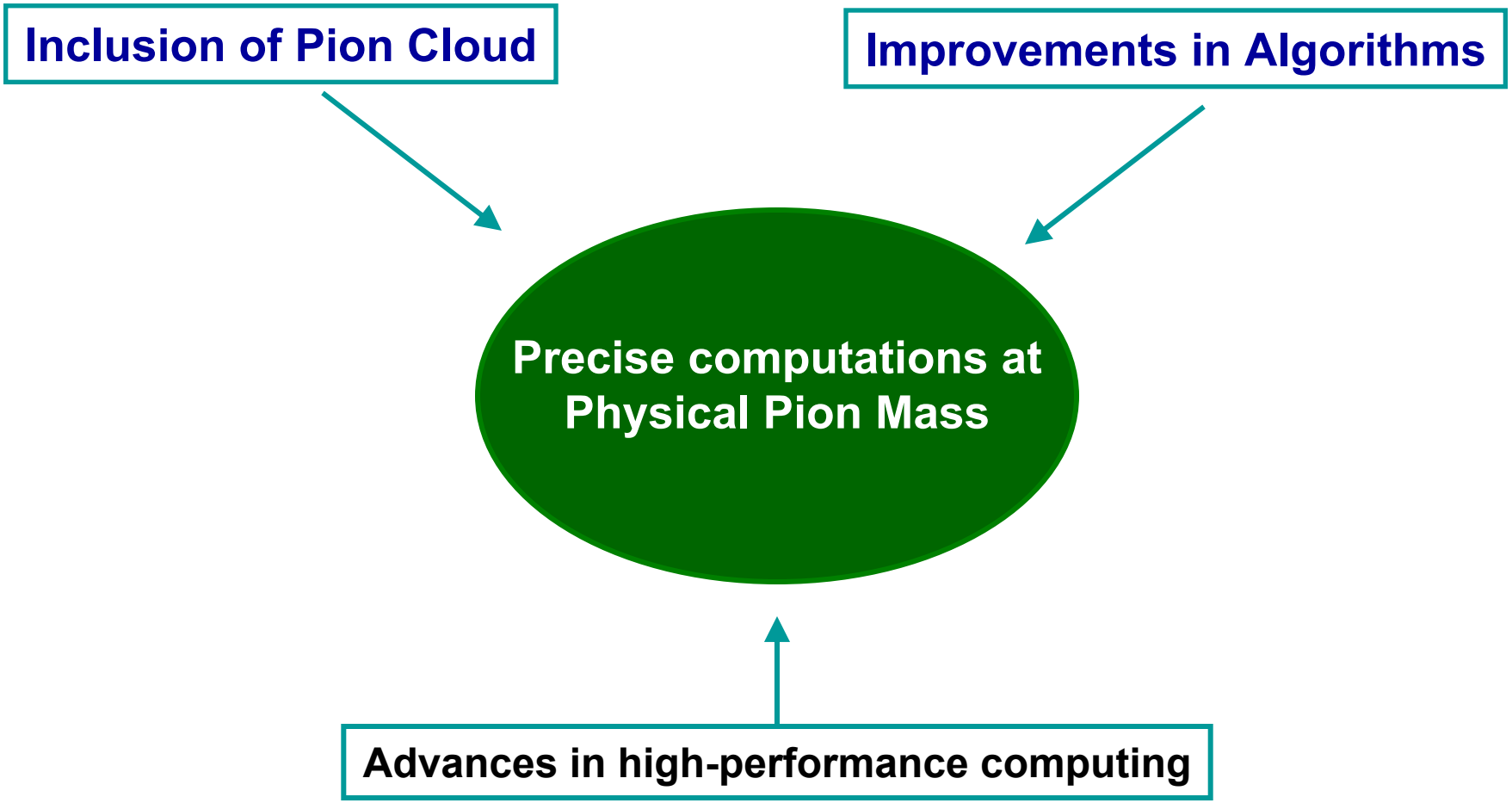
# Proton Structure from Lattice QCD

**David Richards**

***Jefferson Laboratory***

- ***Lattice QCD at JLab***
- ***Proton Sructure***
  - ***Form factors***
  - ***Chiral extrapolations***
  - ***Moments of GPDs***
- ***Outlook***

# Advances in Lattice QCD



# JLab and National Effort

- **Jefferson Laboratory co-equal partner with BNL and FNAL in lattice QCD effort.**

**FNAL**
***Weak matrix elements***

**BNL**
***RHIC Physics***

**JLAB**
***Hadronic Physics***

SciDAC – R&D Vehicle

- Cluster Prototyping – Balint Joo and Roy Whitney
- **Software R&D** – Balint Joo

**Lattice QCD at JLab will have major impact on DOE's Nuclear Physics Program**

# Lattice studies of Hadron Structure

- LHPC
  - 2 + 1 flavours, using DWF valence propagators on an asqtad sea, provided by MILC Collaboration
  - Pion masses down to 350 MeV
- QCDSF/UKQCD
  - 2-flavor Wilson, pion masses down to 318 MeV (?); quenched overlap
- RBCK
  - Quenched and now dynamical DWF
- Twisted-mass QCD

# LHP Collaboration

- Arizona
  - D Renner
- JLAB
  - R Edwards
  - HW Lin
  - DGR
- MIT
  - B Bistrovic, J Bratt, J Negele, A Pochinsky, D Sigaev
- William and Mary/JLab
  - K Orginos
- Yale
  - G Fleming
- Athens
  - A Tsapalis
- Cyprus
  - C Alexandrou
  - G Koutsou
  - Ph Leontiou
- T U Munich
  - Ph Haegler
  - B Musch
- New Mexico
  - Michael Engelhardt
- DESY Zeuthen
  - W Schroers

# Computational Outline

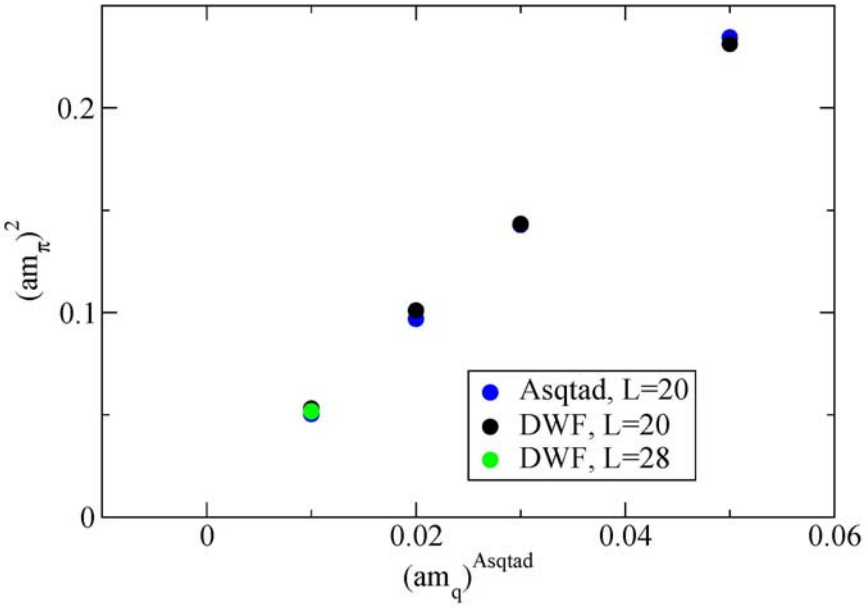
- **Hybrid calculation employing $N_f = 2 + 1$ MILC gauge configurations with *Asqtad* sea quarks, and valence quarks computed using Domain-Wall Fermions (DWF) enabling first investigations of hadron structure in approach to chiral regime.**
- **Menu of computations:**
  - **Nucleon's axial-vector charge**
  - **Isovector Form Factors**
  - **Moments of Generalized Parton Distributions:**
    - **Orbital angular momentum carried by quarks**
    - **Transverse structure of the nucleon**
    - **Parameterizations of GPDs**
- ***Shared Computations with NPLQCD***

# Hybrid Asqtad/DWF Calculation

- MILC Asqtad lattices – staggered sea quarks
- Conventional DWF valence quarks computed on HYP-smeared lattices
- Try to match pion mass
- $a \simeq 0.125$ fm, two volumes at lightest pion mass

PRD64, 034504

| Volume $\Omega$ | $(am_q)^{Asqtad}$ | $m_{PS}$ / MeV |
|---|---|---|
| $20^3 \times 32$ | 0.050 | 790(2) |
| | 0.040 | 693(3) |
| | 0.030 | 594(2) |
| | 0.020 | 492(2) |
| | 0.010 | 354(2) |
| $28^3 \times 32$ | 0.010 | 352(1) |



# Hybrid approach: $f_K/f_\pi$

***NPLQCD, Beane, Bedaque, Orginos, Savage*, hep-lat/0606023**

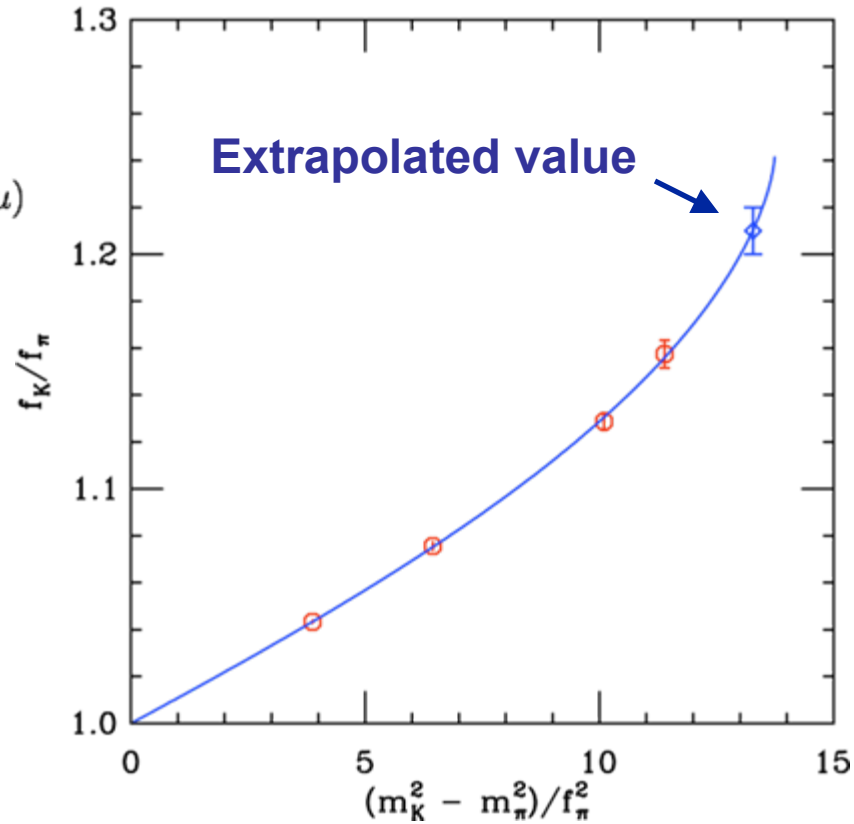
**Gasser-Leutwyler:**

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4} l_\pi(\mu) - \frac{1}{2} l_K(\mu) - \frac{3}{4} l_\eta(\mu) + \frac{8}{f^2}\left(m_K^2 - m_\pi^2\right) L_5(\mu)$$

$$l_i(\mu) = \frac{1}{16\pi^2} \frac{m_i^2}{f^2} \log\left(\frac{m_i^2}{\mu^2}\right)$$

$$\frac{f_K}{f_\pi} = 1.218(2)$$

$$\left.\frac{f_K}{f_\pi}\right|_{\text{exp}} = 1.223(12)$$

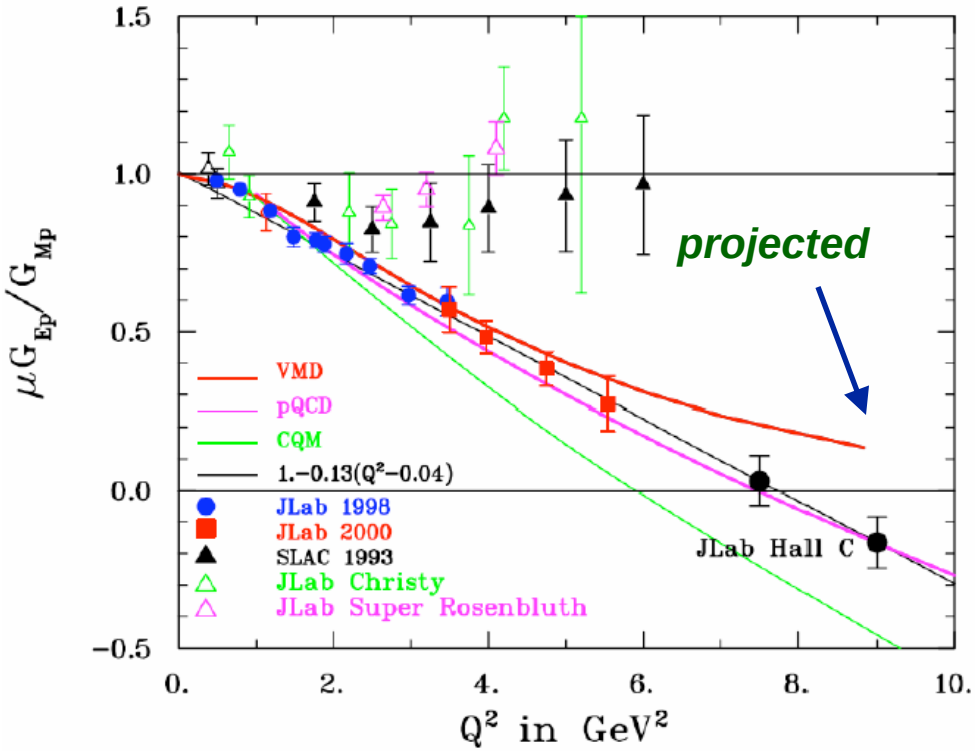



Result comparable with MILC $\left.\frac{f_K}{f_\pi}\right|_{\text{MILC}} = 1.210(4)(13)$

**Need much higher precision to see effects of Mixed χPT Baer et.al.'05**

# Proton EM Form-Factors - I

## EM Form Factors describe the distribution of *charge* and *current* in the proton



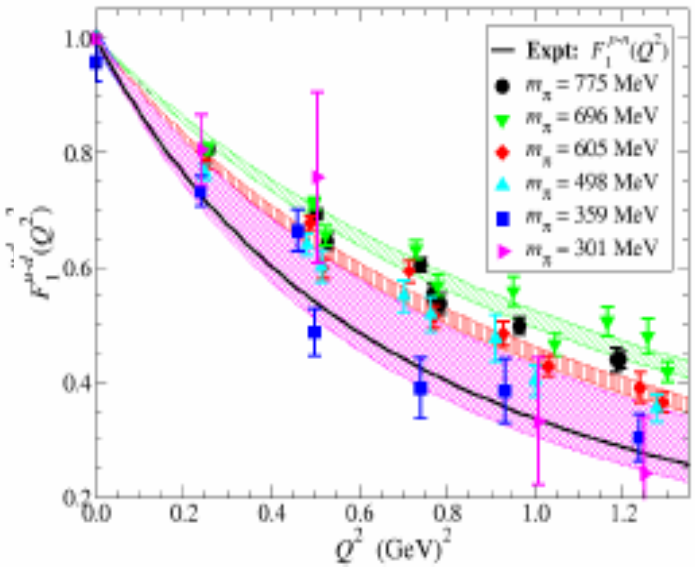

C. Perdrisat (W&M) , JLab Users Group Meeting, June 2005

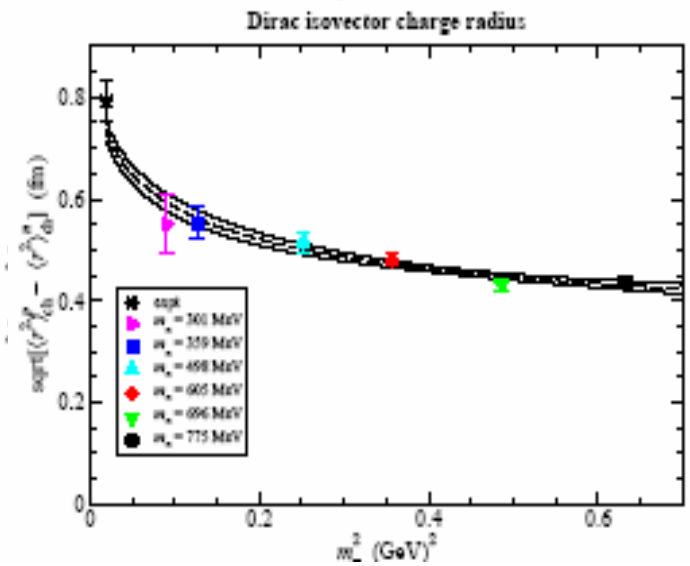
**Important element of current and future program contributing to**

**HP 2010**

- **LT separation disagrees with polarization transfer**
- **New exp. at $Q^2$ = 9 $GeV^2$**
- **Does lattice QCD predict the vanishing of $G_E^p(Q^2)$ around $Q^2$ ~ 8 $GeV^2$ ?**

# Proton EM Form Factors - II




- **Lattice QCD computes the *isovector* form factor**
- **Hence obtain** Dirac charge radius $\langle r^2 \rangle^{u-d}_{ch}$ **assuming dipole form**
- **Chiral extrap. Using LNA and LA terms and finite-range regulator.**

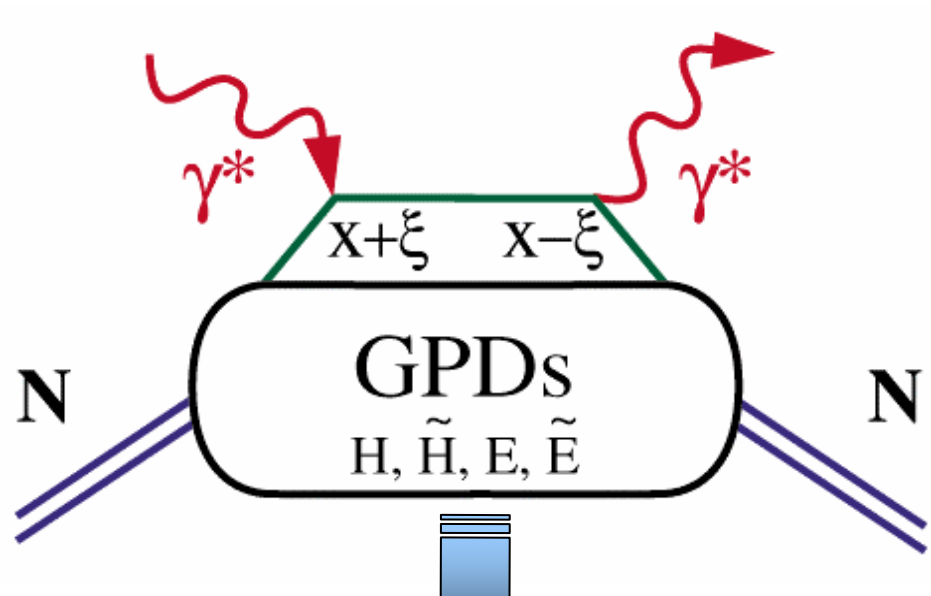
**LHPC, hep-lat/0610007**

**Leinweber, Thomas, Young, PRL86, 5011**

- **As the pion mass approaches the physical value, the size approaches the correct value**

$$\langle r^2 \rangle^{u-d}_{ch} = a_0 - 2\frac{\left(1+5g_A^2\right)}{(4\pi f_\pi)^2}\frac{1}{2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

# Generalized Parton Distributions (GPDs): New Insight into Hadron Structure




**HP 2008**

**D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)**

e.g.

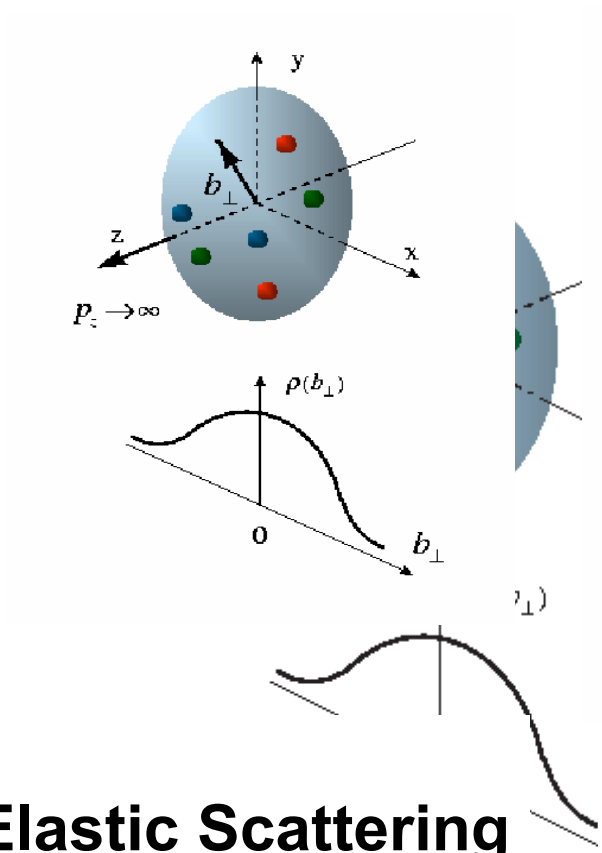
Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x dx \left[ H^q(x,\xi,0) + E^q(x,\xi,0) \right]$$

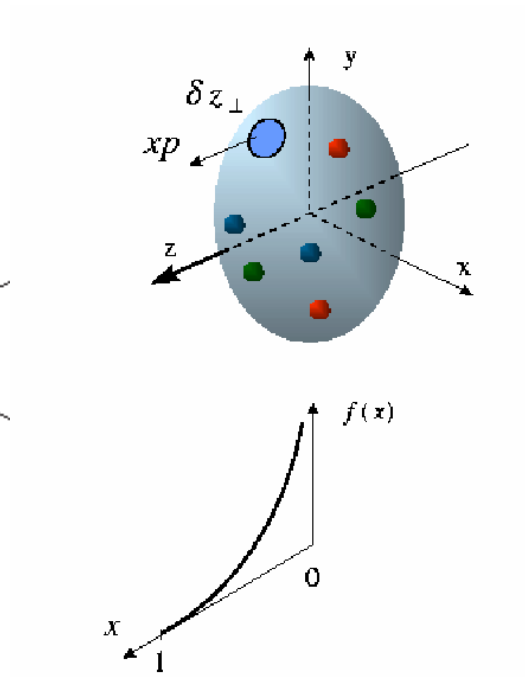
X. Ji, Phy.Rev.Lett.78,610(1997)

**Major review by *Belitsky and Radyushkin*, Phys. Rep. 418 (2005), 1-387**

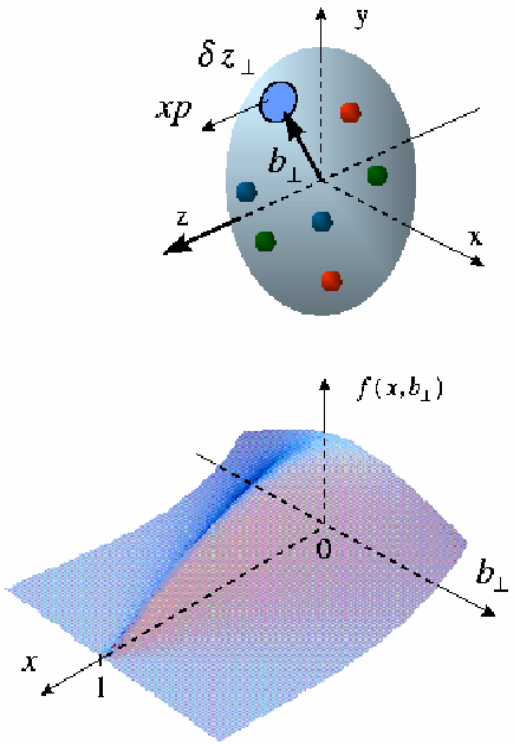
# GPDs: Different Regimes in Different Experiments




**Elastic Scattering**
**transverse quark distribution in Coordinate space**




**DIS**
**longitudinal quark distribution in momentum space**




**DES (GPDs)**
**Fully-correlated quark distribution in both coordinate and momentum space**

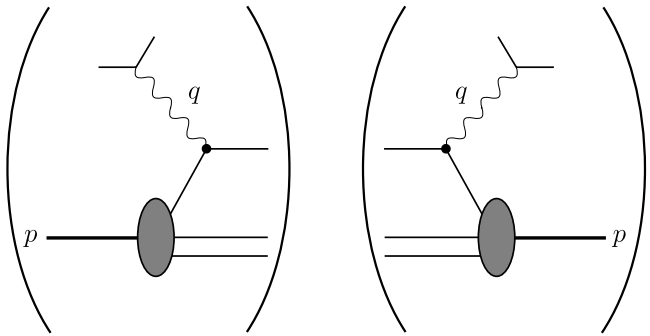
# GPDs in Lattice QCD

- **Measures light-cone correlation functions**

$$O(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) n P e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n.A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

- **DIS gives diagonal matrix element**

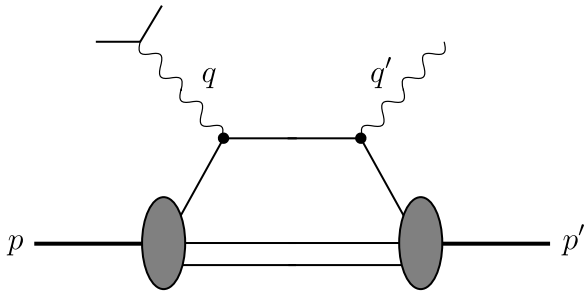
$$< P \,|\, O(x) \,|\, P >= q(x)$$




- **DVCS gives off-diagonal matrix element**

$$< P' | O(x) | P >=< \gamma > H(x,\xi,t) + \frac{i\Delta}{2m} < \sigma > E(x,\xi,t)$$

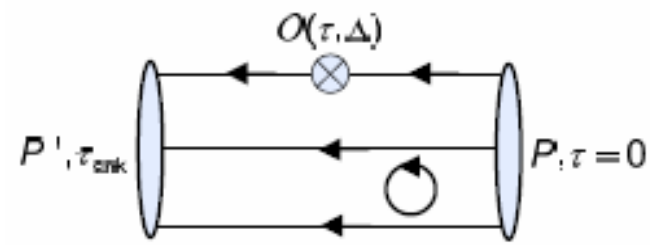
$$\Delta = P - P', t = \Delta^2, \xi = -n.\Delta/2$$



# Moments of Structure Functions and GPD's

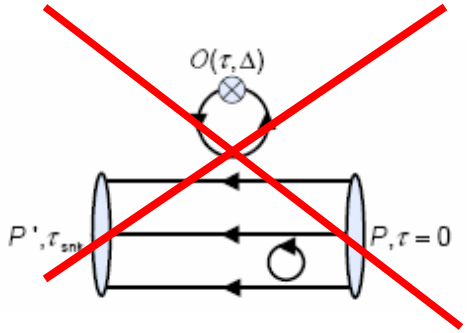
- **Expand *O(x)* around light-cone**

$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} i D^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$




- **Diagonal matrix element**

$$\langle P|O_q^{\{\mu_1\ldots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1} q(x)$$




- **Off-diagonal matrix element**

$$\langle P'|O_q^{\{\mu_1\ldots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1} [H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t)$$

**Axial-vector** $\longrightarrow$ $\tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$

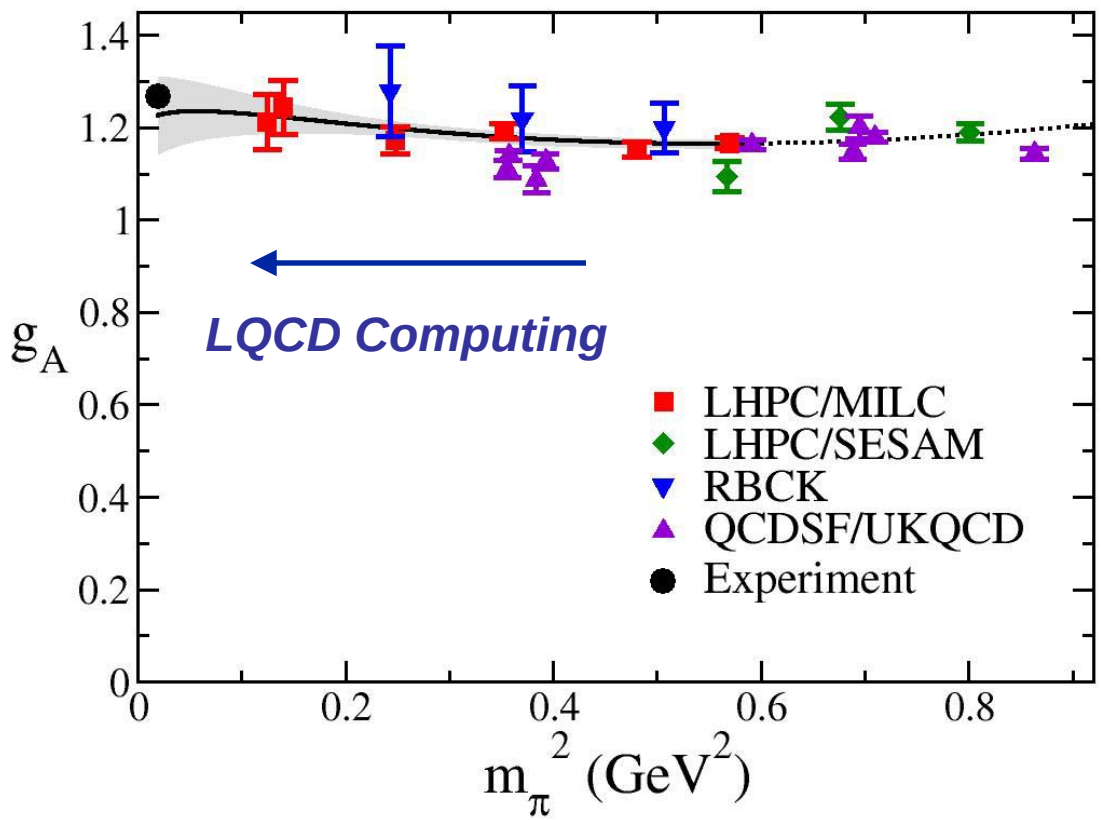
**Use perturbative matching coefficients** ***B. Bistrovic, MIT, PhD***

# Nucleon Axial-Vector Charge

***Nucleon's axial-vector charge $g_A$: benchmark of lattice QCD***

- **Hybrid lattice QCD at $m_\pi$ down to 350 MeV**
- **Finite-volume chiral-perturbation theory**

**LHPC, PRL 96 (2006), 052001**



# Chiral Extrapolation - I

**Example: Unpolarized Moments:**

***LHPC, hep-lat/0610007***

$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_{A,0}+1)}{(4\pi f_{\pi,0})^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right) + b_n'(\mu) m_\pi^2$$

- **Set $\mu = f_{\pi,0}$, and at one loop: $g_{A,0} \rightarrow g_{A,\, m\{pi\}}$ and $f_{\pi,0} \rightarrow f_{\pi,\, m_\pi}$**

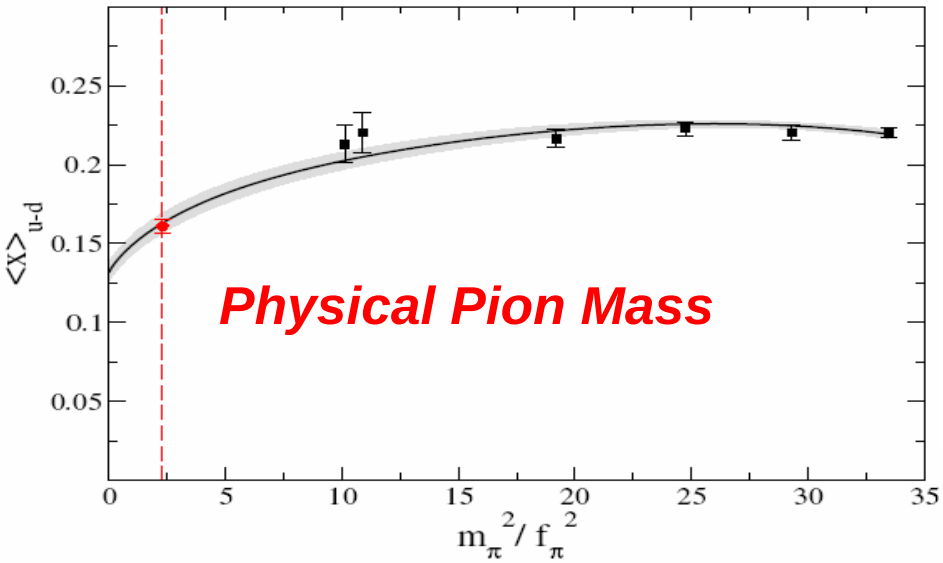
$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_{A,m_\pi}+1)}{(4\pi)^2} \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \ln\left(\frac{m_\pi^2}{f_{\pi,m_\pi}^2}\right)\right) + b_n'(\mu) m_\pi^2$$

- **Now self-consistently set $g_A = g_{A,\, lat}$, $f_\pi = f_{\pi,\, lat}$, $m_\pi = m_{\pi,\, lat}$**

$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_{A,lat}+1)}{(4\pi)^2} \frac{m_{\pi,lat}^2}{f_{\pi,lat}^2} \ln\left(\frac{m_{\pi,lat}^2}{f_{\pi,lat}^2}\right)\right) + b_n'(\mu) m_{\pi,lat}^2$$

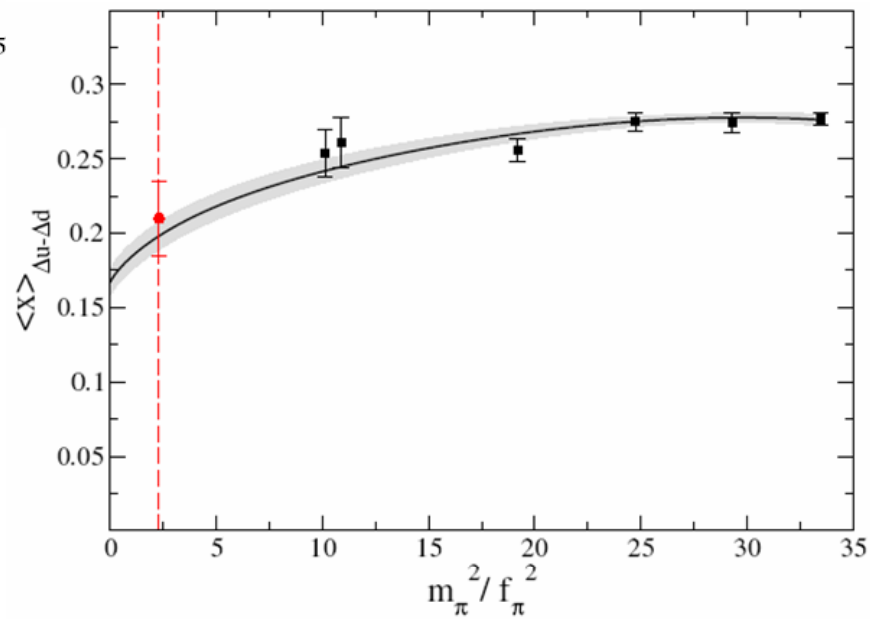
- **Similarly for other moments….**

# Chiral Extrapolation - II




**Helicity fraction**

$\langle x \rangle_{\Delta u - \Delta d}$




**Momentum fraction**

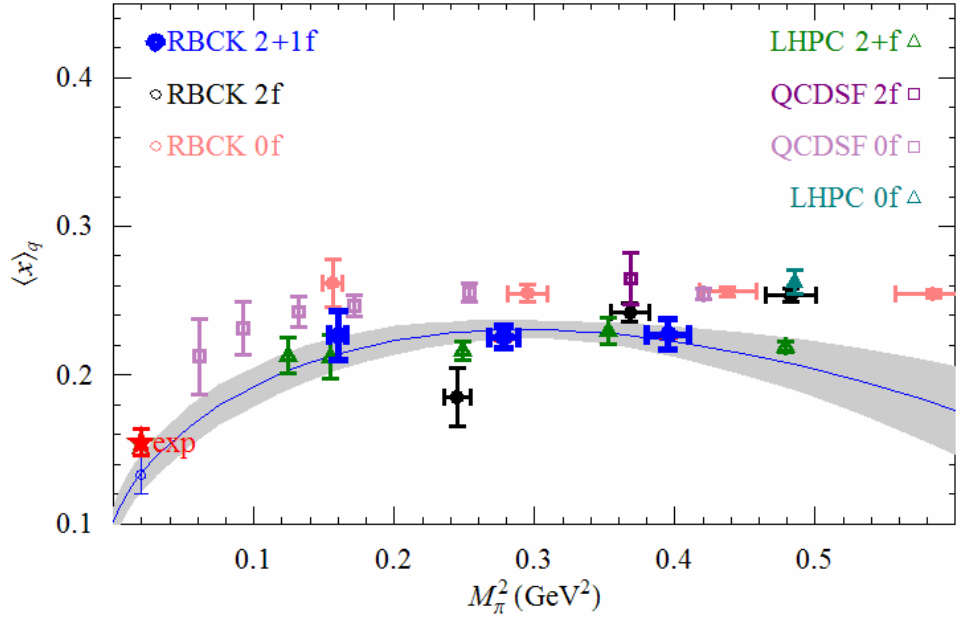
$\langle x \rangle_{u-d}$

# Comparison with RBCK

- **Comparison among calculations of the first moment of the momentum fraction**

*Chen et al., Nucl.Phys. A707, 452 (2002); Phys. Lett. B523, 107 (2001)*

**Slide from H-W Lin**



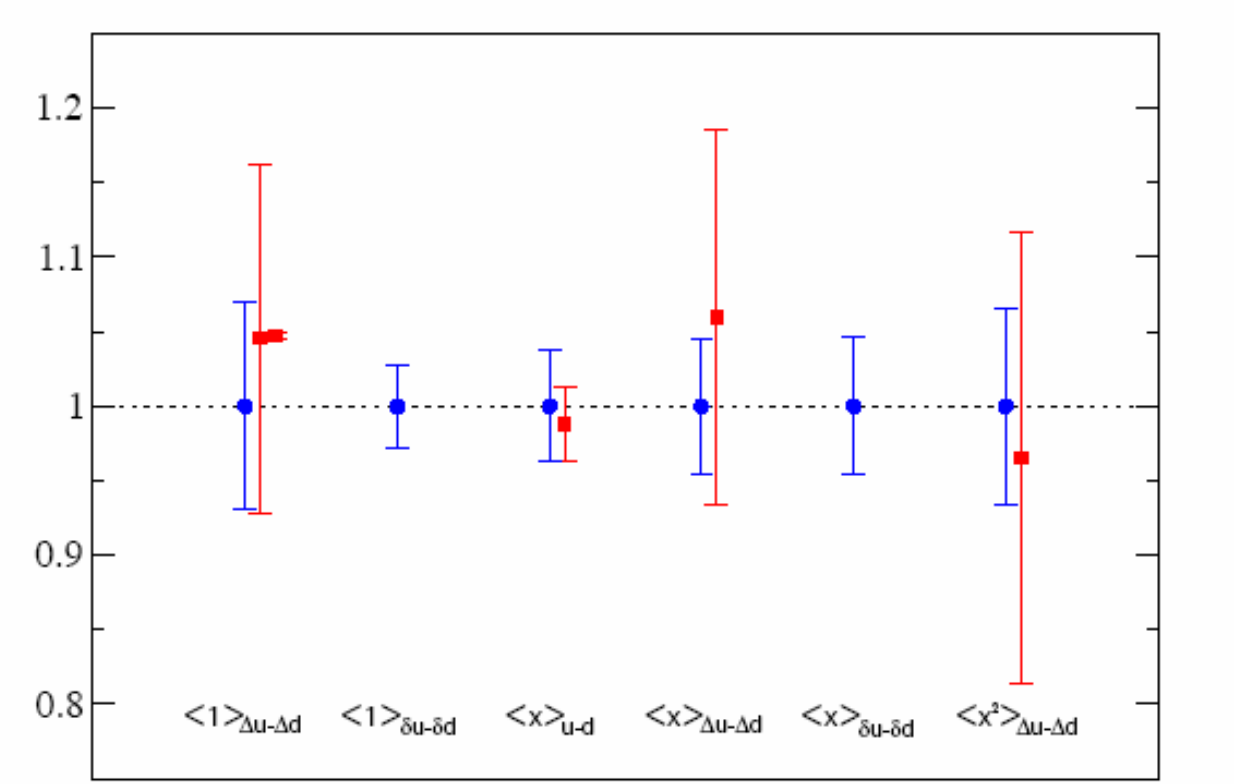

*K. Orginos et al., Phys.Rev.D73:094507, 2005; H.-W. Lin et al., coming soon*

*M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120*

*D. Renner et al., PoS(LAT2006)121*

# Chiral Extrapolation - III

- **Gold plated observables for hadronic physics**



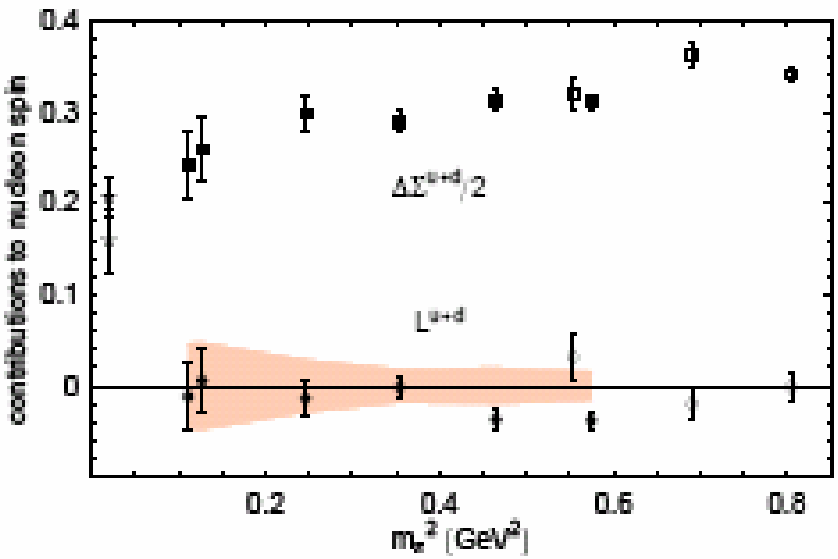

*LHPC, hep-lat/0610007*

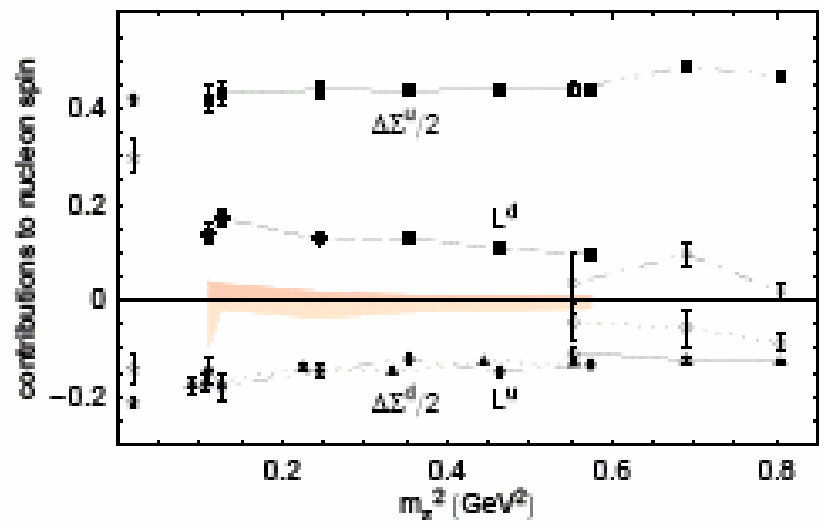
# Origin of Nucleon Spin

- How is the spin of the nucleon divided between quark spin, gluon spin and orbital angular momentum?

$$J_q = \frac{1}{2}(A_{20}(t=0) + B_{20}(t=0))$$
$$\Delta\Sigma^q/2 = \tilde{A}^q_{20}(t=0)$$

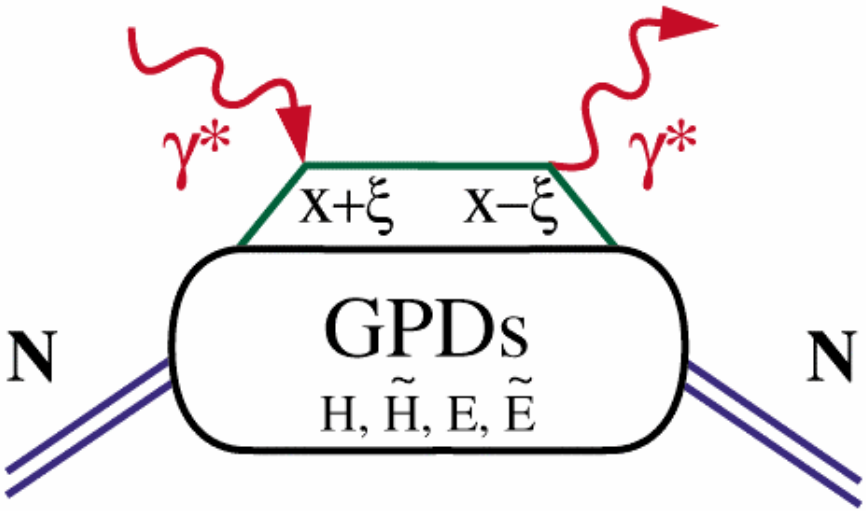
LHPC, Haegler et al., hep-lat/0705.4295




**Total Quark OAM negligible: that of individual flavors substantial.**

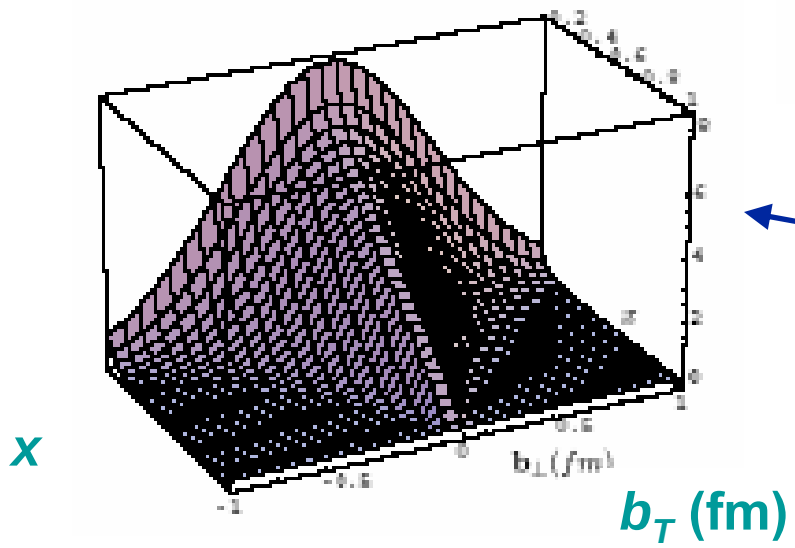
# Transverse Distribution - I




**HP 2008**

- **Lattice QCD can compute moments of GPDs and PDFs, and the *t*-dependence**

$$A^q_{n0}(-\vec{\Delta}_\perp^2) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} \, q(x,\vec{b}_\perp)$$




**Compare to phenomenological models**

***Decrease slope : decreasing transverse size as x → 1***

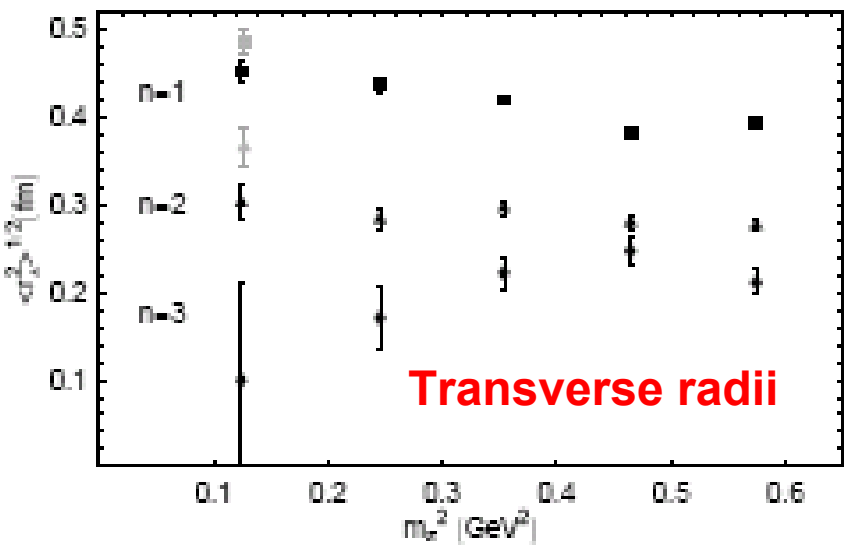
**Burkardt**

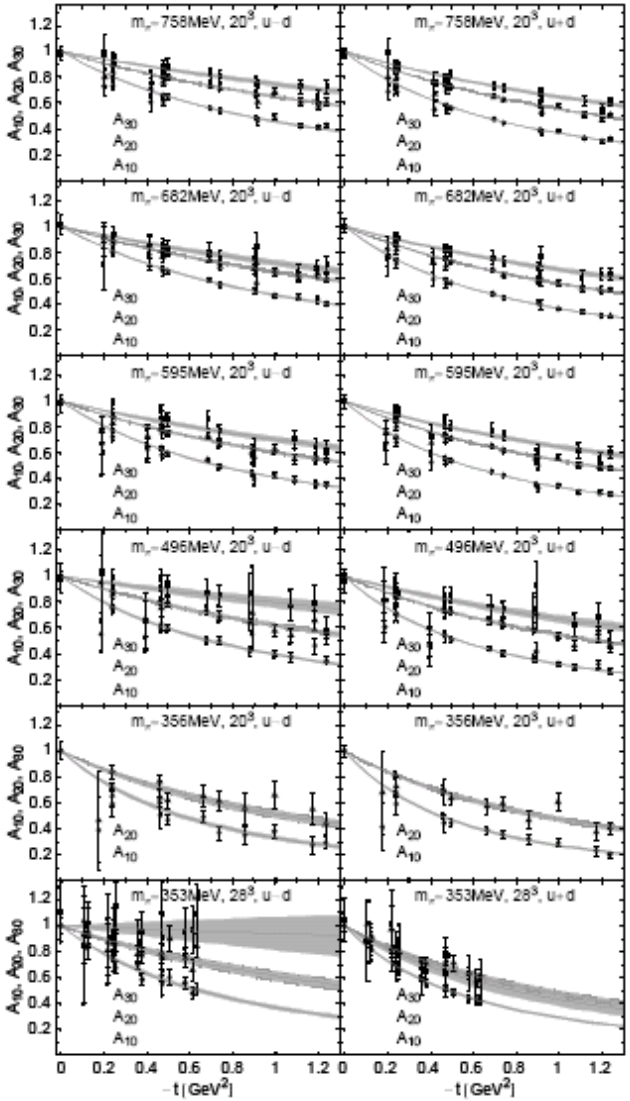
# Transverse Distribution - II

**Lattice results consistent with narrowing of transverse size with increasing x**

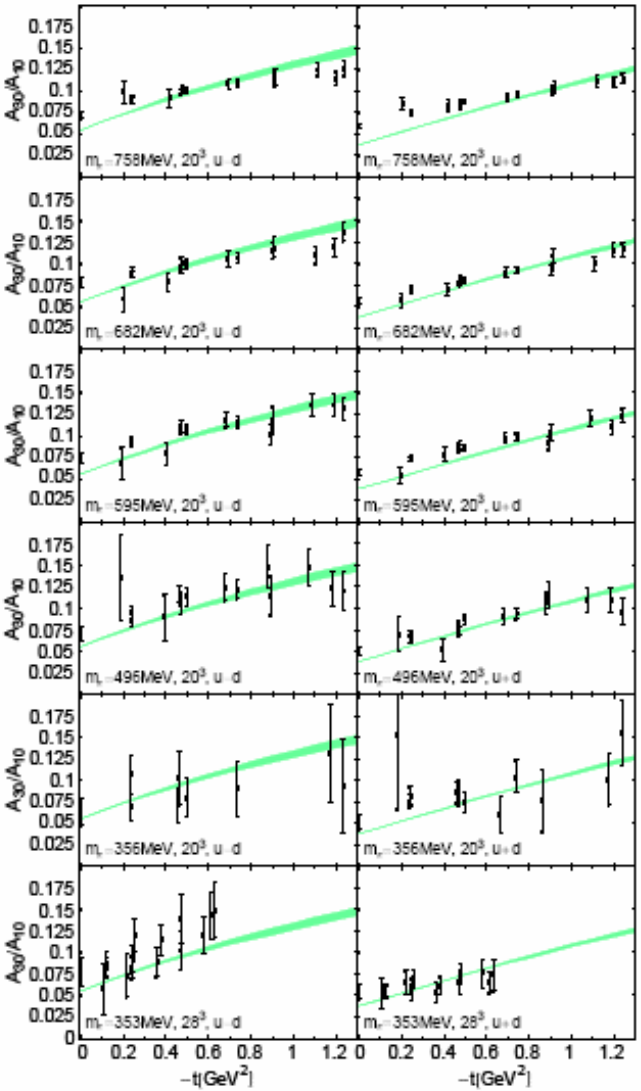
***LHPC, Haegler et al.,*** **hep-lat/0705.4295**

***Flattening of GFFs with increasing n***



# Parametrizations of GPDs



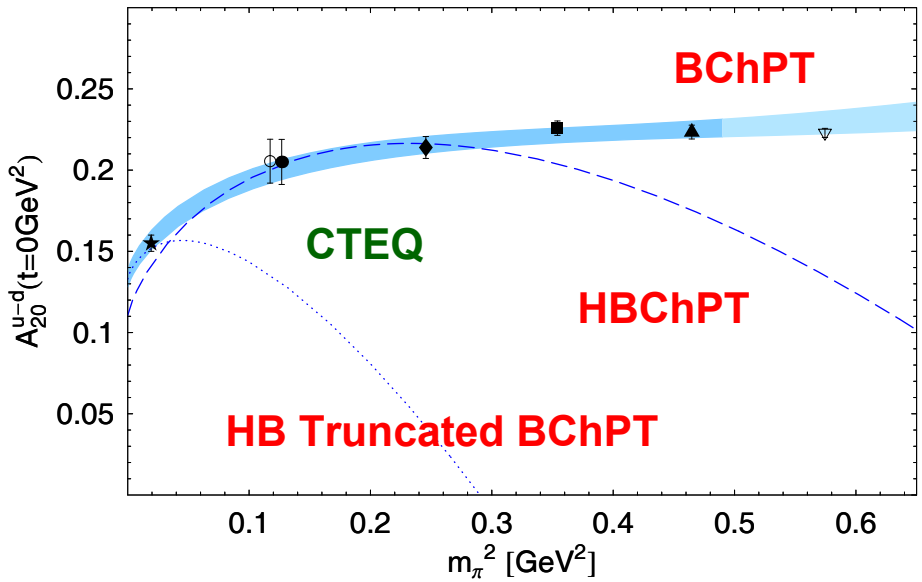

**Provide phenomenological guidance for GPD's**

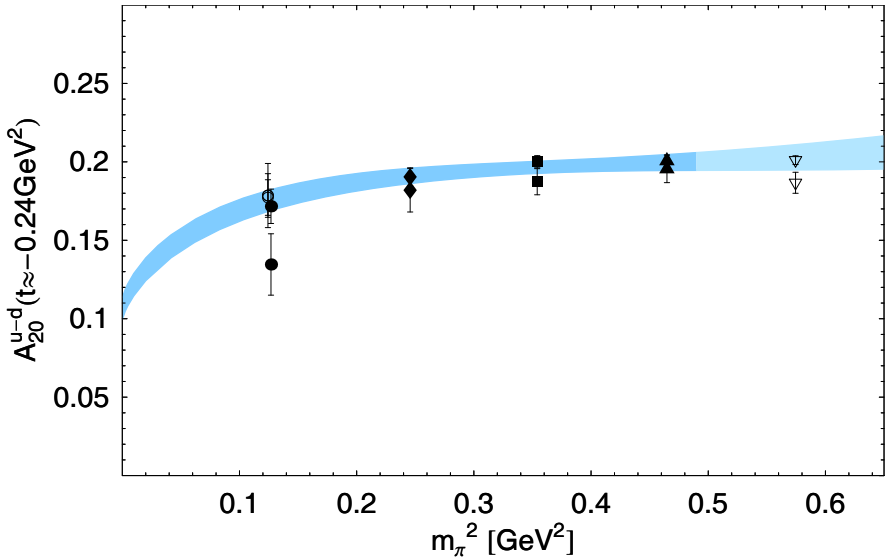
- ***CTEQ, Nucleon Form Factors, Regge***

**Comparison with *Diehl et al*, hep-ph/0408173**

# Covariant BChPT vs. HBChPT

$$A_{20}^{u-d}(t,m_\pi) = A_{20}^{0,u-d}(f_A^{u-d}(m_\pi) + \frac{g_A^2}{192\pi^2 f_\pi^2} h_A(t,m_\pi))$$
$$+\tilde{A}_{20}^{0,u-d} j_A^{u-d}(m_\pi) + A_{20}^{m_\pi,u-d} m_\pi^2 + A_{20}^{t,u-d} t$$

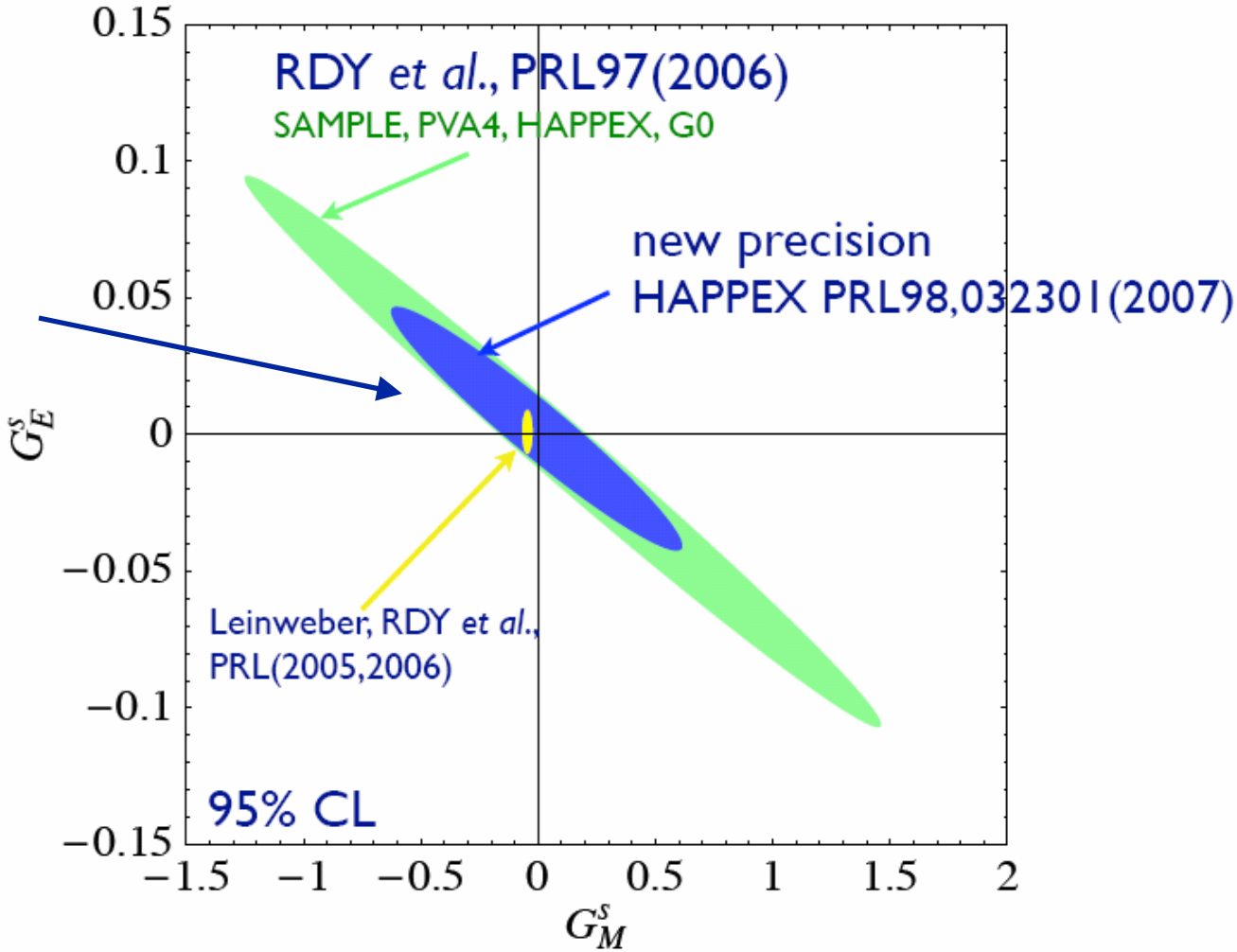



**t = - 0.24 GeV$^2$**

# Disconnected Contributions

**Amalgam of Lattice QCD and Phenomenology by Leinweber *et al.***



# Hadron Structure - Outlook

- Hybrid Asqtad/DWF approach enabling first entre into chiral regime
- Precision computations of range of benchmark quantities, eg $g_A$; CBchPT yields <x> remarkably close to experiment
- Important insight into hadron structure: orbital angular momentum carried by quarks, transverse size of nucleon, pion form factor,…
- *Future*
  - Dramatic algorithmic improvements (RHMC) enabling fully consistent $N_f = 2 + 1$ DWF gauge configurations
  - Joint RBC/LHPC 07-08 proposal to generate DWF configurations and propagators at a = 0.093, with matching by UKQCD
  - Early-use proposal at ANL for more extensive generation of configurations and propagators
  - Range of physics thrusts:
    - *Nucleon structure, including form factors, structure functions, GPDs,…*
    - *Disconnected Diagrams…*
    - *K-meson mixing,…*
  - *Multi-year plan aimed at attaining precise results in chiral regime.*

# Summary - II

- Lattice QCD calculations essential to our understanding of hadronic physics, and to achieving the DOE nuclear-physics goals
- Dramatic algorithmic improvements are enabling us both to measure previously studied quantities to greater precision, and to exploit new opportunities
- Physics measurements, rather than gauge configuration generation, an increasingly computationally demanding part of the effort
- Increasing recognition of lattice QCD as essential tool for hadronic physics among both theory and experimental communities

# BACKUP SLIDES

# Project Milestones - Spectrum

Spectrum project (anisotropic Clover)

- Phase I – a=0.1fm lattice spacing, 2.4fm to 3.2fm boxes
  - Smallest pion mass is 220 MeV
  - Result: first full QCD calculation of $\pi_1$ ($1^{-+}$) hybrid photo-coupling and exotic meson masses
  - Cost of production = 2.7 TF-yr + 10% analysis
- Phase II – a=0.1fm at 2.4fm, 3.2fm, and 4.0fm boxes
  - Smallest pion mass is 181 to 200 MeV
  - Result: several (more than 2 or 3) low-lying excited baryon resonance masses with decay widths, nucleon form-factors, strange FF in nucleon
  - Cost of production = 10 TF-yr + 50% analysis
- Phase III – add a=0.125fm lattice on 2.4fm, 3.2fm and 4.0fm boxes

# GOALS

- Understanding structure, spectroscopy and interactions of hadrons central challenge of nuclear physics
- How are charge, magnetism and spin distributed in the nucleon?
- What are the effective degrees of freedom describing the low-energy spectrum of the theory?
- How does the nucleon-nucleon and hadron-hadron interaction arise from QCD?