

Angular Momentum and Polarization in Hadron Collisions up to LHC Energies

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Abstract

Longstanding puzzles in spin physics can be confronted at the high energies of the LHC. Heavy quarks will be produced with significant polarization, both as single spin asymmetries and through polarization correlations. Lower energy proton accelerator and lepton production data suggest various mechanisms within QCD for polarization phenomena that can be tested at higher energies. Observation of strange and charm hadron polarization reveals important aspects of QCD spin physics. Top quark polarization is predicted to be significant, and polarization correlations will reveal important aspects of the gluon distributions of the hadrons.

Collaborators

- Work in preparation done mostly in collaboration with
Simonetta Liuti
- Experimental analysis also with Pasquale Di Nezza and
Liliet Calero Diaz - ALICE
- Related Aurore Courtoy, Osvaldo Gonzalez Hernandez,
- Kunal Kathuria, Abha Rajan
- Tracy McAskill, Jon Poage

Angular Momentum and Polarization in Hadron Collisions up to LHC Energies: Polarization as a probe of non-perturbative QCD

1. Longstanding puzzles in spin physics can be confronted at the high energies of the LHC.
2. Will heavy quarks be produced with significant polarization, both as single spin asymmetries and through polarization correlations.
3. Lower energy proton accelerator and lepton production data suggest various mechanisms within QCD for polarization phenomena that can be tested at higher energies – LHC & EIC &/or LHeC.
4. Observation of strange and charm hadron polarization reveals important aspects of QCD spin physics.
5. Top quark polarization (SSA) is predicted to be significant.
6. Top polarization correlations will reveal important aspects of the gluon distributions of the hadrons.



Outline

$\Lambda_{s,c,b}$ polarization Puzzles and Uses

I. Large polarization in hadron processes

- I. Very large p+p $\rightarrow A_N, A_{NN}, p's$
- II. Very large Pol'zn for inclusive Λ & Σ
- III. Intriguing Systematics
- IV. Explanations? **Basic evidence** for non-perturbative systematics of hadron structure & formation.
- V. Charmed & heavy hyperons (Fermilab fixed target)
- VI. Will hyperons maintain large Pol'zn?? Need understanding of NPQCD mechanism
- VII. If we do not understand large SSA's we do not understand NPQCD !



Outline

$\Lambda_{s,c,b}$ polarization Puzzles and Uses

- I. Large polarization in hadron processes
- II. Leptoproduction of Λ_s & Σ_s

Not outstanding Single Spin Asymmetries ***yet***

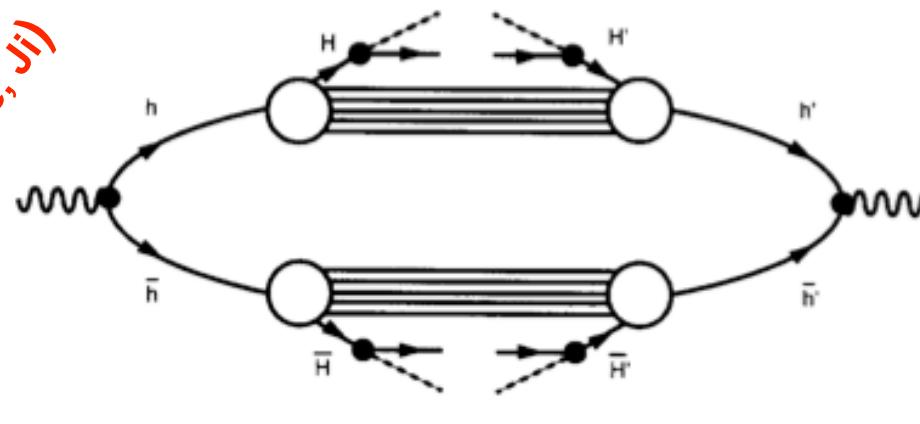
- I. Large double correlations at small Q^2
- II. Analysis more tractable
- III. Which formalism is most useful? TMDs, GPDs, Generalized Fracture Functions?

Outline (cont'd)

- I. Large polarization in hadron processes
- II. Leptoproduction of Λ_s & S_s
- III. Tool to get into transversity -
 - I. Chen, GG, Jaffe, Ji ($e^+e^- \rightarrow \Lambda_s \text{ anti}\Lambda_s X$)
 - II. “off-diagonal” SIDIS via Transversity odd distributions (intrinsic charm?)
 - III. Target fragmentation: GPDs, Fragmentation functions, Fracture Functions (many authors: D.Boer; M. Anselmino. et al.; A. Kotzinian; . . .)
 - IV. Collider production – target or central region (e.g. D. Sivers)
 - V. π^0 , η , K electroproduction \rightarrow Chiral odd GPDs & Transversity: Liuti, GG, et al.
- IV. TMDs, GPDs, Generalized Fracture Functions
 - I. Why GPDs? Phases and transversity - - -
 - II. Preliminary results & relations



Tool for determining
transversity transfer
or $H_1(z)$ (Chen, GG, Jaffe, Ji)



$$\hat{f}_1(z) = \frac{1}{4}z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \not{p} \psi(0) | A(PS) X \rangle \langle A(PS) X | \bar{\psi}(\lambda n) | 0 \rangle ,$$

$$\hat{g}_1(z) = \frac{1}{4}z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \not{p} \gamma_5 \psi(0) | A(PS_{||}) X \rangle \langle A(PS_{||}) X | \bar{\psi}(\lambda n) | 0 \rangle ,$$

$$\hat{h}_1(z) = \frac{1}{4}z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \not{p} \gamma_5 \not{s}_{\perp} \psi(0) | A(PS_{\perp}) X \rangle \langle A(PS_{\perp}) X | \bar{\psi}(\lambda n) | 0 \rangle ,$$

Predicts small back to back transverse spin correlations
ALEPH measurement at Z mass (ave. over $\Lambda\Lambda\bar{b}\bar{a}r$):

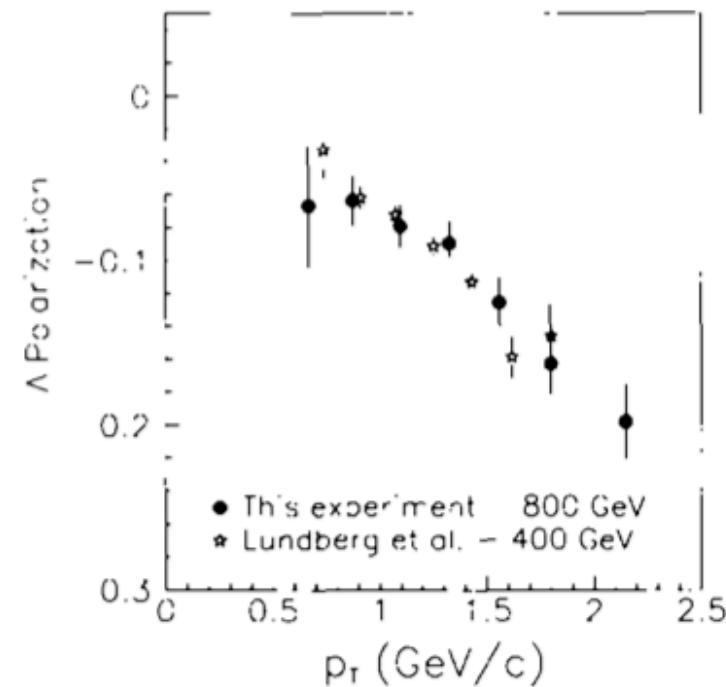
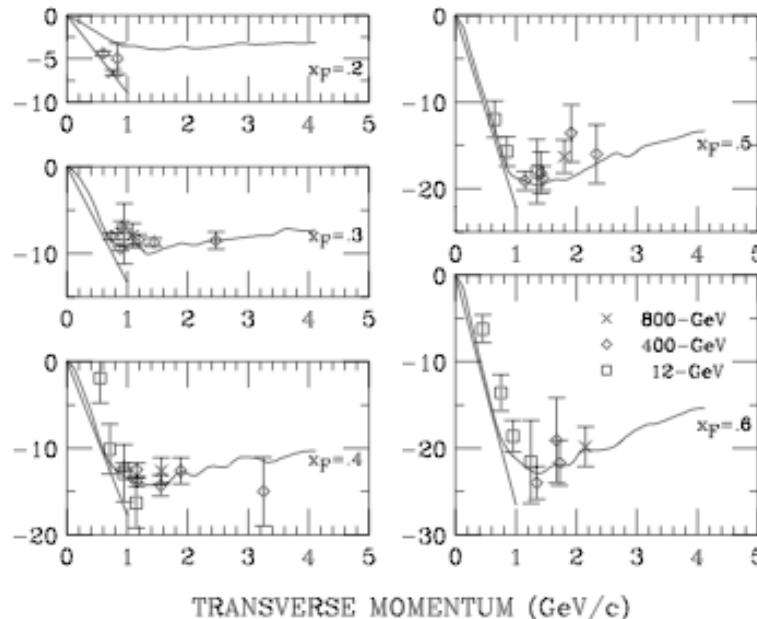
$$P_T^{\Lambda} = 0.016 \pm 0.007 \text{ for } p_T > 0.3 \text{ GeV}/c ,$$

$$P_T^{\Lambda} = 0.019 \pm 0.007 \text{ for } p_T > 0.6 \text{ GeV}/c ,$$



Large polarization in hadron+hadron

POLARIZATION (%)



$p+p \rightarrow \Lambda + X$ Polzn(x_F, p_T)

compiled by K.Heller (1997)

Curves Dharmaratna & GG

Fig. 4. Lambda polarization versus production transverse momentum (p_T). For comparison, data for 400 GeV production (Ref. 10) are also shown.

Ramberg, et al.,(FNAL) PLB338, 403 (1994)



Evolving Ideas about Source of Λ Polarization in Hadrons

- Semi-classical: Lund; Thomas precession; SU(6); Soffer, et al.
- Q Field Th: Single polarization requires interference => Real x Im part & helicity flip
- Kane, Pumplin, Repko: PQCD (PRL41,1689(1978) $\rightarrow P_L \sim \alpha(\hat{s})m_q / \sqrt{\hat{s}}$

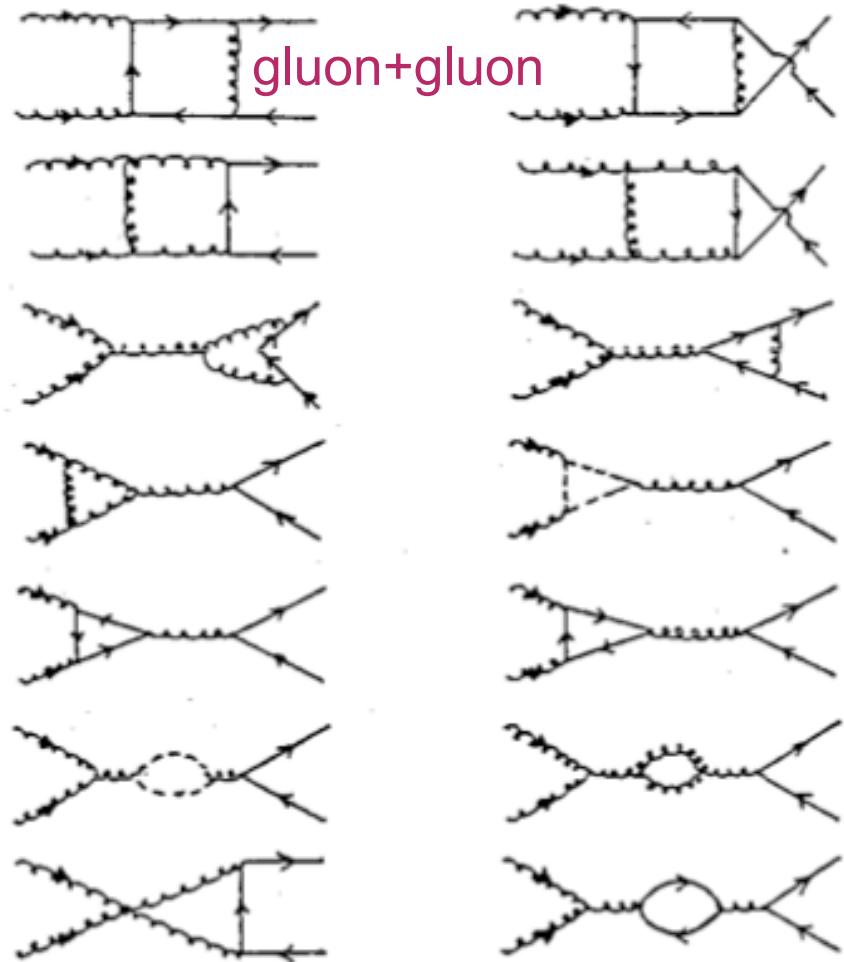
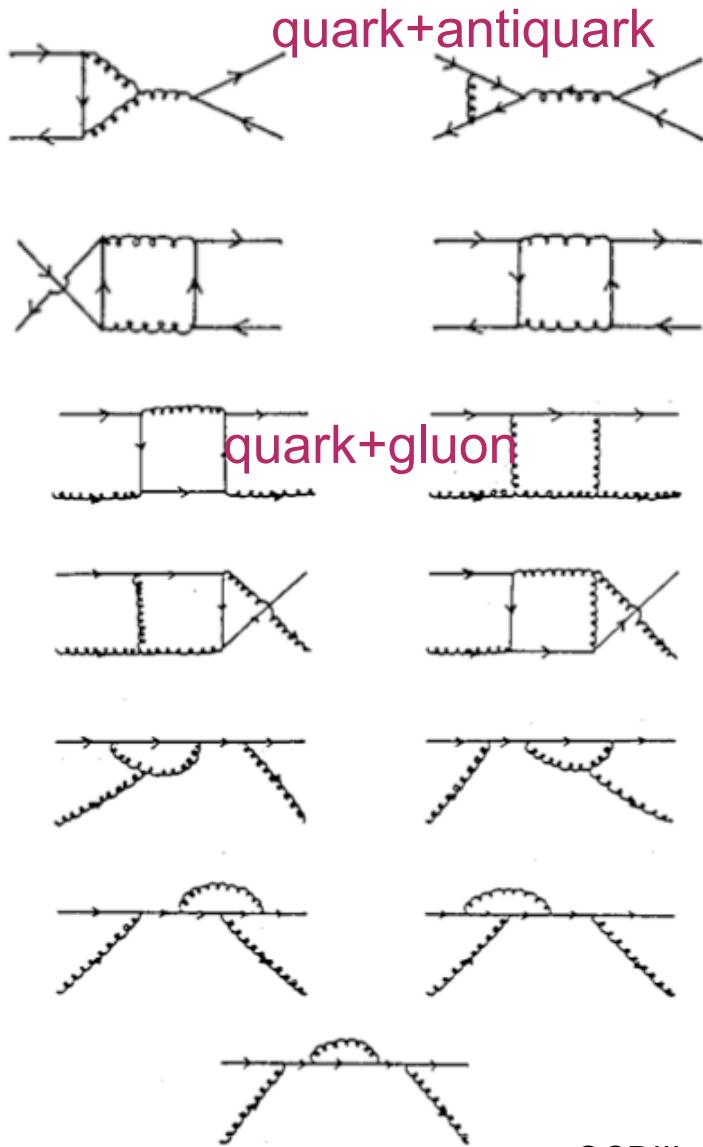


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- Complete order α_s calculation of quark, antiquark, gluon 2-body scattering $\rightarrow s\uparrow + s\bar{q}$ imbedded in hadron+hadron pdf's (but small m_S) (Dharmaratna & GG 1990,1996) How does $s\uparrow$ get translated to $\Lambda\uparrow$ & enhanced?
- NPQCD must play a significant role in our understanding of orbital angular momentum & hadron formation.

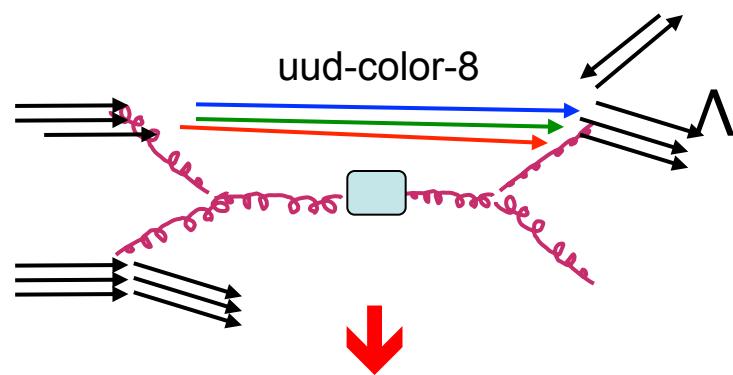
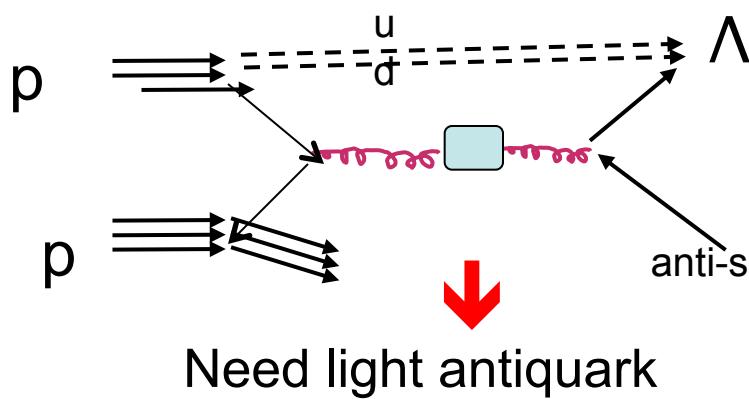
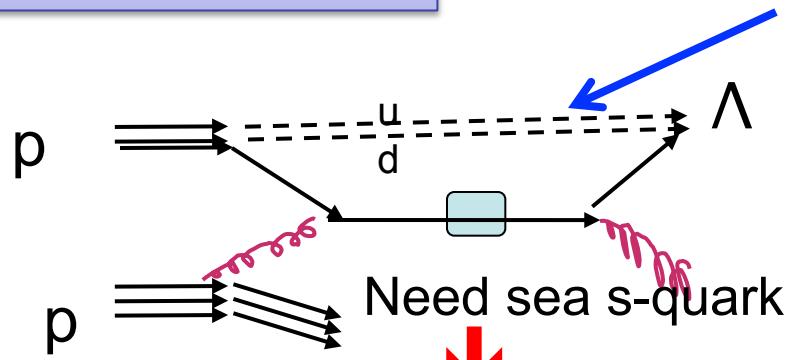
Contributions to order α_s Imaginary Part

(Dharmaratna & GG 1990, 1996)



How to get to hyperon Polz'n?

s quark accelerates toward (ud) remnant of proton



Box represents loop contributions to Im part.
Seen as *GPD* already have Im part!



Model of hyperon polarization

Dharmaratna & GRG (1990,96,99)

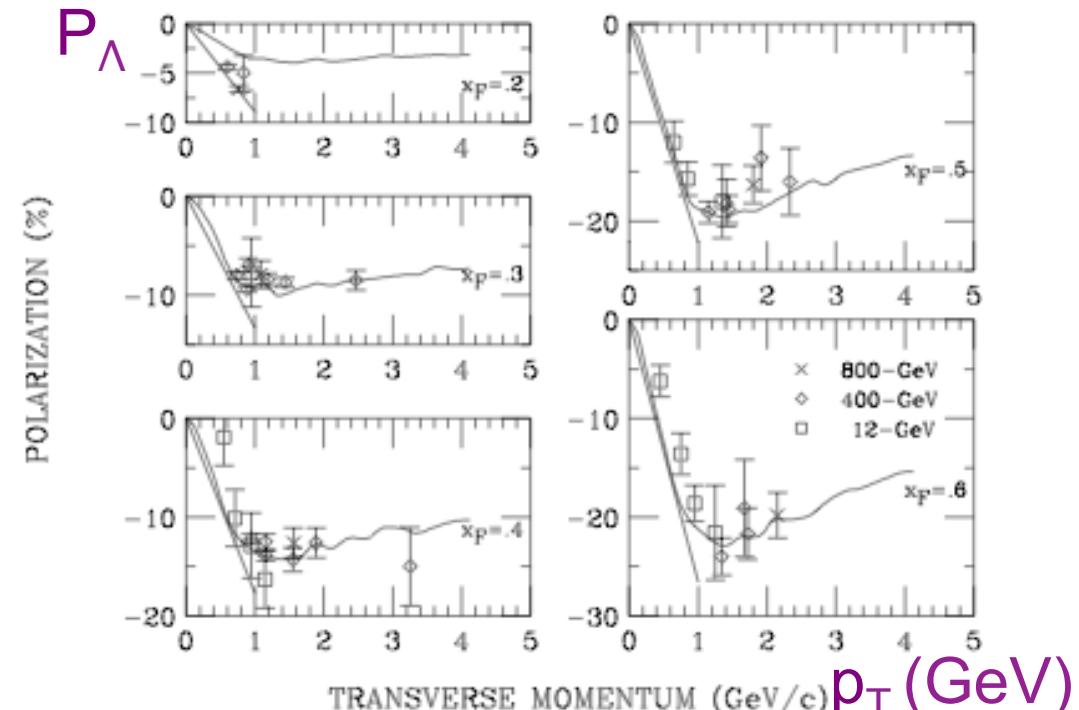
1. $p+p \rightarrow \Lambda+X$ has large negative P_Λ with flat s dependence & growth with p_T (see Heller . . .)

2. Clues: $K^- p \rightarrow \Lambda+X$ at 176 GeV/c or $\sqrt{s}=18\text{GeV}$
Polzn even larger - need s-quark?

3. Simple factorization expectation
Kane, Pumplin, Repko
 $P_\Lambda \sim \alpha(\hat{s}) m_q / \sqrt{\hat{s}}$

helicity flip $\sim m_q/\text{hard energy scale}$
Soft phenomenon?

Dharmaratna & GRG: 1. Gluon fusion dominant mechanism for producing polarized massive quark pair
2. Low p_T phenomenon
3. Acceleration mechanism



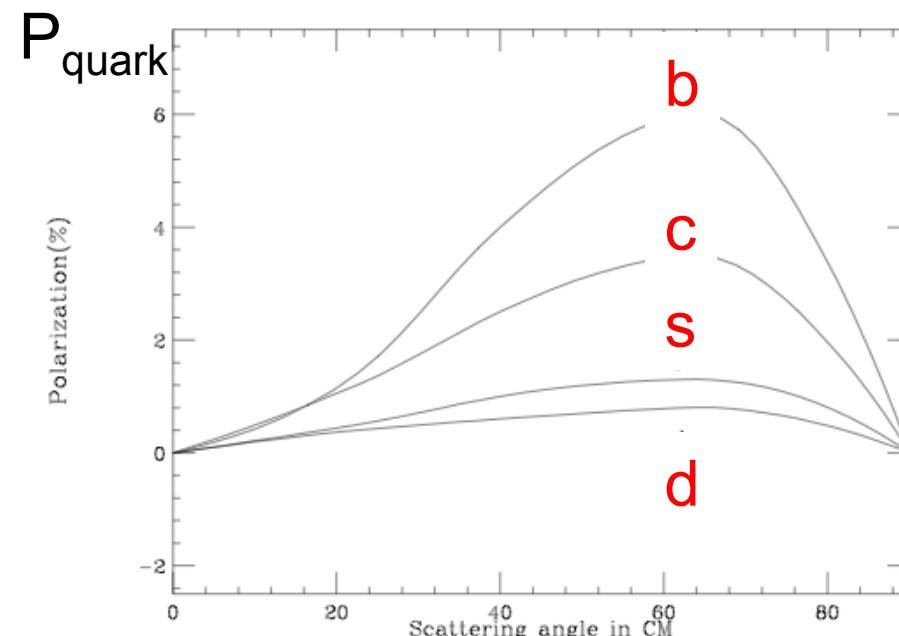
$p+p \rightarrow \Lambda+X$ Polzn(Λ)
compiled by K.Heller (1997)



P_{quark} vs. flavor
from gluon fusion
grows with flavor

Does this give
larger P_{hadron} for
heavier flavor?

What sets scales?
quark “mass” or
hyperon mass



(Dharmaratna & GG 1990,1996)

$$\text{Pol}_n(Q) \sim m_Q/v_s$$



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- Burkardt: impact parameter b distortion of quarks in spinning hadron from FT-GPDs, production overlap region & anomalous moments κ for hyperons

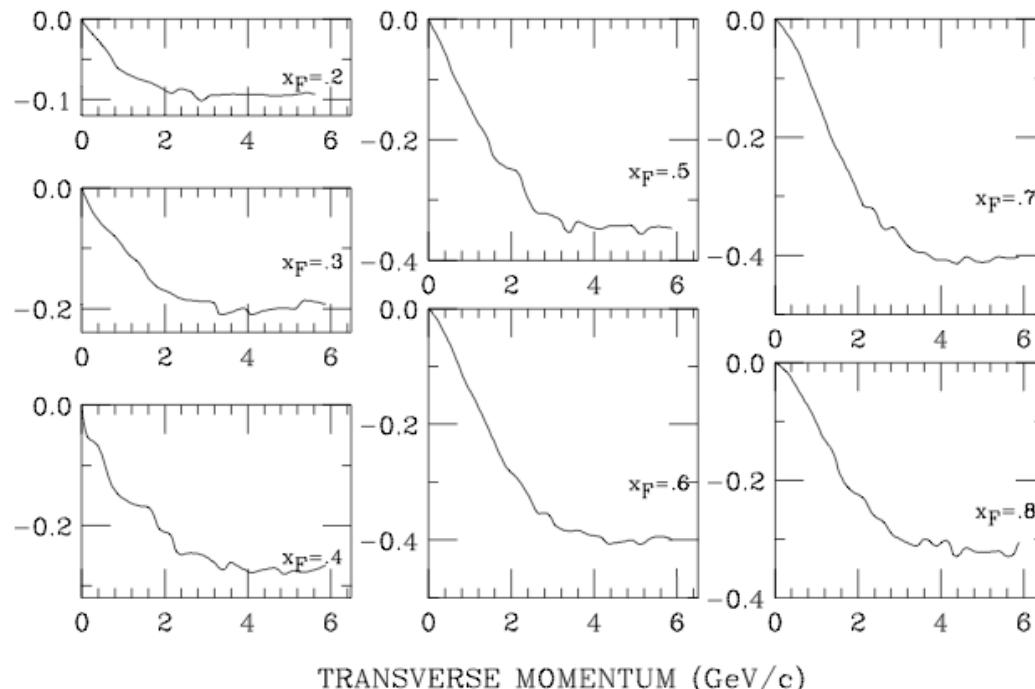


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- S.Liuti, K. Kathuria & GG: Vorticity & b in production (overlap)
 $\rightarrow G_2$: OAM & distortion \rightarrow polarized hyperons



Charmed Hyperon Polarization

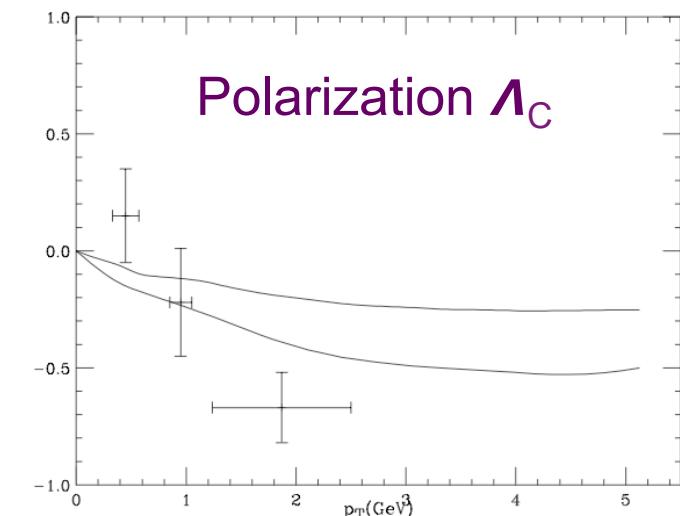


$P(\Lambda_c)$ vs. p_T (GeV) for several x_F values)

E.M. Aitala, et al. (E791 Collaboration) "Multidimensional Resonance Analysis of $\Lambda_c^+ \rightarrow pK^- \pi^+$ ", Fermilab 1999.

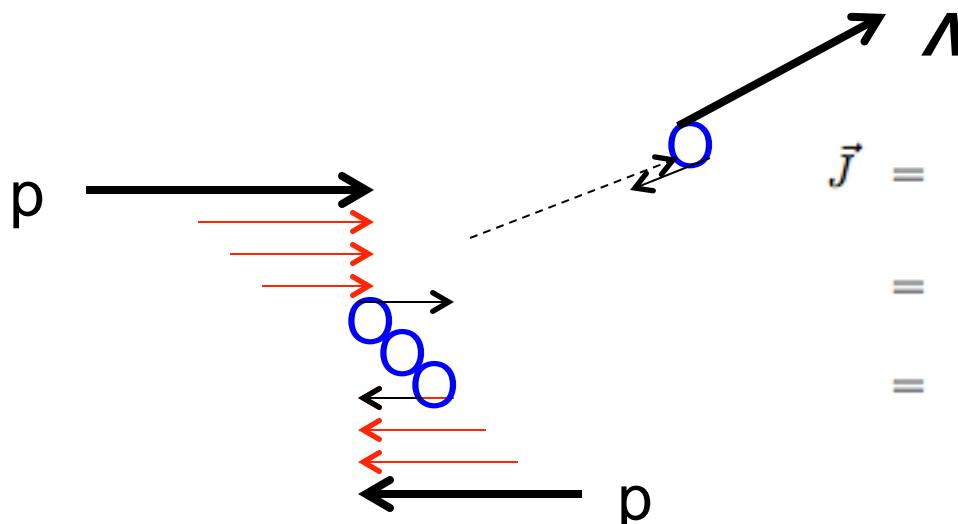
$P(\Lambda_c)$ does not fall off with p_T Trend to be tested?

GG – hep-ph/990757 proceedings FNAL workshop on
Charmed hyperons



Vorticity picture

- High Energy $p + p$ at fixed b has large relative OAM
- Fluid picture with laminar flow in viscous medium



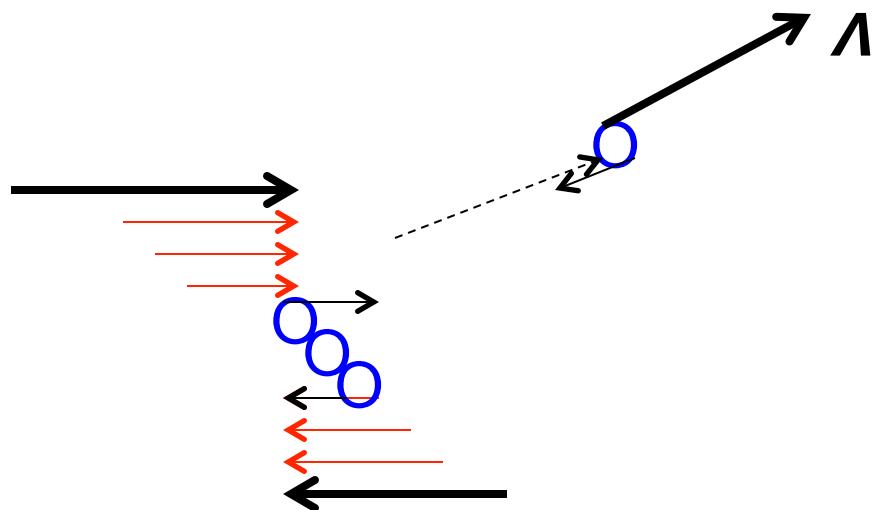
$$\begin{aligned}\vec{J} &= \int d^3x \vec{x} \times \vec{\pi} = \int d^3x \vec{\nabla} \frac{x^2}{2} \times \vec{\pi} \\ &= \int d^3x \vec{\nabla} \times \frac{x^2}{2} \vec{\pi} - \int d^3x \frac{x^2}{2} \vec{\nabla} \times \vec{\pi} \\ &= - \int d^3x x^2 \gamma m_q \vec{\omega} - \dots\end{aligned}$$

$$\vec{\omega} = \frac{1}{2} \vec{\nabla} \times \vec{\pi} / \gamma m_q$$

J contains ω = quark field “vorticity” which transfers to hadrons
see Becattini, et al. PRC77, 024906 (2008) for heavy ion collisions

OAM in hyperon production

Roiling sea of vorticity \rightarrow emerging heavy flavor quark with fraction of OAM represented by $G_2(x, p_T^2, 0)$ (γ^\perp term in quark correlator at twist 3) \rightarrow polarized heavy flavor hadron



Work in progress
See S.Liuti talk re
OAM



How are quark polarizations measured?

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Hadronization of polarized or unpolarized heavy flavor quark mixes Fragmentation Functions (Anselmino, Boer, et al.) with small p_T production mechanisms – factorization?
Initial or Final state interactions?

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Unpolarized quark \rightarrow hadron $\uparrow\downarrow + X$ for larger p_T ?

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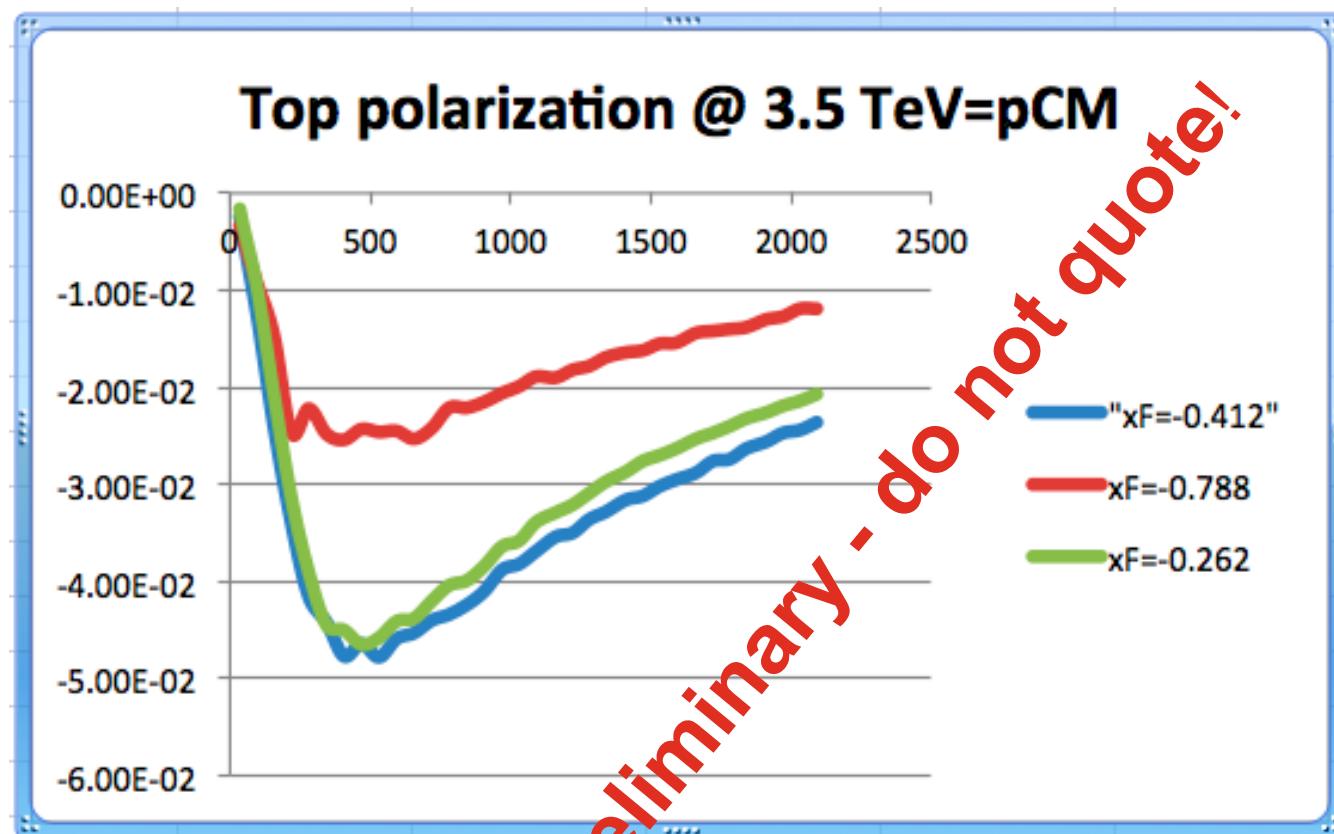
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Top quarks decay before hadronizing \Rightarrow
decays are “self analyzing”
Unique feature of heavy flavors
 \Rightarrow provide window into heavy flavor QCD



Direct measure of hard process - top polarization
Top decays weakly before hadronizing \Rightarrow decay “self-analyzing”



Analyze $t \rightarrow W^+ b$



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- How does $s\uparrow$ get translated to $\Lambda\uparrow$?
- Consider electroproduction of Λ 's. Prelude to hadron production. QCD more under control.
 - Soft matrix elements from TMDs & SIDIS or GPDs &/or Fracture Functions



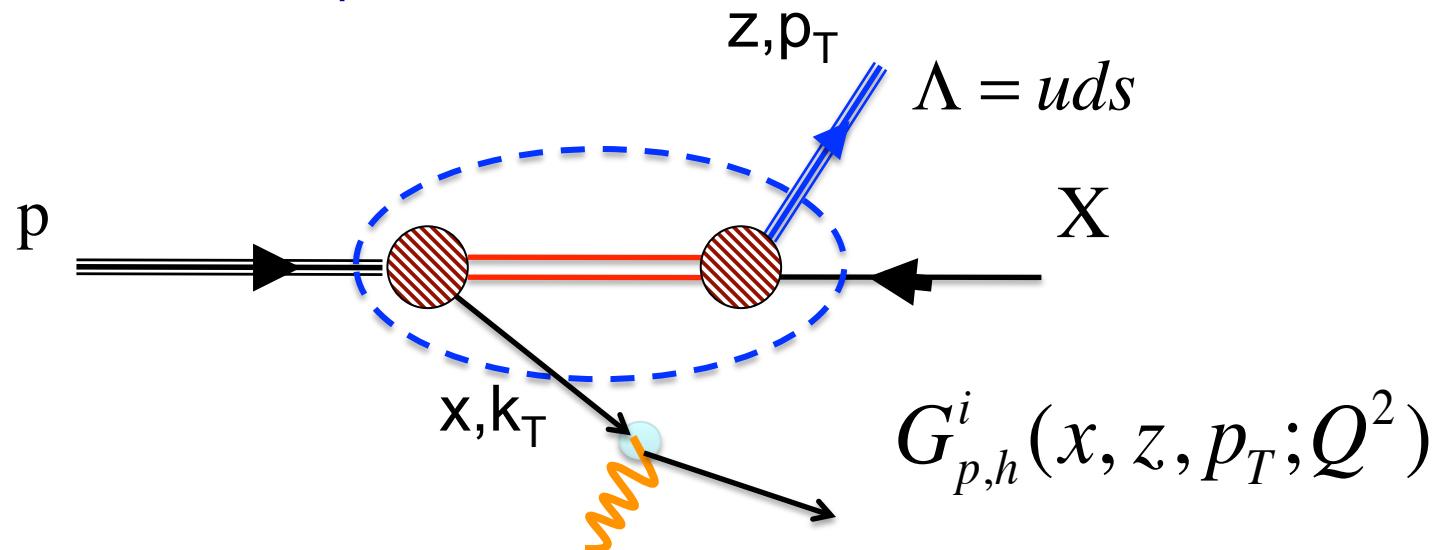
Electroproduction of Λ

Simple tree level model for extended fracture function
(Trentadue & Veneziano)

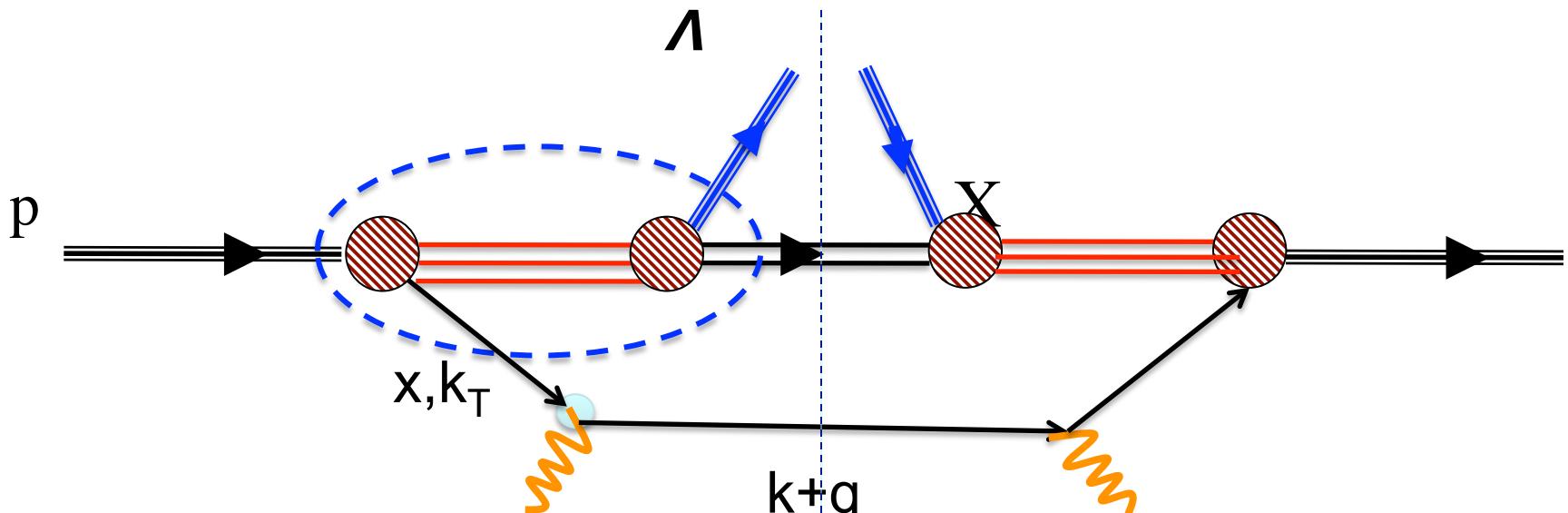
Diquark spectator & fragmentation

“ $d\sigma$ ” squares & sums over X states with anti-s flavor

Diquark $\rightarrow \Lambda + s\bar{s}$ simple vertex



$z = E_L / (1-x) E_{gP\text{ CM}}$ for target fragment
or $P_L^+ = z(1-x) P^+$



$$(P - k)^2 = m_s^2 + \frac{M_\Lambda^2}{z} + \frac{1-z}{z} \left(\vec{P}_{\Lambda T} - \frac{z}{1-z} \vec{P}_{XT} \right)^2$$

Dipole form factors dampen $P \rightarrow u + \text{diquark}$ vertex
 Λ , diquark, struck quark all on shell

$$k^2 = x M^2 - \frac{\vec{k}_T^2}{(1-x)} - \frac{x}{(1-x)} (P - k)^2$$



diquark model extended fracture function

$$(P - k)^2 = m_s^2 + \frac{M_\Lambda^2}{z} + \frac{1-z}{z} \left(\vec{P}_{\Lambda T} - \frac{z}{1-z} \vec{P}_{XT} \right)^2$$

$$k^2 = x M^2 - \frac{\vec{k}_T^2}{(1-x)} - \frac{x}{(1-x)} (P - k)^2$$

$$\mathcal{F}_{\Lambda_N, \Lambda_\Lambda}^{\lambda_q}(x, k_T, z, p_T, Q^2) = \sum_{\Lambda_X} \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \int \frac{d^4 \xi}{(2\pi)^4} e^{ik \cdot \xi} \langle P \mid \bar{\psi}(\xi) \mid P_h; X \rangle \langle P_h; X \mid \psi(0) \mid P \rangle.$$

quark correlator for Extended Fracture Functions
helicity labels $\langle P, \Lambda_N |$ & $| P_h, \Lambda_\Lambda ; X \rangle$
For unpolarized $d\sigma$, sum over all helicity labels.
For polarized Λ , keep floating

diquark model extended fracture function

In the spectator model

$$\mathcal{F}_{\lambda_q, \Lambda_N}^{\Lambda_\Lambda, \Lambda'_\Lambda}(x, k_T, z, p_T, Q^2) = A_{\Lambda_N, \lambda_q} \sum_{\Lambda_X} B_{\Lambda_X}^{\Lambda_\Lambda, \Lambda'_\Lambda}$$

where

$$A_{\Lambda_N, \lambda_q} = |\phi_{\lambda, \Lambda}(k, P)|^2,$$

with

$$\phi_{\Lambda, \lambda}(k, P) = \Gamma(k) \frac{\bar{u}(k, \lambda_q) U(P, \Lambda_N)}{k^2 - m^2},$$

and

$$k = P - P_X - P_\Lambda \Rightarrow k^2 = k^2(x, \mathbf{k}_T, z, \mathbf{p}_T)$$

whereas

$$B_{\Lambda_X}^{\Lambda_\Lambda, \Lambda'_\Lambda} = \tilde{\phi}_{\Lambda_X, \Lambda'_\Lambda}^*(P_X, P_h) \tilde{\phi}_{\Lambda_X, \Lambda_\Lambda}(P_X, P_h),$$

with

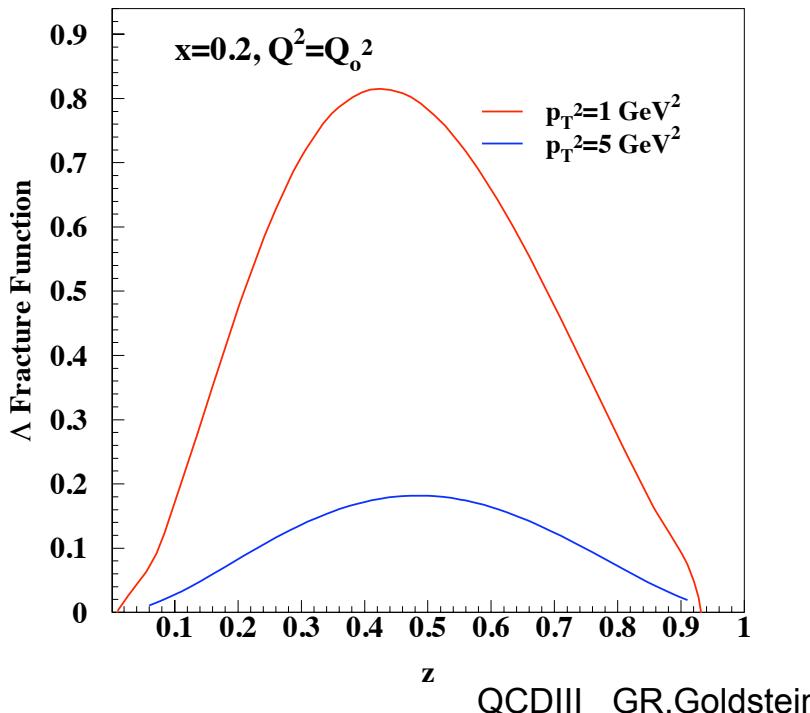
$$\tilde{\phi}_{\Lambda_X, \Lambda_\Lambda}(P_X, P_h) = \Gamma(P_X) \bar{v}(P_X, \Lambda_X) U(P_h, \Lambda_\Lambda)$$

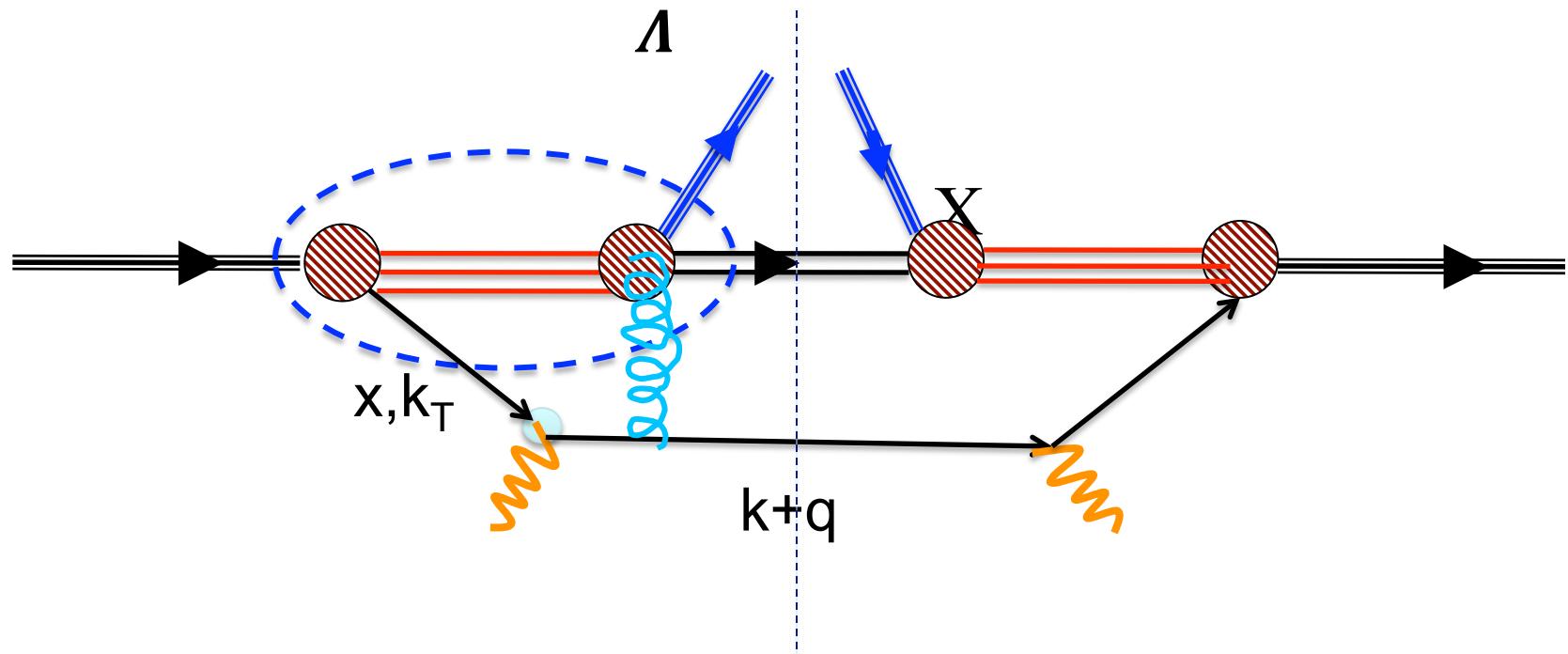
diquark model extended fracture function

$$\sum_{\Lambda_X} B_{\Lambda_X}^{\Lambda_\Lambda, \Lambda'_\Lambda} = (1-x)^2 \left([-zM_X + (1-z)M_\Lambda]^2 + p_T^2 \right) \delta_{\Lambda_\Lambda, \Lambda'_\Lambda}$$

A_{Λ_N, λ_q} is squared & summed over

$$\rightarrow f(x, k_T) / (k^2 - m_{dipole}^2)^4 \Big|_{P_X^2 = (P - k - P_\Lambda)^2 = m_s^2}$$





Spin dependence?

- a. non-trivial quark or proton- Λ spin correlation \rightarrow axial diquark
- b. SSA need phase \rightarrow beyond tree

Figure shows final state interaction contribution to $\Lambda \uparrow$

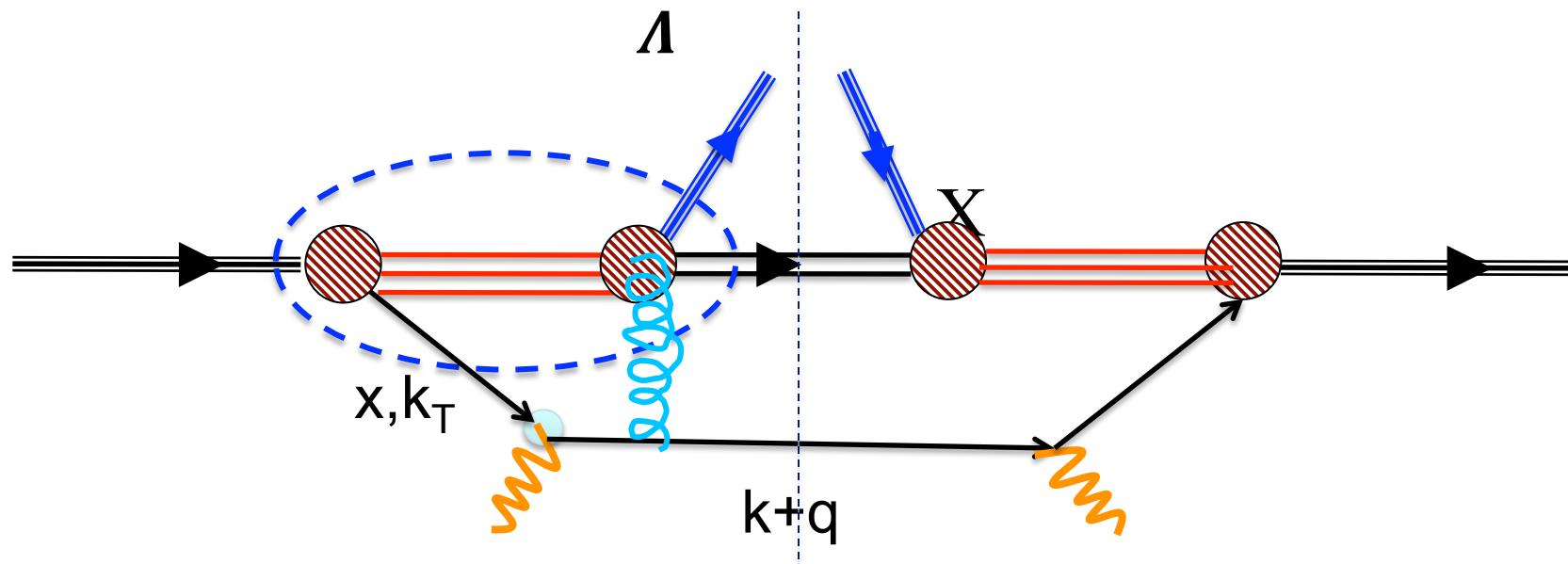
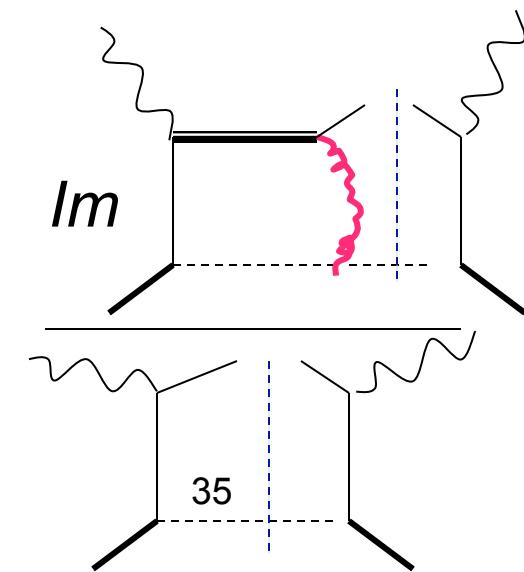


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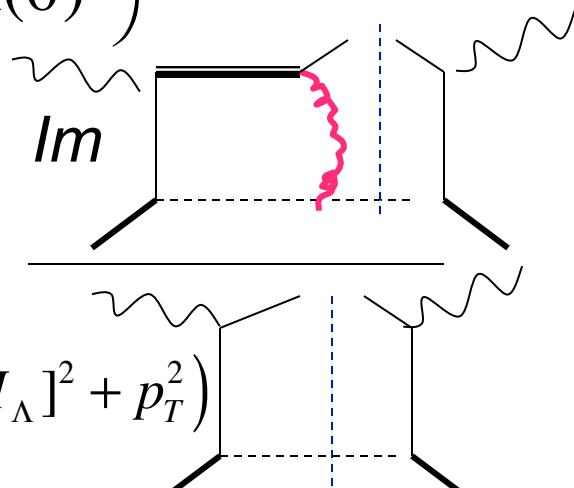


Final State Interactions or gauge links . . .

Recall f.s.i. (e.g. Brodsky, Hwang & Schmidt; Gamberg & Goldstein, etc.)

$$P_y = C_F \alpha_s(m^2) \frac{(xM + m)k_x}{[(xM + m)^2 + \vec{k}_\perp^2]} \frac{\Lambda(\vec{k}_\perp^2)}{\vec{k}_\perp^2} \ln \left(\frac{\Lambda(\vec{k}_\perp^2)}{\Lambda(0)} \right)$$

$$\Lambda(\vec{k}_\perp^2) = \vec{k}_\perp^2 + x(1-x) \left(-M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x} \right)$$



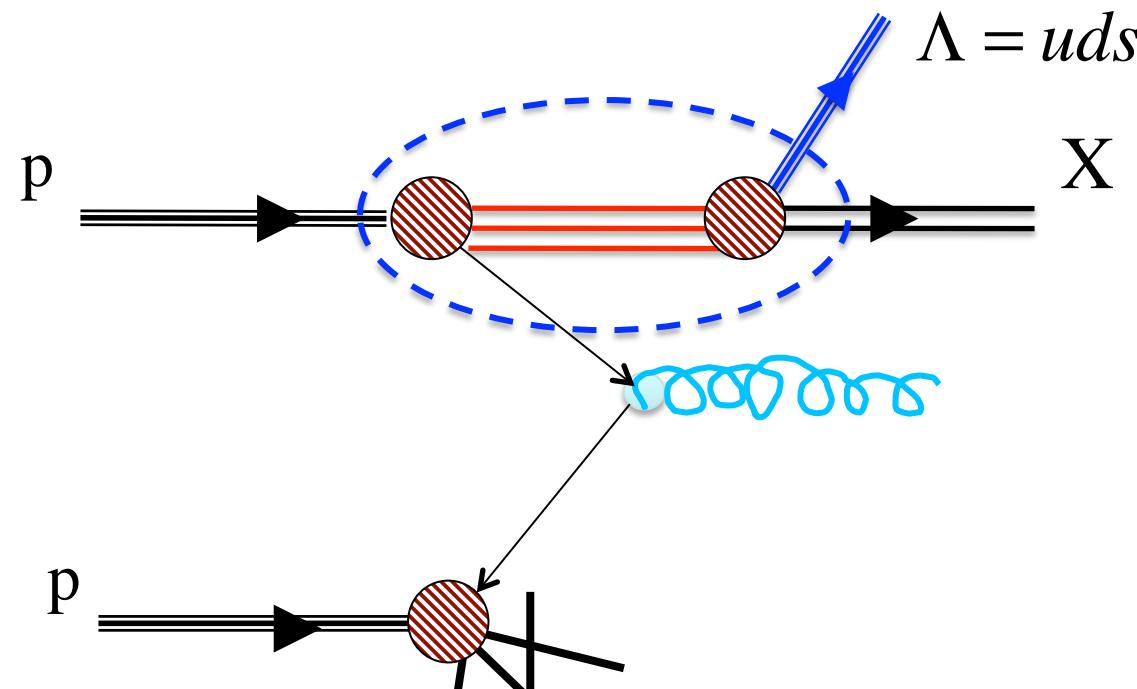
For Frac.Fn. model replace denom with $([-zM_X + (1-z)M_\Lambda]^2 + p_T^2)$

$$\Omega(\vec{k}_T^2, \vec{p}_T^2) = xM^2 - \left\{ \vec{k}_T^2 + x[m_s^2 + \frac{M_\Lambda^2}{z} + \frac{1-z}{z} (\vec{p}_T - \frac{z}{1-z} \vec{p}_{XT})^2] \right\} / (1-x)$$

numerator with $(1-x)^2 k_T (zM_X - (1-z)M_\Lambda^2)$ from Im flip \times non-flip



PDFs + Fracture functions $\langle p | \bar{\psi} | \Lambda X \rangle$
Incorporated into P + P

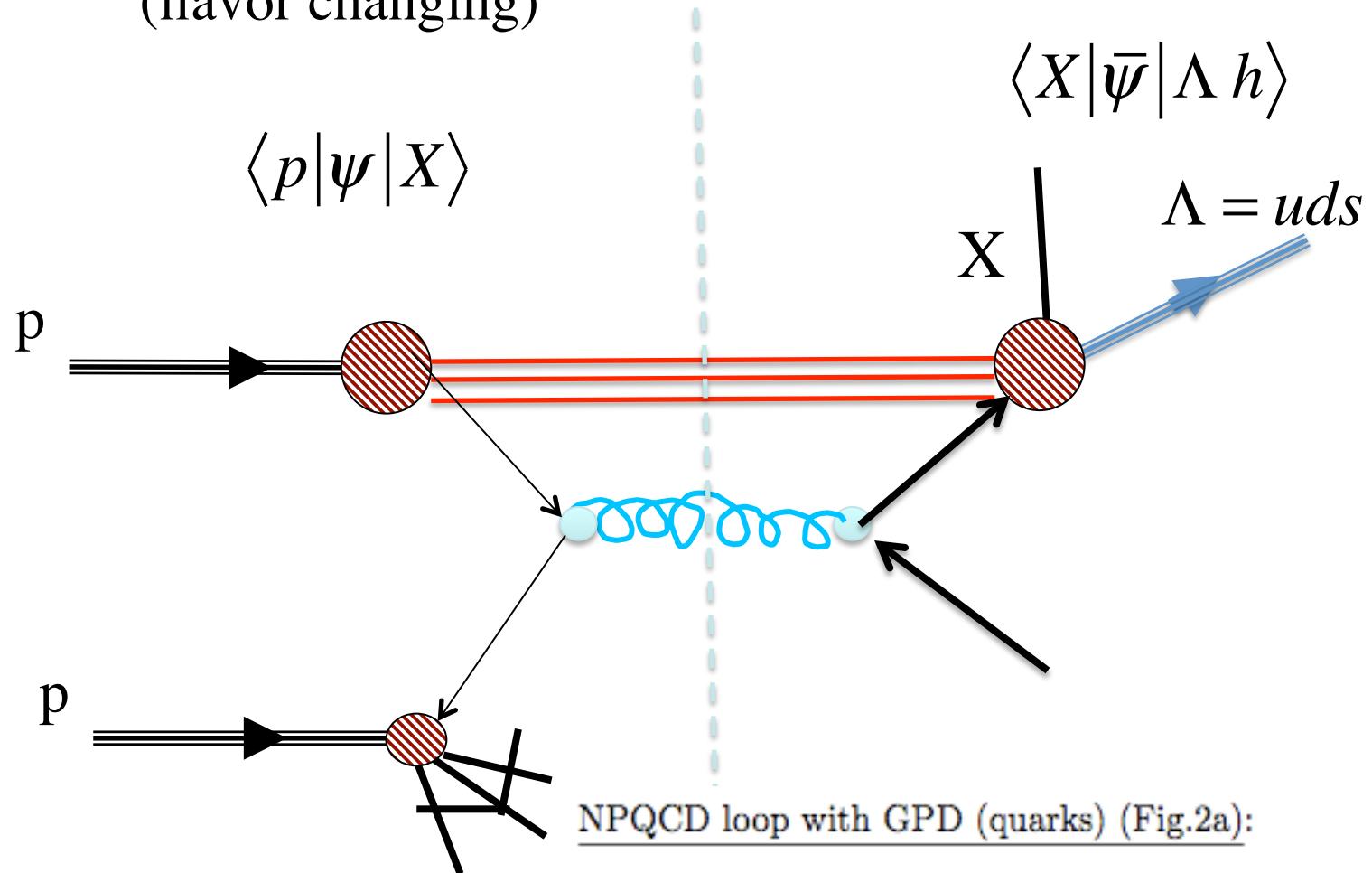


Formally, for a quark-nucleon FF,

$$\begin{aligned} \mathcal{F}(x, k_T, z, p_T, Q^2) = & \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \int \frac{d^4 \zeta}{(2\pi)^4} e^{ik \cdot \zeta} \\ & \times \langle P | \bar{\psi}(\zeta) | P_{hadron}; X \rangle \\ & \times \langle P_{hadron}; X | \psi(0) | P \rangle. \quad (6) \end{aligned}$$

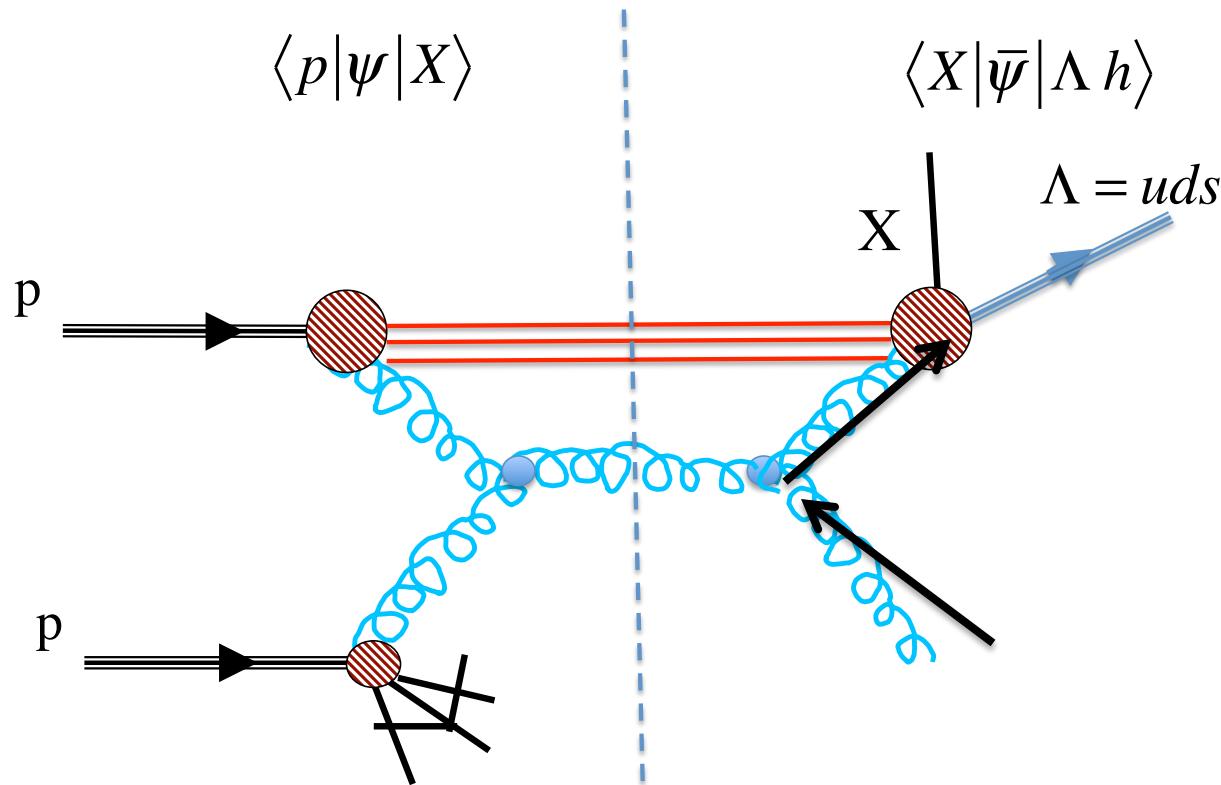


Generalized Fracture Function → extended GPDs (flavor changing)



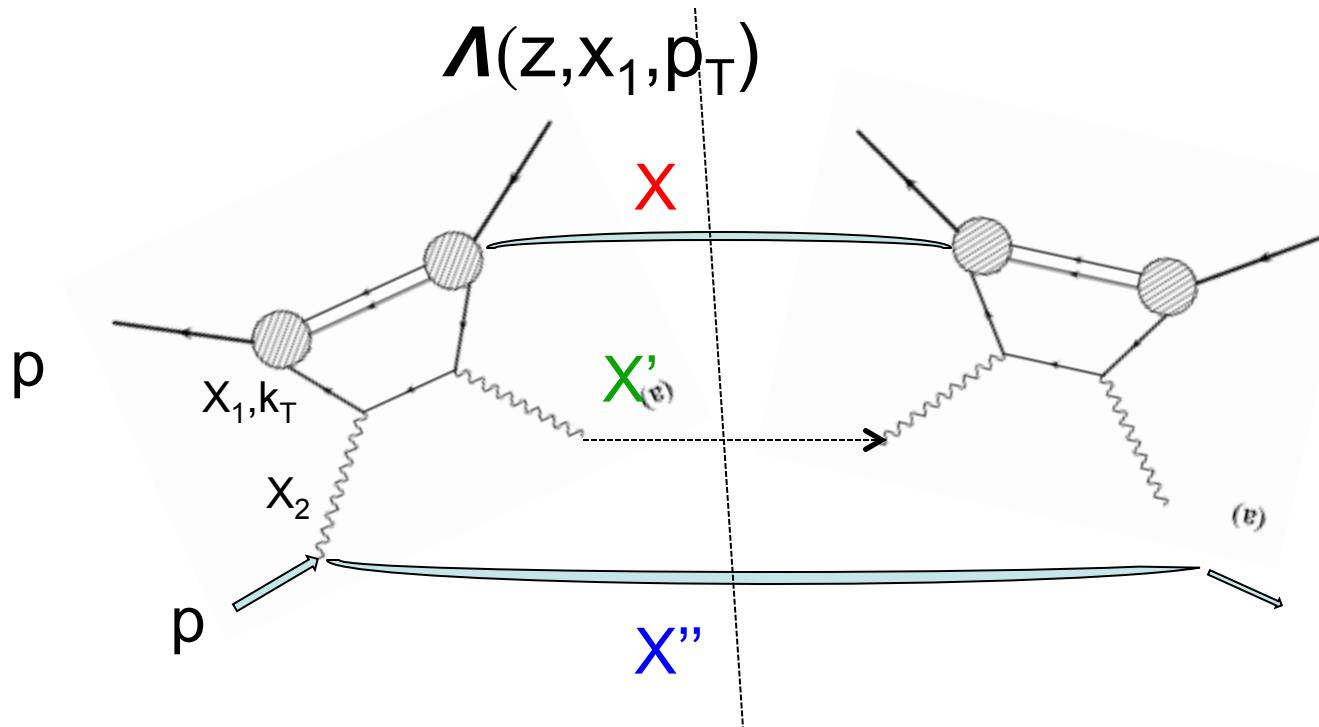
$$\int dx_1 dx_2 [\mathcal{H}_{N \rightarrow Y}^* \mathcal{H}_{N \rightarrow Y}] (x_1, \zeta, t, Q^2) \\ \times f(x_2, Q^2) \hat{\sigma}_{12 \rightarrow sX}^{LO}(x_1, x_2, x_F^s, p_T)$$

Generalized Fracture Function



Gluon fusion is largest source of polarized quarks & gluons . . .
Gluons will be plentiful at LHC. Move toward Gluon GPDs

GPD source of Λ
CFF \rightarrow Re & Im parts



Each line has helicity summed over except Λ_L & Λ'_L
Real & Im CFF multiplied for Polzn

3 . Polarized top quark production and spin correlations

Top decays vs. mass

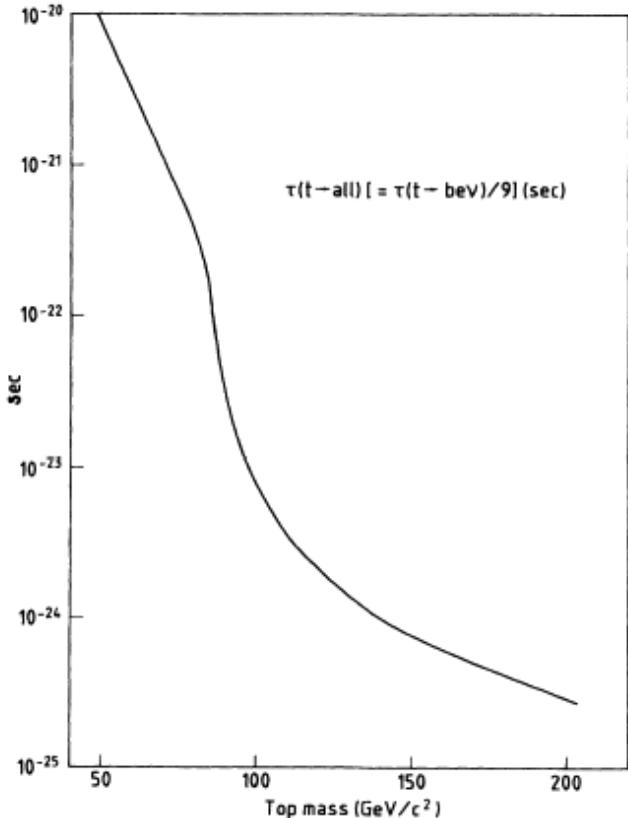


FIG. 1. Total top-quark lifetime as a function of its mass m_t .

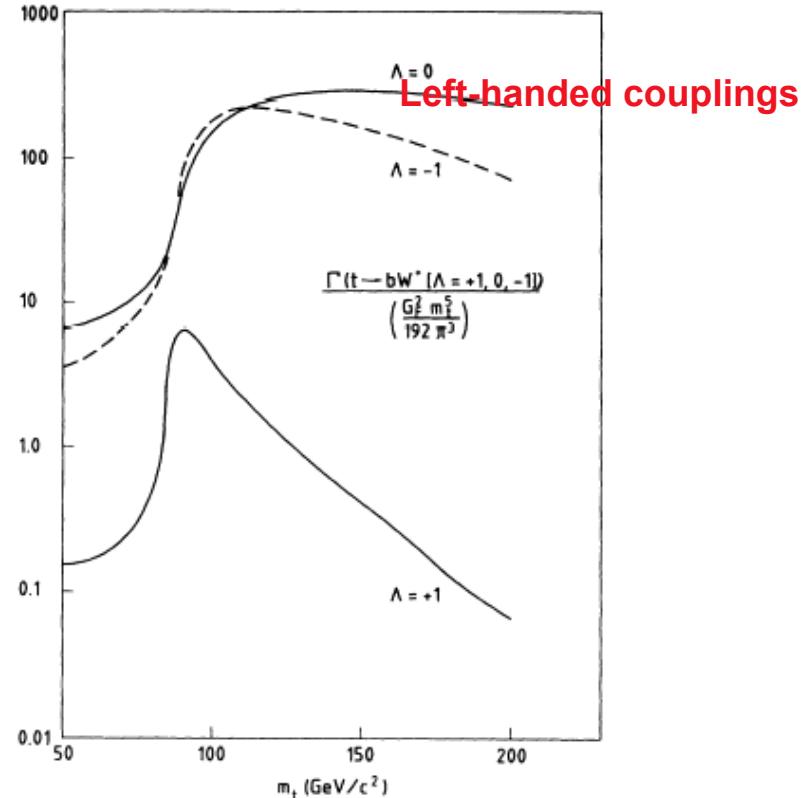


FIG. 2. Partial rates for top-quark decay to bW^+ , for W helicity $\Lambda = +1, 0$, and -1 along its momentum in the top-quark rest frame.

- R.H. Dalitz and G.R. Goldstein, “Decay and Polarization Properties of the Top Quark”, Phys. Rev. D45, 1531 (1992);
 R.H. Dalitz and G.R. Goldstein, “The Analysis of Top-Antitop Production and Dilepton Decay Events and the Top Quark Mass”, Phys. Lett. B287, 225 (1992).
 R.H. Dalitz and G.R. Goldstein, “Test of analysis for top--antitop production and decay events”, Proc. Royal Soc. of London, A455, 2803 (1999).

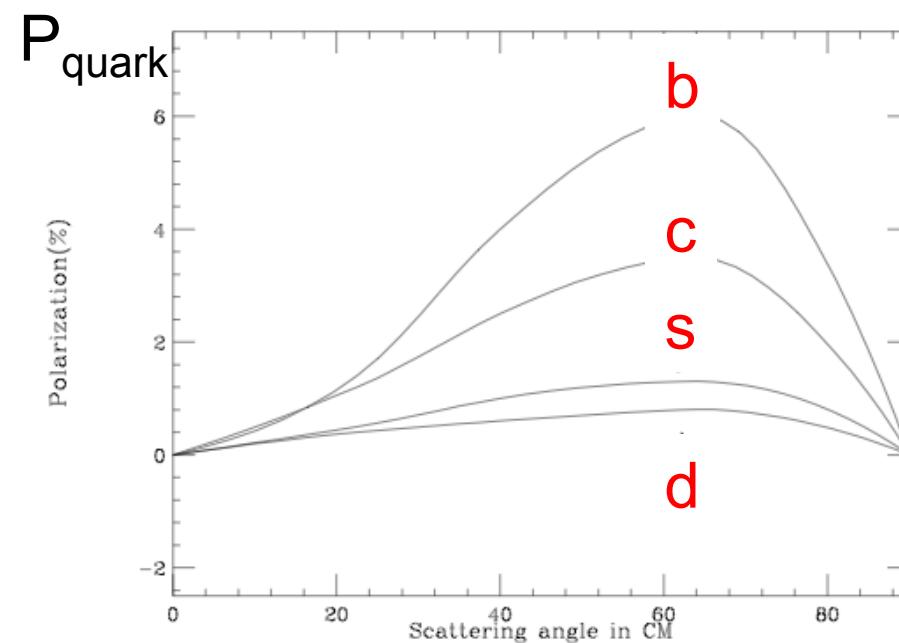


P_{quark} vs. flavor
from gluon fusion
grows with flavor

Does this give
larger P_{hadron} for
heavier flavor?

What sets scales?
quark “mass” or
hyperon mass

Recall previous slide –
Perturbative calculation

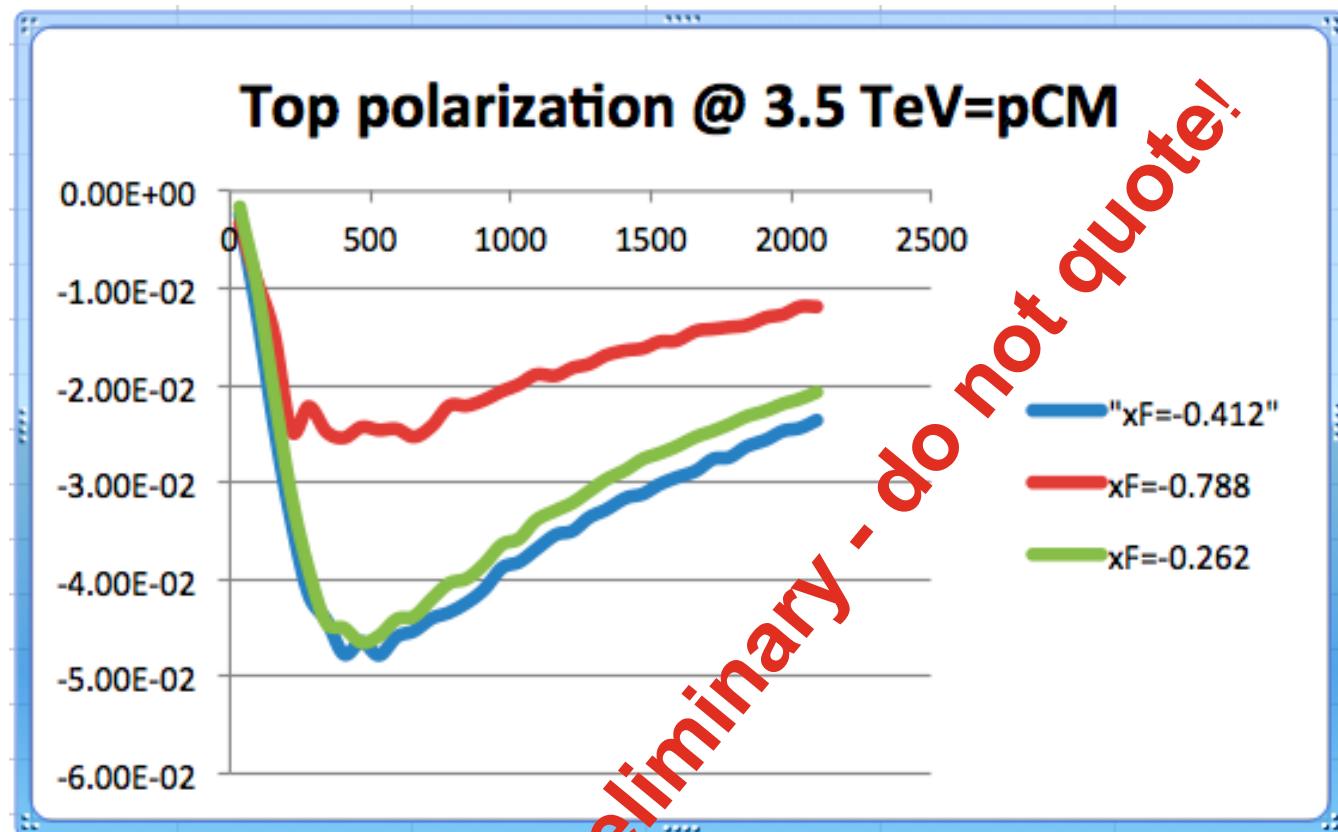


$$\text{Pol}_{\text{zn}}(Q) \sim m_Q/\sqrt{s}$$



Direct measure of hard process - top polarization preliminary predictions of D&G PQCD

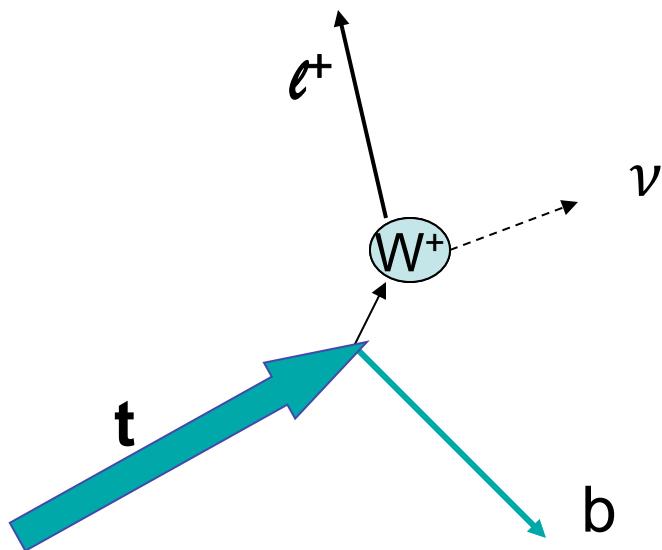
(GG & S.Liuti, arXiv:1201:0193)



Analyze $t \rightarrow W^+ b$

preliminary - do not quote!

How is actual top polarization determined?
Its decay is good analyzer.

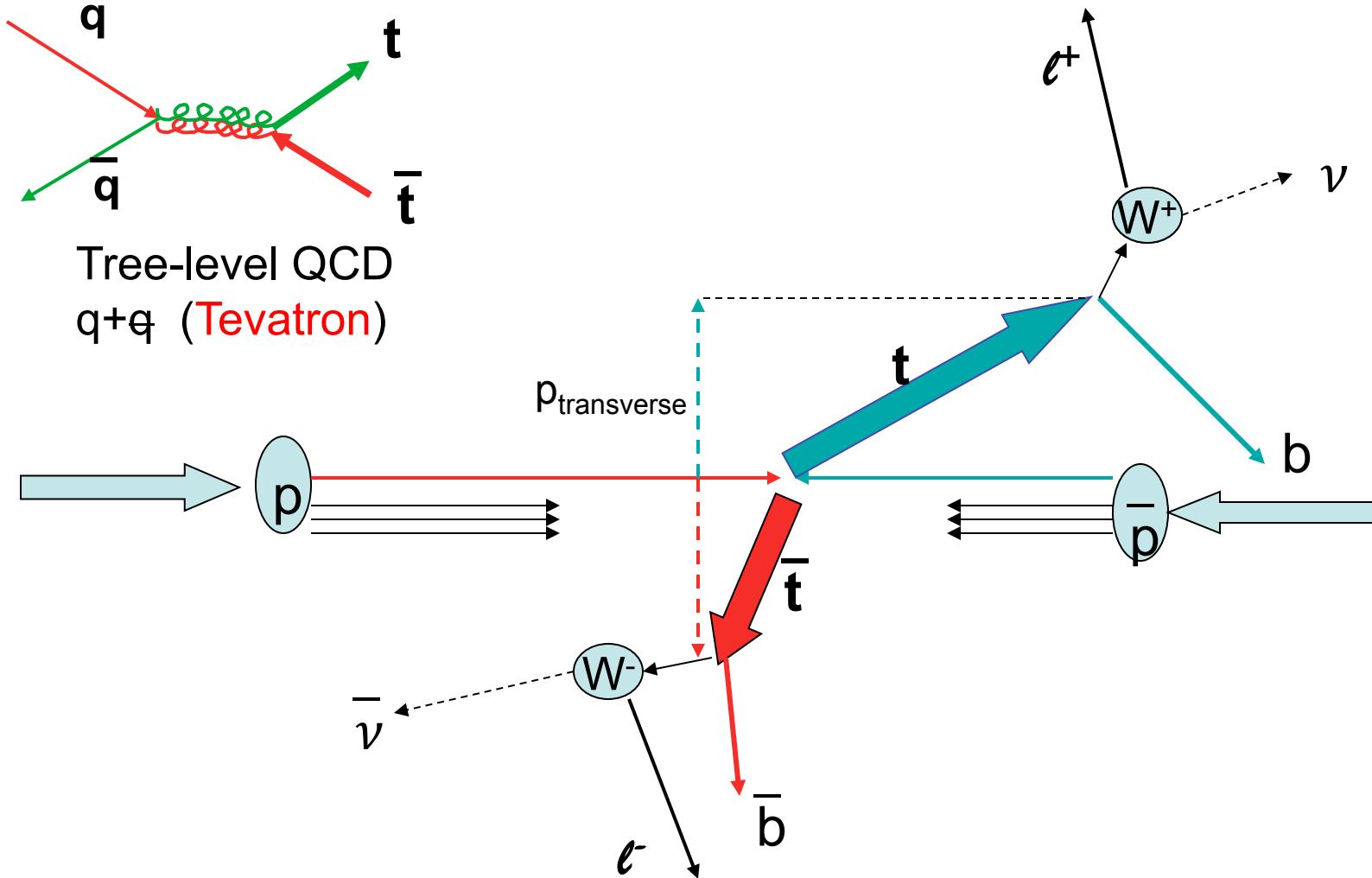


$$U_{t,\bar{t}} = \sum_{\lambda_b} B_{\lambda_b, \bar{t}}^* B_{\lambda_b, t}$$
$$\propto (I + \vec{p}_{\bar{t}} \cdot \vec{\sigma}_t / p_{\bar{t}})_{t,\bar{t}} (p_b \cdot p_\nu),$$

Polarized top pair production and polarized gluon distributions

Double spin asymmetries are “naïve-T” even
→ get “tree-level”
QCD contributions ⇒ good test of BSM

Dilepton events



Polarized top pair production in p+p and polarized gluon distributions

p→g(polzn x or y,p_{1T}) :f₁g(p₁) or h_{1T} ⊥g(p₁)

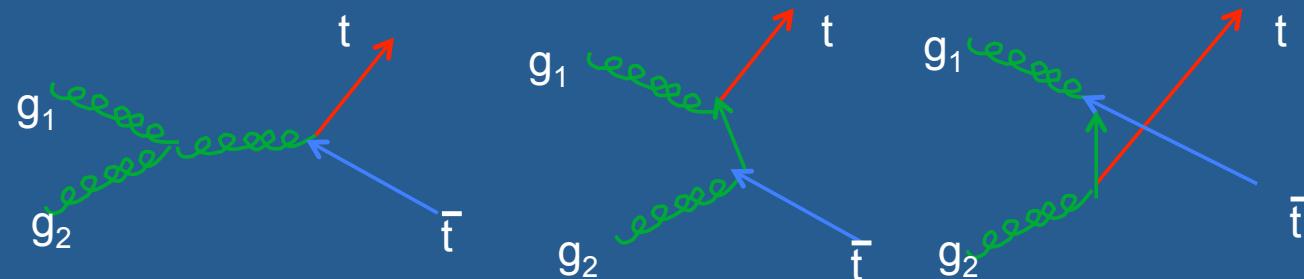
p→g(polzn x or y,p_{2T}) :f₁g(p₂) or h_{1T} ⊥g(p₂)

g₁(p₁) ↑↑ + g₂(p₂) ↓↓ → t(k₁) + tbar (k₂)

$$A_{\Lambda_{g1}, \Lambda_{g2}; t, \bar{t}}|_{(j, k)}^{(b, c)} = \left[\frac{(\lambda^b \lambda^c)_{jk}}{(m_t^2 - \hat{t})} a_{\Lambda_{g1}, \Lambda_{g2}; t, \bar{t}}^t + \frac{(\lambda^c \lambda^b)_{jk}}{(m_t^2 - \hat{u})} a_{\Lambda_{g1}, \Lambda_{g2}; t, \bar{t}}^u \right]$$

At LHC:

Gluon fusion tree level mechanism
(Color gauge invariance)



g_1, g_2 carry helicity $\Lambda_1 \Lambda_2 = \pm 1$

$t, t\bar{}$ carry helicity $\lambda_t \lambda_{t\bar{}} = \pm \frac{1}{2}$

Introduced in:

G.R.Goldstein, ``Spin Correlations in Top Quark Production and the Top Quark Mass'' in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

R.H. Dalitz, G.R. Goldstein and R. Marshall, "Heavy Quark Spin Correlations in e^+e^- -annihilations", Phys. Lett. B215, 783 (1988);

R.H. Dalitz, G.R. Goldstein and R. Marshall, "On the Helicity of Charm Jets", Zeits.f. Phys. C42, 441 (1989).

Amplitudes for $p_1 + p_2 \rightarrow t + X_1 + \bar{t} + X_2$

$$g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda_{g1}}^{(1)} g_{\Lambda_{N2}, \Lambda_{X2}, \Lambda_{g2}}^{(2)} A_{\Lambda_{g1}, \Lambda_{g2}; t, \bar{t}}$$

Implicitly convoluted over k_{T1} & k_{T2}

Differential cross section for inclusive $p_1 + p_2 \rightarrow t + X_1 + \bar{t} + X_2$

$$\sum \int_{X_1, X_2} g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda_{g1}}^{(1)} g_{\Lambda_{N2}, \Lambda_{X2}, \Lambda_{g2}}^{(2)} A_{\Lambda_{g1}, \Lambda_{g2}; t, \bar{t}} g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda'_{g1}}^{(1)*} g_{\Lambda_{N2}, \Lambda_{X2}, \Lambda'_{g2}}^{(2)*} A_{\Lambda'_{g1}, \Lambda'_{g2}; t', \bar{t}'}$$

Regrouping terms gives gluon distributions

$$\begin{aligned} & \sum_{\Lambda_{g1}, \Lambda_{g2}, \Lambda'_{g1}, \Lambda'_{g2}} \left(\sum_{\Lambda_{N2}, \Lambda_{X2}} \int_{X_2} g_{\Lambda_{N2}, \Lambda_{X2}, \Lambda'_{g2}}^{(2)*} g_{\Lambda_{N2}, \Lambda_{X2}, \Lambda_{g2}}^{(2)} \right) \\ & \times \left(\sum_{\Lambda_{N1}, \Lambda_{X1}} \int_{X_1} g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda'_{g1}}^{(1)*} g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda_{g1}}^{(1)} \right) A_{\Lambda'_{g1}, \Lambda'_{g2}; t', \bar{t}'}^* A_{\Lambda_{g1}, \Lambda_{g2}; t, \bar{t}} \end{aligned}$$

Polarized hard gluon fusion forms t+t̄ amplitudes & density matrices in terms of helicities

$$G_{\Lambda_{N1}, \Lambda_{g1}, \Lambda'_{g1}}^{(1)} = \sum_{\Lambda_{X1}} \int_{X_1} g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda'_{g1}}^{(1)*} g_{\Lambda_{N1}, \Lambda_{X1}, \Lambda_{g1}}^{(1)}$$

(off)diagonal gluon distributions

$$\begin{aligned} \rho_{t', \bar{t}'; t, \bar{t}} = & \sum_{\Lambda_{N1}, \Lambda_{N2}} \{ G_{\Lambda_{N2}, UP}^{(2)} \rho_{t', \bar{t}'; t, \bar{t}}^{UP, UP} G_{\Lambda_{N1}, UP}^{(1)} + G_{\Lambda_{N2}, UP}^{(2)} \rho_{t', \bar{t}'; t, \bar{t}}^{UP, LP} G_{\Lambda_{N1}, LP}^{(1)} \\ & + G_{\Lambda_{N2}, LP}^{(2)} \rho_{t', \bar{t}'; t, \bar{t}}^{LP, UP} G_{\Lambda_{N1}, UP}^{(1)} + G_{\Lambda_{N2}, LP}^{(2)} \rho_{t', \bar{t}'; t, \bar{t}}^{LP, LP} G_{\Lambda_{N1}, LP}^{(1)} \} \end{aligned}$$

$$\begin{aligned} \rho_{t', \bar{t}'; t, \bar{t}}^{UP, UP} &= [A_{RR, t', \bar{t}}^* A_{RR, t, \bar{t}} + A_{RL, t', \bar{t}}^* A_{RL, t, \bar{t}} + A_{LR, t', \bar{t}}^* A_{LR, t, \bar{t}} + A_{LL, t', \bar{t}}^* A_{LL, t, \bar{t}}] \\ \rho_{t', \bar{t}'; t, \bar{t}}^{UP, LP} &= [A_{LR, t', \bar{t}}^* A_{RR, t, \bar{t}} + A_{LL, t', \bar{t}}^* A_{RL, t, \bar{t}} + A_{RR, t', \bar{t}}^* A_{LR, t, \bar{t}} + A_{RL, t', \bar{t}}^* A_{LL, t, \bar{t}}] \\ \rho_{t', \bar{t}'; t, \bar{t}}^{LP, UP} &= [A_{RL, t', \bar{t}}^* A_{RR, t, \bar{t}} + A_{RR, t', \bar{t}}^* A_{RL, t, \bar{t}} + A_{LL, t', \bar{t}}^* A_{LR, t, \bar{t}} + A_{LR, t', \bar{t}}^* A_{LL, t, \bar{t}}] \\ \rho_{t', \bar{t}'; t, \bar{t}}^{LP, LP} &= [A_{LL, t', \bar{t}}^* A_{RR, t, \bar{t}} + A_{LR, t', \bar{t}}^* A_{RL, t, \bar{t}} + A_{RL, t', \bar{t}}^* A_{LR, t, \bar{t}} + A_{RR, t', \bar{t}}^* A_{LL, t, \bar{t}}] \end{aligned}$$

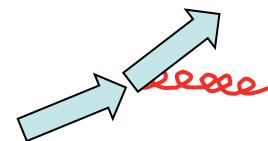
Combine g distributions with hard gluon fusion t+tbar amplitudes

$$\begin{aligned} G_{\Lambda_{N1},R,R}^{(1)} + G_{\Lambda_{N1},L,L}^{(1)} &= G_{\Lambda_{N1},XX}^{(1)} + G_{\Lambda_{N1},YY}^{(1)} = G_{\Lambda_{N1},UP}^{(1)} \\ G_{\Lambda_{N1},R,L}^{(1)} + G_{\Lambda_{N1},L,R}^{(1)} &= G_{\Lambda_{N1},YY}^{(1)} - G_{\Lambda_{N1},XX}^{(1)} = G_{\Lambda_{N1},LP}^{(1)} \end{aligned}$$

Linearly polarized gluon distributions arise naturally
in the heavy pair production
Simple spin structure

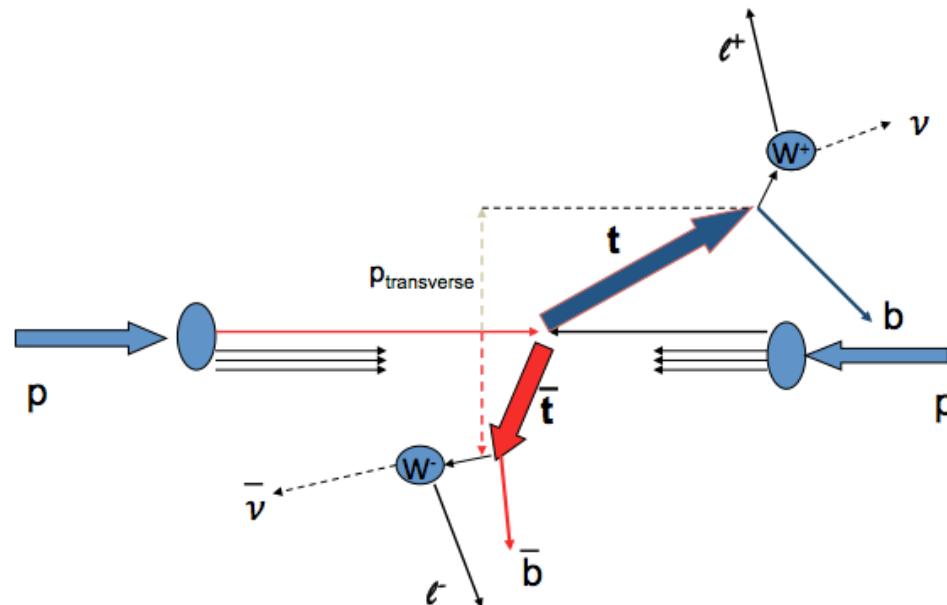
$$\mathcal{G}_{\Lambda'; \Lambda_g, \Lambda}(x, \vec{k}_T^2) = \Gamma(\vec{k}_T^2) \bar{U}_{\Lambda'}(p') \epsilon_{\Lambda_g}^j(k) \gamma_j U_{\Lambda}(p)$$

$$\rho_{\Lambda'_g, \Lambda_g} = \sum_{\Lambda', \Lambda} \mathcal{G}_{\Lambda'; \Lambda'_g, \Lambda}^* \mathcal{G}_{\Lambda'; \Lambda_g, \Lambda}$$

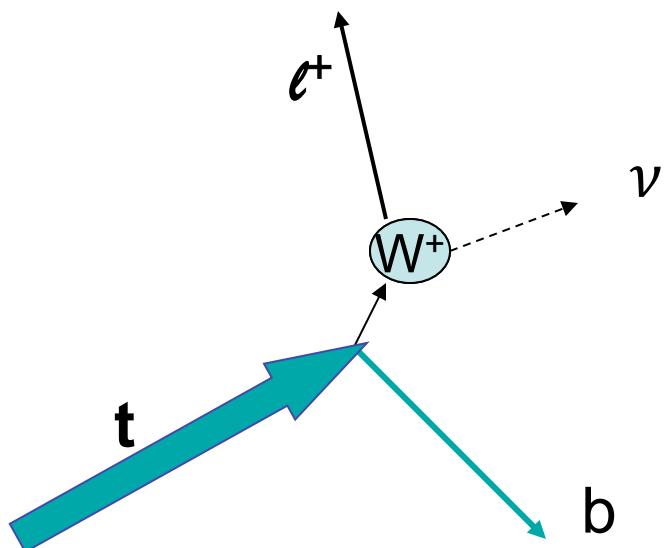


$$q+q\text{-bar} \rightarrow t + t\text{-bar}$$

- The quark spin correlations are transmitted to the decay products.
- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.
- Correlations expressed as a weighting factor.



How is actual top polarization determined?
Its decay is good analyzer.



$$U_{t,\bar{t}} = \sum_{\lambda_b} B_{\lambda_b, \bar{t}}^* B_{\lambda_b, t}$$
$$\propto (I + \vec{p}_{\bar{t}} \cdot \vec{\sigma}_t / p_{\bar{t}})_{t,\bar{t}} (p_b \cdot p_\nu),$$

Calculated in top rest frame

q+q-bar → t + t-bar

- The light quark-antiquark annihilation mechanism gives rise to the angular distribution between opposite charge lepton pairs,

$$W(\theta, p, p_{\bar{l}}, p_l) = \frac{1}{4} \left\{ 1 + [sin^2\theta([p^2 + m^2](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2 - m^2](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}}) - 2mpcos\theta sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}) + ([p^2 - m^2] + [p^2 + m^2]cos^2\theta)(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}]/[(p^2 + m^2) + (p^2 - m^2)cos^2\theta] \right\}$$

m = top mass, θ = t production angle in q+q-bar CM

p= light quark 3-momentum in CM

Unit vectors p-hat are anti-lepton⁺ and lepton⁻ 3-momenta directions in the top and anti-top rest frames.

See G.R.Goldstein, ``Spin Correlations in Top Quark Production and the Top Quark Mass''

in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

$g_1 + g_2 \rightarrow t + t\text{-bar}$ Spin correlations

Correlations expressed as a weighting factor **for unpolarized gluons**.

- The **gluon fusion mechanism** gives rise to a higher order angular distribution due to the combination of two spin 1 gluons.

$$\begin{aligned} W(\theta, p, p_{\bar{l}}, p_l) = & \frac{1}{4} - \frac{1}{4} \{ [p^4 \sin^4 \theta + m^4] (\hat{p}_{\bar{l}})_x (\hat{p}_l)_{\bar{x}} + [p^2(p^2 - 2m^2) \sin^4 \theta - m^4] (\hat{p}_{\bar{l}})_y (\hat{p}_l)_{\bar{y}} \\ & + [p^4 \sin^4 \theta - 2p^2(p^2 - m^2) \sin^2 \theta + m^2(2p^2 - m^2)] (\hat{p}_{\bar{l}})_z (\hat{p}_l)_{\bar{z}} \\ & + 2mp^2 \sqrt{p^2 - m^2} \cos \theta \sin^3 \theta [(\hat{p}_{\bar{l}})_x (\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z (\hat{p}_l)_{\bar{x}}] \} \\ & / [p^2(2m^2 - p^2) \sin^4 \theta + 2p^2(p^2 - m^2) \sin^2 \theta] + m^2(2p^2 - m^2) \end{aligned}$$

Use these to test SM vs. BSM – Integrated version agrees –
with big errors

GG – also Mahlon & Parke

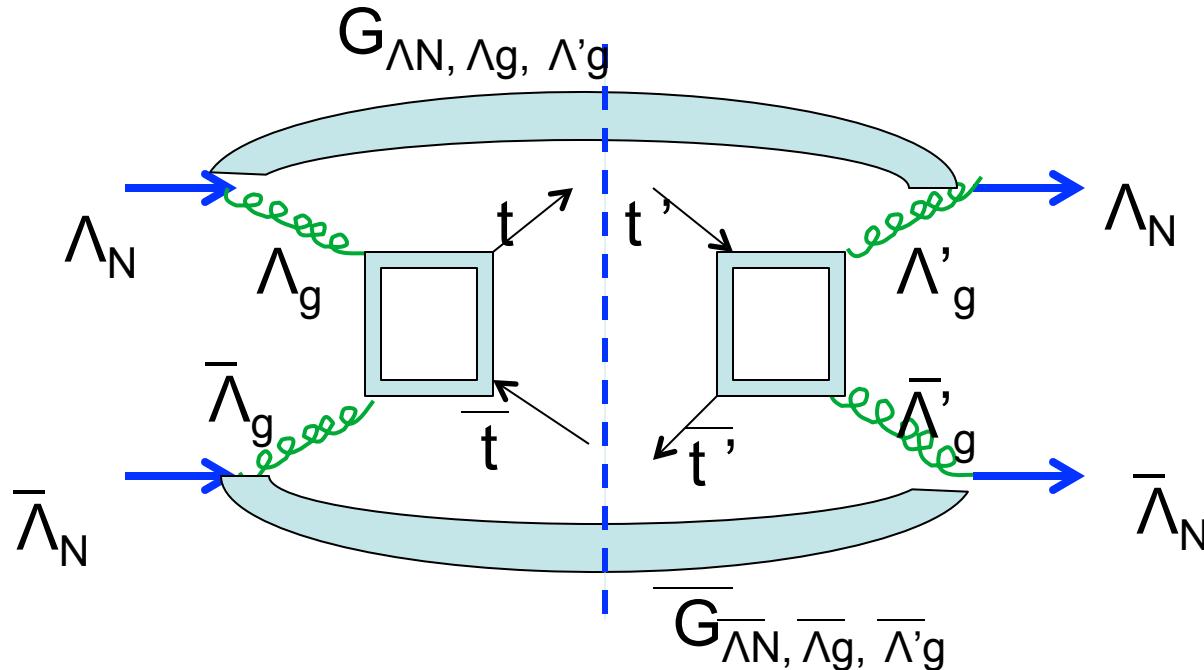
$$\rho_{t',\bar{t}';t,\bar{t}} \propto \sum_{all-helicities-not-tops} \bar{G}_{\bar{\Lambda}_N \bar{\Lambda}_g \bar{\Lambda}'_g} A^*_{\Lambda'_g \bar{\Lambda}'_g; t', \bar{t}'} A_{\Lambda_g \bar{\Lambda}_g; t, \bar{t}} G_{\Lambda_N \Lambda_g \Lambda'_g}$$

- The gluon spin correlations are transmitted to (determine the spin of) the decay products.
- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.
- The **gluon fusion mechanism** gives rise to a higher order (wrt quark antiquark) angular distribution due to the combination of two spin 1 gluons.

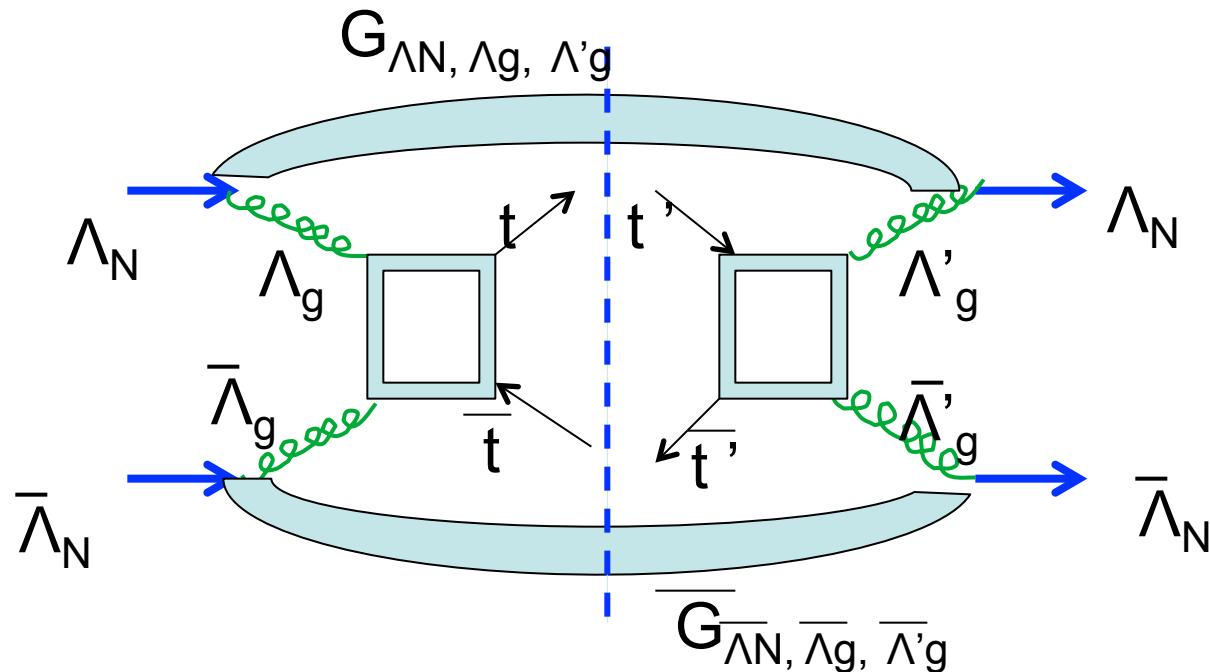
G.R.Goldstein, ``Spin Correlations in Top Quark Production and the Top Quark Mass'' in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328

Helicity structure of polarized top-anti-top production

$$\sum_{\text{all-helicities-not-tops}} \bar{G}_{\bar{\Lambda}_N \bar{\Lambda}_g \bar{\Lambda}'_g} A_{\Lambda'_g \bar{\Lambda}'_g; t', \bar{t}'}^* A_{\Lambda_g \bar{\Lambda}_g; t, \bar{t}} G_{\Lambda_N \Lambda_g \Lambda'_g} \\ \propto \rho_{t', \bar{t}'; t, \bar{t}}$$



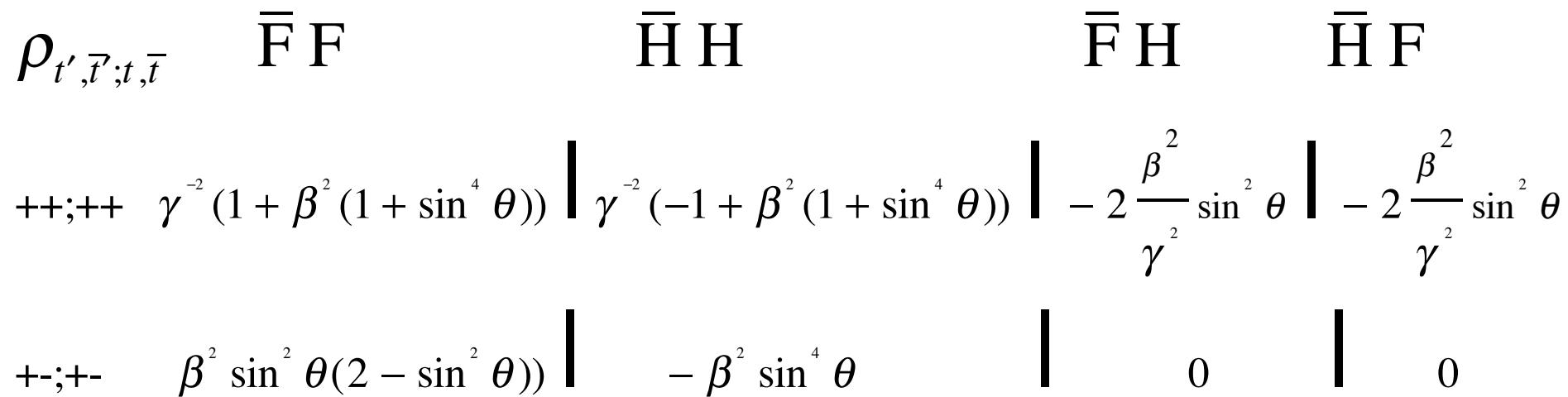
Observable cross section with polarized $t + t\text{-bar}$



*Use top pair polarization to determine
the polarized gluon distributions*

Gluon linear polarization with like and unlike t-tbar helicities (work in progress S.Liuti & GG)

F~G_{XX}+G_{YY}, H~G_{XX}-G_{YY} or linear polarization



Top spin correlations & gluon polarizations

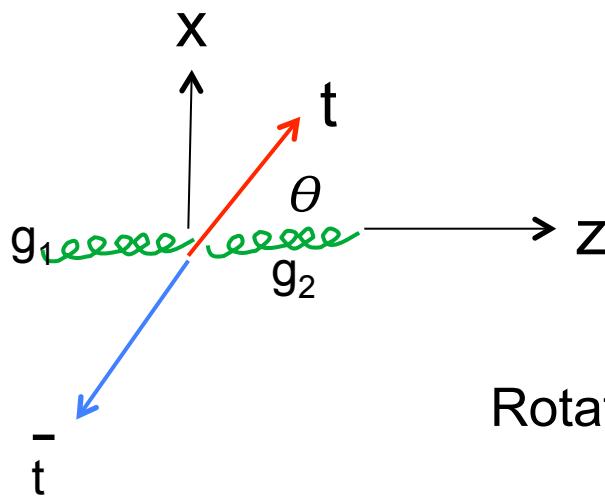
$\rho_{t',\bar{t}';t,\bar{t}}$	UP,UP	LP,LP	UP,LP + LP,UP
++, ++	$\gamma^{-2}(1 + \beta^2(1 + \sin^4\theta))$	$\gamma^{-2}(-1 + \beta^2(1 + \sin^4\theta))$	$-4\gamma^{-2}\beta^2\sin^2\theta$
+-, +-	$\beta^2\sin^2\theta(2 - \sin^2\theta)$	$-\beta^2\sin^4\theta$	0
++, ---	$\gamma^{-2}(-1 + \beta^2(1 + \sin^4\theta))$	$\gamma^{-2}(+1 + \beta^2(1 + \sin^4\theta))$	$+4\gamma^{-2}\beta^2\sin^2\theta$
+-, -+	$\beta^2\sin^4\theta$	$-\beta^2\sin^2\theta(2 - \sin^2\theta)$	0
++, +-	$-2\gamma^{-1}\beta^2\sin^3\theta$	$-2\gamma^{-1}\beta^2\sin^3\theta$	$-4\gamma^{-1}\beta^2\sin\theta\cos\theta$
++, -+	$2\gamma^{-1}\beta^2\sin^3\theta\cos\theta$	$2\gamma^{-1}\beta^2\sin^3\theta\cos\theta$	$4\gamma^{-1}\beta^2\sin\theta\cos\theta$

TABLE I: Values of double density matrix elements ρ for combinations in Eq. 10 using values of helicity amplitudes from Eq. 12

UP = unpolarized, LP = Linearly polarized gluon distributions
assuming $g+g \rightarrow t + t\bar{}$ in single plane CM

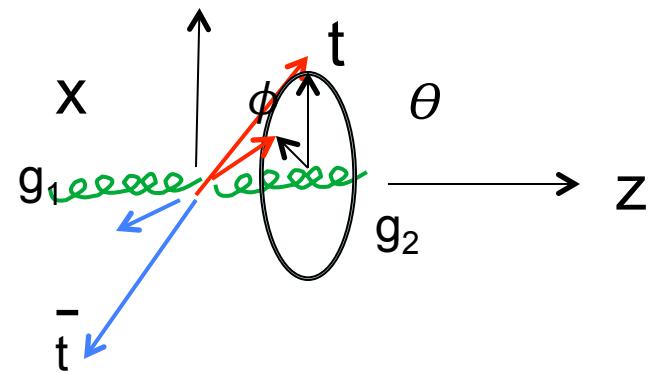
Taking X-Z plane for $p+p \rightarrow (t+t\bar{})_{CM} + X$ gives ϕ dependence to $t+t\bar{}$ plane for opposite helicities: $Re(e^{\pm(1\text{or}2)i\phi} \cdot e^{\pm(-i(1\text{or}2)\phi)})$
leading to $\cos 2\phi$ for UP,LP and LP,UP and $\cos 4\phi$ modulations for LP,LP.

Azimuthal dependence



Evaluated $g+g \rightarrow t+t\bar{t}$
in CM X-Z plane

Rotate by ϕ



Rotate 2-spinors
Matrix element rotates
Or gluon x & y rotate.
Amplitude phases

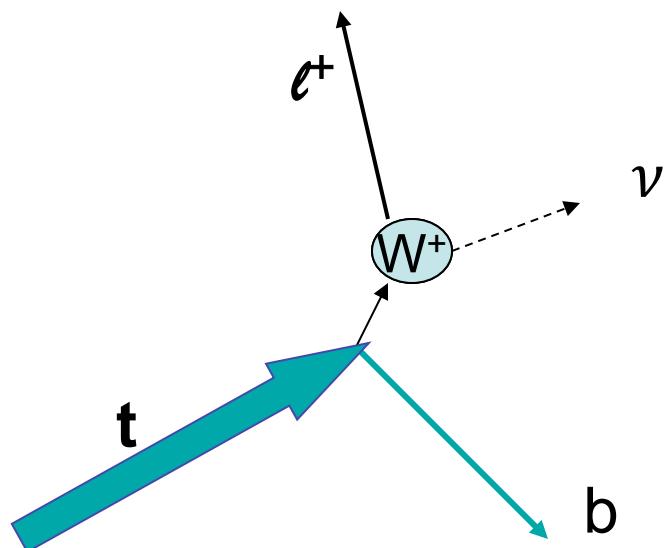
$$\chi_\lambda(k, \theta, \phi) = e^{-i\sigma_z \frac{\phi}{2}} e^{-i\sigma_y \frac{\theta}{2}} e^{+i\sigma_z \frac{\phi}{2}} \chi_\lambda(k, 0, 0)$$

$$\chi_t^\dagger(k, \theta, \phi) \mathcal{M} \chi_{\bar{t}}(k, \pi - \theta, \pi + \phi),$$

$$e^{-i\sigma_z \phi} \sigma^\pm e^{+i\sigma_z \phi} = \pm e^{\mp 2i\phi} \sigma^\pm.$$

→ $\cos 2\phi$ & $\cos 4\phi$
modulations of $t+t\bar{t}$ angular distributions, depending on helicities

Multiply each configuration of t & tbar by actual top polarization decay as a good analyzer.



$$U_{t,\bar{t}} = \sum_{\lambda_b} B_{\lambda_b, \bar{t}}^* B_{\lambda_b, t}$$
$$\propto (I + \vec{p}_{\bar{t}} \cdot \vec{\sigma}_t / p_{\bar{t}})_{t,\bar{t}} (p_b \cdot p_\nu),$$

Provides unique method
to decompose gluon
distributions.
Each row in table of top-antitop
density matrix elements gets
distinct kinematic variation

...also....Boer, Brodsky, C. Pisano (PRL 2012, JHEP2013) have considered the determination of polarized gluon distributions from azimuthal distributions of unpolarized $Q\bar{Q}$ production in SIDIS and p+p.

This amounts to summing the columns of the table

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However, we consider the **polarizations** of the $t-t\bar{t}$ as levers to differentiate between like pairs of gluon polarizations and unlike pairs

Separating rows AND columns

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However, we consider the **polarizations** of the $t\bar{t}$ as levers to differentiate between like pairs of gluon polarizations and unlike pairs

Separating rows AND columns

Our method of separating rows is new. Mahlon and Parke , (PRD81, 074024 (2010)) took integrated spin correlation measure which was confirmed by D0 & now at ATLAS. We single out gluon distributions by separating different $t+t\bar{t}$ helicities (instead of just overall polarizations)

Work being completed

Summary

- Hyperon polarization is touchstone for understanding transversity & hence NPQCD
- Several ways to begin to explain phenomena
 - “Upside down” TMDs with f.s.i.
 - Extended GPDs → Extended Fracture Functions
 - Vorticity & OAM: Work in progress
- How to see quark or gluon polarization?
- top quarks as a probe of SSA sources
- & $t+t\bar{t}$ spin correlations - window into linearly polarized gluon distributions