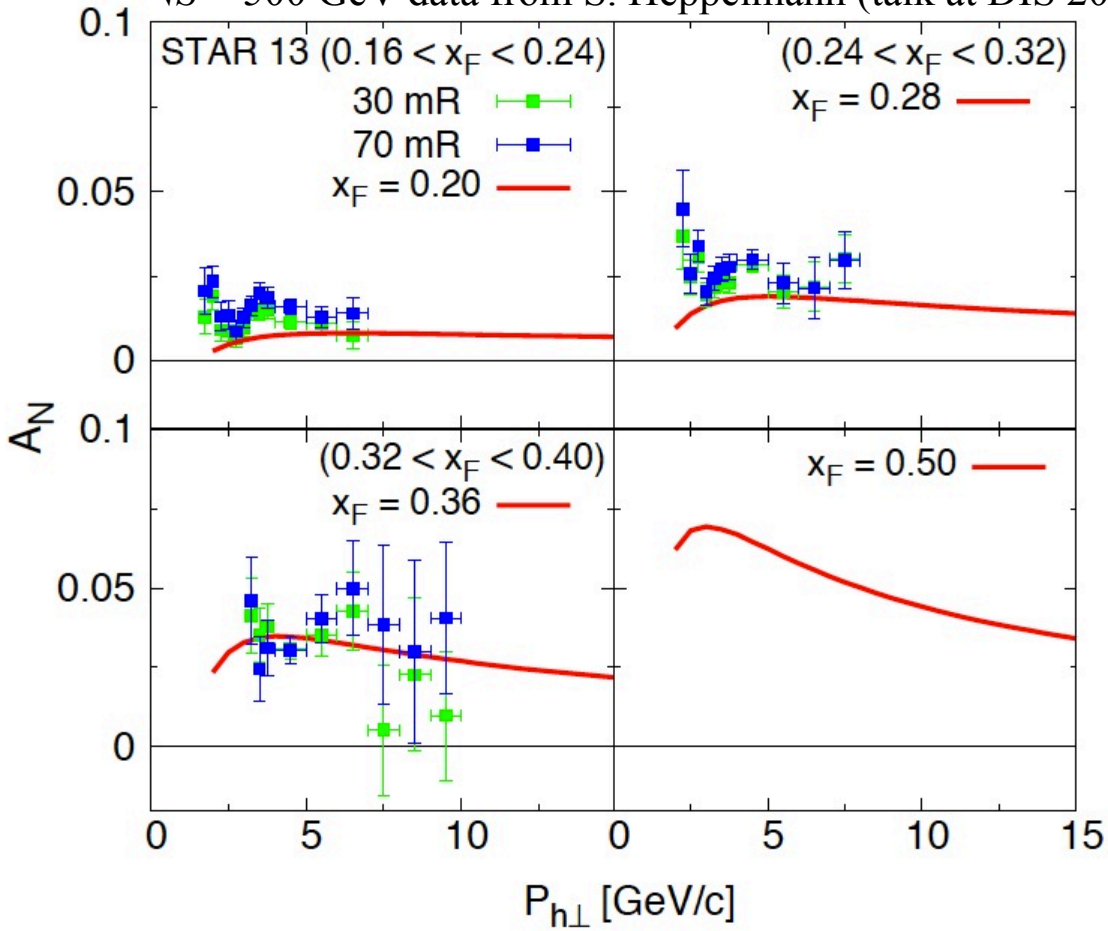




- Favored and disfavored (chiral-odd) collinear twist-3 FFs are roughly equal in magnitude but opposite in sign ➔ similar to Collins FF
- A_N for π^+ (π^-) dominated by favored (disfavored) fragmentation

- Flat P_T dependence thought to be an issue for collinear twist-3 approach ➔ $A_N \sim 1/P_T$
- First shown by Kanazawa and Koike (2011) that this does not have to be the case

$\sqrt{S}$ = 500 GeV data from S. Heppelmann (talk at DIS 2013)



Theory

Note: 500 GeV data was NOT included in the fit

- Our analysis also shows a flat P_T dependence for A_N seen so far at RHIC ➔ remains flat even to larger P_T values

Summary and outlook

- For many years it was unclear what mechanism causes large TSSAs in hadron production from *pp* collisions
- Twist-3 fragmentation could finally give us an explanation
  - ➡ Full analytical pQCD result now available
  - ➡ Including this term allows for a very good description of the RHIC data, in particular the rise in A_N towards large x_F and flat P_T dependence
  - ➡ Our analysis provides a consistency between spin/azimuthal asymmetries in *pp* (collinear) and SIDIS, e^+e^- (TMD)
  - ➡ Future work: include SFPs (can help with charged pions) and proper evolution of the 3-parton FF

- Global analysis involving several reactions will be needed in order to extract all the collinear twist-3 distribution and fragmentation functions in $p^{\uparrow}p \to hX$
  - ➡ Measurement of $p^{\uparrow}p \to jet\ X$ by the AnDY Collaboration (Bland, et al. (2013))
  - ➡ Measurements of Drell-Yan in $p^{\uparrow}p$ and $p^{\uparrow}p \to \gamma\ X$ at RHIC (also DY experiment planned at COMPASS for $\pi p^{\uparrow}$)
  - ➡ Large $P_{h\perp}$ measurement of Sivers and Collins asymmetries in SIDIS should also be possible at JLab12, COMPASS, or a future EIC
  - ➡ HERMES (Airapetian, et al. (2013)) / JLab (Allada, et al. (2013)) have recently published data on $ep^{\uparrow} \to hX$ / $en^{\uparrow} \to hX$
  - ➡ Can one consistently describe all of these reactions?

Backup slides

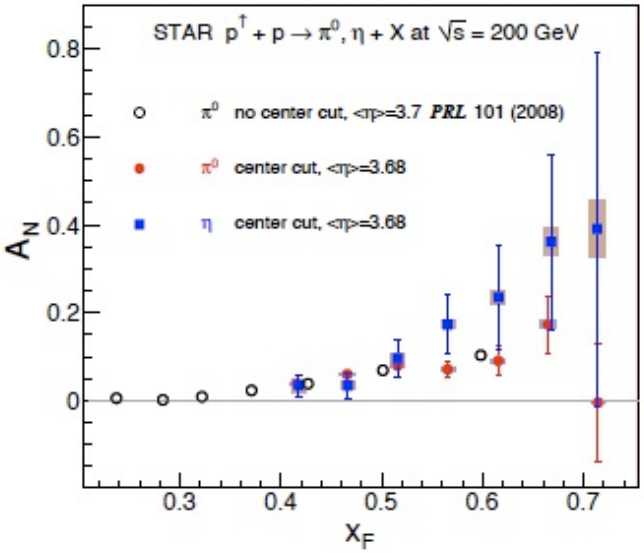
- Large TSSAs observed in the mid-1970s in the detection of hyperons from proton-beryllium collisions (Bunce, et al. (1976))
- Initially thought to contradict pQCD (Kane, Pumplin, Repko (1978)) – within the naïve collinear parton model:

$$A_N \sim \alpha_s m_q / P_{h\perp}$$

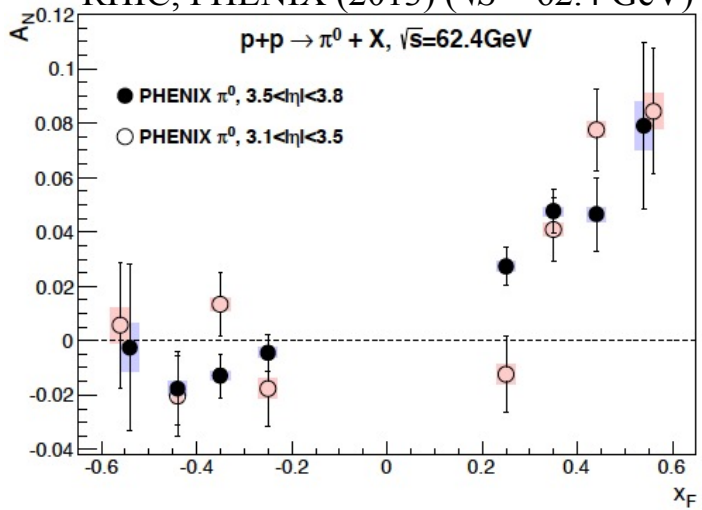
- Higher-twist approach to calculating TSSAs in *pp* collisions introduced in the 1980s (Efremov and Teryaev (1982, 1985))
- Benchmark calculations performed starting in the early 1990s (Qiu and Sterman (1992, 1999); Kouvaris, et al. (2006); Koike and Tomita (2009), etc.)
- RHIC (BRAHMS, STAR, PHENIX) has provided the most recent experimental data on proton-proton TSSAs (also FNAL (E704) in the 1990s)

➢ Experimental data

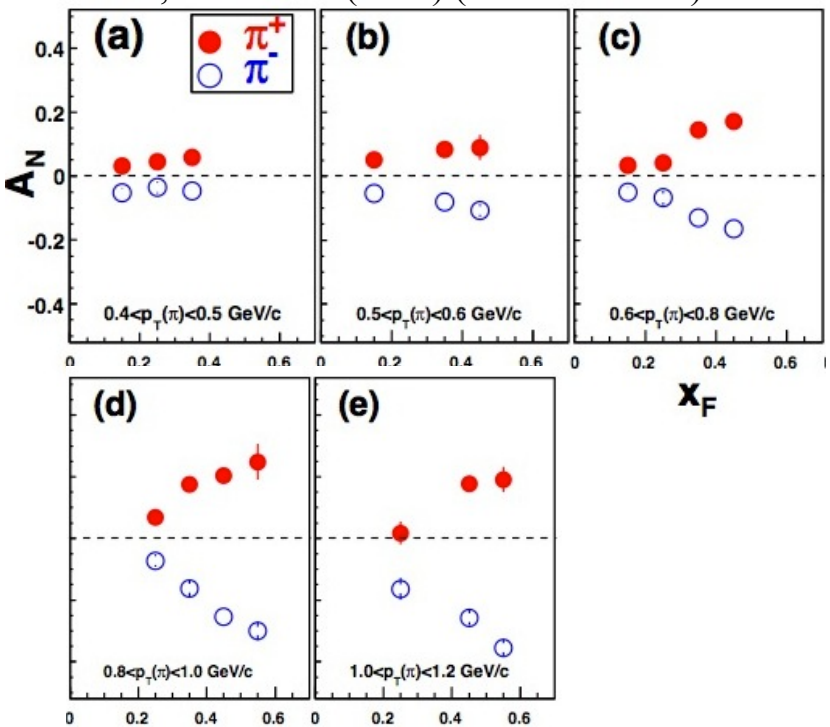
RHIC, STAR (2012) ($\sqrt{S}$ = 200 GeV)



RHIC, PHENIX (2013) ($\sqrt{S}$ = 62.4 GeV)

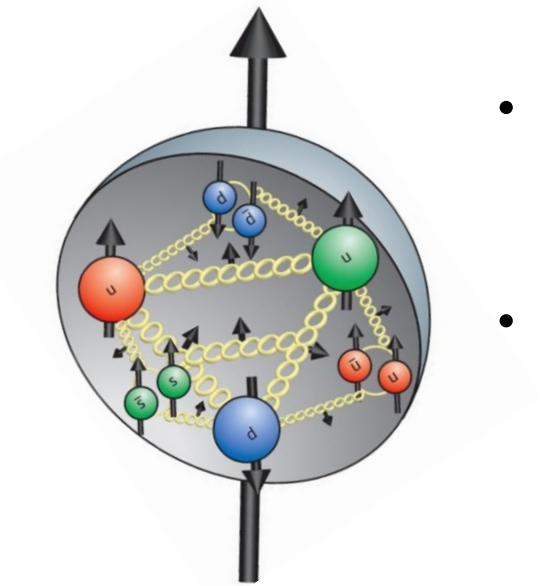


RHIC, BRAHMS (2008) ($\sqrt{S}$ = 62.4 GeV)



Also preliminary data from BRAHMS at $\sqrt{S}$ = 200 GeV

$$x_F = 2p_z/\sqrt{S}$$

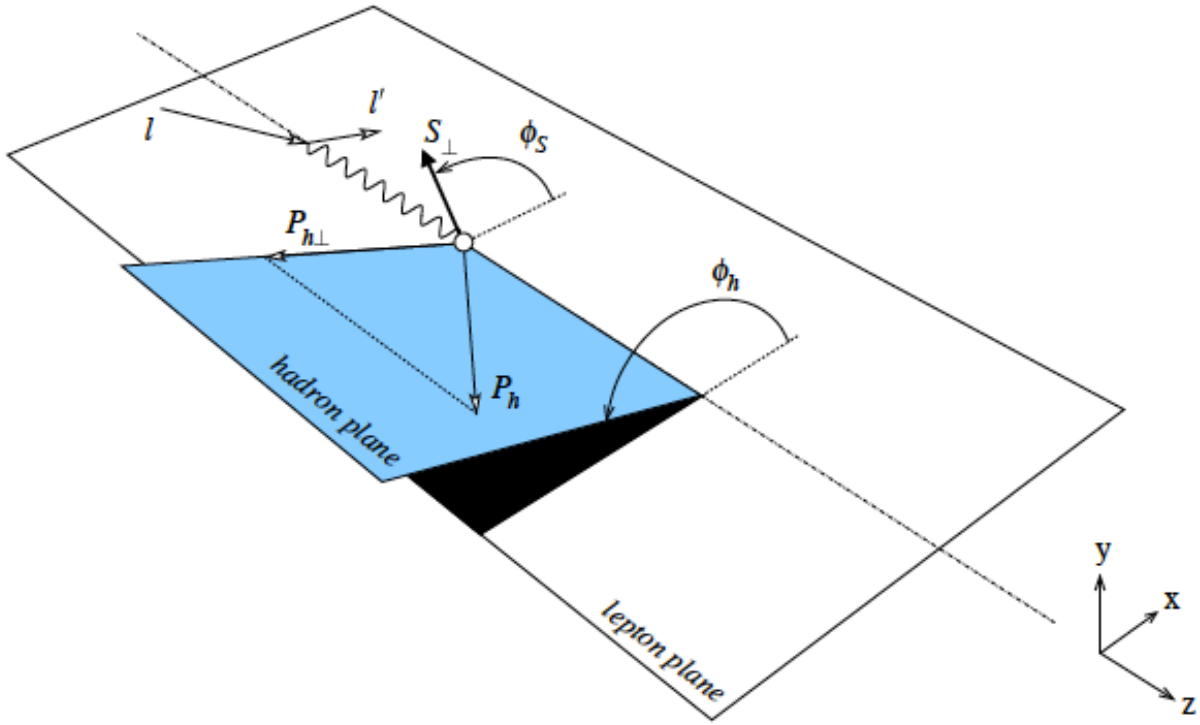


$$A_N = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + \sigma_R}$$

- Data tells us (if fragmentation mechanism dominates) that the pions care about the transverse spin of the fragmenting quark ➔ fragment in a particular direction (left or right)
- Small and negative x_F ➔ probe sea quarks and gluons in $p^\uparrow$
  - ➡ $gg \to gg$ channel gives large contribution to unpolarized cross section, but NO gluon "transversity" ➔ no such channel in spin-dependent cross section
  - ➡ Little information on sea quark "transversity" ➔ might speculate sea quarks, on average, are less likely to emerge from $p^\uparrow$ with a transverse spin in a certain direction
- Large x_F ➔ probe valence quarks in $p^\uparrow$
  - ➡ From SIDIS we know *u* quarks (*d* quarks) are more likely emerge from $p^\uparrow$ with their transverse spin aligned (anti-aligned) with $p^\uparrow$ ➔ pions more likely to fragment in a particular direction (left or right)
  - ➡ $gg \to gg$ channel dies out in this region ➔ unpolarized cross section becomes smaller

- An aside: TSSAs in SIDIS and the TMD formalism

$$A_{UT}^{\sin(\phi_h-\phi_s)} = \frac{\int d\phi_h d\phi_S \, \sin(\phi_h - \phi_S)\, d\sigma}{\int d\phi_h d\phi_S \, d\sigma}$$



(Figure from Bacchetta, et al. (2007))

Sivers function

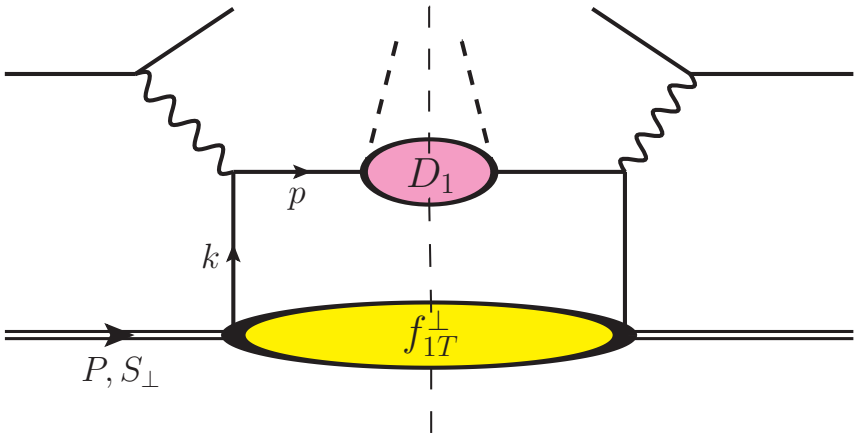
$$A_{UT}^{\sin(\phi_h-\phi_s)} = \frac{w(k_\perp)\, f_{1T}^{\perp,q}(x,\vec{k}_\perp) \otimes D_1^{h/q}(z,\vec{p}_\perp)}{f_1^q(x,\vec{k}_\perp) \otimes D_1^{h/q}(z,\vec{p}_\perp)}$$

Sivers asymmetry

TMD approach

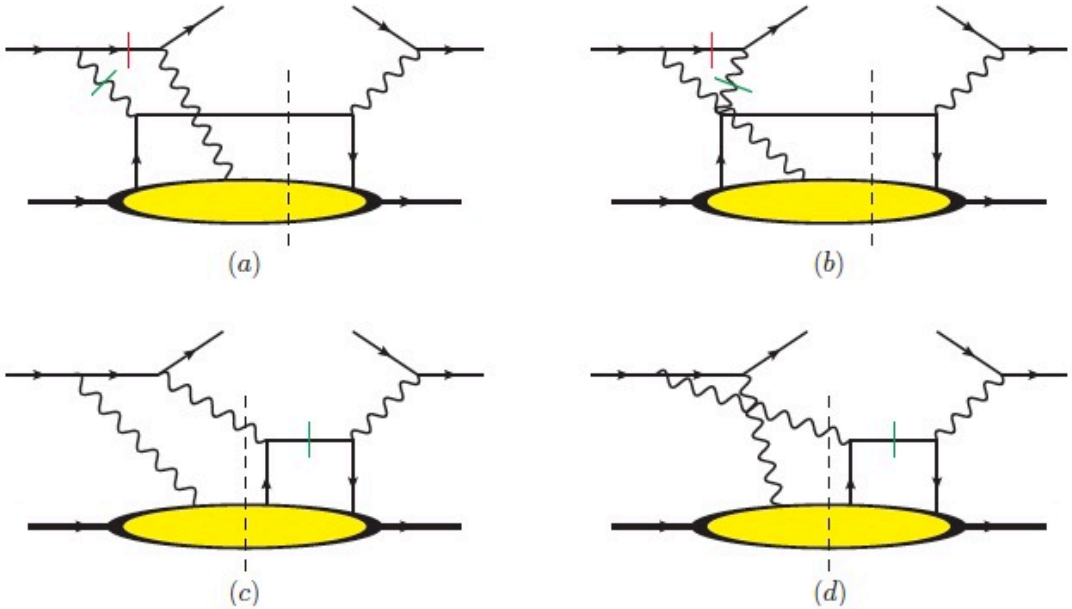
(Sivers (1990, 1991); Collins (1993))

$Q >> P_{hT} \geq \Lambda_{QCD}$



- T-odd effect ➡ imaginary phase is generated by "Wilson line"
  ➡ multiple re-interactions of the quark with the target remnants
- Process dependence: $f_{1T}^{\perp}(x,\vec{k}_\perp^2)\big|_{SIDIS} = -f_{1T}^{\perp}(x,\vec{k}_\perp^2)\big|_{DY}$ (Collins (2002))

- TSSA in inclusive DIS (Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou, PRD **86** (2012))



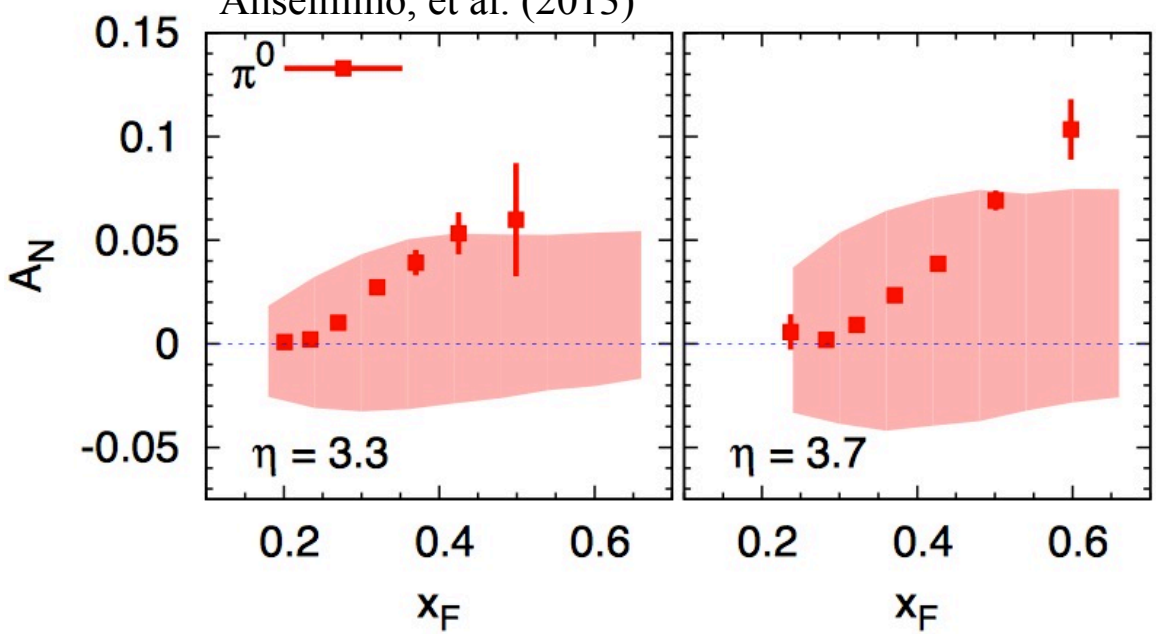
$$k'^0 \frac{d\sigma^N_{pol}}{d^3\vec{k}'} = \frac{8\pi\alpha_{em}^2\, x y^2\, M}{Q^8} \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\left(2+\frac{\hat{u}}{\hat{t}}\right)\varepsilon^{S_N P k k'} \sum_q e_q^2\, x \tilde{F}_{FT}^{q/N}(x,x)$$

$$\text{with} \quad \tilde{F}_{FT}(x,x) = F_{FT}(x,x) - x\frac{d}{dx}F_{FT}(x,x)$$

(Work has also been done on both photons coupling to the same quark: Metz, Schlegel, Goeke (2006); Afanasev, Strikman, Weiss (2007); Schlegel (2012))

- A note on the TMD approach to TSSAs in *pp* collisions
  - ➡ Only a phenomenological model, since there is no proof such a formalism holds in processes with only one (large) scale
  - ➡ Use Sivers function extracted from SIDIS ➔ large uncertainties due to unknown large *x* behavior ➔ cannot draw any definite conclusions

Anselmino, et al. (2013)



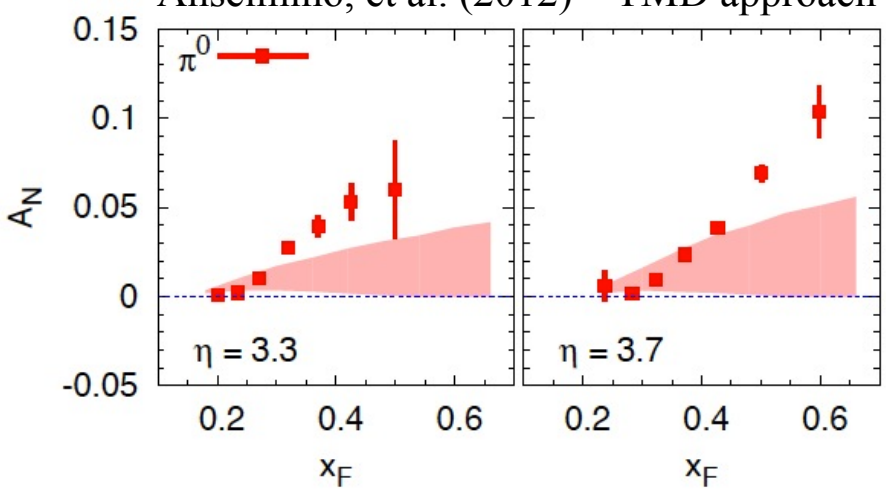
  - ➡ NO sign mismatch problem, but if one takes the re-scattering picture seriously then the issue cannot be avoided

$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$
$$+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}$$
$$+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}$$

Negligible (Kanazawa and Koike (2000))

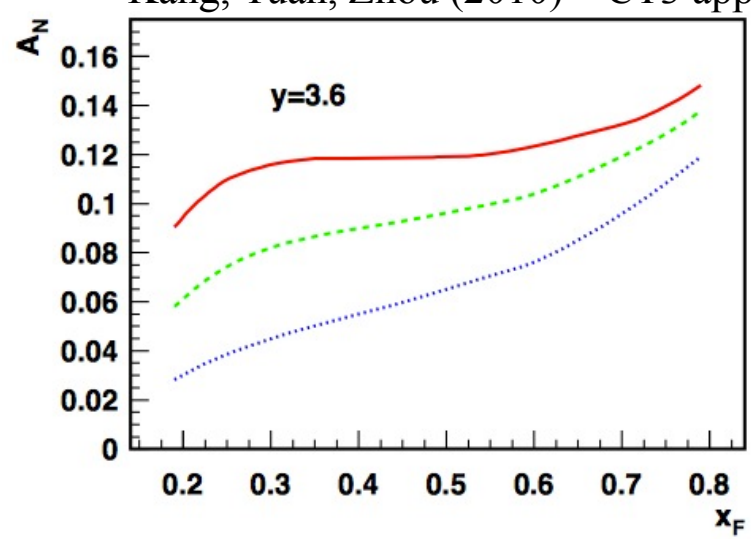
- Collinear twist-3 fragmentation term:

Anselmino, et al. (2012) – TMD approach



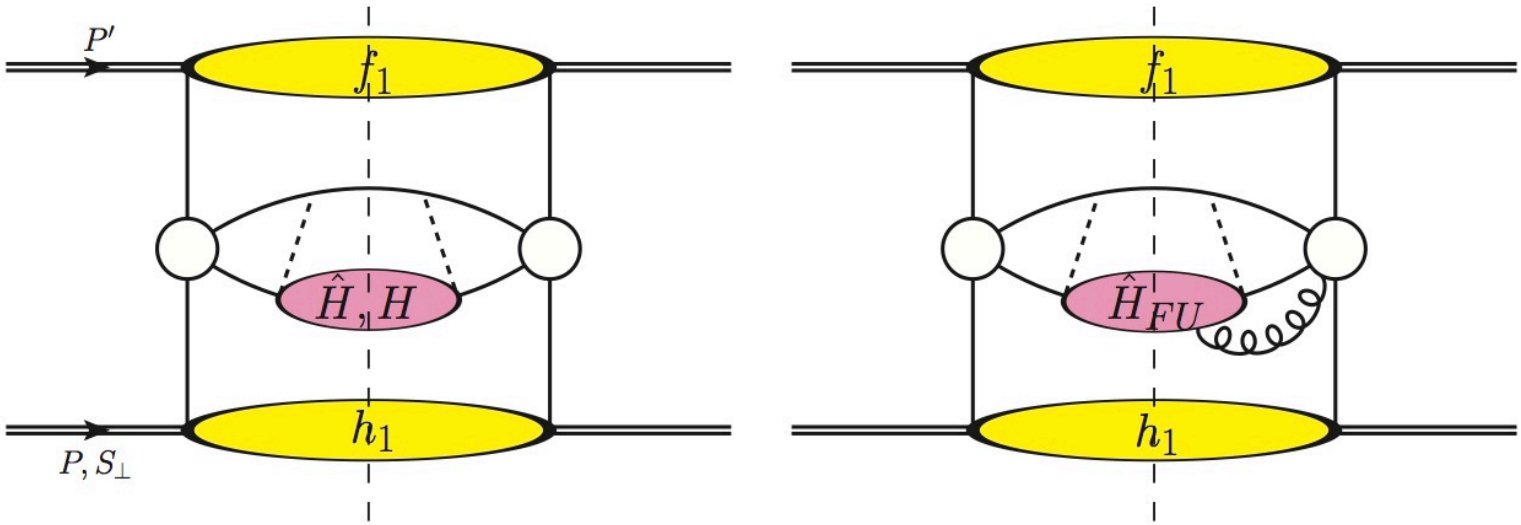
- Uses Collins function extracted from e^+e^- and SIDIS

Kang, Yuan, Zhou (2010) – CT3 approach



- Only looks at "derivative term" using simple parameterization

– Could at the very least give a contribution comparable to SGP term



$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{2M_h^2} H_1^{\perp\, h/q}(z, z^2\vec{k}_\perp^{\,2})$$ Collins-type function

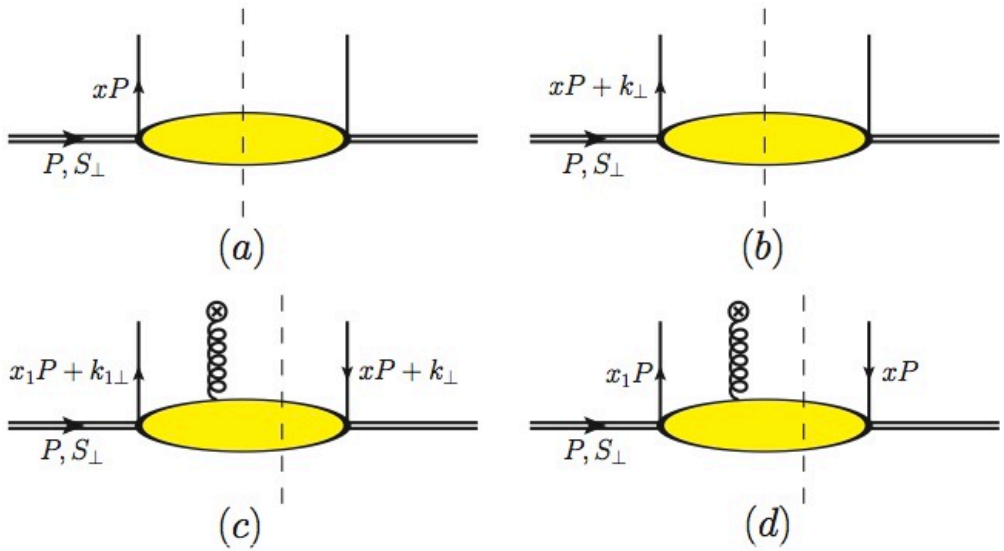
$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1) = H^{h/q}(z) + 2z\hat{H}^{h/q}(z)$$ 3-parton correlator

➡ There are 2 independent (unpolarized) collinear twist-3 FFs

Collinear twist-3 fragmentation structure is richer than that for the TMD formalism

➢ Theoretical description: collinear twist-3 formalism

Lightcone gauge



(a) ➡ $\Phi^q_{ij}(x; P, S_\perp) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S_\perp | \bar{\psi}^q_j(0)\psi^q_i(\xi^-) | P, S_\perp \rangle \xrightarrow{\gamma^i\gamma_5} g^q_T(x)$

(b) ➡ $\Phi^{q,\mu}_{\partial_\perp,ij}(x; P, S_\perp) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S_\perp | \bar{\psi}^q_j(0)\partial^\mu_\perp\psi^q_i(\xi^-) | P, S_\perp \rangle \xrightarrow{\gamma^+\gamma_5} \tilde{g}^q(x) \left(= g^{q(1)}_{1T}(x)\right)$

(d) ➡ $\Phi^{q,\mu}_{A,ij}(x, x_1; P, S_\perp) = \int \frac{d\xi^-}{2\pi} \int \frac{d\zeta^-}{2\pi} e^{ix_1P^+\xi^-} e^{i(x-x_1)P^+\zeta^-}$
$\times \langle P, S_\perp | \bar{\psi}^q_j(0) A^\mu_\perp(\zeta^-) \psi^q_i(\xi^-) | P, S_\perp \rangle \xrightarrow{\gamma^+/\gamma^+\gamma_5} \begin{cases} F^q_{FT}(x, x_1) \\ G^q_{FT}(x, x_1) \\ F^q_{DT}(x, x_1) \\ G^q_{DT}(x, x_1) \end{cases}$

Rewrite in terms of F or D

Twist-3 collinear PDFs for a transversely polarized p

(c) gives a twist-4 contribution

(see, e.g., Zhou, Yuan, Liang (2010))

- Symmetry properties

$$F^q_{FT}(x,\, x_1) = F^q_{FT}(x_1,\, x) \;\text{ and }\; G^q_{FT}(x,\, x_1) = -G^q_{FT}(x_1,\, x)$$

$$F^q_{DT}(x,\, x_1) = -F^q_{DT}(x_1,\, x) \;\text{ and }\; G^q_{DT}(x,\, x_1) = G^q_{DT}(x_1,\, x)$$

- Relations between F-type and D-type functions (see, e.g., Eguchi, et al. (2006))

$$F^q_{DT}(x, x_1) = PV \frac{1}{x - x_1} F^q_{FT}(x, x_1)$$

$$G^q_{DT}(x, x_1) = PV \frac{1}{x - x_1} G^q_{FT}(x, x_1) + \delta(x - x_1)\, \tilde{g}^q(x)$$

- g_T can be related to D-type functions through the EOM (see, e.g., Efremov and Teryaev (1985); Jaffe and Ji (1992); Boer, Mulders, Teryaev (1998)):

$$x\, g^q_T(x) = \int dx_1 \left[G^q_{DT}(x,\, x_1) - F^q_{DT}(x,\, x_1)\right]$$

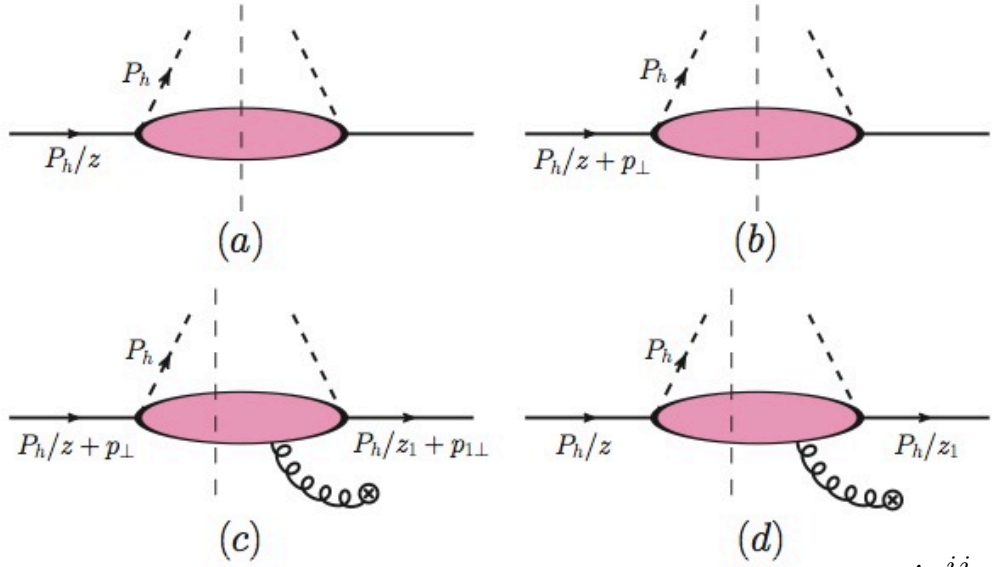
There are 3 independent collinear twist-3 functions relevant for a transversely polarized p

$$\tilde{g},\; F_{FT},\; G_{FT}$$

$$or$$

$$\tilde{g},\; F_{DT},\; G_{DT}$$

Lightcone gauge



(a) $\Delta^{h/q}_{ij}(z;P_h) = \sum\!\!\!\!\!\!\int_X z \int \frac{d\xi^+}{2\pi} e^{i\frac{P_h^-}{z}\xi^+} \langle 0|\psi^q_i(\xi^+)|P_h;X\rangle\langle P_h;X|\bar\psi^q_j(0)|0\rangle \xrightarrow{i\sigma^{ij}\gamma_5/\mathbb{1}} \begin{cases} H^{h/q}(z) \\ E^{h/q}(z)\end{cases}$

(b) $\Delta^{h/q,\mu}_{\partial_\perp,ij}(z;P_h) = \sum\!\!\!\!\!\!\int_X z \int \frac{d\xi^+}{2\pi} e^{i\frac{P_h^-}{z}\xi^+} \langle 0|\partial^\mu_\perp\psi^q_i(\xi^+)|P_h;X\rangle\langle P_h;X|\bar\psi^q_j(0)|0\rangle \xrightarrow{i\sigma^{i-}\gamma_5} \hat H^{h/q}(z) = \left(H_1^{\perp\, h/q(1)}(z)\right)$

(d) $\Delta^{h/q,\mu}_{A,ij}(z,z_1;P_h) = \sum\!\!\!\!\!\!\int_X \frac{1}{z}\int\frac{d\xi^+}{2\pi}\int\frac{d\zeta^+}{2\pi} e^{i\frac{P_h^-}{z_1}\xi^+} e^{i\left(\frac{1}{z}-\frac{1}{z_1}\right)P_h^-\zeta^+} \times \langle 0|A^\mu_\perp(\zeta^+)\psi^q_i(\xi^+)|P_h;X\rangle\langle P_h;X|\bar\psi^q_j(0)|0\rangle \xrightarrow{i\sigma^{i-}\gamma_5} \begin{cases}\hat H^{h/q}_{FU}(z,z_1)\\ \hat H^{h/q}_{DU}(z,z_1)\end{cases}$

Twist-3 collinear FFs for an unpolarized h

Rewrite in terms of F or D

Note: $\hat H_{FU}$ and $\hat H_{DU}$ have real and imaginary parts.

(c) gives a twist-4 contribution

- Relations between F-type and D-type function

$$\hat{H}_{DU}^{h/q,\Im}(z,z_1) = PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1) - \frac{1}{z^2}\hat{H}^{h/q}(z)\,\delta\left(\frac{1}{z}-\frac{1}{z_1}\right)$$

$$\hat{H}_{DU}^{h/q,\Re}(z,z_1) = PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Re}(z,z_1)$$

- H (E) can be related to the imaginary (real) part of the D-type function through the EOM:

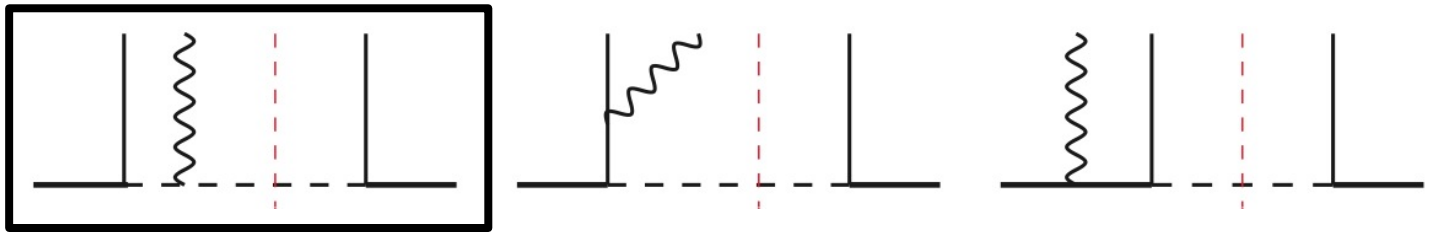
$$H^{h/q}(z) = 2z^3\int\frac{dz_1}{z_1^2}\hat{H}_{DU}^{h/q,\Im}(z,z_1)$$

$$E^{h/q}(z) = -2z^3\int\frac{dz_1}{z_1^2}\hat{H}_{DU}^{h/q,\Re}(z,z_1)$$

There are 2 independent collinear twist-3 functions relevant for the fragmentation of a quark into an unpolarized h

$$\hat{H},\ \hat{H}_{FU}$$
$$or$$
$$\hat{H},\ \hat{H}_{DU}$$

- Involves F_{FT} in a QED process ($q\gamma q$ correlator) ➡ relate to F_{FT} in a QCD process (qgq correlator) through a diquark model

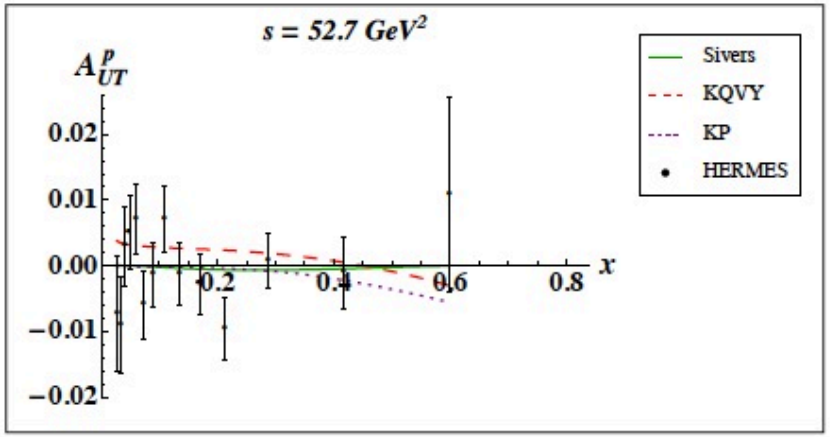


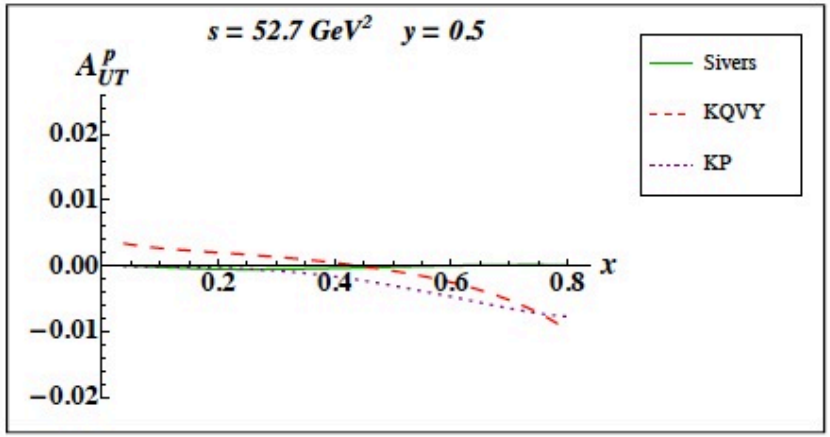
$$(F_{FT}^{u/p})_{QED} = \frac{\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{u/p})_{QCD} \qquad (F_{FT}^{d/p})_{QED} = \frac{4\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{d/p})_{QCD}$$

$$(F_{FT}^{u/n})_{QED} = -\frac{2\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{d/p})_{QCD} \qquad (F_{FT}^{d/n})_{QED} = \frac{\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{u/p})_{QCD}$$

- Use 3 different inputs for F_{FT} in a QCD process:
  1) Sivers: fit from Anselmino, et al. (2008) of Sivers asymmetry from SIDIS data
  2) KQVY: fit from Kouvaris, et al. (2006) for SSAs in *pp* collisions
  3) KP: simultaneous fit from Kang and Prokudin (2012) of *pp* and SIDIS data

- Proton SSA:





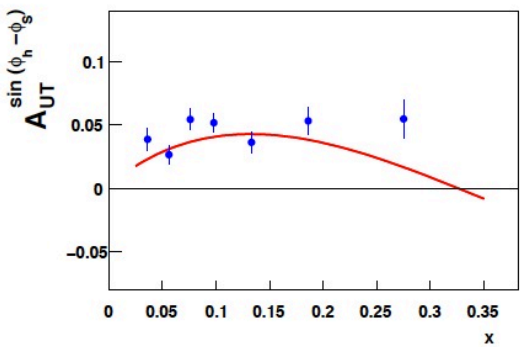
Sivers input agrees exactly with the HERMES data (Airapetian, et al. (2009))

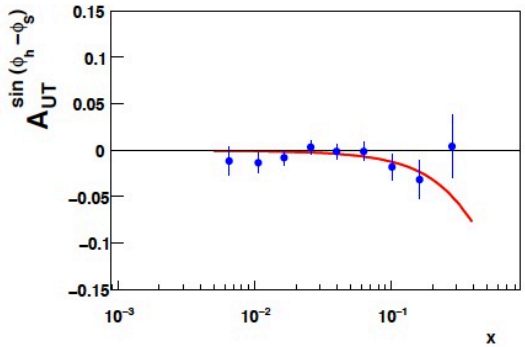
KP input appears to become too large at large x (result of the node in x for the up quark Sivers function)

➡ Node in x in the Sivers function is not preferred, although it cannot be definitively excluded by the current data → need more accurate data at larger x

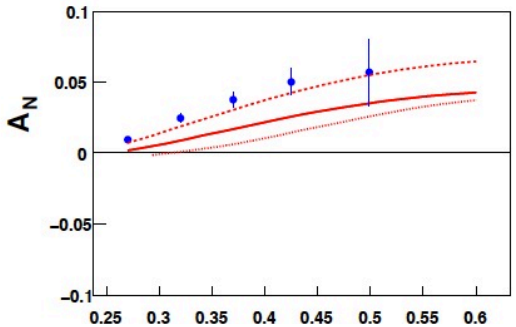
KQVY input also appears to become too large at large x and actually diverges as $x \to 1$

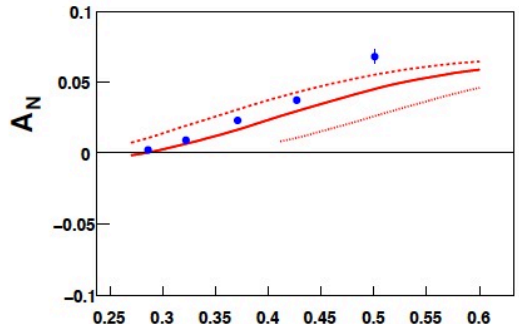
- Node in x or k_T in the Sivers function:
  - Attempt to simultaneously fit SIDIS and pp data (Kang and Prokudin (2012))



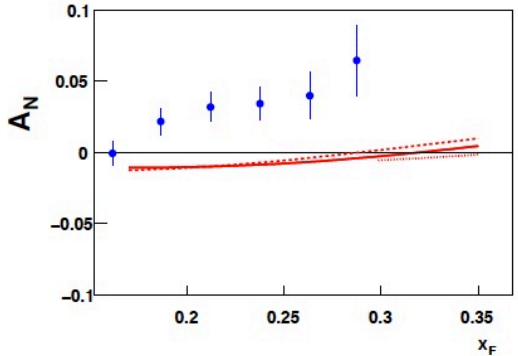


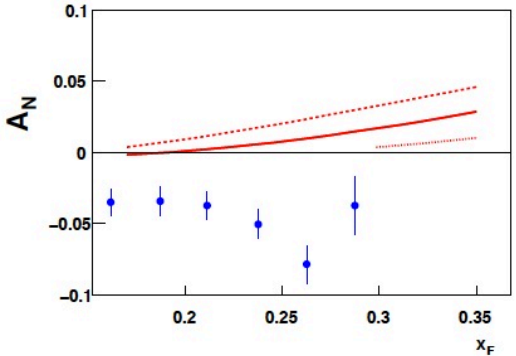
SIDIS data from HERMES (left) and COMPASS (right)





Proton-proton data from STAR at $y = 3.3$ (left) and $y = 3.7$ (right)





Proton-proton data from BRAHMS for π^+ (left) and π^- (right)