

Quark spin-orbit correlations

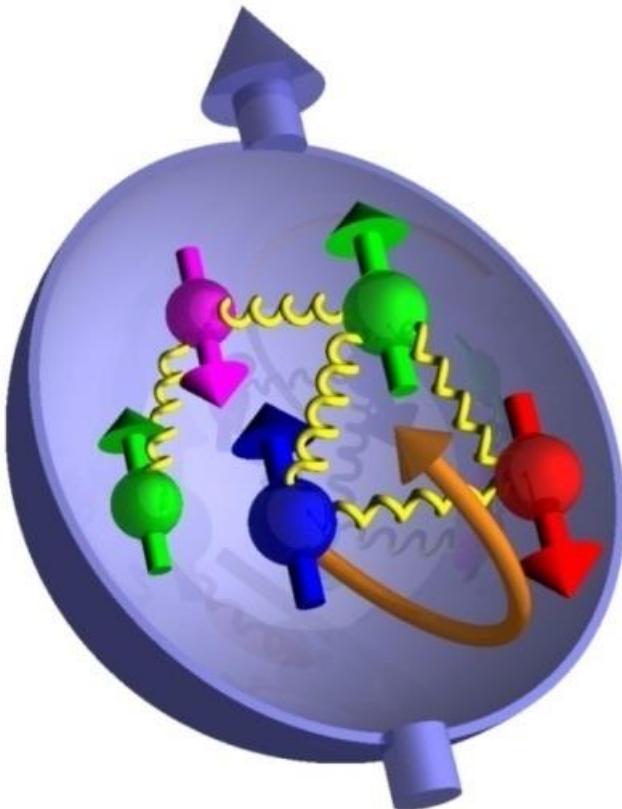
Based on : [C.L., arXiv:[1401.7784](#)]
[C.L., Pasquini (in preparation)]

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Outline



- Energy-momentum tensor
- Quark spin-orbit correlations
- Phase-space transverse modes
- Conclusions

Energy-momentum tensor

A lot of interesting physics is contained in the EM tensor

[Polyakov, Shubaev (2002)]

[Polyakov (2003)]

[Goeke *et al.* (2007)]

[Cebulla *et al.* (2007)]

$$T^{\mu\nu} = \begin{bmatrix} \text{Energy density} & \text{Momentum density} \\ T^{00} & T^{01} \quad T^{02} \quad T^{03} \\ T^{10} & T^{11} \quad T^{12} \quad T^{13} \\ T^{20} & T^{21} \quad T^{22} \quad T^{23} \\ T^{30} & T^{31} \quad T^{32} \quad T^{33} \end{bmatrix}$$

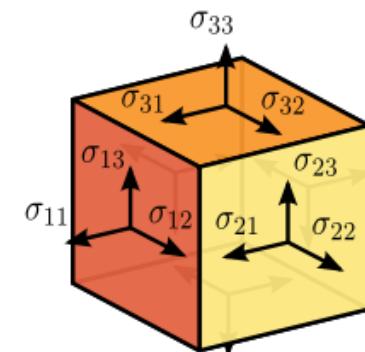
Shear stress

Normal stress (pressure)

Energy flux Momentum flux

E.g. $M = \int d^3r T^{00}(\vec{r})$

$$L^i = \int d^3r \epsilon^{ijk} r^j T^{0k}(\vec{r})$$



Energy-momentum tensor

Momentum-space expression

$$\begin{aligned} j^\mu(r) &= \text{F.T. } \langle p + \frac{\Delta}{2} | \hat{j}^\mu | p - \frac{\Delta}{2} \rangle & \hat{j}^\mu &= \bar{\psi} \gamma^\mu \psi \\ T^{\mu\nu}(r) &= \text{F.T. } \langle p + \frac{\Delta}{2} | \hat{T}^{\mu\nu} | p - \frac{\Delta}{2} \rangle & \hat{T}^{\mu\nu} &= \bar{\psi} \gamma^\mu iD^\nu \psi \end{aligned}$$

General parametrization

$$\begin{aligned} \langle p' | \hat{T}^{\mu\nu} | p \rangle &= \bar{u}(p') \left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2} A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}}\Delta}{4M} B(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C(t) \right. \\ &\quad \left. + Mg^{\mu\nu} \bar{C}(t) + \frac{P^{[\mu}\gamma^{\nu]}}{2} D(t) \right] u(p) \end{aligned}$$

Non-conservation **Asymmetry**

Angular momentum

$$J_z = \frac{1}{2} [A(0) + B(0)] \qquad \qquad \qquad [\text{Ji (1997)}]$$

$$L_z = \frac{1}{2} [A(0) + B(0) + D(0)] \qquad \qquad \qquad [\text{Shore, White (2000)}]$$

$$\hookrightarrow -2S_z$$

Link with GPDs

« Trick »

Local		Non-local
$\bar{\psi} \gamma^\mu i D^+ \psi$	$= \int \frac{dz^-}{2\pi} 2\pi \delta(z^-) \bar{\psi}(-\frac{z^-}{2}) \gamma^\mu \mathcal{W}[-\frac{z^-}{2}, 0] i \overset{\leftrightarrow}{D}^+ \mathcal{W}[0, \frac{z^-}{2}] \psi(\frac{z^-}{2})$	
	$= \int \frac{dz^-}{2\pi} \int dx P^+ e^{ixP^+ z^-} \bar{\psi}(-\frac{z^-}{2}) \gamma^\mu \mathcal{W}[-\frac{z^-}{2}, 0] i \overset{\leftrightarrow}{D}^+ \mathcal{W}[0, \frac{z^-}{2}] \psi(\frac{z^-}{2})$	
	$= P^+ \int dx \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} i \partial_z^+ [\bar{\psi}(-\frac{z^-}{2}) \gamma^\mu \mathcal{W}[-\frac{z^-}{2}, \frac{z^-}{2}] \psi(\frac{z^-}{2})]$	
		$= 2(P^+)^2 \int dx x \left[\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \bar{\psi}(-\frac{z^-}{2}) \gamma^\mu \mathcal{W}[-\frac{z^-}{2}, \frac{z^-}{2}] \psi(\frac{z^-}{2}) \right]$
		GPD operator

Twist-2
 $\mu = +$

$$A(t) + B(t) = \int dx x [H(x, \xi, t) + E(x, \xi, t)]$$

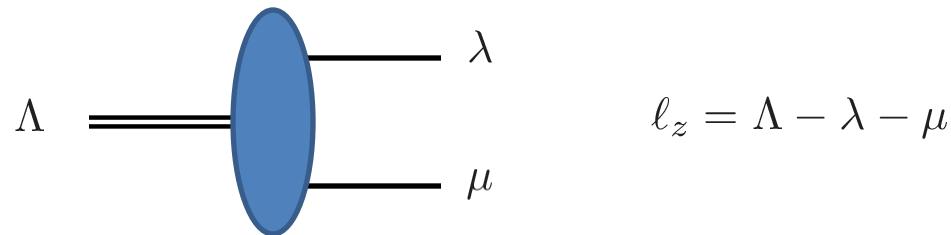
[Ji (1997)]

Twist-3
 $\mu = \perp$

$$A(t) + B(t) + D(t) = -2 \int dx x G_2(x, \xi, t)$$

[Penttinen *et al.* (2000)]
[Kiptily, Polyakov (2004)]
[Hatta, Yoshida (2012)]

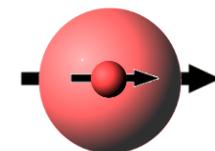
Proton spin structure



« Quark spin »

$$\langle\langle S_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda \lambda |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N S_z^q \rangle$$

$$\bar{\psi} \gamma^+ \gamma_5 \psi$$



« Quark OAM »

$$\langle\langle L_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N L_z^q \rangle$$

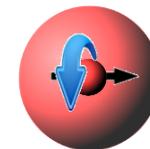
$$\bar{\psi} \gamma^+ (\vec{r}_\perp \times i \vec{D}_\perp)_z \psi$$



Quark spin-orbit correlation

$$\langle\langle C_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^q L_z^q \rangle$$

$$\bar{\psi} \gamma^+ \gamma_5 (\vec{r}_\perp \times i \vec{D}_\perp)_z \psi$$



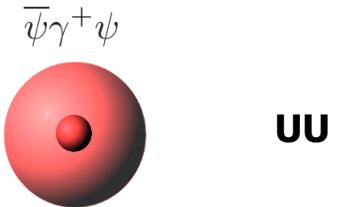
Not involved in proton spin sum rule !

Proton spin structure

Quark number

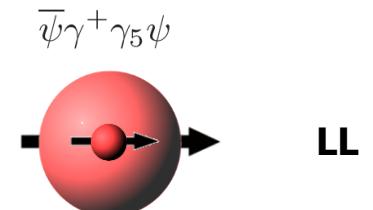
$$\langle\langle N^q \rangle\rangle \sim \rho_{\uparrow\uparrow} + \rho_{\uparrow\downarrow} + \rho_{\downarrow\uparrow} + \rho_{\downarrow\downarrow}$$

Spin OAM



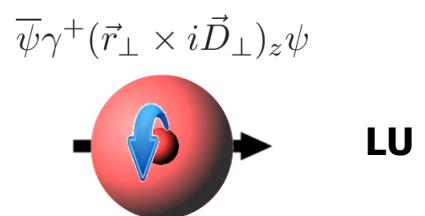
« Quark spin »

$$\langle\langle S_z^q \rangle\rangle \sim \rho_{\uparrow\uparrow} + \rho_{\uparrow\downarrow} - \rho_{\downarrow\uparrow} - \rho_{\downarrow\downarrow}$$



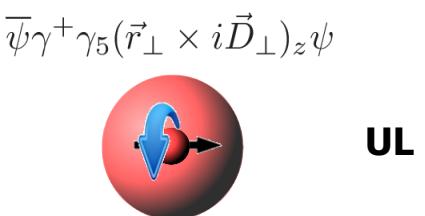
« Quark OAM »

$$\langle\langle L_z^q \rangle\rangle \sim \rho_{\uparrow\uparrow} - \rho_{\uparrow\downarrow} + \rho_{\downarrow\uparrow} - \rho_{\downarrow\downarrow}$$



Quark spin-orbit correlation

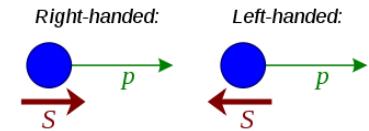
$$\langle\langle C_z^q \rangle\rangle \sim \rho_{\uparrow\uparrow} - \rho_{\uparrow\downarrow} - \rho_{\downarrow\uparrow} + \rho_{\downarrow\downarrow}$$



P-odd energy-momentum tensor

Chiral decomposition

$$\begin{aligned}\bar{\psi} \gamma^\mu i D^\nu \psi &= \hat{T}^{\mu\nu} = \hat{T}_R^{\mu\nu} + \hat{T}_L^{\mu\nu} \\ \bar{\psi} \gamma^\mu \gamma_5 i D^\nu \psi &= \hat{T}_5^{\mu\nu} = \hat{T}_R^{\mu\nu} - \hat{T}_L^{\mu\nu}\end{aligned}$$



$$\begin{aligned}\hat{T}_R^{\mu\nu} &= \bar{\psi}_R \gamma^\mu i D^\nu \psi_R \\ \hat{T}_L^{\mu\nu} &= \bar{\psi}_L \gamma^\mu i D^\nu \psi_L\end{aligned}$$

General parametrization

$$\begin{aligned}\langle p' | \hat{T}_5^{\mu\nu} | p \rangle &= \bar{u}(p') \left[\frac{P^{\{\mu} \gamma^{\nu\}} \gamma_5}{2} \tilde{A}(t) + \frac{P^{\{\mu} \Delta^{\nu\}} \gamma_5}{4M} \tilde{B}(t) \right. \\ &\quad \left. + \frac{P^{[\mu} \gamma^{\nu]} \gamma_5}{2} \tilde{C}(t) + \frac{P^{[\mu} \Delta^{\nu]} \gamma_5}{4M} \tilde{D}(t) + M i \sigma^{\mu\nu} \gamma_5 \tilde{F}(t) \right] u(p)\end{aligned}$$

$$C_z = \frac{1}{2} [\tilde{A}(0) + \tilde{C}(0)]$$

Twist-2
 $\mu = +$

$$= \frac{1}{2} \int dx [x \tilde{H}(x, \xi, 0) - H(x, \xi, 0)] + \mathcal{O}(\frac{m_q}{M})$$

Twist-3
 $\mu = \perp$

$$= - \int dx x [\tilde{G}_2(x, \xi, 0) + 2 \tilde{G}_4(x, \xi, 0)]$$

Some figures

Valence number

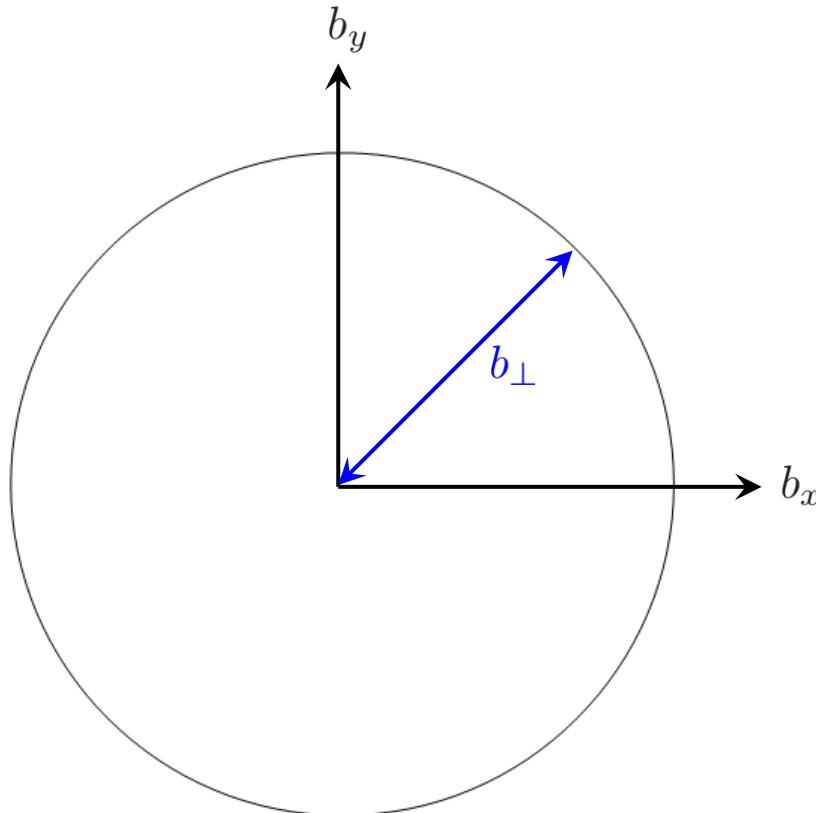
$$F_1^u(0) = 2 \quad F_1^d(0) = 1$$

	$\int dx x \Delta u$	$\int dx x \Delta d$	C_z^u	C_z^d
NQM	4/9	-1/9	-7/9	-5/9
LFCQM	0.34	-0.09	-0.83	-0.54
LF χ QSM	0.39	-0.10	-0.80	-0.55
LSS2010	0.19	-0.06	-0.90	-0.53

Spin and kinetic OAM of valence quarks are anti-correlated !

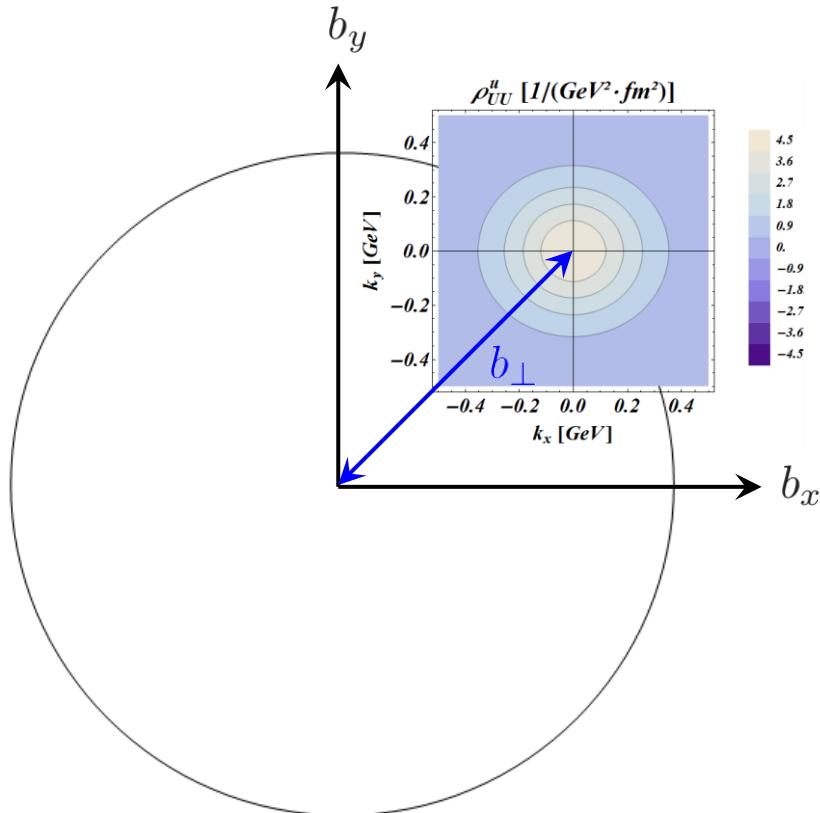
Phase-space transverse modes

Parametrization of a correlator is not unique  Natural modes ?



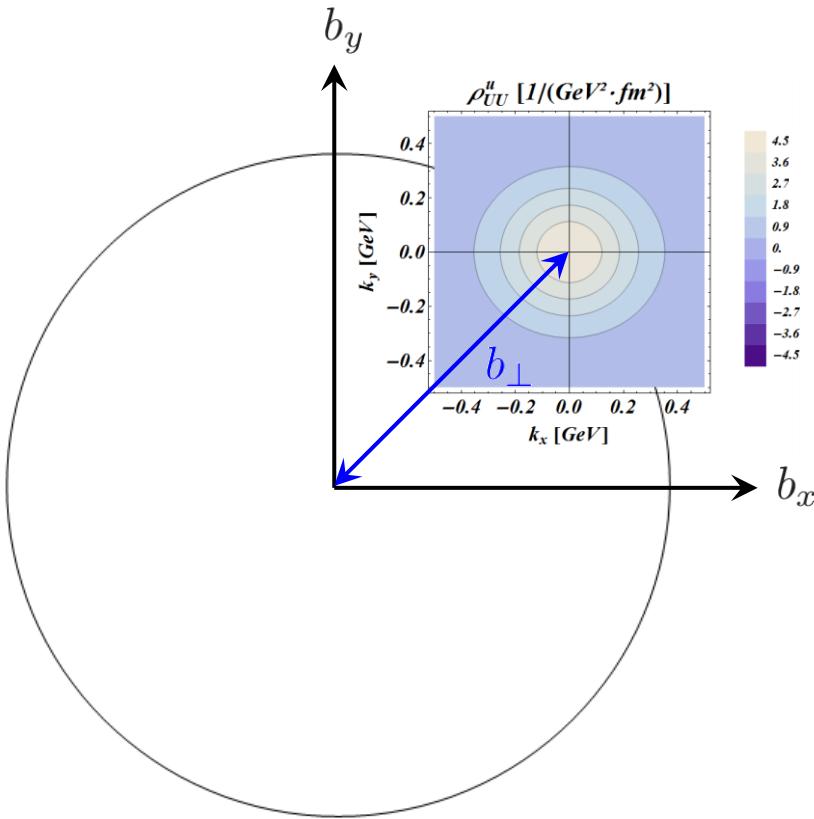
Phase-space transverse modes

Parametrization of a correlator is not unique  Natural modes ?



Phase-space transverse modes

Parametrization of a correlator is not unique → Natural modes ?



Properties under
parity and time-reversal

$$\begin{array}{ll} \vec{a}_P = -c_P \vec{a} & \times_P = c_P \times \\ \vec{a}_T = c_T \vec{a} & \times_T = c_T \times \end{array}$$

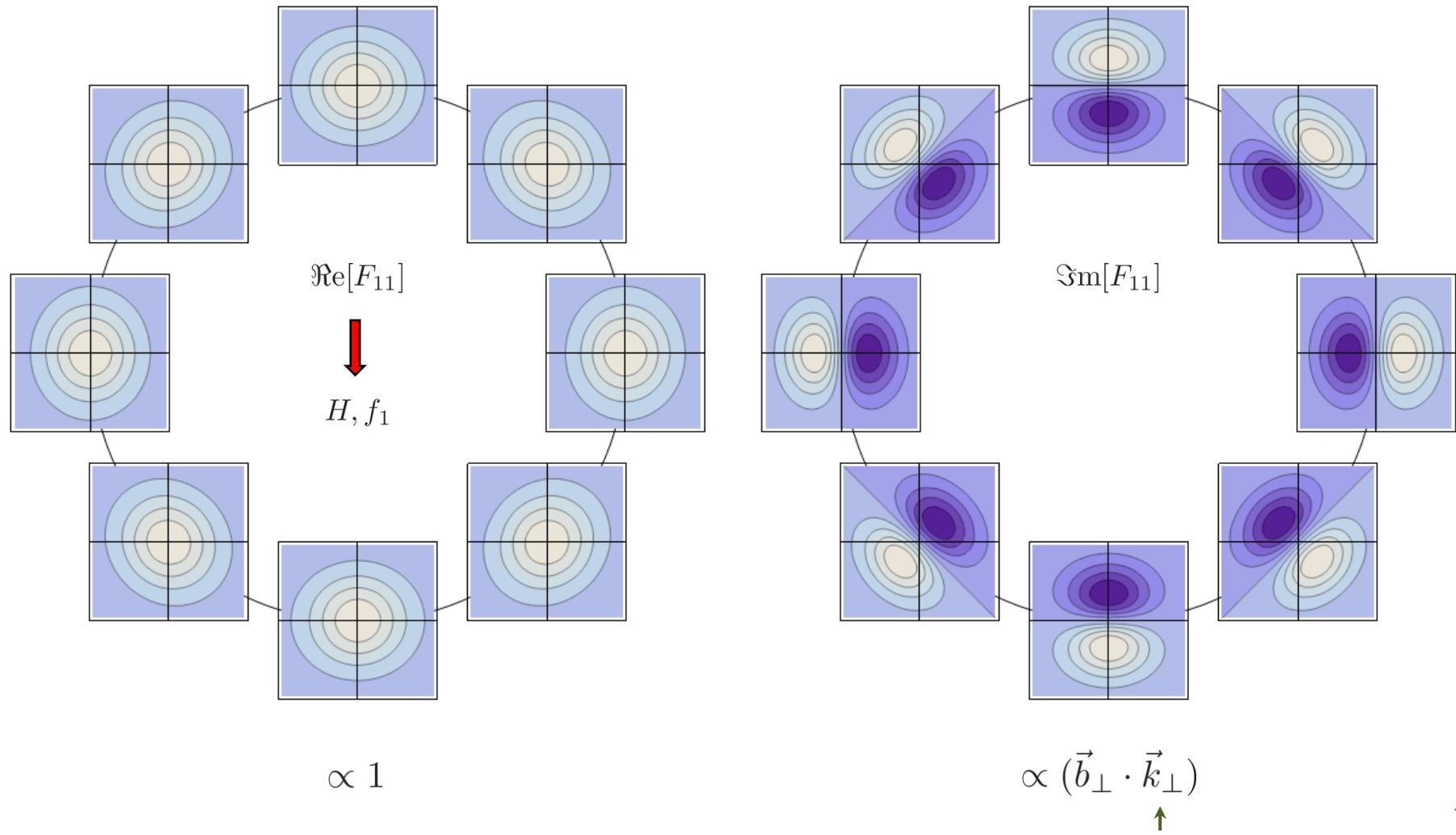
	\vec{b}_\perp	\vec{k}_\perp	$\vec{e}_z \equiv \frac{\vec{P}}{P}$	\vec{S}	\times
c_P	+	+	+	-	-
c_T	+	-	-	-	+

Phase-space transverse modes

[C.L., Pasquini (in preparation)]

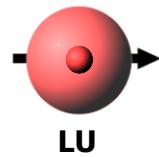


Unpolarized quark in unpolarized target

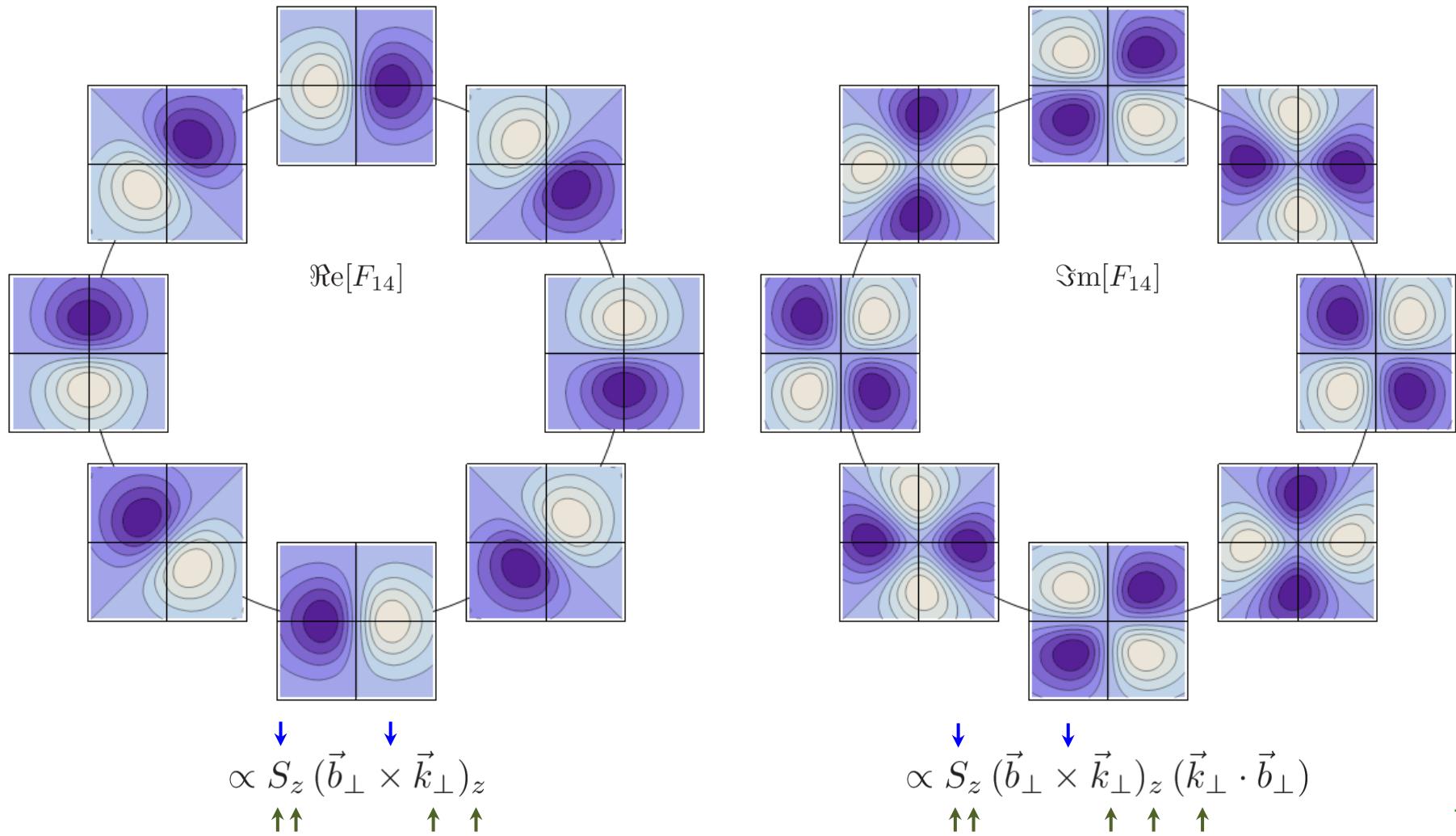


Phase-space transverse modes

[C.L., Pasquini (in preparation)]



Unpolarized quark in longitudinally polarized target



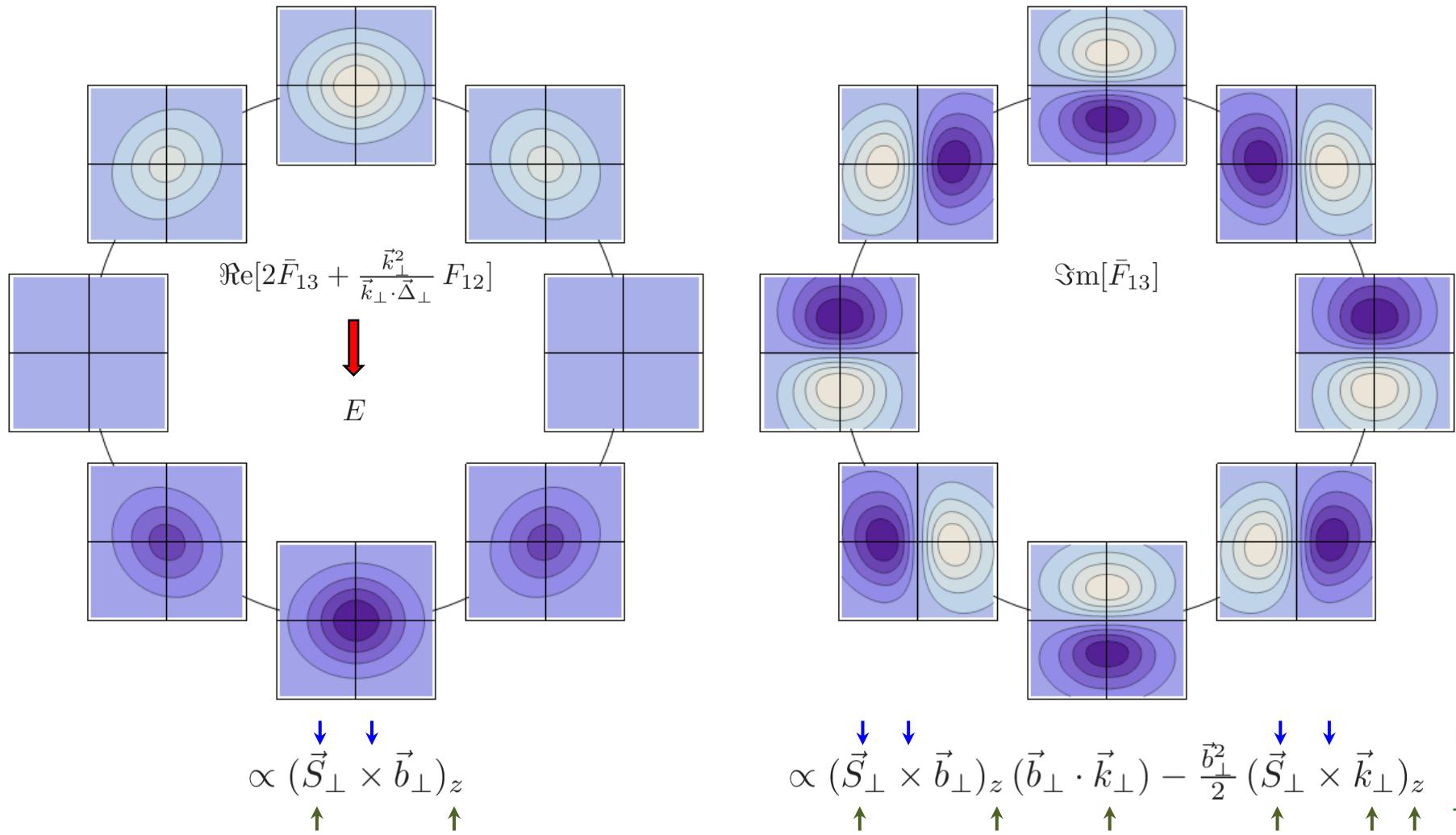
Phase-space transverse modes

[C.L., Pasquini (in preparation)]



Unpolarized quark in transversely polarized target (1)

$$\bar{F}_{13} \equiv F_{13} - \frac{1}{2} F_{11}$$



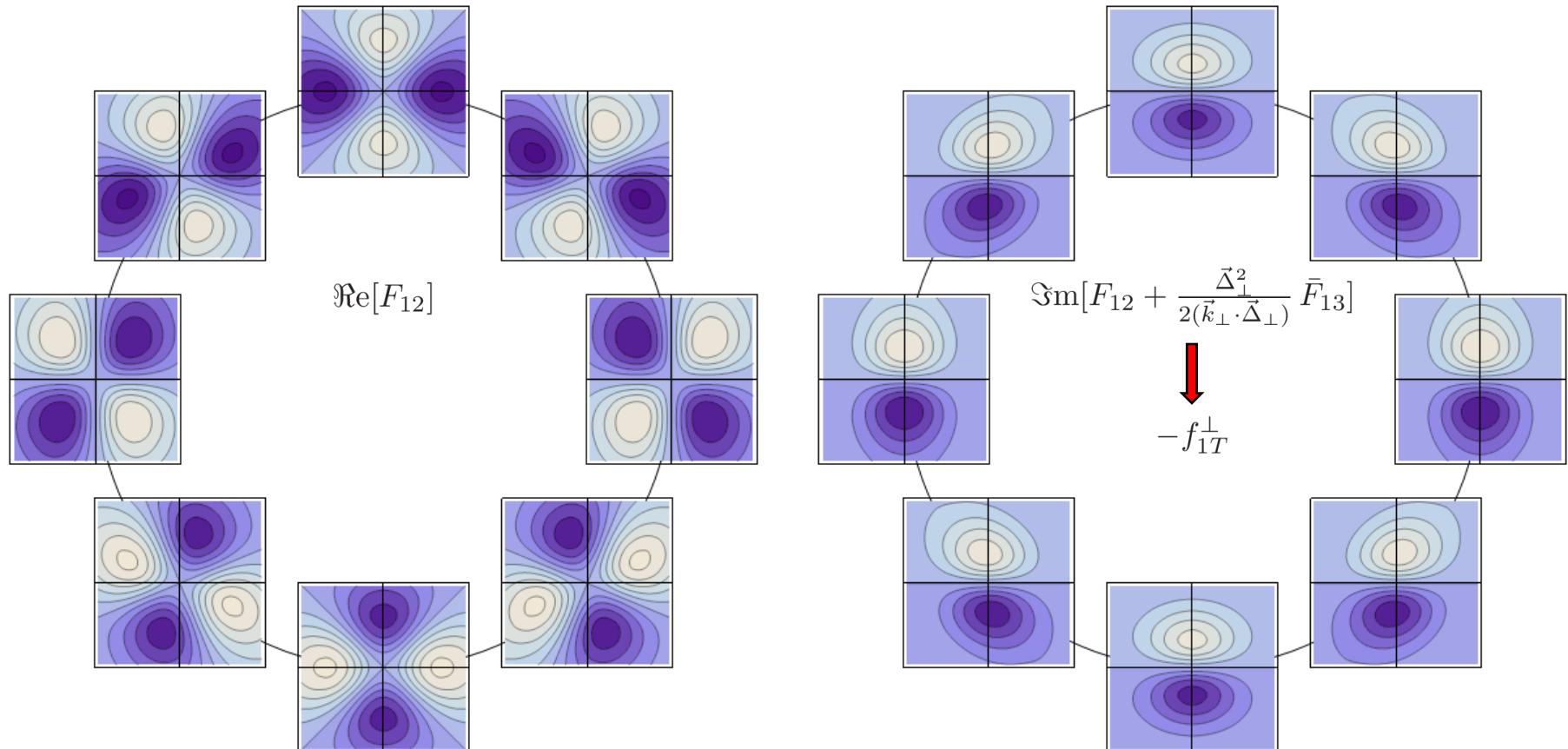
Phase-space transverse modes

[C.L., Pasquini (in preparation)]



Unpolarized quark in transversely polarized target (2)

$$\bar{F}_{13} \equiv F_{13} - \frac{1}{2} F_{11}$$



$$\propto (\vec{S}_\perp \times \vec{k}_\perp)_z (\vec{k}_\perp \cdot \vec{b}_\perp) - \frac{\vec{k}_\perp^2}{2} (\vec{S}_\perp \times \vec{b}_\perp)_z$$

↑ ↓
↑ ↑ ↑ ↑

$$\propto (\vec{S}_\perp \times \vec{k}_\perp)_z$$

↑ ↓
↑ ↑ ↑

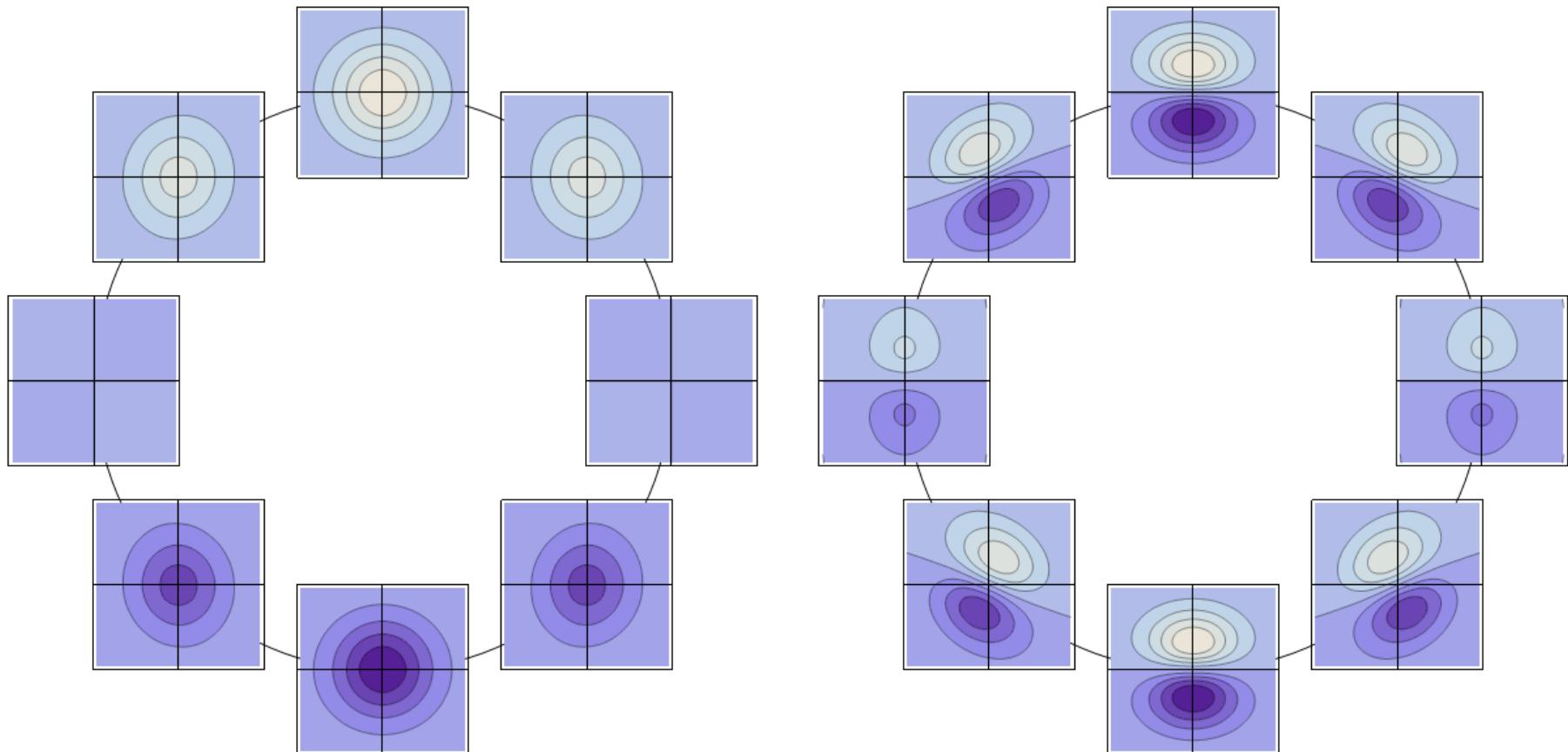
P
T

Phase-space transverse modes

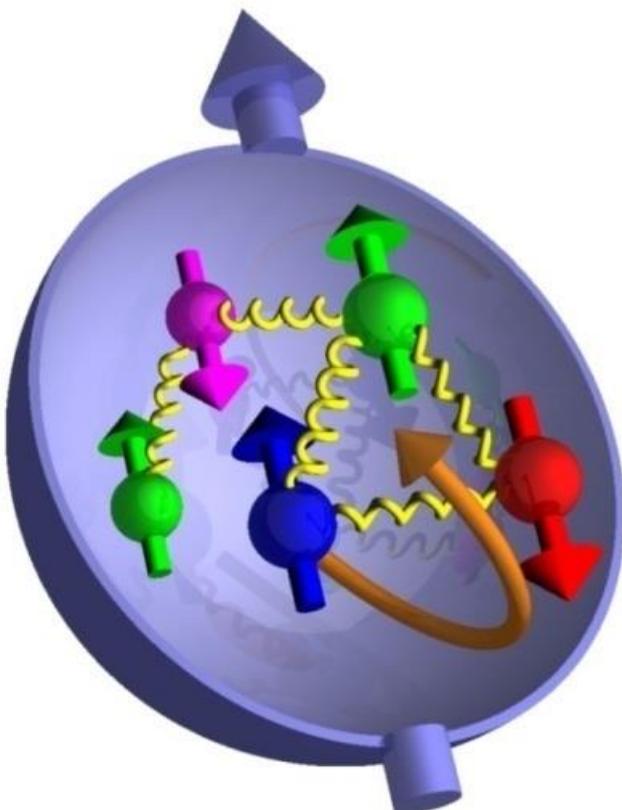
[C.L., Pasquini (in preparation)]



Unpolarized quark in transversely polarized target (1+2)



Conclusions



- Complete characterization of spin structure requires spin-orbit correlation
- Like quark OAM, quark spin-orbit correlation is given by moments of measurable parton distributions
- Natural phase-space transverse modes help to understand the physics contained in parton distributions

Backup slides

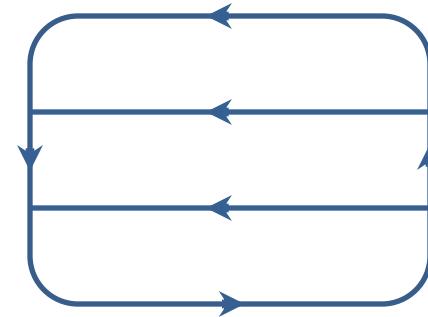
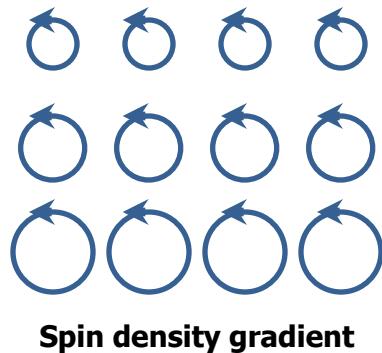
Energy-momentum tensor

In presence of spin density

$$T^{0i} \neq T^{i0}$$

Belinfante
« improvement »

$$\begin{aligned} T_B^{\mu\nu} &\equiv T^{\mu\nu} + \frac{1}{2}\partial_\lambda[S^{\lambda\mu\nu} + S^{\mu\nu\lambda} + S^{\nu\mu\lambda}] \\ &= T_B^{\nu\mu} \end{aligned}$$



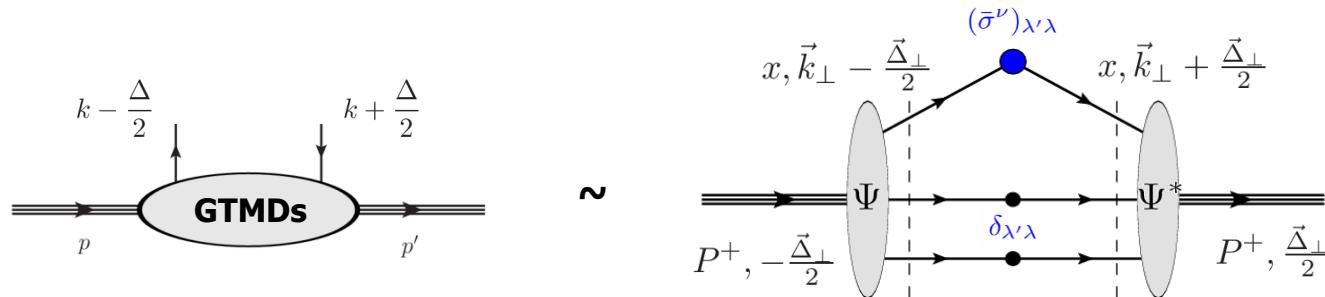
In rest frame

$$M = \int d^3r T_B^{00}(\vec{r})$$

$$J^i = \int d^3r \epsilon^{ijk} r^j T_B^{0k}(\vec{r})$$

No « spin » contribution !

Light-front overlap representation



$$W_{\Lambda'\Lambda}^{[\Gamma]}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp) = \frac{1}{\sqrt{1 - \xi^2}} \sum_{\beta', \beta} \int [dx]_3 [d^2 k_\perp]_3 \bar{\delta}(\tilde{k}) \psi_{\Lambda' \beta'}^*(r') \psi_{\Lambda \beta}(r) M^{[\Gamma] \beta' \beta}$$

Momentum **Polarization**

Light-front quark models

$$\psi_{\Lambda \beta}(r) = \mathcal{N} \Psi(r) \sum_{\sigma_i} \Phi_{\Lambda}^{\sigma_1 \sigma_2 \sigma_3} \prod_{i=1}^3 D_{\lambda_i \sigma_i}(\tilde{k}_i)$$

Wigner rotation

$$D(\tilde{k}) = \frac{1}{|\vec{K}|} \begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}, \quad K_{R,L} = K_x \pm i K_y$$

Model	$\Psi(r)$	K_z	\vec{K}_\perp	κ_z
LFCQM	$\tilde{\psi}(r)$	$m + y\mathcal{M}_0$	$\vec{\kappa}_\perp$	$y\mathcal{M}_0 - \omega$
LFχQSM	$\prod_{i=1}^3 \vec{K}_i $	$f_{\parallel}(y, \kappa_\perp)$	$\vec{\kappa}_\perp f_\perp(y, \kappa_\perp)$	$y\mathcal{M}_N - E_{\text{lev}}$