Quark and Glue Components of Proton Spin

- Status of nucleon spin components
- Momentum and angular momentum sum rules
- Lattice results
- Quark spin from anomalous Ward identity
- Glue helicity ΔG

χ QCD Collaboration:
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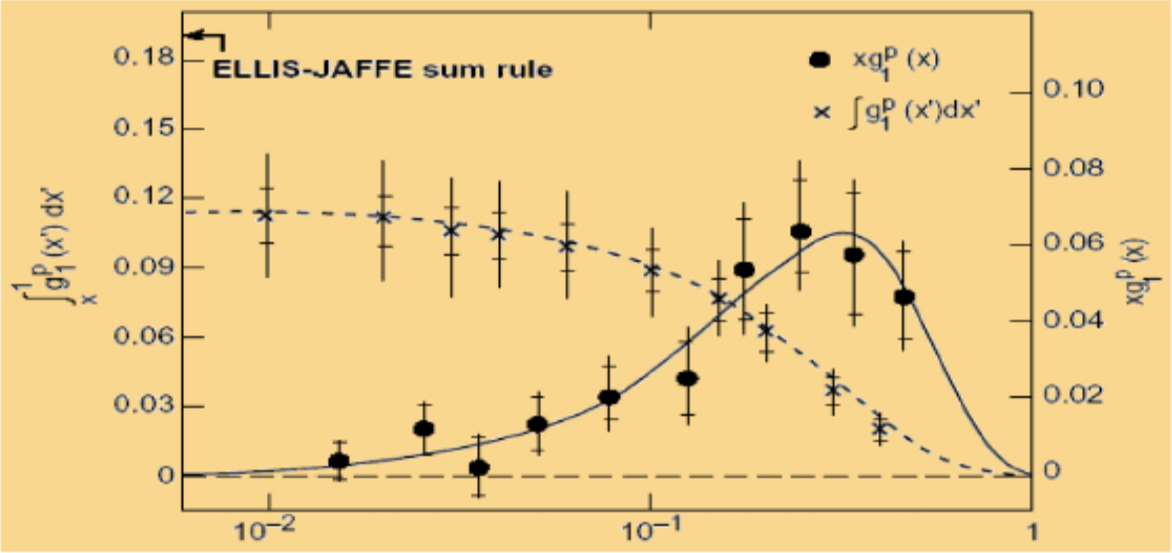
QCD Evolution Workshop
Santa Fe, May 13, 2014

Where does the spin of the proton come from?

Twenty years since the "spin crisis"

- EMC experiment in 1988/1989 – "the plot":



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\|}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{\|}\rangle$$

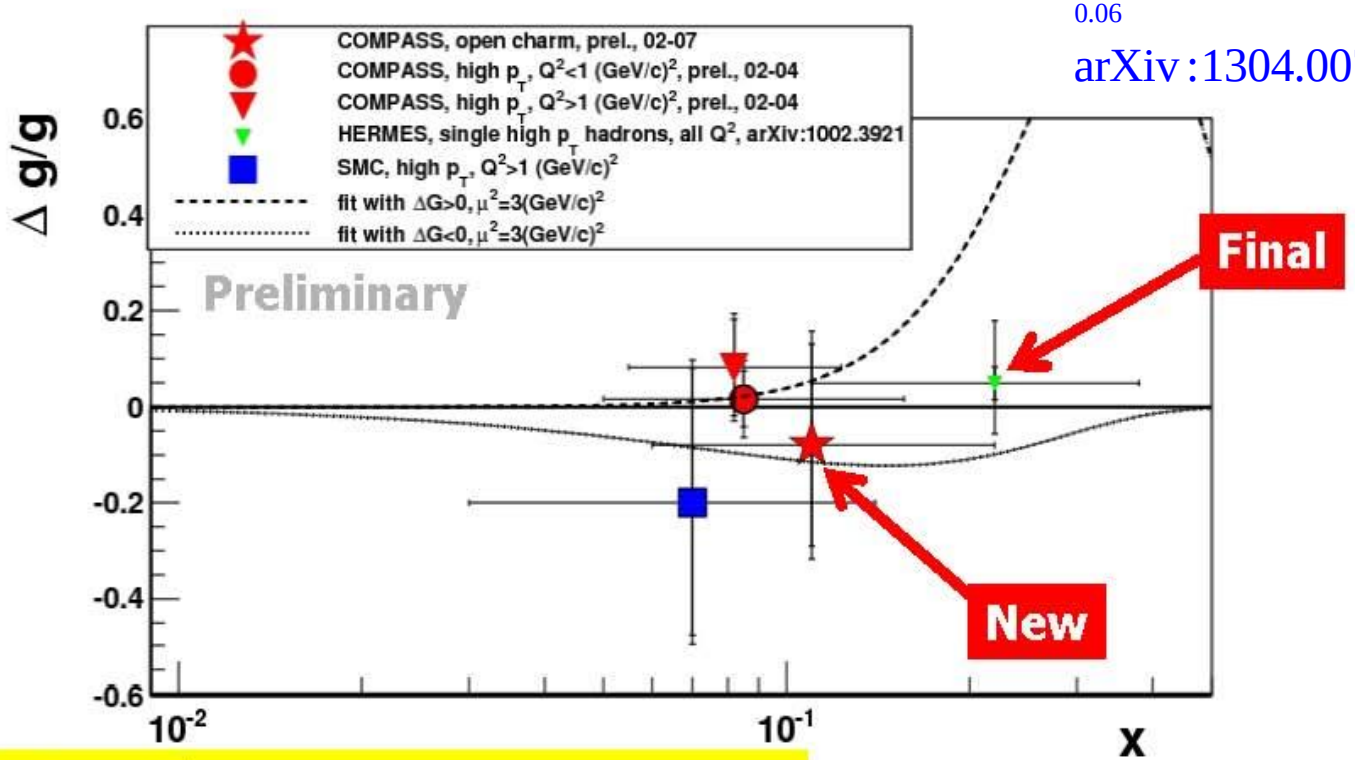
- "Spin crisis" or puzzle: $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

Summary Gluon Polarization

Presently all Analysis in LO only

$$\int_{0.06}^{0.2} Dg(x)dx = 0.1 \pm^{0.06}_{0.07}$$

arXiv:1304.0079



COMPASS Open Charm:

$\Delta G/G$=-0.08+- 0.21(stat) +- 0.11(sys.)
(Systematic error still under investigations)

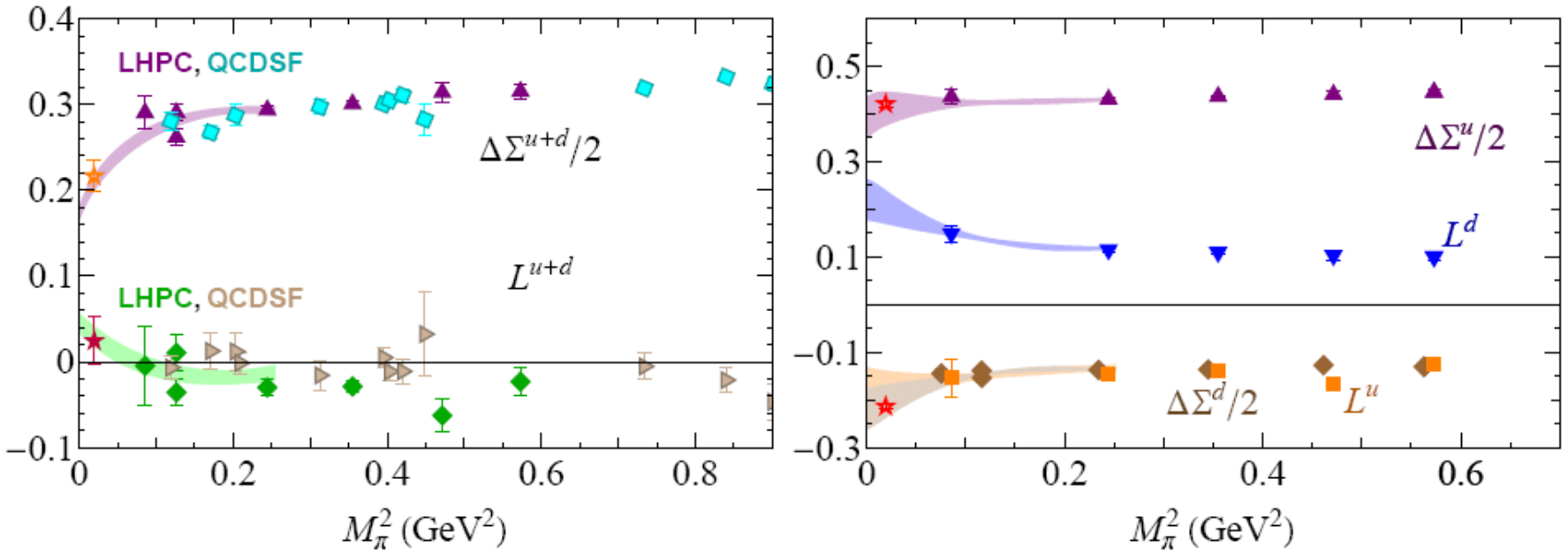
(Value supersedes previous publication)

C.Franco

See Talk 1193 by F. Kunne

Horst Fischer DIS2010

Quark Orbital Angular Momentum from connected insertion (~ valence)



Status of Proton Spin

- Quark spin $\Delta\Sigma \sim 20 - 30\%$ of proton spin (DIS, Lattice)
- Quark orbital angular momentum? (lattice calculation (LHPC,QCDSF)→ ~ 0)
- Glue spin $\Delta G/G$ small (COMPASS, STAR) ?
- Glue orbital angular momentum is zero (Brodsky and Gardner) ?

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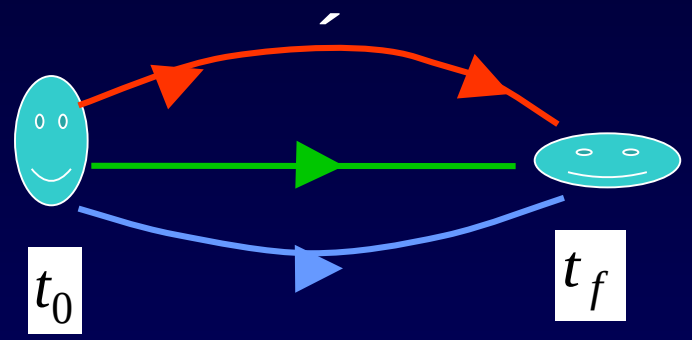
→ Dark Spin

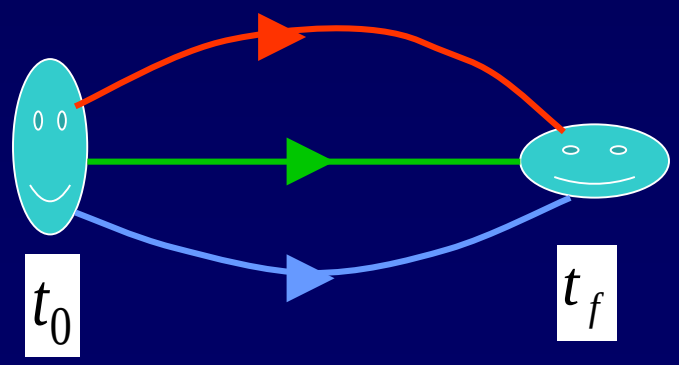
Hadron Structure with Quarks and Glue

- Quark and Glue Momentum and Angular Momentum in the Nucleon

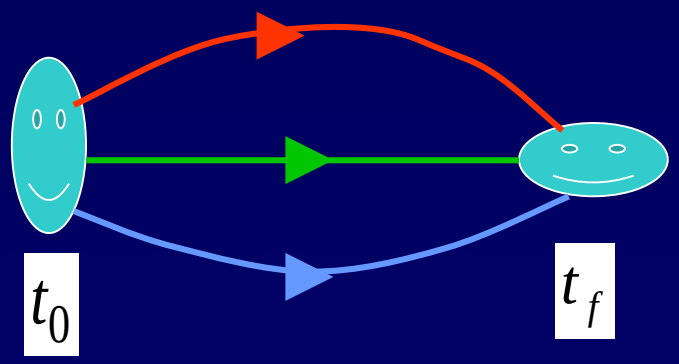
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$



$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$



$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$



Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{J}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \qquad \rightarrow \vec{J}_g = \int d^3x \left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\langle p,s|T_{\mu\nu}|p's'\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu \bar{p}_\nu - T_2(q^2)\bar{p}_\mu \sigma_{\nu\alpha}q_\alpha / 2m$$

$$-iT_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ [\mathrm{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2}$$

$$\Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

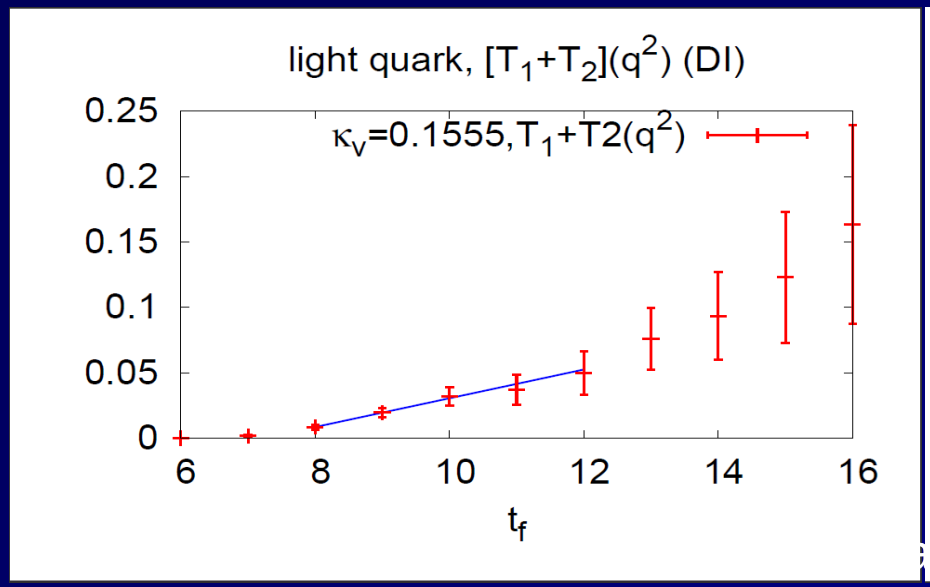
$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

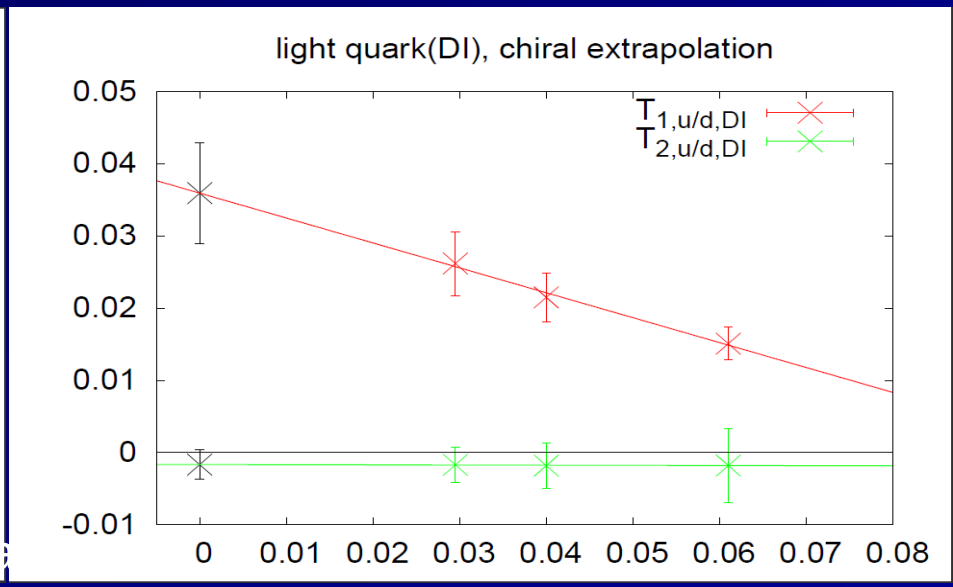
M. Glatzmaier, KFL
arXiv:1403.7211

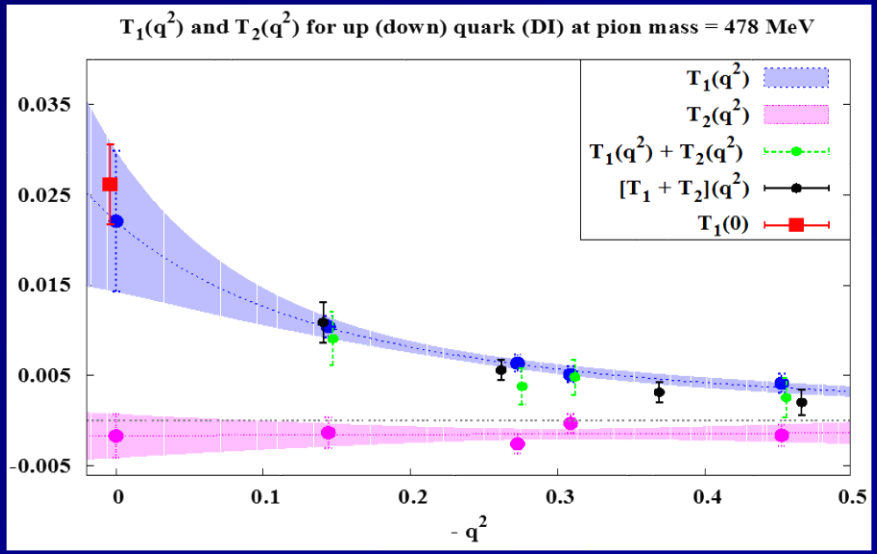
Lattice Parameters

- Quenched $16^3 \times 24$ lattice with Wilson fermion
- Quark spin and <x> were calculated before for both the C.I. and D.I.
- $\kappa = 0.154, 0.155, 0.1555$ ($m_\pi = 650, 538, 478$ MeV)
- 500 gauge configurations
- 400 noises (Optimal Z_4 noise with unbiased subtraction) for DI
- 16 nucleon sources

Disconnected Insertions of T_1 (q^2) and T_2 (q^2) for u/d Quarks







Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \;\; H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = -Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

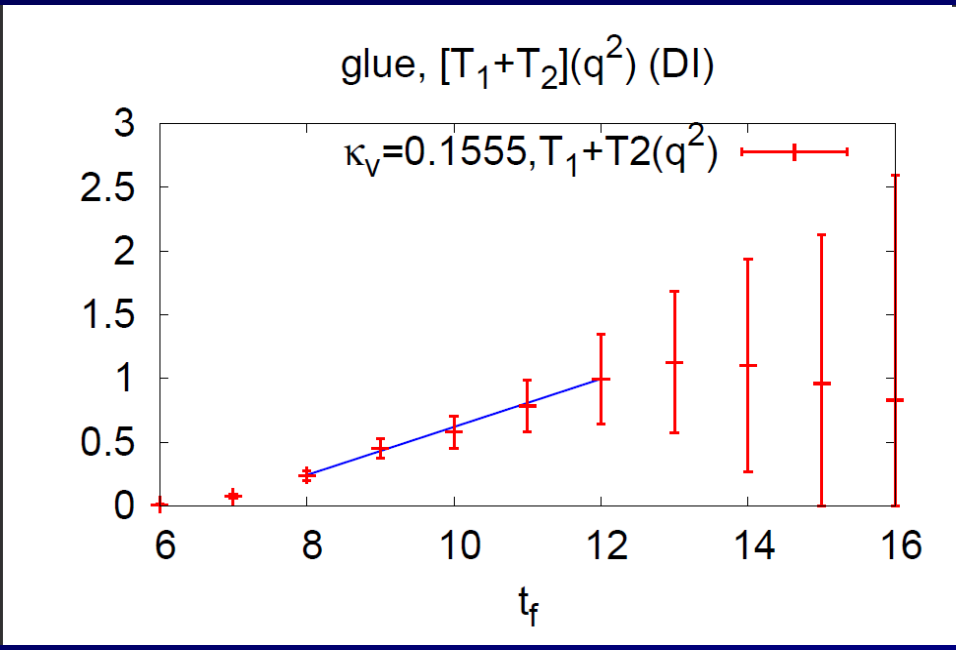
$$q_L(x) = -tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

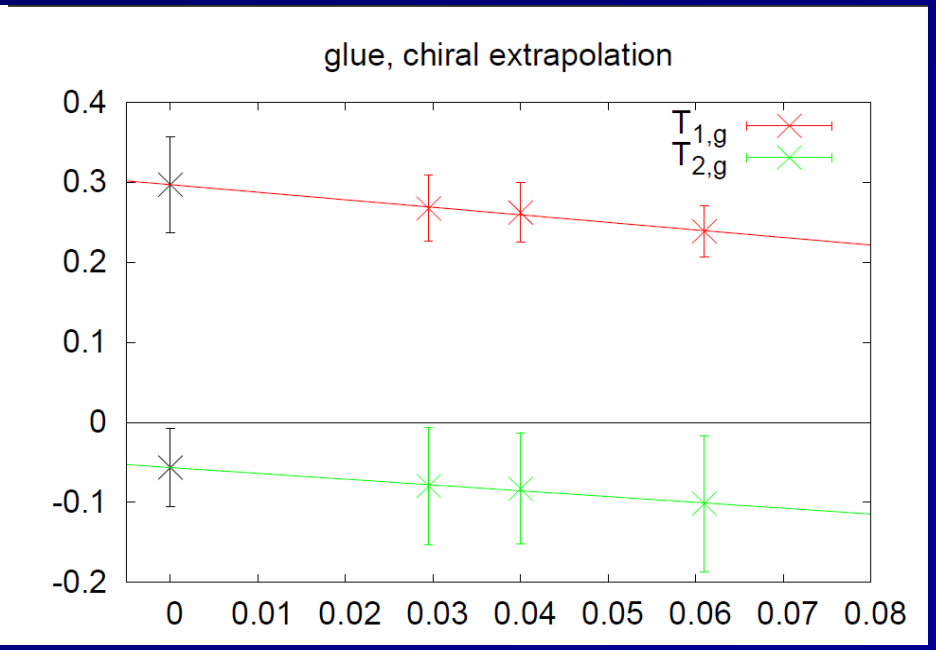
- Gauge field tensor

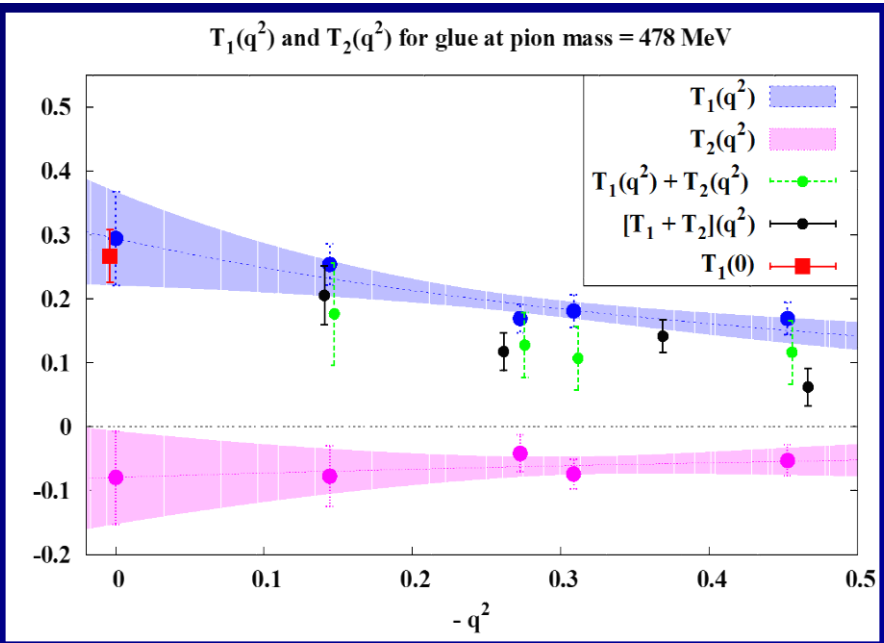
$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu}(x) + O(a^3),$$

Liu, Alexandru, Horvath – PLB 659, 773 (2007)

Glue $T_1(q^2)$ and $T_2(q^2)$







S.J. Dong, J.-F. Lagae, and KFL, PRL 75, 2096 (1995)

$$g_A^0 = (\Delta u + \Delta d)_{con} + (\Delta u + \Delta d + \Delta s)_{dis} = 0.62(9) + 3(-0.12(1)) = 0.25(12)$$

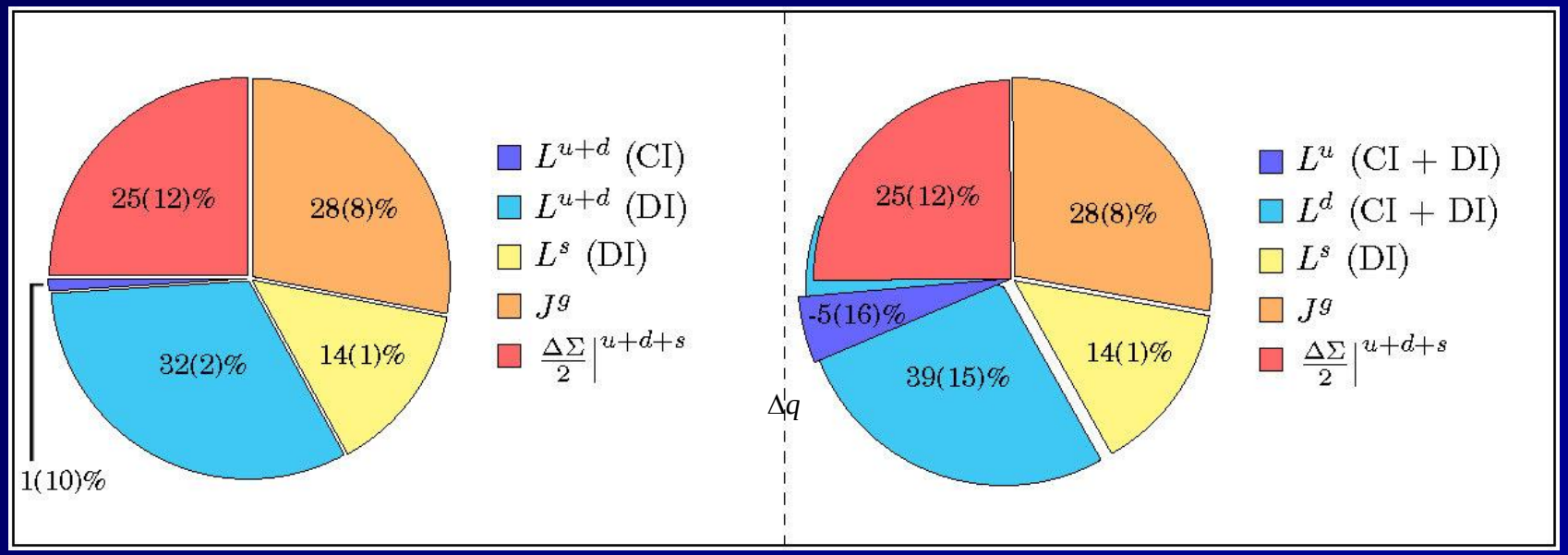
	Lattice	Expt. (SMC)	NRQM	RQM
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.25(12)	0.22(10)	1	0.75
$g_A^3 = \Delta u - \Delta d$	1.20(10)	1.2573(28)	5/3	1.25
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.61(13)	0.579(25)	1	0.75
Δu	0.79(11)	0.80(6)	1.33	1
Δd	-.42(11)	-0.46(6)	-0.33	-0.25
Δs	-.12(1)	-0.12(4)	0	0
F_A	0.45(6)	0.459(8)	0.67	0.5
D_A	0.75(11)	0.798(8)	1	0.75
F_A / D_A	0.60(2)	0.575(16)	0.67	0.67

$$F_A = (\Delta u - \Delta s)/2; \quad D_A = (\Delta u - 2\Delta d + \Delta s)/2$$

Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.416 (40)	0.151 (20)	0.567 (45)	0.037 (7)	0.023 (6)	0.334 (56)
$T_2(0)$	0.283 (112)	-.217 (80)	0.061 (22)	-0.002 (2)	-.001 (3)	-0.056 (52)
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)
g_A	0.91 (11)	-0.30 (12)	0.62 (9)	-0.12 (1)	-0.12 (1)	
2 L	-0.21 (16)	0.23 (15)	0.01 (10)	0.16 (1)	0.14 (1)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum



$\Delta q \approx 0.25$;

$2\ L_q \approx 0.47$ (0.01(valence)+0.46(sea));

$2\ J_g \approx 0.28$

M. Deka et al., 1312.4816

Summary of Quenched Lattice Calculations

- Complete calculation of momentum fractions of quarks (both valence and sea) and glue have been carried out for a quenched lattice:
  - Glue momentum fraction is ~ 33%.
  - g_A^0 ~ 0.25 in agreement with expt.
  - Glue angular momentum is ~ 28%.
  - Quark orbital angular momentum is large for the sea quarks (~ 47%).
- These are quenched results so far.

Uncertainty of Quark Spin Calculation

- Recent calculation of strange quark spin with dynamical fermions
  - R. Babich et al. (1012.0562)

    $$\Delta s = -0.019(11)$$

  - QCDSF (G. Bali et al. 1206.4205) gives

    $$\Delta s = -0.020(10)(4)$$

  much smaller than that of of quenched result.

  - C. Alexandrou et al. (arXiv:1310.6339)

    $$\Delta s \sim -0.0227(34)$$

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MeV

32^3 x 64, a =0.12 fm

($O(a^2)$ extrapolation)

La ~ 2.8 fm
m_π ~ 330 MeV

24^3 x 64, a =0.115 fm

La ~ 5.5 fm
m_π ~ 140 MeV

48^3 x 96, a =0.115 fm

La ~ 2.7 fm
m_π ~ 295 MeV

32^3 x 64, a =0.085 fm

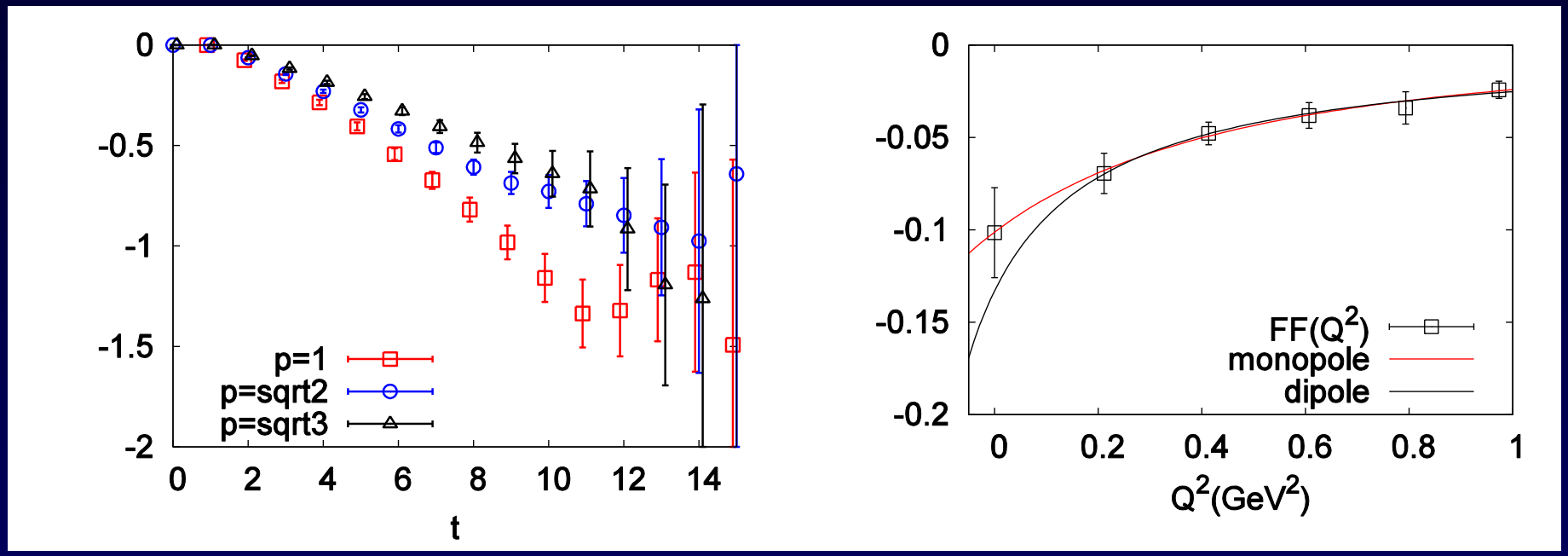
La ~ 5.5 fm
m_π ~ 140 MeV

64^3 x 128, a =0.085 fm

Quark Spin from Anomalous Ward Identify

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $Z_A \partial_\mu A_\mu^0 = 2mP + \frac{N_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
  - Overlap fermion --> mP is RGI.
  - Overlap operator for $q(x) = -1/2\,\mathrm{Tr}\gamma_5 D_{ov}(x,x)$ is RGI.
  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and is dominated by high eigenmodes.
  - Directly check the origin of `proton spin crisis'.

Preliminary Results



Overlap on 2+1 flavor domain-wall fermion on $24^3 \times 64$ lattice

Valence at strange quark mass (~ 100 MeV)

Anomaly contribution < - 0.1 for physical u/d mass

2mP contribution is small for u/d and charm is large enough cancel with that of anomaly.

Eventual understanding of `proton spin crisis' ?

Glue Helicity ΔG

- Jaffe and Manohar

$$S_g = \int d^3x\, \vec{E} \times \vec{A} \text{ in } A^+ = 0 \text{ gauge and IMF frame.}$$

- Collins, Soper; Manohar

$$\Delta G\, S^+ = \int dx\, \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle PS \,|\, F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) \,|\, PS \rangle$$

- X.S. Chen, T. Goldman, F. Wang; Wakamatsu; Hatta, etc.

$$S_g = \int d^3x\, \vec{E} \times \vec{A}_{phys},$$

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{phys} \rightarrow U^\dagger A^\mu_{phys} U, \quad A^\mu_{pure} \rightarrow U^\dagger A^\mu_{pure} U - \frac{i}{g} U^\dagger \partial^\mu U$$

$$or\ A^\mu_{phys} \rightarrow A^\mu_{Coul}$$

Glue Helicity ΔG

- X. Ji, J.H. Zhang, Y. Zhao; Y. Hatta, X. Ji, Y. Zhao

$$\Delta G\ S^+ = \int dx\ \frac{i}{2xP^+}\int \frac{d\xi^-}{2\pi}\ e^{-ixP^+\xi^-}\langle PS\,|\,F_a^{+\alpha}(\xi^-)L^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)\,|\,PS\rangle$$

$$= \frac{1}{2P^+}\langle PS\,|\,\varepsilon^{ij}F^{i+}A^j_{phys}\,|\,PS\rangle;$$

where $A^\mu_{phys} \equiv \frac{1}{D^+}F^{+\mu}$

However, at IMF

$$A^\mu_{phys} \equiv \frac{1}{D^+}F^{+\mu} = A^\mu - \frac{1}{D^+}\partial^\mu A^+ \to A^\mu - A^\mu_{\parallel} = A^\mu_\perp \text{ (i.e. } A^\mu_C\text{)}$$

- Therefore,

$$\Delta G\ S_z = \frac{\langle PS\,|\int d^3x\ (\vec{E}\times\vec{A}_\perp)_z\,|\,PS\rangle}{2E_P}$$

Challenges and Summary

- Continuum limit at physical pion mass and with large lattice volume (5.5 fm) with chiral fermions are being carried out.
- $48^3 \times 96$ and $64^3 \times 128$ lattices with large number of eigenvectors (~ 2000)
- Decomposition of proton spin into quark spin, quark orbital angular momentum, glue helicity, and glue orbital angular momentum on the lattice is feasible.

Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

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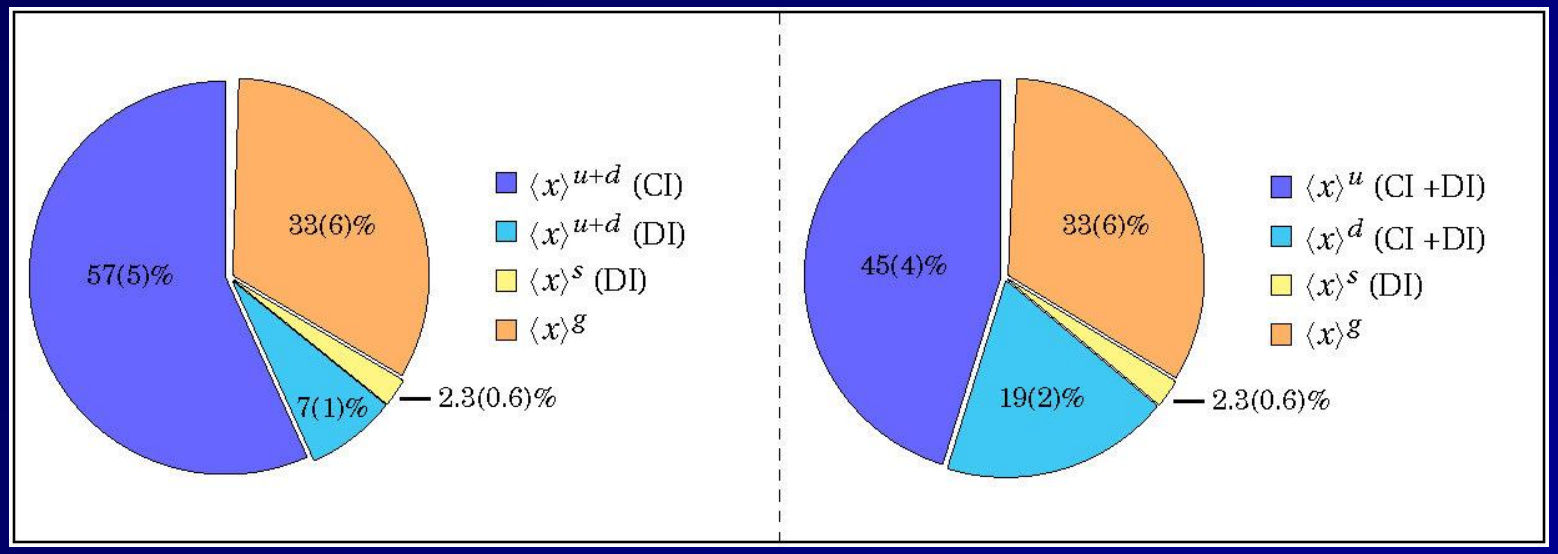
$$T_2(0)^R_{CI} + T_2(0)^R_{DI} + T_2(0)^R_g = 0$$

I.Yu. Kobzarev, L.B. Okun, Zh. Eksp. Teor. Fiz. 43, 1904 (1962) [Sov. Phys. JETP 16, 1343 (1963);

S. Brodsky et al. NPB 593, 311(2001) → no anomalous gravitomagnetic moment

Momentum fractions $\langle x \rangle^q$, $\langle x \rangle^g$



Angular Momentum fractions J^q, J^g