What is breaking the Wandzura-Wilczek relation?

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Outline

- **Introduction**
  - WW in the OPE
- **WW in QCD factorization**
  - Lorentz invariance relations
  - Equations of motion
  - What is breaking the WW? And the BC?
- **Size of the breaking terms**
  - DIS on a quark target
  - Experimental data
- **Experimental outlook**
- **Conclusions**

Introduction

Issues at low Q^2

- At large Q^2, DIS cross section scale with $x_B = Q^2/(2m_N\nu)$ modulo logarithmic corrections (QCD evolution)

- At low Q^2, power suppressed contributions become important:
  1) higher twist terms (quark-gluon correlations) $\propto \Lambda/Q$ ← **this talk**
  2) target mass corrections (TMC) $\propto x_B^2\, m_N^2/Q^2$
  3) jet mass corrections (JMC) $\propto m_j^2/Q^2$
  4) ...

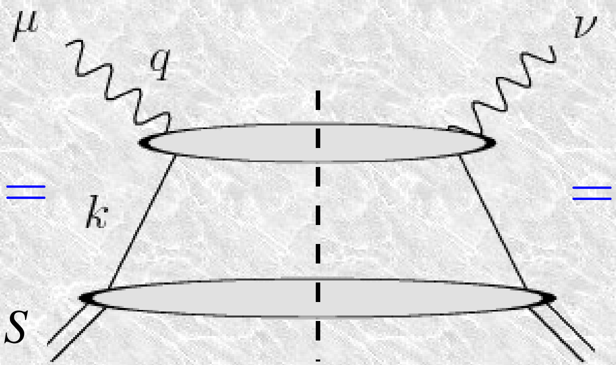
  [Accardi,Qiu & Accardi,Melnitchouk '08] (for items 2–3)

- Higher-twist terms need to be identified in experimental data:
  - verify quark-hadron duality
  - measure the (twist-2) parton distributions at low-Q^2, large-x
    [e.g., d/u and $\Delta d/d$ at $x \to 1$]
  - measure multiparton correlations: hadron structure beyond PDFs
    [e.g., d_2 – see Burkhardt's talk]

The g_2 structure function

- DIS cross section determined by the hadronic tensor

$$W^{\mu\nu}(q,P,S) =$$



$$= \frac{1}{8\pi}\int d^4z\, e^{-iq\cdot z}\langle P,S|j^{\dagger\mu}(z)j^{\nu}(0)|P,S\rangle$$

- Inclusive DIS structure functions:

$$\begin{aligned} W^{\mu\nu}(p,q) = &\Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big)F_1(x_B,Q^2) \\ &+ \Big(p^\mu - q^\mu\frac{p\cdot q}{q^2}\Big)\Big(p^\nu - q^\nu\frac{p\cdot q}{q^2}\Big)\frac{F_2(x_B,Q^2)}{p\cdot q} \\ &+ \frac{1}{p\cdot q}\varepsilon^{\mu\nu\rho\sigma}q_\rho\Big[S_\sigma g_1(x_B,Q^2) + \Big(S_\sigma - \frac{S\cdot q}{p\cdot q}p_\sigma\Big)g_2(x_B,Q^2)\Big] \end{aligned}$$

The g_2 structure function

- g_2 is a special structure function:
  - it is the only one with twist-3 contributions that can be measured in inclusive DIS
  - in the OPE analysis, its twist-3 term can be isolated:

$$g_2(x) = \underbrace{g_2^{WW}(x)}_{\text{“Wandzura-Wilczek relation”}} + \underbrace{\delta(x)}_{\text{“pure twist-3 term”}}$$

$$g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$

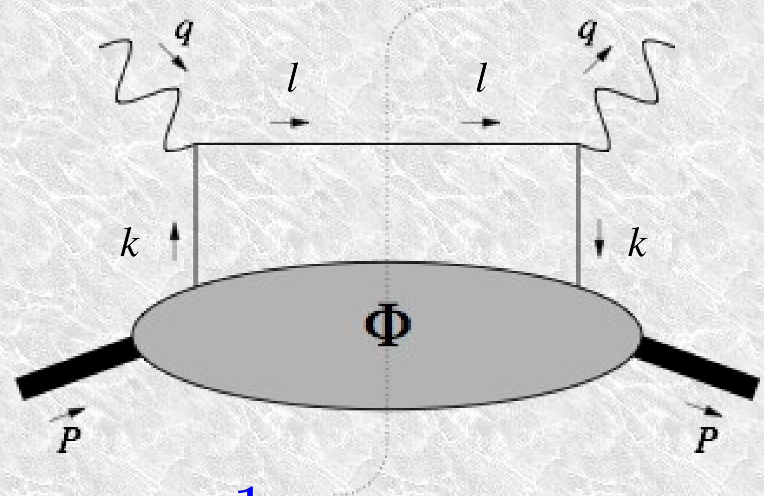
moments are matrix elements of twist-3 operators

WW in QCD factorization

Background material:
“Transverse thinking: an introduction to TMDs"
mini lecture series by A.Bacchetta

Correlation functions in DIS

- At Leading Order in the strong coupling constant,

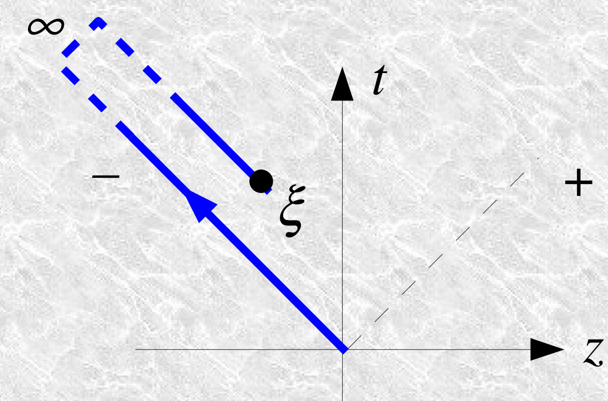


$$W^{\mu\nu}(q,P,S) = \frac{1}{8\pi}\sum_q e_q^2 \mathrm{Tr}\left[\Phi(x_B,S)\gamma^\mu\gamma^+\gamma^\nu\right]$$

- Quark correlator:

$$\Phi_{ij}^a(k,P,S,n_-) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\langle P,S\,|\,\bar{\psi}^a{}_j(0)\,\mathcal{W}(0,\xi|n_-)\,\psi_i^a(\xi)\,|\,P,S\rangle$$

Wilson line
(n_- = light-cone vector)



Correlation functions in DIS

- Lorentz / Dirac decomposition of the quark correlator
  - available vectors: $k^\mu,\ P^\mu,\ S^\mu,\ n_-^\mu$
  - available Dirac matrices: $1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, i\sigma^{\mu\nu}\gamma_5$ $\qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$
  - Hermiticty, parity constraints

$$\begin{aligned}\Phi^a(k,P,S,n_-) = {} & M \not{S}\gamma_5 A_6 + \frac{(k\cdot S)}{M}\not{P}\gamma_5 A_7 + \frac{(k\cdot S)}{M}\not{k}\gamma_5 A_8 \\ & + M\frac{(S\cdot n_-)}{(P\cdot n_-)}\not{P}\gamma_5 B_{11} + M\frac{(S\cdot n_-)}{(P\cdot n_-)}\not{k}\gamma_5 B_{12} \\ & + M\frac{(k\cdot S)}{(P\cdot n_-)}\not{n}_-\gamma_5 B_{13} + M^3\frac{(S\cdot n_-)}{(P\cdot n_-)^2}\not{n}_-\gamma_5 B_{14} \\ & + \ldots\end{aligned}$$

see, e.g., Mulders, Tangerman, NPB 461 (96)
Goeke, Metz, Schlegel, PLB 618 (05)

A and *B* are called “parton correlation functions”.

- Note: the *B* terms appear in the n_--dependent part of the decomposition
  - in the past (before 2005) they have been neglected [see Mulders et al.]

Parton distributions

- Transverse momentum dependent (TMD) quark distributions are parts of suitable Dirac projections of the k^--integrated correlator:

$$\Phi^a(x,\vec{k}_T) = \int dk^- \, \Phi(k,P,S,n_-)$$
$$= \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} \, e^{ik\cdot\xi} \langle P,S\,|\,\bar{\psi}^a(0)\,\mathcal{W}(0,\xi|n_-)\,\psi^a(\xi)\,|\,P,S\rangle\Big|_{\xi^+=0}$$

where $x = k\cdot n_-/P\cdot n_-$ is the parton's fractional momentum

- Define $\Phi^{[\Gamma]} = \mathrm{Tr}[\Gamma\,\Phi]/2$, then the relevant TMDs are

$$\Phi^{a[\gamma^+\gamma_5]}(x,\vec{k}_T) = S_L\, g_1^a(x,\vec{k}_T^2) + \frac{\vec{k}_T\cdot\vec{S}_T}{M}\, g_{1T}^a(x,\vec{k}_T^2)$$
$$\Phi^{a[\gamma^i\gamma_5]}(x,\vec{k}_T) = \frac{M}{P^+} S_T^i\, g_T^a(x,\vec{k}_T^2) + \dots$$

- Collinear PDFs, are defined by transverse momentum integration

$$g_\sharp(x) = \int d^2k_T\, g_\sharp(x,\vec{k}_T) \qquad g_\sharp^{(1)}(x) = \int d^2k_T\, \frac{\vec{k}_T^2}{2M} g_\sharp(x,\vec{k}_T)$$

Parton distributions

- In particular, in terms of the A and B correlations,

$$g_1^a(x,\vec{k}_T^2) = \int dk^- \Big(- A_6^a - B_{11}^a - xB_{12}^a - \tfrac{k\cdot P - 2xM^2}{2M^2}(A_7^a + xA_8^a)\Big)$$

$$g_{1T}^a(x,\vec{k}_T^2) = \int dk^- \Big(A_7^a + xA_8^a\Big)$$

$$g_T^a(x,\vec{k}_T^2) = \int dk^- \Big(-A_6^a - \frac{\vec{k}_T^2}{2M^2}A_8^a\Big)$$

- **Note:**
  - $g_1(x)$ is the only collinear PDF where B terms survive k_T integration
  - from now on we will only discuss collinear PDFs

Lorentz Invariance Relation

- There is a relation between $g_1(x)$, $g_{1T}(x)$ and $g_T(x)$:

$$g_T^a(x) = \underbrace{g_1^a(x) + \frac{d}{dx}\, g_{1T}^{a(1)}}_{\text{“Lorentz Invariance Relation”}}(x) + \underbrace{\hat{g}_T^a(x)}_{\text{LIR breaking term}}$$

where

$$\hat{g}_T^a(x) = \int dk^- d^2k_T \left[B_{11}^a + x B_{12}^a - \frac{\mathsf{k}_T^2}{2M^2}\left(\frac{\partial A_7^a}{\partial x} + x\frac{\partial A_8^a}{\partial x}\right)\right]$$

$$+ \pi \int dk^- k_T^2 \frac{k\cdot P - 2xM^2}{2M^2}\left(A_7^a + x A_8^a\right)\Big|_{\vec{k}_T^2\to 0}^{\vec{k}_T^2\to\infty}$$

- **Note:** the above LIR was so named in the pre-2005 era
  - The *B* where neglected, as well as the explicit dependence of *A* on *x* and the surface terms – then $\hat{g}_T = 0$
  - but in a perturbative computation both *B* and surface terms are non-zero (see later) !

Lorentz Invariance Relation

- The operator definition of $\hat{g}_T$ was in fact obtained 25 years ago:
  Bukhvostov, Kuraev, Lipatov, '83-'84 (see refs. in Belitsky, hep-ph/9703432)

$$\hat{g}_T(x) = \int dx' \, \frac{\overline{D}(x,x') + \overline{D}(x',x)}{x'-x}$$

where

$$\overline{D}(x,x') = \tfrac{1}{2}\left[\overline{D}_1(x,x') + \overline{D}_2(x',x)\right]$$

and, in light-cone gauge,

$$\frac{M}{P^+} S_T^i \overline{D}_1(x,x') = -\frac{g_s}{8}\int \frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}\, e^{ik\cdot\xi - ik'\cdot\eta} \langle P,S|\bar{\psi}(\eta)\gamma^+ \not{A}_T(0)\gamma^i\gamma_5\psi(\xi)|P,S\rangle$$

$$\frac{M}{P^+} S_T^i \overline{D}_2(x',x) = -\frac{g_s}{8}\int \frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}\, e^{ik'\cdot\eta - ik\cdot\xi} \langle P,S|\bar{\psi}(\xi)\gamma^+ \gamma^i \not{A}_T(0)\gamma_5\psi(\eta)|P,S\rangle$$

- Hence $\hat{g}_T$ is a pure twist-3 function

Equations Of Motion relation

- The Dirac equation of motion $\not{D}\psi - m\psi = 0$ implies

$$g_{1T}^{a(1)}(x) = x g_T^a(x) - x\tilde{g}_T^a(x) - \frac{m}{M}\, h_1^a(x)$$

where $\hat{g}_T$ is a pure twist-3 function: in light-cone gauge,

$$\begin{aligned}\tilde{g}_T^a(x) &= \frac{1}{x}\int dx'\, D(x,x') \\ &= \frac{g_s}{2x}\frac{P^+}{M S_T^i}\int \frac{d\xi^-}{2\pi}\langle P,S|\bar{\psi}^a(0)\gamma^+\Big(\not{A}_T(0) - \not{A}_T(\xi)\Big)\gamma_5\,\psi^a(\xi)|P,S\rangle\end{aligned}$$

- **Note:** $\hat{g}_T, \tilde{g}_T$ are different projection of the D(x,x') quark-gluon correlator

The Wandzura-Wilczek relation

- The LIR and the EOM imply the Wandura-Wilczek relation:
  - eliminate $\hat{g}_{1T}^{a(1)}$ between LIR and EOM, solve for g_T^a
  - use $g_1 = \frac{1}{2}\sum_a e_a^2 g_1^a$, $g_1 + g_2 = \frac{1}{2}\sum_a g_T^a$

$$g_2(x) = g_2^{WW}(x) + \tilde{\delta}(x) + \hat{\delta}(x) + \frac{m_q}{\Lambda}\delta_m(x)$$

can be neglected for light quarks

where the pure twist-3 term has 2 contributions:

$$\tilde{\delta}(x) = \frac{1}{2}\sum_a e_a^2 \int_x^1 \frac{dy}{y}\frac{d}{dy}\left[y\tilde{g}_T^a(y)\right]$$

$$\hat{\delta}(x) = \frac{1}{2}\sum_a e_a^2 \int_x^1 \frac{dy}{y}\hat{g}_T^a(y)$$

- **The WW relation is broken by 2 "pure twist-3" terms**
  - can in principle be large and canceling: need to measure separately
  - it is a first principles, model independent decomposition (Lorentz invariance, Dirac equations of motion)

The Burkhardt Cottingham sum rule

- Define the BC integral

$$\Gamma = \int_0^1 dx\, g_2 = \Gamma_{\rm WW} + \Gamma_{\text{twist-3}} + \dots$$

- **BC is satisfied by each twist separately**
  - if integrands are regular enough
  - at least up to twist-3

$$\Gamma = 0$$

with

$$\Gamma_{\rm WW} = 0$$
$$\Gamma_{\text{twist-3}} = 0$$

Size of the breaking terms

Experimental WW breaking

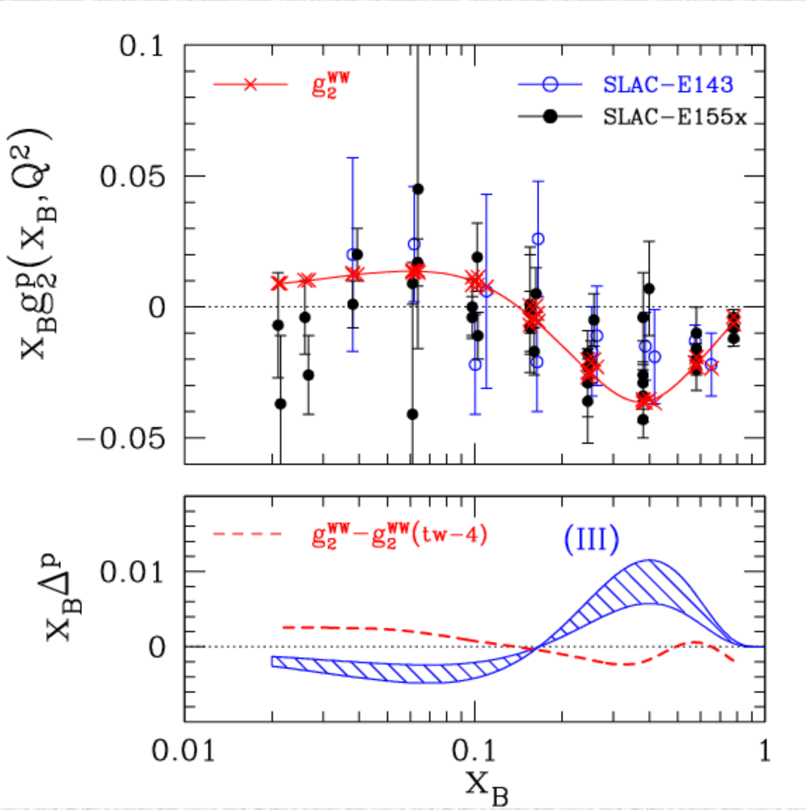
- g_2^{WW} is computed with the twist-2 part of g_1 from the LSS06 fit

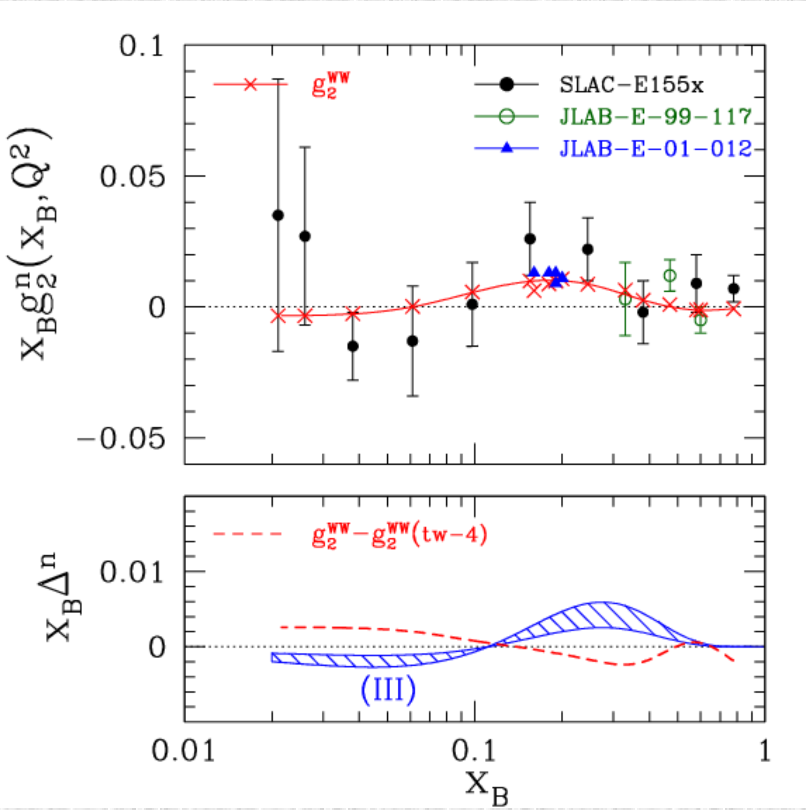
$$g_2^{\rm WW}(x,Q^2) = -g_1^{\text{tw-2}}(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^{\text{tw-2}}(y,Q^2)$$

with breaking term $\delta^{\rm exp.}(x,Q^2) = g_2^{\rm exp.}(x,Q^2) - g_2^{\rm WW}(x,Q^2)$

- Fit δ to functions that integrate to 0 [C free param, choose n to minimize χ^2]

$$F^{(n)}(x) = C\int_x^1 \frac{dy}{y}\frac{d}{dy}\left[y f^{(n)}(y)\right] \quad \text{with} \quad f^{(n)}(y) = y(1-y)^n$$





Experimental WW breaking

- Relative size of δ and g_2^{WW} (sign-changing function...)

$$r^2 = \frac{\int_{x_{min}}^{x_{max}} dy\, y\, \delta_{\rm th}^2(y)}{\int_{x_{min}}^{x_{max}} dy\, y\, \left[g_2^{\rm WW}\right]^2(y)}$$

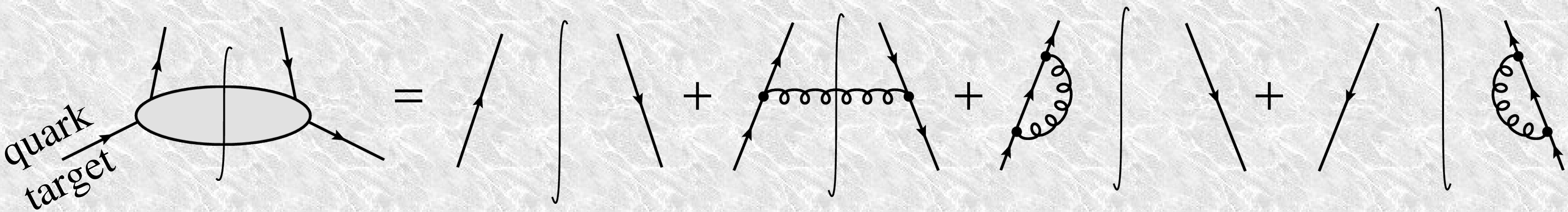
  - if $\delta_{th} = C\, g_2^{WW}$ then $r = C$

	proton	χ^2/d.o.f.	$r_{\rm tot}$	$r_{\rm low}$	$r_{\rm hi}$
(I)	$\Delta_{\rm th} = 0$	1.22			
(III)	$\Delta_{\rm th} = \alpha(1-x)^4(6x-1)$				
	$\alpha = 0.12 \pm 0.04$	1.03	16-29%	18-34%	15-28%
	neutron				
(I)	$\Delta_{\rm th} = 0$	1.66			
(III)	$\Delta_{\rm th} = \alpha(1-x)^7(9x-1)$				
	$\alpha = 0.10 \pm 0.04$	1.30	27-57%	21-30%	45-64%

- The **WW is broken at the 15-30% (30-60%) level** on the proton (neutron)
  - this is not small, contrary to standard claims in the literature
  - due to use of twist-2 part of g_1 in g_2^{WW}

DIS on a quark target

[Harindranath,Zhang, PLB409 ('97) 347, Kundu,Metz, PRD65 ('02) 014009]



- The pure twist-3 terms in the LIR and EOM $\left[\, C = \frac{\alpha_s}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \,\right]$

$$\tilde{g}_T(x) + \frac{m_q}{\Lambda}\frac{h(x)}{x} = \delta(1-x) + C\Big[\frac{1+x^2}{(1-x)_+} + 1 + x + \frac{1}{2}\delta(1-x)\Big]$$

$$\hat{g}_T(x) = C[1 - \delta(1-x)]$$

are of the same size of the other terms:

$$g_1(x) = \delta(1-x) + C\Big[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\Big]$$

$$g_T(x) = \delta(1-x) + C\Big[\frac{1+x^2}{(1-x)_+} + 2x + \frac{1}{2}\delta(1-x)\Big]$$

$$\hat{g}_{1T}^{(1)}(x) = -Cx(1-x)$$

- No a priori reason to assume either of them are small

How interesting are $\tilde{g}_T$ and $\hat{g}_T$?

Lorentz Invariance: $$g_T^a(x) = g_1^a(x) + \frac{d}{dx}\, g_{1T}^{a(1)}(x) + \hat{g}_T^a(x)$$

Eqs. of motion: $$g_{1T}^{a(1)}(x) = x g_T^a(x) - x\tilde{g}_T^a(x) - \frac{m}{M}\, h_1^a(x)$$

WW relation: $$g_2(x) = g_2^{WW}(x) + \tilde{\delta}(x) + \hat{\delta}(x)$$

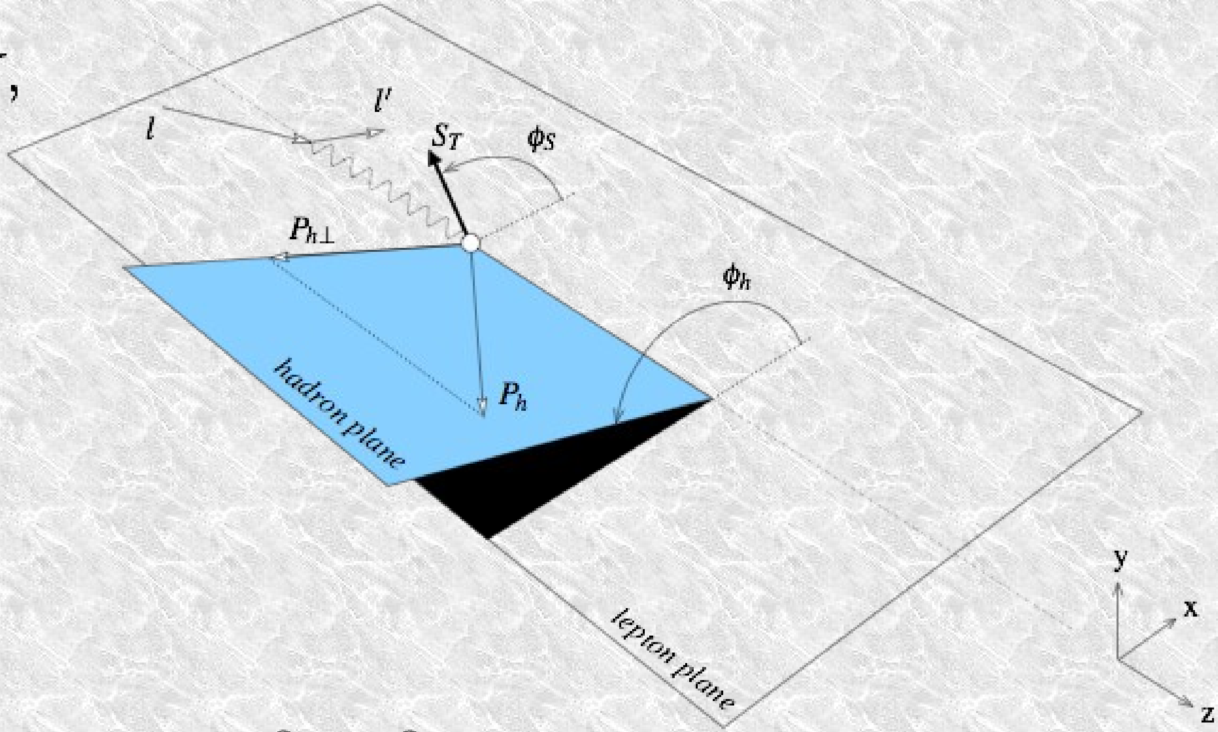
- $\hat{g}_T, \tilde{g}_T$ access different "projections" of $D(x,x')$
  - important for evolution of twist-3 quark-gluon correlators
  - The WW breaking term provides a sort of weighted combination

- 3 independent measurements (g_T, g_1, $g_{1T}^{(1)}$) for 2 independent relations:
  - check the whole picture / verify Lorentz-invariance, eqs. of motion
  - study physically interesting moments, e.g., d_2

How can we measure $\tilde{g}_T$ and $\hat{g}_T$?

- We need to measure ${g_{1T}}^{(1)}$: **double L-T spin asymmetry in SIDIS**

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$



$$x_B = \frac{Q^2}{2\,P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot l}$$

$$\gamma = \frac{2Mx_B}{Q} \qquad z_h = \frac{P\cdot P_h}{P\cdot q}$$

Long. pol. beam
Transv. pol. target

$$d\sigma_{LT} \propto y\sqrt{1-y}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}(x_B, z_h, P_{h\perp}^2, Q^2)$$
$$+\,y(1-y/2)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}(x_B, z_h, P_{h\perp}^2, Q^2)$$
$$+\,y\sqrt{1-y}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}(x_B, z_h, P_{h\perp}^2, Q^2)$$

$$\int \frac{|P_{h\perp}|}{z_h M}\,F_{LT}^{\cos(\phi_h-\phi_S)}(x_B, z_h, P_{h\perp}^2, Q^2) = \sum_q e_q^2\,x_B\,g_{1T}^{(1)q}(x_B)\,D_1^q(z_h)$$

Summary

- **WW is experimentally broken at the 20-60% level – not small**

- **Breaking term decomposes in 2 due to Lorentz Invariance and E.O.M.**

- **$\tilde{g}_T$ and $\hat{g}_T$ are both interesting**
  - give info on different parts of the quark-gluon correlator
  - needed to verify model indipendent features of the theory (LI & EOM)
  - complement the WW relation

- **Experimentally**
  - need higher precision g_2 data (${g_1}$ is OK for now)
  - need to measure $g_{1T}^{(1)}$: LT double asymmetry in SIDIS

The end

How can we measure $\tilde{g}_T$ and $\hat{g}_T$?

- We need to measure $g_{1T}{}^{(1)}$: **double L-T spin asymmetry in SIDIS**

$$\ell(l) + N(P) \to \ell(l') + X$$

$$x_B = \frac{Q^2}{2\,P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot l}$$

$$\gamma = \frac{2Mx_B}{Q}$$

$$d\sigma_{LT} \propto y\sqrt{1-y-\gamma^2 y^2/4}\,\cos\phi_S\, 2x_B\gamma\Big(g_1(x_B,Q^2)+g_2(x_B,Q^2)\Big)$$

Longitudinally pol. beam
Transversely pol. target