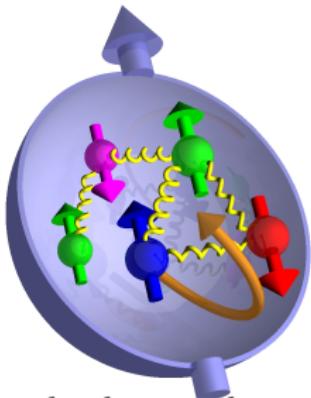


# GPDs and Quark Orbital Angular Momentum

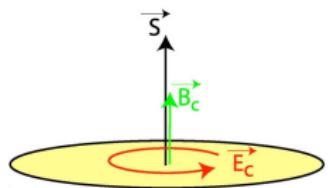
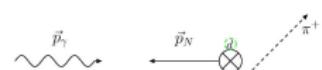
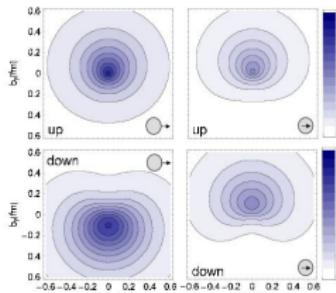
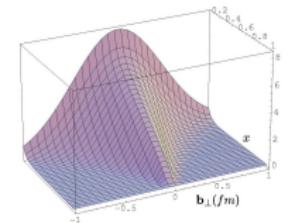
Matthias Burkardt

NMSU

May 14, 2014

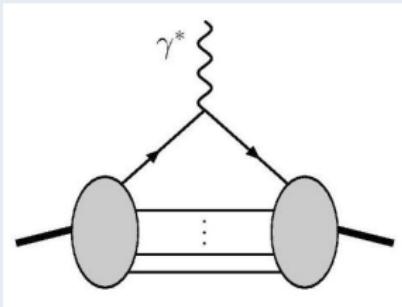


- background
- proton spin puzzle
- 3D imaging of the nucleon
- Single-Spin Asymmetries (SSAs)
- Quark orbital angular momentum
- angular momentum decompositions (Jaffe v. Ji)
- quark-gluon correlations
- Summary



## form factor

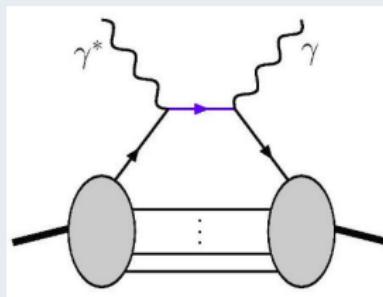
- electron hits nucleon, nucleon remains intact



- study amplitude that nucleon remains intact as function of momentum transfer  $\rightarrow F(q^2)$
- $F(q^2) = \int dx GPD(x, q^2)$
- $\hookrightarrow$  GPDs provide momentum dissected form factors

## Compton scattering

- electron hits nucleon, nucleon remains intact & photon gets emitted



- study both energy &  $q^2$  dependence
  - $\hookrightarrow$  additional information about momentum fraction  $x$  of active quark
  - $\hookrightarrow$  generalized parton distributions  $GPD(x, q^2)$

MB, PRD62, 071503 (2000)

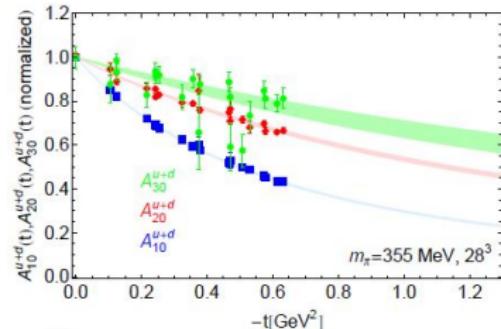
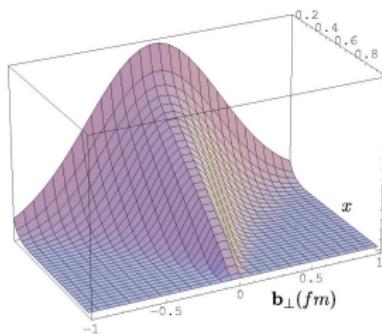
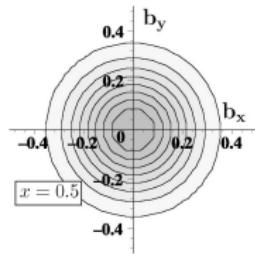
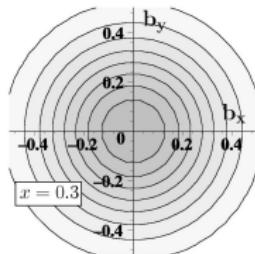
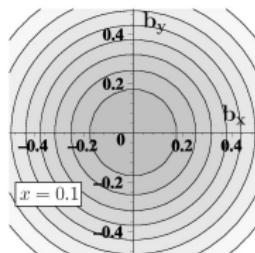
- form factors:  $\xleftarrow{FT} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$ : form factor for quarks with momentum fraction  $x$ 
  - ↪ suitable FT of  $GPDs$  should provide spatial distribution of quarks with momentum fraction  $x$

### Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} GPD(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

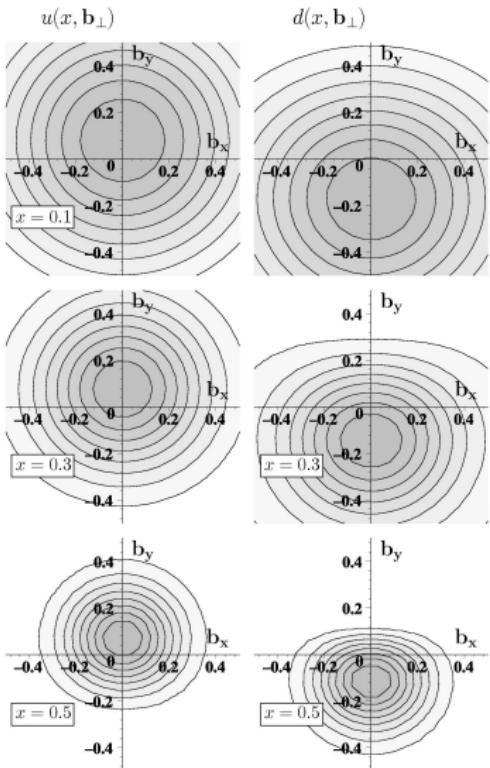
$q(x, \mathbf{b}_\perp)$  = parton distribution as a function of the separation  $\mathbf{b}_\perp$  from the transverse center of momentum  $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$

- probabilistic interpretation!
- no relativistic corrections: Galilean subgroup! (MB,2000)
  - ↪ corollary: interpretation of 2d-FT of  $F_1(Q^2)$  as charge density in transverse plane also free from relativistic corrections (MB,2003;G.A.Miller, 2007)

$q(x, \mathbf{b}_\perp)$  for unpol. p

## unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$
  - $F_1(-\Delta_\perp^2) = \int dx H(x, 0, -\Delta_\perp^2)$
  - $x$  = momentum fraction of the quark
  - $\mathbf{b}_\perp$  relative to  $\perp$  center of momentum
  - small  $x$ : large 'meson cloud'
  - larger  $x$ : compact 'valence core'
  - $x \rightarrow 1$ : active quark becomes center of momentum
- ↪  $\vec{b}_\perp \rightarrow 0$  (narrow distribution) for  $x \rightarrow 1$



proton polarized in  $+\hat{x}$  direction  
no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

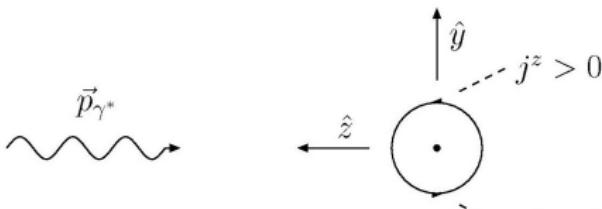
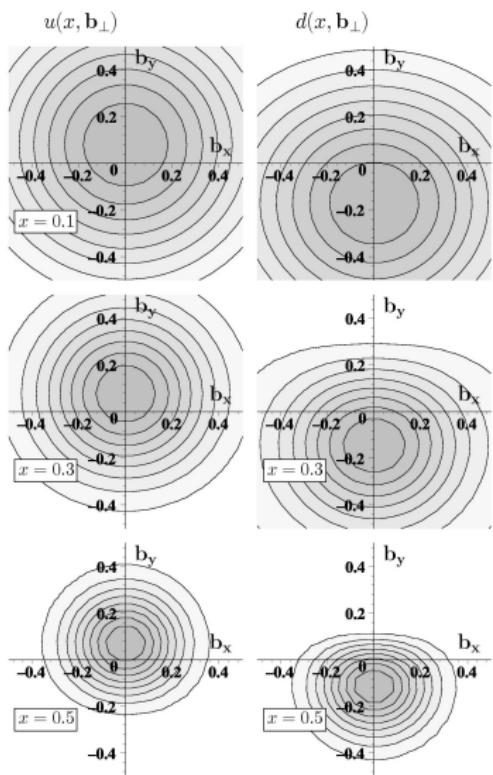
Physics: relevant density in DIS is  
 $j^+ \equiv j^0 + j^z$  and left-right asymmetry  
from  $j^z$

### intuitive explanation

- moving Dirac particle with anomalous magnetic moment has electric dipole moment  $\perp$  to  $\vec{p}$  and  $\perp$  magnetic moment
- $\gamma^*$  'sees' flavor dipole moment of oncoming nucleon

# Impact parameter dependent quark distributions

6

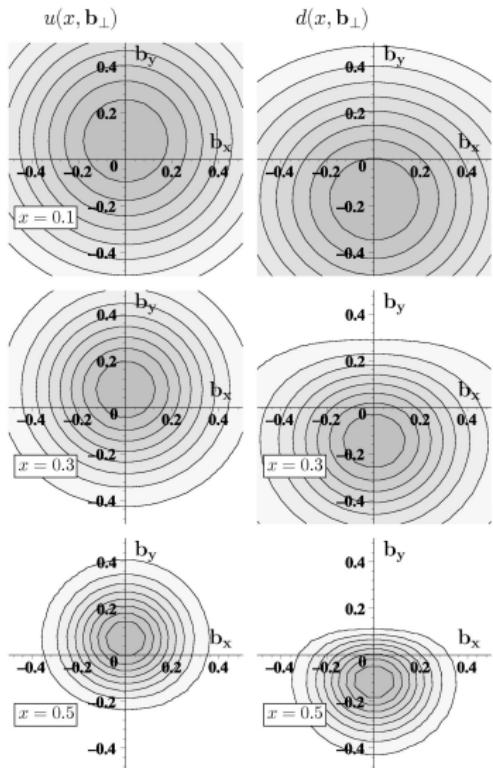


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$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

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 $j^+ \equiv j^0 + j^z$  and left-right asymmetry  
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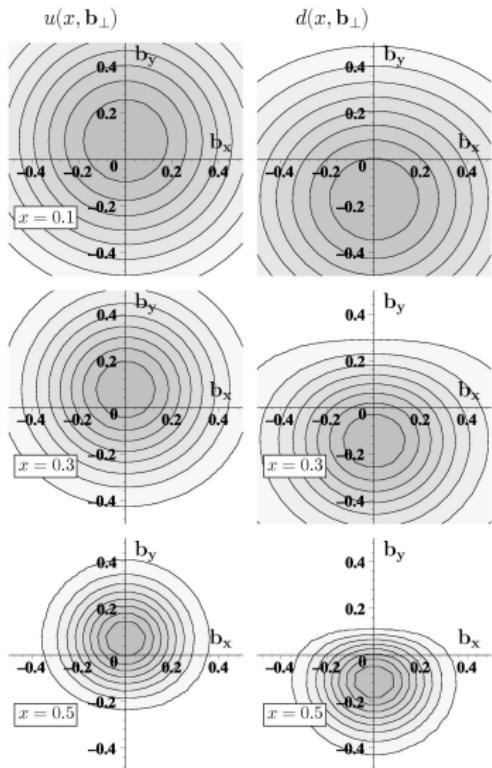
$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

sign & magnitude of the average shift

model-independently related to p/n  
anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$



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$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$

$$\kappa^p = 1.913 = \frac{2}{3} \kappa_u^p - \frac{1}{3} \kappa_d^p + \dots$$

- $u$ -quarks:  $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

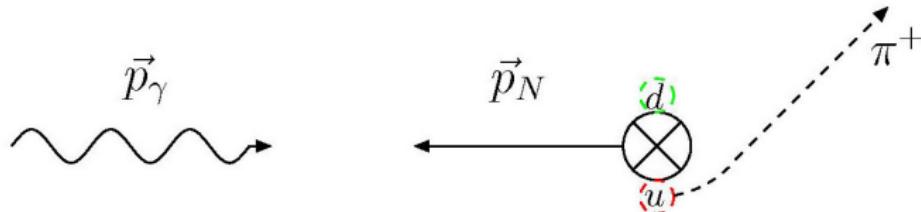
↪ shift in  $+\hat{y}$  direction

- $d$ -quarks:  $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

↪ shift in  $-\hat{y}$  direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 \text{ fm})$  !!!!

example:  $\gamma p \rightarrow \pi X$



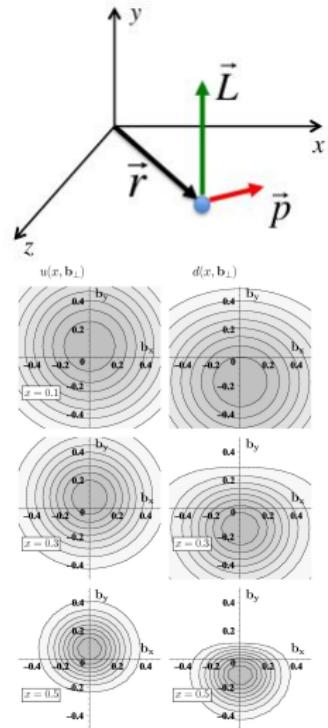
- $u, d$  distributions in  $\perp$  polarized proton have left-right asymmetry in  $\perp$  position space (T-even!); sign “determined” by  $\kappa_u$  &  $\kappa_d$
- attractive final state interaction (FSI) deflects active quark towards the center of momentum
- ↗ FSI translates position space distortion (before the quark is knocked out) in  $+\hat{y}$ -direction into momentum asymmetry that favors  $-\hat{y}$  direction → **chromodynamic lensing**

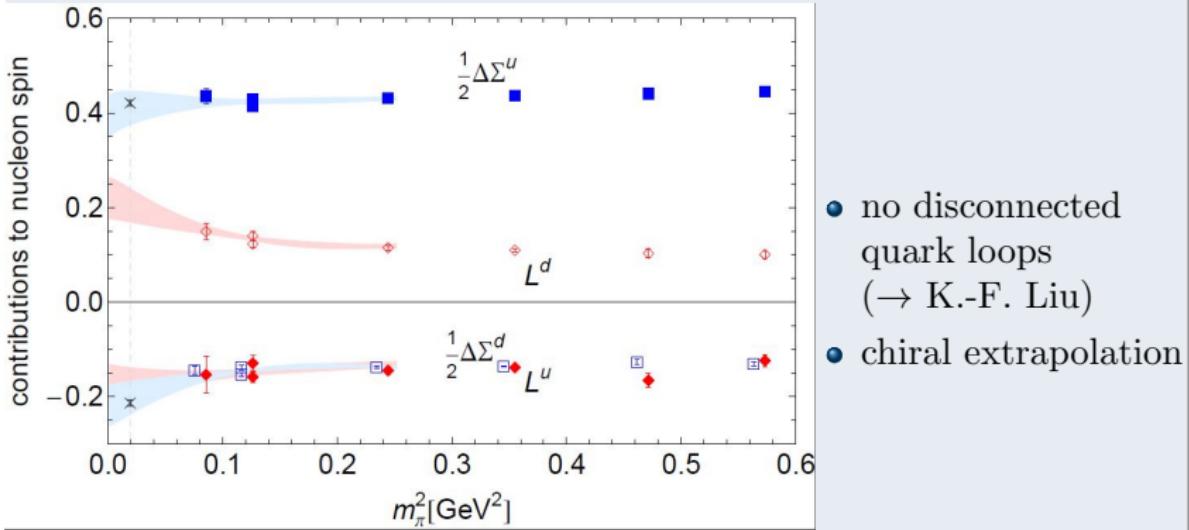
$\Rightarrow$

$\kappa_p, \kappa_n \longleftrightarrow$  sign of SSA!!!!!!! (MB,2004)

- confirmed by HERMES & COMPASS data

- $L_x = yp_z - zp_y$
- if state invariant under rotations about  $\hat{x}$  axis then  $\langle yp_z \rangle = -\langle zp_y \rangle$
- ↪  $\langle L_x \rangle = 2\langle yp_z \rangle$
- GPDs provide simultaneous information about  $p_z$  &  $\mathbf{b}_{\perp}$
- ↪ use quark GPDs to determine angular momentum carried by quarks
- ↪  $J_q^x = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$   
(X.Ji, 1996)
- interpretation in terms of 3D distribution (MB,2001,2005)

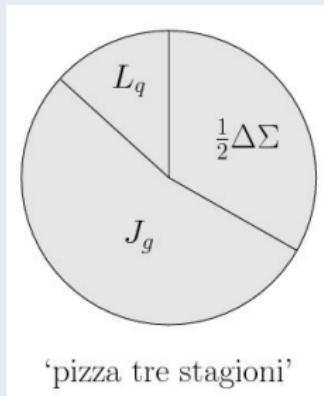


lattice results for  $J^q$  (LHPC)

$$J^q = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$$

$$L^q = J^q - \frac{1}{2} \Delta\Sigma^q$$

## Ji decomposition



$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + J_g$$

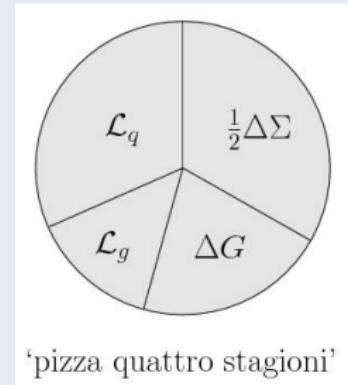
$$\frac{1}{2} \Delta q = \frac{1}{2} \int d^3x \langle P, S | q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) | P, S \rangle$$

$$\mathcal{L}_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D})^3 q(\vec{x}) | P, S \rangle$$

$$J_g = \int d^3x \langle P, S | [\vec{x} \times (\vec{E} \times \vec{B})]^3 | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

## Jaffe-Manohar decomposition



light-cone framework & gauge  $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P, S | \bar{q}(\vec{r}) \gamma^+( \vec{r} \times i\vec{\partial})^z q(\vec{r}) | P, S \rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle$$

$$\mathcal{L}_g = 2 \int d^3r \langle P, S | \text{Tr} F^{+j} (\vec{x} \times i\vec{\partial})^z A^j | P, S \rangle$$

manifestly gauge invariant definition  
for each term exists ( $\rightarrow$  Hatta)

## QED with electrons

$$\begin{aligned}
 \vec{J}_\gamma &= \int d^3r \vec{r} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{r} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})] \\
 &= \int d^3r [E^j (\vec{r} \times \vec{\nabla}) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla}) \vec{A}] \\
 &= \int d^3r [E^j (\vec{r} \times \vec{\nabla}) A^j + (\vec{r} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}]
 \end{aligned}$$

- replace 2<sup>nd</sup> term (eq. of motion  $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger \psi$ ), yielding

$$\vec{J}_\gamma = \int d^3r [\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j (\vec{r} \times \vec{\nabla}) A^j + \vec{E} \times \vec{A}]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$  cancels similar term in electron OAM  $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing  $\vec{J}_\gamma$  into spin and orbital also shuffles angular momentum from photons to electrons!

- can also be done for only part of  $\vec{A}$  → Chen/Goldman, Wakamatsu

## Ji decomposition

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + L_q + J_g$$

$$\frac{1}{2} \Delta q = \frac{1}{2} \int d^3x \langle P, S | q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) | P, S \rangle$$

$$L_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D})^3 q(\vec{x}) | P, S \rangle$$

$$J_g = \int d^3x \langle P, S | [\vec{x} \times (\vec{E} \times \vec{B})]^3 | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

## Jaffe-Manohar decomposition

light-cone framework & gauge  $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i\vec{\partial})^z q(\vec{r}) | P, S \rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle$$

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manifestly gauge invariant definitions  
for each term exist ( $\rightarrow$  Hatta)

- GPDs  $\longrightarrow L^q$
- $\overleftrightarrow{p \cdot p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i \in q,g} \mathcal{L}^i$
- QED:  $\mathcal{L}^e \neq L^e$  [M.B. + Hikmat BC, PRD **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$ 
  - can we calculate/predict/measure the difference?
  - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ q(\xi) | PS \rangle.$$

- (quasi) probability distribution for  $\mathbf{b}_\perp$  and  $\mathbf{k}_\perp$
- $f(x, \mathbf{k}_\perp) = \int d^2 \mathbf{b}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
- $q(x, \mathbf{b}_\perp) = \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$

OAM from Wigner (Lorcé, Pasquini, ...)

$$\begin{aligned} L_z &= \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x) \\ &= \int d^3 r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i\vec{\partial})^z q(\vec{r}) | P, S \rangle = \mathcal{L}^q \end{aligned}$$

Gauge Invariance?

need to include Wilson-line gauge link to connect 0 and  $\xi$   
 (Ji, Yuan; Hatta; Lorcé;...)

Wigner Functions with gauge link  $\mathcal{U}_{0\xi}$  (Ji, Yuan; Hatta)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle.$$

$\langle \vec{k}_\perp \rangle \equiv \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) \vec{k}_\perp$  depends on choice of path!

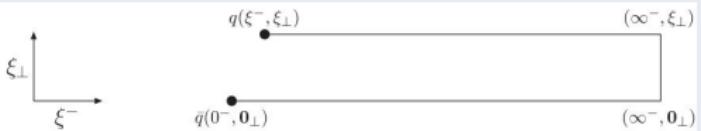
## straight-line gauge link

$$\langle \vec{k}_\perp \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(\vec{x})$

- $\langle \vec{k}_\perp \rangle = 0$  (T-odd !)

## light-cone staple



- correct choice for  $\mathbf{k}_\perp$  distributions relevant for SIDIS

$$\langle \vec{\mathcal{K}}_\perp \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{\mathcal{D}} q(\vec{x}) | P, S \rangle$$

- $i \vec{\mathcal{D}} = i \vec{\partial} - g \vec{A}(x^- = \infty, \mathbf{x}_\perp)$   $A^+ = 0$

- $\langle \vec{\mathcal{K}}_\perp \rangle \neq 0$  (FSI! Brodsky, Hwang, Schmidt)

difference  $\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle$

$$\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_{\perp}) q(\vec{x}) | P, S \rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

### straight-line gauge link

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- $i \vec{\mathcal{D}} = i \vec{\partial} - g \vec{A}(x^- = \infty, \mathbf{x}_{\perp}) + g \int_{x^-}^{\infty} dr^- \vec{\partial} A^+$
- $i \mathcal{D}^j = i \partial^j - g A^j(x^-, \mathbf{x}_{\perp}) - g \int_{x^-}^{\infty} dr^- F^{+j}$

difference  $\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle$

$$\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_{\perp}) q(\vec{x}) | P, S \rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left( \vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1)$$

### Impulse due to FSI

$\Delta \vec{k}_{\perp}^q \equiv \langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle$   
 = (average) change in  $\perp$  momentum due to FSI!

### straight-line gauge link

$$\langle \vec{k}_{\perp} \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(\vec{x})$

- $\langle \vec{k}_{\perp} \rangle = 0$  (T-odd !)

### light-cone staple



- correct choice for  $\mathbf{k}_{\perp}$  distributions relevant for SIDIS

$$\langle \vec{\mathcal{K}}_{\perp} \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(x^- = \infty, \mathbf{x}_{\perp}) + g \int_{x^-}^{\infty} dr^- \vec{\partial} A^+$

- $i \mathcal{D}^j = i \partial^j - g A^j(x^-, \mathbf{x}_{\perp}) - g \int_{x^-}^{\infty} dr^- F^{+j}$

difference  $\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$

$$\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x_-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x}) | P, S \rangle$$

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Corollary:  $d_2 \leftrightarrow$  average  $\perp$  force on quark in DIS from  $\perp$  pol target  
polarized DIS: MB, PRD 88 (2013) 114502

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$       •  $\sigma_{LT} \propto g_T \equiv g_1 + g_2$
- 'clean' separation between  $g_2$  and  $\frac{1}{Q^2}$  corrections to  $g_1$
- $g_2 = g_2^{WW} + \bar{g}_2$  with  $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S | \bar{q}(0) \gamma^+ gF^{+y}(0) q(0) | P, S \rangle$$

matrix element defining  $d_2$

$\leftrightarrow$  1<sup>st</sup> integration point in QS-integral

difference  $\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$

$$\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x_-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x}) | P, S \rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left( \vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1)$$

Corollary:  $d_2 \leftrightarrow$  average  $\perp$  force on quark in DIS from  $\perp$  pol target  
polarized DIS: MB, PRD 88 (2013) 114502

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$       •  $\sigma_{LT} \propto g_T \equiv g_1 + g_2$
- 'clean' separation between  $g_2$  and  $\frac{1}{Q^2}$  corrections to  $g_1$
- $g_2 = g_2^{WW} + \bar{g}_2$  with  $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S | \bar{q}(0) \gamma^+ gF^{+y}(0) q(0) | P, S \rangle$$

sign of  $d_2^q$  opposite Sivers  $f_{1T}^{\perp q}$      $\leftrightarrow$      $\perp$  deformation of quark distributions

Wigner Functions with gauge link  $\mathcal{U}_{0\xi}$  (Ji, Yuan; Hatta)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle.$$

$W$  and thus  $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$  may depend on choice of path!

straight line (Ji et al.)

straigth Wilson line from 0 to  $\xi$  yields

$$L^q = \int d^3 x \langle P, S | q^\dagger(\vec{x}) \left( \vec{x} \times i \vec{D} \right) \overset{z}{\vec{q}}(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(\vec{x})$
- same as Ji-OAM
- $f(x, \mathbf{k}_\perp) = \int d^2 \mathbf{b}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$   
not the TMDs relevant for SIDIS  
(missing FSI!)

Wigner Functions with gauge link  $\mathcal{U}_{0\xi}$  (Ji, Yuan; Hatta)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | PS \rangle.$$

$W$  and thus  $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$  may depend on choice of path!

Light-Cone Staple for  $\mathcal{U}_{0\xi}^{\pm LC}$  (Hatta)

- want Wigner function that yields TMDs relevant for SIDIS when integrated  $d^2 \mathbf{b}_\perp$
- ↪ path for gauge link → 'light-cone staple' →  $\mathcal{U}_{0\xi}^{+LC}$

$$\mathcal{L}_+^q = \int d^3 x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{\mathcal{D}})^z q(\vec{x}) | P, S \rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp) \quad (A^+ = 0)$$



Wigner Functions with gauge link  $\mathcal{U}_{0\xi}$  (Ji, Yuan; Hatta)

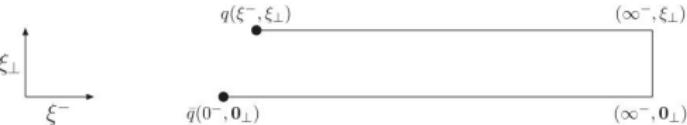
$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle.$$

$W$  and thus  $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$  may depend on choice of path!

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$$\begin{aligned} \mathcal{L}_+^q &= \int d^3 x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{\mathcal{D}})^z q(\vec{x}) | P, S \rangle \\ i\vec{\mathcal{D}} &= i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp) + g \int_{x_-^-}^\infty dr^- \vec{\partial} A^+ \\ i\mathcal{D}^j &= i\partial^j - g A^j(x^-, \mathbf{x}_\perp) - g \int_{x_-^-}^\infty dr^- F^{+j} \end{aligned}$$



straight line ( $\rightarrow J_i$ )

$$L_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ( $\rightarrow$  Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

$$i\vec{D} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference  $\mathcal{L}^q - L^q$ 

$$\mathcal{L}^q - L^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

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color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

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$$L_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

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$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

Torque along the trajectory of  $q$ 

$$T^z = \left[ \vec{x} \times \left( \vec{E} - \hat{\vec{z}} \times \vec{B} \right) \right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^\infty dr^- \left[ \vec{x} \times \left( \vec{E} - \hat{\vec{z}} \times \vec{B} \right) \right]^z$$

straight line ( $\rightarrow J_i$ )

$$L_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

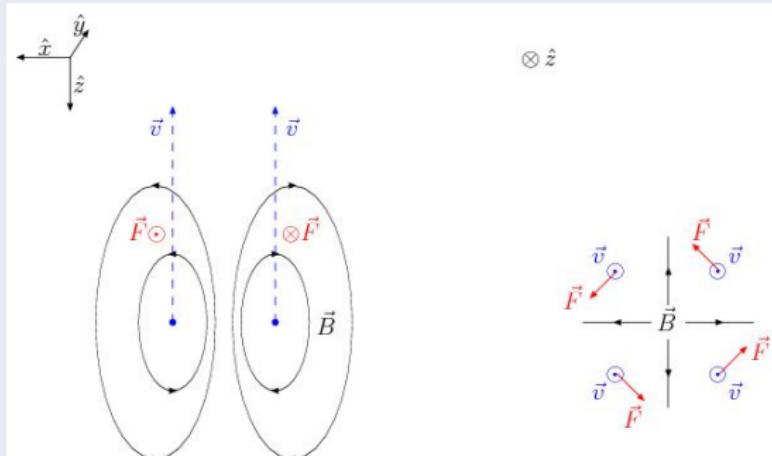
light-cone staple ( $\rightarrow$  Jaffe-Manohar)

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- $i\vec{D} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference  $\mathcal{L}^q - L^q$  ( $\rightarrow$  Wakamatsu:  $L_{pot}^q$ ) $\mathcal{L}^q - L^q = \Delta L_{FSI}^q =$  change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



antisymm. boundary condition

- $A^+ = 0$
- fix residual gauge inv.  $A^\mu \rightarrow A^\mu + \partial^\mu \phi(\vec{x}_\perp)$   
by imposing  $\vec{A}_\perp(\infty, \vec{x}_\perp) = -\vec{A}_\perp(-\infty, \vec{x}_\perp)$
- $\vec{A}_\perp(\infty, \vec{x}_\perp) - \vec{A}_\perp(-\infty, \vec{x}_\perp) = \int dx^- F^{+\perp}$  gauge inv.
- $\mathcal{L}_+$  involves  $i\vec{\mathcal{D}}_+ = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_-$  involves  $i\vec{\mathcal{D}}_- = i\vec{\partial} - g\vec{A}(-\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_+ = \mathcal{L}_- \rightarrow$  no contribution from  $\vec{A}(\infty, \mathbf{x}_\perp)$   
 $\hookrightarrow$  'naive' JM OAM  $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

alternative: Bashinsky-Jaffe

- $A^+ = 0$
- $\vec{x} \times i\vec{\partial} \rightarrow \vec{x} \times [i\vec{\partial} - g\vec{A}(\vec{x}_\perp)]$
- $\vec{A}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

antisymm. boundary condition

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- ↪ 'naive' JM OAM  $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

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- $A^+ = 0$
  - $\vec{x} \times i\vec{\partial} \rightarrow \vec{x} \times [i\vec{\partial} - g\vec{A}(\vec{x}_\perp)]$
  - $\vec{A}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2} (\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp))$
- ↪  $\mathcal{L}_{JB} = \frac{1}{2} (\mathcal{L}_+ + \mathcal{L}_-) = \mathcal{L}_+ = \mathcal{L}_-$

# Nucleon Spin Decompositions

20

The Difference  $\mathcal{L}_q - L_q$  [MB, PRD88, 056009 (2013)]

$$\mathcal{L}^q - L^q = -\int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^\infty dr^- g F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(0) | P, S \rangle / \mathcal{N}$$

- in QCD, additional Wilson lines (along  $r^-$ )

compare  $\langle \mathbf{k}_\perp \rangle \equiv \int dx \int d^2 \mathbf{k}_\perp f(x, \mathbf{k}_\perp) \mathbf{k}_\perp$

$$f(x, \vec{k}_\perp) \equiv \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle$$

- Wilson line  $\mathcal{U}_{0\xi}$  along  $x^-$  to properly account for FSI acting on ejected quark, i.e.  $f(x, \mathbf{k}_\perp)$  momentum distribution incl. FSI
- relevant for SIDIS (JLab, EIC) and DY (RHIC)

$$\langle \mathbf{k}_\perp \rangle = \langle P, S | \bar{q}(0) \gamma^+ \int_0^\infty dr^- g F^{+\perp}(r^-, \mathbf{0}_\perp) q(0) | P, S \rangle$$

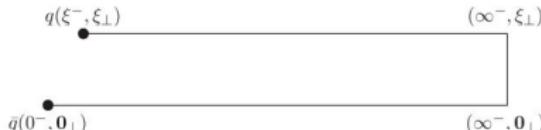
$\langle \mathbf{k}_\perp \rangle$  = (average) change in  $\perp$  momentum due to FSI as quark leaves target (Qiu, Sterman)

Color Lorentz Force

$$\begin{aligned}\sqrt{2} F^{+y} &= F^{0y} + F^{zy} \\ &= -E^y + B^x \\ &= -(\vec{E} + \vec{v} \times \vec{B})^y\end{aligned}$$

for  $\vec{v} = (0, 0, -1)$

Light-Cone Staple for  $\mathcal{U}_{0\xi}^{\pm LC}$



# Nucleon Spin Decompositions

20

The Difference  $\mathcal{L}_q - L_q$  [MB, PRD88, 056009 (2013)]

$$\mathcal{L}^q - L^q = - \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^\infty dr^- g F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(0) | P, S \rangle / \mathcal{N}$$

$\mathcal{L}_q - L_q = (\text{average}) \text{ change in OAM due to FSI as quark leaves target}$

compare  $\langle \mathbf{k}_\perp \rangle \equiv \int dx \int d^2 \mathbf{k}_\perp f(x, \mathbf{k}_\perp) \mathbf{k}_\perp$

$$f(x, \vec{k}_\perp) \equiv \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle$$

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Color Lorentz Force

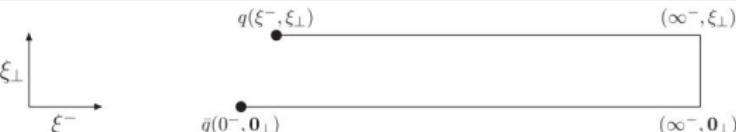
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$$\langle \mathbf{k}_\perp \rangle = \langle P, S | \bar{q}(0) \gamma^+ \int_0^\infty dr^- g F^{+\perp}(r^-, \mathbf{0}_\perp) q(0) | P, S \rangle$$

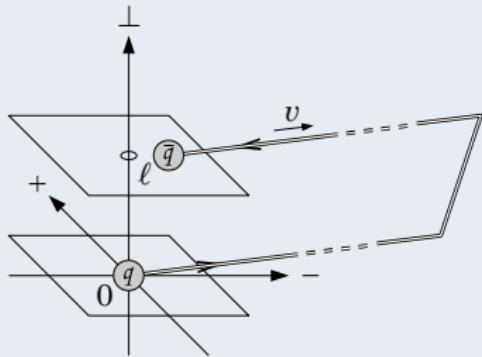
$\langle \mathbf{k}_\perp \rangle = (\text{average}) \text{ change in } \perp \text{ momentum due to FSI as quark leaves target}$  (Qiu, Sterman)

Light-Cone Staple for  $\mathcal{U}_{0\xi}^{\pm LC}$

for  $\vec{v} = (0, 0, -1)$



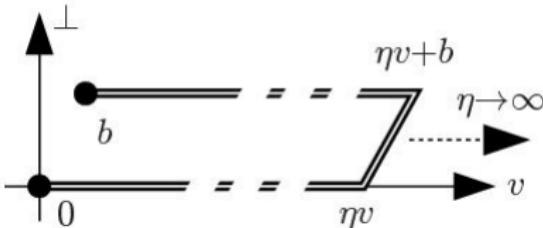
## challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

## TMDs in lattice QCD

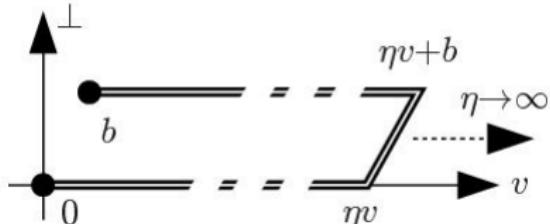
B. Musch, P. Hägler, M. Engelhardt



- calculate space-like staple-shaped Wilson line pointing in  $\hat{z}$  direction; length  $L \rightarrow \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios
- extrapolate/evolve to  $P_z \rightarrow \infty$
- talk by M. Engelhardt tomorrow...

## TMDs in lattice QCD

B. Musch, P. Hägler, M. Engelhardt



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- talk by M. Engelhardt tomorrow...

## next: Orbital Angular Momentum

- same operator as for TMDs, only nonforward matrix elements:
  - momentum transfer provides position space information ( $\rightarrow \mathbf{r}_\perp \times \mathbf{k}_\perp$ )
  - staple with long side in  $\hat{z}$  direction
  - (large) nucleon momentum in  $\hat{z}$  direction
  - small momentum transfer in  $\hat{y}$  direction
- generalized TMD  $F_{14}$  (Metz et al.)
- quark OAM
- renormalization same as  $f_{1T}^\perp$
- study ratios...

- generalized parton distributions and 3d imaging
- final state interactions in deep-inelastic scattering
- GPDs and OAM
- TMDs and OAM from Wigner distributions
- straight-(Wilson)line gauge link  $\rightarrow L^q$  ('Ji-OAM')
- light-cone staple- gauge link  $\rightarrow \mathcal{L}_+^q$  ('JM-OAM')
- $\mathcal{L}_+^q - L^q =$  change in OAM as quark leaves nucleon (torque due to FSI)
- $A^+ = 0$  gauge (with anti-symmetric boundary condition)  $\mathcal{L}_+^q \rightarrow$  canonical OAM (Jaffe-Manohar-OAM)
- JM-OAM from lattice QCD

