Pion Form Factor in Holographic QCD Backgrounds

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Herry Kwee

Florida State University

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Outline

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- Bulk/Louiville coordinate z $\leftrightarrow$ inverse energy scale ($Q \sim \frac{1}{z}$)
- one boundary at $z = 0$, our "brane".
- modified boundary $z = \epsilon$ with $\epsilon \equiv$ UV cutoff $\rightarrow$ "UV brane"

Conformal Field Theory

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- we address this issue in our model.

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- practically:
  every CFT operator $\mathcal{O}(x)$
  is associated with
  $\Psi(x)|_{z=\epsilon}$ a bulk field at the "UV brane"

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- $\mathcal{O}$: operator with ϕ_0 as source (the corresponding operator), constructed from the ψ's fields.
- can work out two-point correlation function:

$$\langle 0|\mathcal{O}(x)\mathcal{O}(y)|0\rangle = \frac{\delta^2 \, Z_S[\phi_0]}{\delta\phi_0(x)\delta\phi_0(y)} \, .$$

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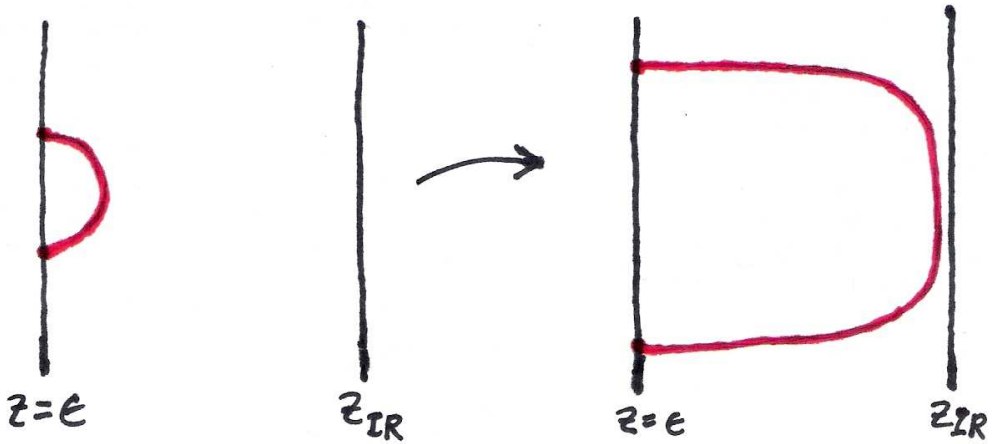
  3. holographic version QCD sum rule.

Holographic QCD continue



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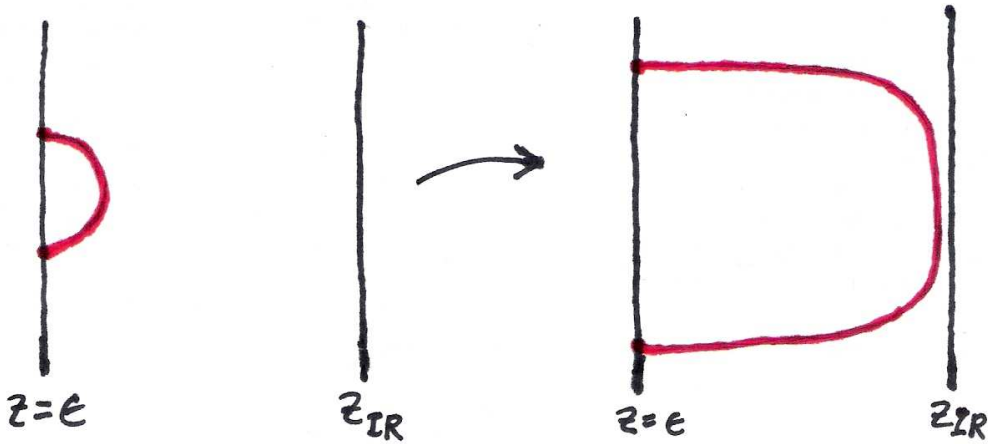

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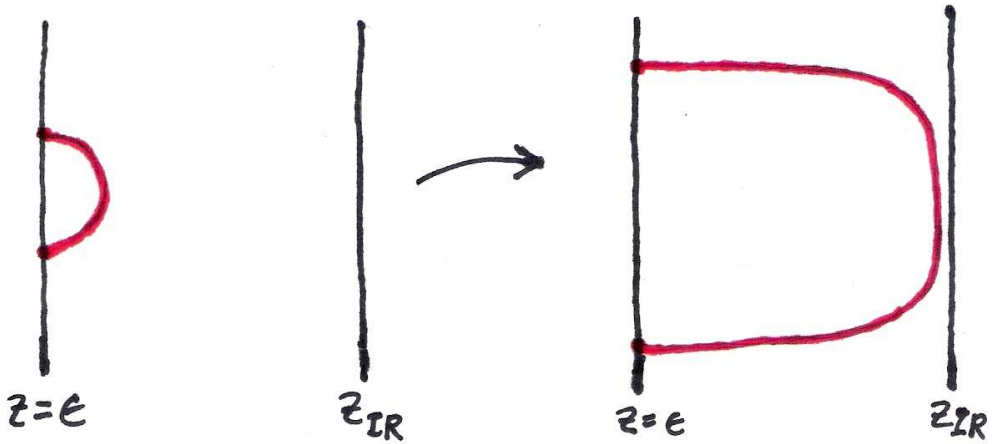

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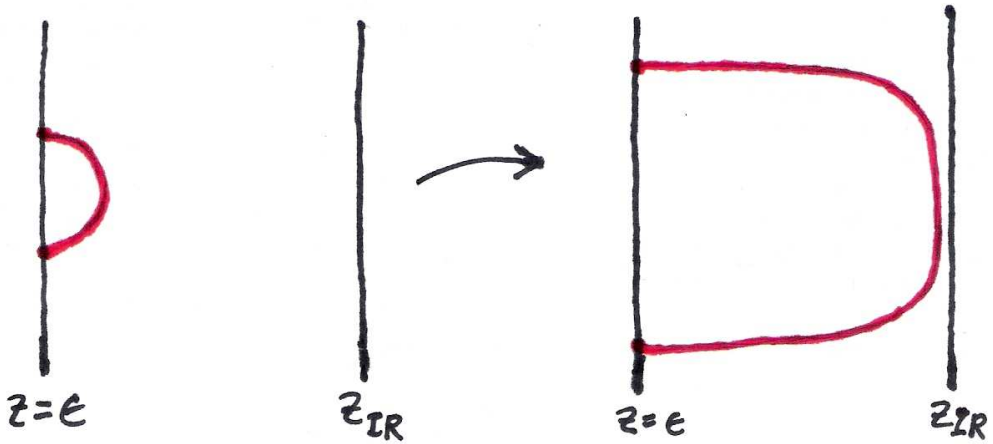

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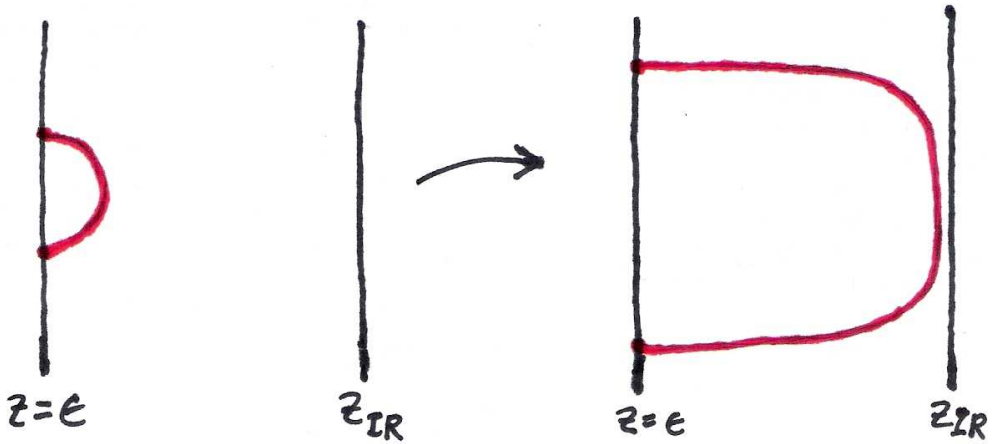

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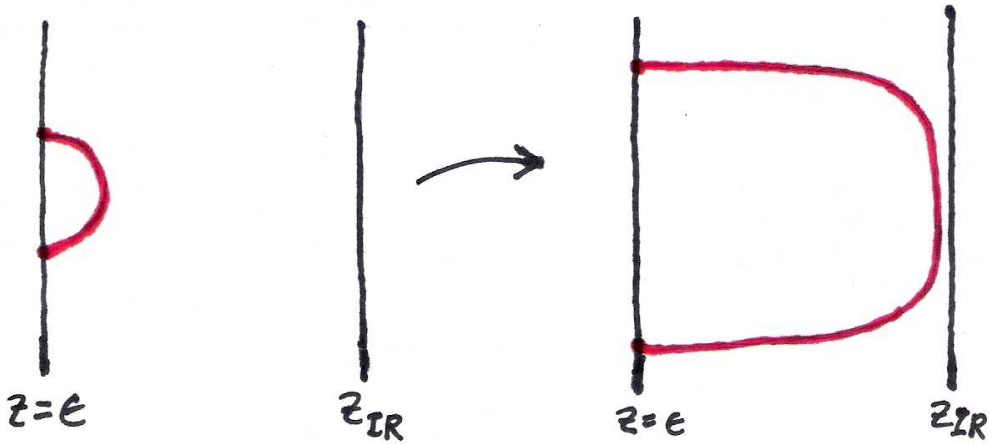

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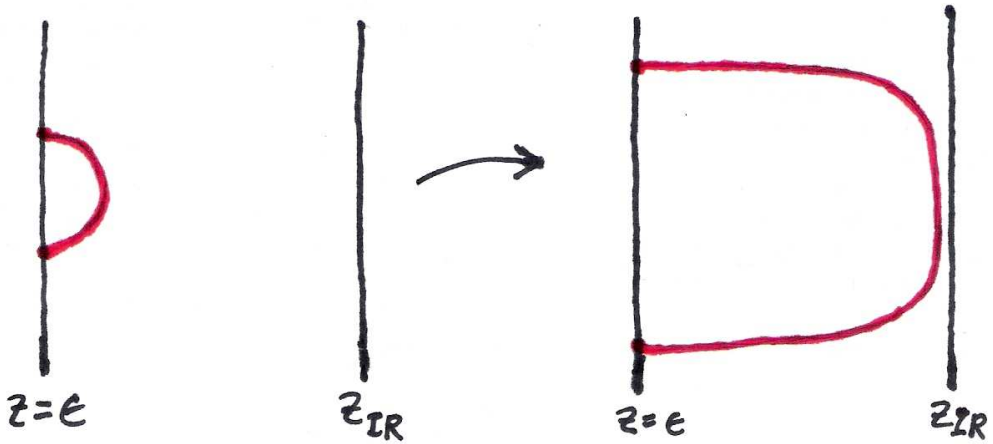

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  3. Vector Meson Dominance.
  4. No running of QCD coupling constant.

Holographic QCD Action

1. The full 5-dimensional action:

$$S = \int d^5x\, e^{-\Phi(z)} \sqrt{g}\, \mathrm{Tr} \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\},$$

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3. $M = m_q 1$: explicit χ_{SB} (nonnormalizable solution).
4. $\Sigma = \sigma 1$: spontaneous χ_{SB} (normalizable solution).
5. Σ: actually determined by the IR BC′s, but in this model it is chosen to be an input parameter.

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4. Pions:
   - $X = X_0 \exp(2i\pi^a t^a)$ (chiral perturbation style).
   - pion field π^a: dimensionless, related to $\tilde{\pi}^a$ of chiral Lagrangians via $\pi^a = \tilde{\pi}^a / f_\pi$, with $f_\pi = 93$ MeV.

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   - A_μ: decomposed into transverse piece $A_{\mu\perp}$ and longitudinal piece φ, $A_\mu = A_{\mu\perp} + \partial_\mu \varphi$

Recap of the Model

1. The Action in physical fields:

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2. The model free parameters:
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   - chiral symmetry breaking: m_q and σ
3. the low-energy QCD observables (to be calculated): m_ρ,F_ρ, m_{a1}, F_{a1}, m_π, f_π, $g_{\rho\pi\pi}$ and π electromagnetic form factor (F_π)

Three different backgrounds

Table: Comparison of hard-wall, soft-wall soft-wall with modified scalar field $X(z)$ and Saxon-Woods hybrid model.

hard-wall	soft-wall	Saxon-Woods hybrid
cut-off at $z = z_0$	no cutoff	no cutoff
$\Phi(z) = 0$	$\Phi(z) = \kappa^2 z^2$	$e^{-\Phi(z)} = \frac{e^{\lambda^2 z_0^2} - 1}{e^{\lambda^2 z_0^2} + e^{\lambda^2 z^2} - 2}$
$\partial_z \Psi(x,z)\|_{z=z_0} = 0$	$\Psi(x,z)\|_{z\to\infty} = 0$	$\Psi(x,z)\|_{z\to\infty} = 0$
$X_0(z) = \frac{1}{2} v(z)$	$X_0(z) = \frac{1}{2} v(z)$	$X_0(z) = \frac{1}{2} v(z)$

with $v(z) = m_q z + \sigma z^3$

Equations of motion of the fields

1. Equation of motion of the Fourier transformed $\Psi(q,z)$:

$$\partial_z\left(\frac{1}{z}\,\partial_z V^a_\mu\right)+\frac{q^2}{z}V^a_\mu=0$$

$$\left[\partial_z\left(\frac{1}{z}\,\partial_z A^a_\mu\right)+\frac{q^2}{z}A^a_\mu-\frac{g_5^2\,v(z)^2}{z^3}A^a_\mu\right]_\perp=0$$

$$\partial_z\left(\frac{1}{z}\,\partial_z\varphi^a\right)+\frac{g_5^2\,v(z)^2}{z^3}(\pi^a-\varphi^a)=0$$

$$-\,q^2\partial_z\varphi^a+\frac{g_5^2\,v(z)^2}{z^2}\,\partial_z\pi^a=0$$

where $v(z)=2X_0(z)=m_q z+\sigma z^3$.

Solving the Vector EOM and Obtaining g_5

1. Equation of motion of the Fourier transformed $V_\mu(q, z)$:

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   $\tilde{V}^a_\mu(q)$: Fourier transform of the source of $J^a_\mu = \bar{q}\gamma_\mu t^a q$ at $z = \epsilon$
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3. Boundary Conditions:
   - axial-like gauge, $V_z(q,z)=0$.
   - z_0 boundary condition.
4. $V(q,z)$: Bessel functions.

Surface Term

1. Referring to the 5th-D surface ($z = \epsilon$) term, the 4D surface term $\longrightarrow 0$.
2. Substituting EOM to the F_V^2 portion of the action:

$$S_{\rm Surf} = -\frac{1}{2g_5^2}\int d^4x \;\; \tilde{V}^a_\mu(-q)\tilde{V}^{\mu a}(q)\,\frac{1}{z}\partial_z V(q,z)\Big|_{z=\epsilon}.$$

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3. Isospin conservation ($q_\mu V^\mu = 0$):
   - $\tilde{V}_\mu^a \tilde{V}^{\mu\, a} \rightarrow \tilde{V}_\mu^a \tilde{V}_\nu^b \Pi^{\mu\nu}\delta^{ab}$.
   - $\Pi^{\mu\nu} \equiv \eta^{\mu\nu} - q^\mu q^\nu / q^2$.

Two-point Correlation Function

Quadratic variation of the action with respect to the source $\tilde{V}$ produces the vector current two-point function:

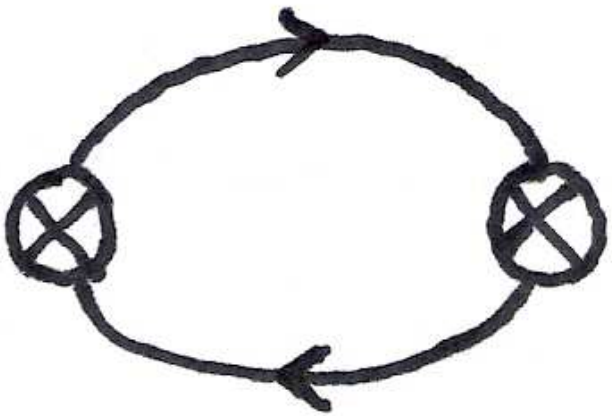
$$\int d^4x\, e^{iqx} \langle J^a_\mu(x) J^b_\nu(0) \rangle = \delta^{ab}\, \Pi_{\mu\nu}\, \Sigma_V(q^2)$$

$$\overset{AdS/CFT}{=} \left(\frac{\delta}{\delta \tilde{V}^a_\mu(-q)} \right) \left(\frac{\delta}{\delta \tilde{V}^b_\nu(q)} \right) S[\tilde{V}^c_\alpha] \,.$$

$$\Rightarrow \Sigma_V(q^2) = -\frac{1}{g_5^2} \frac{\partial_z V(q,z)}{z} \bigg|_{z=\epsilon} \,.$$

And in the large Euclidean $Q^2 = -q^2$ limit:

$$\Sigma_V(-Q^2) = \frac{Q^2}{2g_5^2} \ln Q^2 \,.$$



Matching to the original QCD sum rule result for currents J_μ, Shifman *et al.* (1979):

$$\Sigma_V(-Q^2) = \frac{Q^2 N_c}{24\pi^2} \ln Q^2 \,,$$

$$\Rightarrow g_5^2 = \frac{12\pi^2}{N_c} \to 4\pi^2 \,.$$

Pion Electromagnetic Form Factor

1. Pion EM FF: obtained from $V\pi\pi$, VAA and $VA\pi$ terms in the action: longitudinal modes of A contribute.
   $|DX|^2 \supset (\partial\pi)(V\pi)$ and $(V\pi)(A)$.
   $F_V^2 \supset (\partial V)(AA)$.
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   $F_V^2 \supset (\partial V)(AA)$.
   F_A^2: does not contribute because the longitudinal modes cancel each other.
2. The complete related action:

$$S_{\mathrm{AdS}}^{V\pi\pi} = \epsilon_{abc} \int d^4x \int dz \left[\frac{1}{g_5^2 z} \left(\partial_z \partial^\mu \varphi^a\right) V_\mu^b \left(\partial_z \varphi^c\right) + \frac{v(z)^2}{z^3} \left(\partial^\mu \pi^a - \partial^\mu \varphi^a\right) V_\mu^b \left(\pi^c - \varphi^c\right) \right] .$$

Form Factor and 3-point Correlation Function

1. The 3-point correlation function:

$$\begin{aligned}\langle J_\pi^a(p_1) J_V^{\mu,b}(q) J_\pi^c(-p_2)\rangle &= \epsilon^{abc} F(p_1^2, p_2^2, q^2)\,(p_1+p_2)^\mu \\ &\times\; i(2\pi)^4 \delta^{(4)}(p_1 - p_2 + q)\,.\end{aligned}$$

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2. The dynamical form factor $F(p_1^2, p_2^2, q^2)$ in terms of transition form factors:

$$F(p_1^2, p_2^2, q^2) = \sum_{n,k=1}^{\infty} \frac{f_n f_k F_{nk}(q^2)}{\left(p_1^2 - M_n^2\right)\left(p_2^2 - M_k^2\right)}\,,$$

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3. the pion form factor $F_\pi(q^2) \equiv F_{11}(q^2)$.

AdS QCD and 3-point Correlation Function

1. Trilinear variation of the action with respect to the source $\tilde{V}\tilde{\pi}\tilde{\pi}$ produces the 3-point function:

$$\langle J_\pi^a(p_1) J_V^{\mu,b}(q) J_\pi^c(-p_2)\rangle \overset{AdS/CFT}{=} \left(\frac{\delta}{\delta\tilde{\pi}^a(p_1)}\right)\left(\frac{\delta}{\delta\tilde{V}^{b,\mu}(q)}\right)\left(\frac{\delta}{\delta\tilde{\pi}^c(-p_2)}\right) S_{\mathrm{AdS}}^{V\pi\pi}\,.$$

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2. Pion EM FF:

$$F_\pi(q^2) = \int_0^{z_{IR}} dz\, \frac{V(q,z)}{f_\pi^2}\left\{\frac{1}{g_5^2 z}[\partial_z\varphi(z)]^2 + \frac{v(z)^2}{z^3}\,[\pi(z)-\varphi(z)]^2\right\}.$$

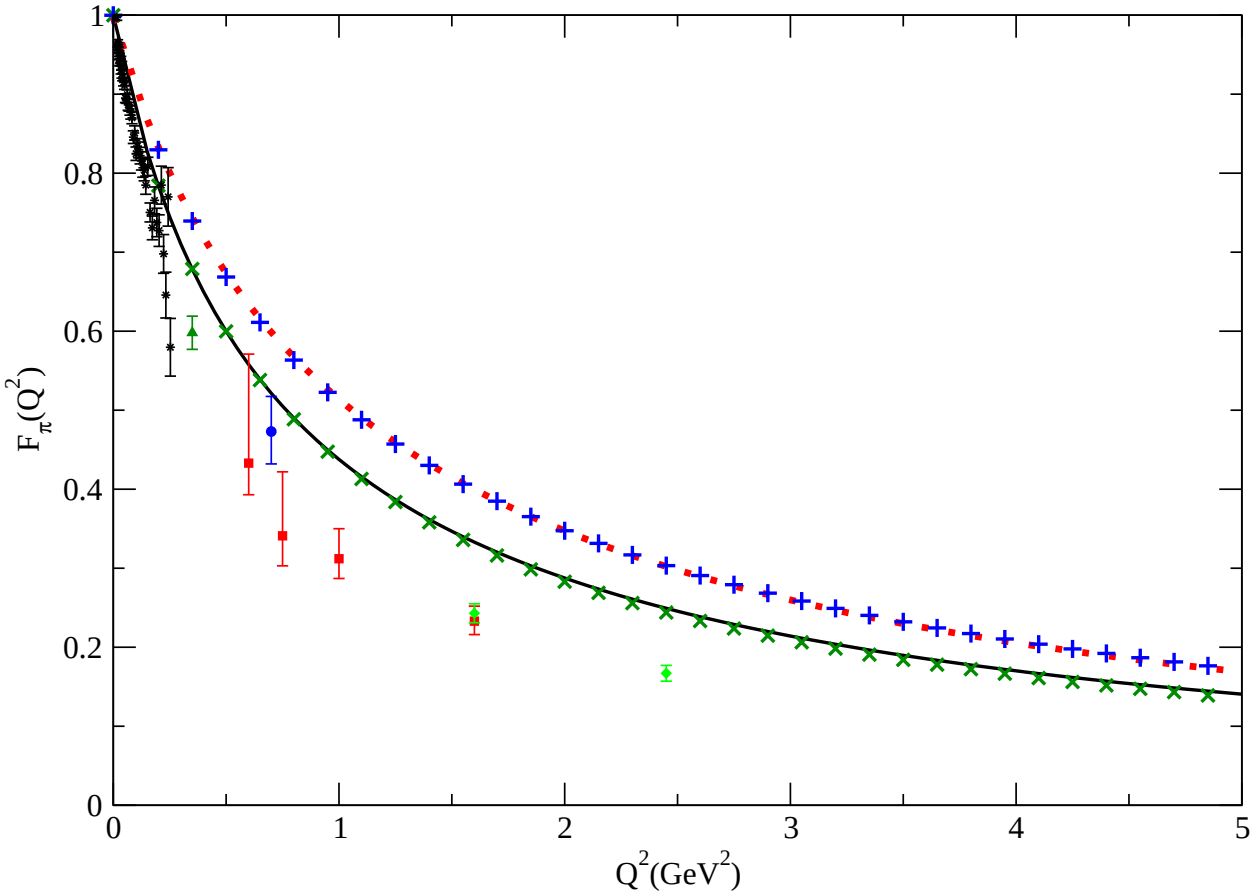

Figure: Spacelike scaling behavior for $F_\pi(Q^2)$ as a function of $Q^2 = -q^2$. The continuous line is the prediction of the hard-wall model with $1/z_0 = 323$ MeV. The dotted line is the prediction of the soft-wall model with $\kappa = m_\rho/2$. The crosses use modified $e^{-\Phi(z)}$ with $\lambda z_0 = 2.1$, and the pluses use $\lambda z_0 = 1$. The black stars are from a data compilation from CERN, the blue circles are from DESY, reanalyzed by Tadevosyan *et al.*, the green triangle is data from DESY, and the red boxes and green diamonds are from Jefferson Lab.

Predictions for QCD observables

Table: Comparison of soft-wall model to modified $e^{-\Phi(z)}$ with $\lambda z_0 = 1$; values in MeV (except for $g_{\rho\pi\pi}$).

Observable	Experiment	Soft-wall	$\lambda z_0 = 1$
m_π	139.6 ± 0.0004	139.6	139.6
m_ρ	775.5 ± 0.4	777.4	779.2
m_{a_1}	1230 ± 40	1601	1596
f_π	92.4 ± 0.35	87.0	92.0
$f_\rho^{1/2}$	346.2 ± 1.4	261	283
$f_{a_1}^{1/2}$	433 ± 13	558	576
$g_{\rho\pi\pi}$	6.03 ± 0.07	3.33	3.49

Predictions for QCD observables

Table: Comparison of hard-wall model to the modified $e^{-\Phi(z)}$ with $\lambda z_0 = 2.1$; values in MeV (except for $g_{\rho\pi\pi}$).

Observable	Experiment	Hard wall	$\lambda z_0 = 2.1$
m_π	139.6 ± 0.0004	139.6	139.6
m_ρ	775.5 ± 0.4	775.3	777.5
m_{a_1}	1230 ± 40	1358	1343
f_π	92.4 ± 0.35	92.1	88.0
$f_\rho^{1/2}$	346.2 ± 1.4	329	325
$f_{a_1}^{1/2}$	433 ± 13	463	474
$g_{\rho\pi\pi}$	6.03 ± 0.07	4.48	4.63

Discussions

1. the Saxon-Woods hybrid model interpolate between hard-wall and soft-wall model.

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2. the SW hybrid model with parameter matching it to hard-wall model produces higher-order vector meson poles exhibiting Regge trajectory behavior $m_n^2 \sim n$

Table: Comparison of vector meson masses and decay constants (in MeV) for the hard-wall model and the model using modified $e^{-\Phi(z)}$ with $\lambda z_0 = 2.1$.

Original hard wall		modified $e^{-\Phi(z)}$ with $\lambda z_0 = 2.1$	
m_ρ	$F_\rho^{1/2}$	m_ρ	$F_\rho^{1/2}$
775.6	392	777.5	325
1780.2	734	1608.1	528
2790.8	1029	2226.8	611
3802.8	1298	2637.5	644
4815.2	1549	2986.6	683

Conclusions

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Conclusions

1. Motivated AdS/CFT correspondence, proposed by Maldacena in 1997, a five-dimensional framework for modeling low-energy properties of QCD is proposed by Erlich *et al.* in 2005.
2. This holographic QCD model naturally incorporates properties of QCD such as confinement and chiral symmetry breaking.
3. Prediction for low-energy QCD observables are off only by 10%
4. Possible improvement on the model and future work:
   - including baryons in the model via Chern-Simons terms.
   - inclusion of strange quark (SU(3)$\times$SU(3) chiral symmetry).
   - including running of the gauge coupling by considering logarithmic corrections to the AdS geometry.

AdS/CFT Dictionary

AdS	CFT
Field	Operator
Mass	Scaling Dimension
Non-Normalizable Mode	Source
Normalizable Mode	State $\leftrightarrow$ VEV
Gauge Field	Global Symmetry

Holographic QCD Dictionary

1. fields $\leftrightarrow$ operators:

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\overline{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Table: Operators/fields of the model

2. masses $\leftrightarrow$ scaling dimensions:
   $(\Delta - p)(\Delta + p - 4) = m_5^2$ (Witten, Gubser *et al.*),
   m_5: 5D masses of the fields $A^a_{L\mu}$, $A^a_{R\mu}$, and X
   Δ: the dimension of the corresponding p-form operator.

Four different backgrounds

Table: Comparison of hard-wall, soft-wall soft-wall with modified scalar field $X(z)$ and hard-wall-soft-wall hybrid model.

hard-wall	soft-wall	modified soft-wall	hard-wall-soft-wall hybrid
cut-off at $z = z_0$	no cutoff	no cutoff	no cutoff
$\phi(z) = 0$	$\phi(z) = \kappa^2 z^2$	$\phi(z) = \kappa^2 z^2$	$e^{-\phi(z)} = \frac{e^{\lambda^2 z_0^2} - 1}{e^{\lambda^2 z_0^2} + e^{\lambda^2 z^2} - 2}$
$\partial_z \Psi(x,z)\|_{z=z_0} = 0$	$\Psi(x,z)\|_{z\to\infty} = 0$	$\Psi(x,z)\|_{z\to\infty} = 0$	$\Psi(x,z)\|_{z\to\infty} = 0$
$X_0(z) = \frac{1}{2} v(z)$	$X_0(z) = \frac{1}{2} v(z)$	$X_0(z) = \frac{1}{2} v(z)(1 - e^{-A_c/\kappa^4 z^4})$ $+ B_c e^{-3/\kappa^2 z^2}$	$X_0(z) = \frac{1}{2} v(z)$

with $v(z) = m_q z + \sigma z^3$