Generalized Gerasimov-Drell-Hearn Sum Rule for the Proton-Neutron Difference in Chiral Perturbation Theory

Volker D. Burkert* Thomas Jefferson National Accelerator Facility Newport News, VA 23606

Abstract

Applications of the chiral expansion to generalize the Gerasimov-Drell-Hearn sum rule for finite Q^2 are discussed. The observation of several authors, that the corrections to the leading order contributions are large and limit the applicability to a very small range in Q^2 , are only valid when considering the generalization of the sum rule for protons and neutrons, separately. When using the protonneutron difference, the chiral expansion may be valid to considerably higher Q^2 where hadronic degrees of freedom at large distances may connect up with quark and gluon degrees of freedom. This could mark the first time that nucleon structure is described by fundamental theory from large to small distances.

The spin structure of the nucleon has been of central interest for more than two decades. Most studies have focused on the deep inelastic regime to measure the spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$, and their respective first moments [1]. In recent years the interest has shifted towards the lower Q^2 domain and the resonance region [2, 3, 4, 5], and measurements are being untertaken to study the transition from the scaling regime to the regime of strong QCD [6, 7, 8]. These advances in experiments made it urgent to study theoretically the connection between these different domains of physics. While perturbative techniques and higher twist expansion approaches seem appropriate at $Q^2 > 0.5 \text{GeV}^2$ and invariant masses above the resonance region ($W > 2.5 \text{ GeV}/c^2$), new approaches are needed to study the low Q^2 and low W regions. Numerous phenomenological models have been constructed to describe the resonance regime and the connection with the deep inelastic regime [2, 3, 4, 5, 9, 10, 11]. Models that explicitly include resonances show that the resonance region, especially the $\Delta(1232)$, plays an important role in the helicity dependence of the inclusive cross section at small Q^2 . At $Q^2 = 0$, the sum rule by Gerasimov [12], Drell, and Hearn [13] (GDH SR) relates the energy-weighted integral of the helicity-dependent cross section to the anomalous magnetic moment of the target nucleon:

$$I = \int \frac{\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)}{\nu} d\nu = -\frac{2\pi^2 \alpha}{M^2} \kappa^2$$
(1)

where κ is the anomalous magnetic moment of the target nucleon, and M is the nucleon mass.

Recently, attempts have been made to evolve this sum rule into the regime of finite Q^2 using heavy baryon chiral perturbation theory (HBChPT) [14, 15]. Ji and

^{*}e-mail: burkert@jlab.org

Osborne [15] constructed a generalized sum rule using a dispersion relation for the invariant photon-nucleon Compton amplitude $S_1(\nu, Q^2 = 0)$. At non-zero values of Q^2 the connection is given by the equation

$$\int G_1(\nu, Q^2) \frac{d\nu}{\nu} = \frac{1}{4} \bar{S}_1(0, Q^2)$$
(2)

where $G_1(Q^2, \nu)$ is the spin-dependent structure function and $\bar{S}_1(0, Q^2)$ is the Compton amplitude, with the overline meaning that the elastic contribution has been subtracted. The right-hand side is then calculated in HBChPT. In leading order $\bar{S}_1(0, Q^2)$ is independent of Q^2 [15]. From power counting one expects that the GDH SR could be evolved to a four-momentum transfer $Q^2 = 0.2 \text{ GeV}^2$. Unfortunately, the next-toleading order (NLO) calculation results in a strong Q^2 dependence [16] with the slope at $Q^2 = 0$ given by

$$\frac{d\bar{S}_1(Q^2)}{dQ^2} = \frac{g_A^2 \pi}{12(4\pi f_\pi)^2 M m_\pi} [1 + 3\kappa_V + 2(1 + 3\kappa_S)\tau^3]$$
(3)

where $\kappa_V = 3.706$ and $\kappa_S = -0.120$ are the experimental values of the isovector and isoscalar anomalous magnetic moments of the nucleon, and τ^3 is +1 for the proton and -1 for the neutron, respectively. When converted to the often used dimensionless quantity

$$\bar{I}(Q^2) = M^2 \int G_1(\nu, Q^2) \frac{d\nu}{\nu}$$
(4)

the low Q^2 evolutions for the proton and neutron are given by [†]

$$\bar{I}^{p}(Q^{2}) = -\frac{\kappa_{p}^{2}}{4} + 6.85Q^{2}(GeV^{2}) + h.o.$$
(5)

$$\bar{I}^n(Q^2) = -\frac{\kappa_n^2}{4} + 5.54Q^2(GeV^2) + h.o.$$
(6)

The very large Q^2 variation in (3), (5), and (6) is much larger than what is expected from simple power counting. This fact will limit the usefulness of the chiral expansion to very small Q^2 values at best. The Q^2 evolution predicted by Ji et al. [16] for protons and neutrons as represented in (5) and (6) is shown in fig. 1 compared to data from SLAC. For neutrons the NLO calculation predicts a sign change at $Q^2 \approx$ $0.16 GeV^2$, pointing in the direction opposite to the high Q^2 data, while for the proton the sign is correct, however with a very steep slope. If the chiral expansion can be applied to the individual isospin channels, it may be in a very limited range of $Q^2 < 0.05$ GeV² only. This effectively eliminates the possibility of using HBChPT to connect the GDH SR for the proton and neutron at the photon point to the deep inelastic spin integrals.

It is well known that at small Q^2 the GDH integrals of both proton and neutron are dominated by the excitation of the $\Delta(1232)$ resonance. The Delta contribution, as well as contributions of higher resonances are difficult to treat in HBChPT. Since the contribution of the $\Delta(1232)$ is very important for the individual isospin channels, we

[†]Ji et al. use $\Gamma(Q^2)$ for $\overline{I}(Q^2)$

eliminate that contribution, as well as the ones of other isopin 3/2 resonance, by taking the proton-neutron difference. This will also reduce contributions by other resonances. While in (3) the values in the bracket are 13.4 and 10.84 for proton and neutron, respectively, the proton-neutron difference is 2.56, yielding a five times smaller slope at $Q^2 = 0$ compared to the neutron and proton case. For the proton-neutron difference of the generalized GDH SR one obtains

$$\bar{I}^{p-n} = \frac{\kappa_n^2 - \kappa_p^2}{4} + 1.31Q^2 + h.o.$$
(7)

In comparison with (5) and (6), a much reduced Q^2 dependence is predicted for this quantity compared to the proton and neutron, separately.

Taking the proton-neutron difference is also quite natural in analogy with the deep inelastic regime. While the Bjorken sum sule [18] for the proton-neutron difference has been tested experimentally [20] with good accuracy, the corresponding Ellis-Jaffe sum rule for proton and neutron separately [19] is significantly violated [20]. Only the Bjorken sum rule may thus be used to provide a reliable theoretical constraint in the deep inelastic region.

In order to compare with existing data we convert (7) to the usual first moment

$$\Gamma_1^{p-n}(Q^2) = \frac{Q^2}{2M^2} \bar{I}^{p-n} \tag{8}$$

In fig. 2, $\Gamma_1^{p-n}(Q^2)$ is shown with the data from SLAC and the pQCD evolution of the Bjorken sum rule to order α_s^3 [17]. The NLO term in the chiral expansion for the proton-neutron difference has now the correct sign and reproduces better the trend of the data.

A similar conclusion can be drawn for the NLO expansion by Ji et al. [16] as applied to the generalized GDH sum rule proposed by Bernard et al. [14]. A much reduced Q^2 dependence is obtained for the proton-neutron difference in this case as well.

In order to better understand the convergence of the chiral expansion at finite Q^2 , it is essential to evaluate the next-to-next-to-leading order (NNLO) corrections for the proton-neutron difference. With the accurate data expected at small and medium Q^2 from experiments at Jefferson Lab, stringent tests of these predictions will be possible. From the higher Q^2 end, the higher twist operator product expansion of pQCD may be used to extend the range down to possibly $Q^2 = 0.5 \text{ GeV}^2$ [16]. If the remaining gap between the chiral expansion and the higher twist expansion of QCD can be bridged using QCD lattice calculations it would mark the first time that nucleon structure is described within fundamental theory from small to large distances, a worthwhile goal.

In conclusion, the chiral expansion may be used successfully to expand the generalized Gerasimov-Drell-Hearn Sum Rule to finite Q^2 if the proton-neutron difference is used rather than proton and neutron separately. In the latter cases the NLO terms are sufficiently large to limit the expansion to $Q^2 < 0.05 \text{ GeV}^2$, while in the former case a five times larger Q^2 range is obtained. It is essential to calculate the NNLO terms to see whether this trend is continued in higher orders of the chiral expansion.

Acknowledgments

I gratefully acknowledge encouraging comments by Xiangdong Ji and Ulf Meissner regarding this report.

References

- For an overview see: B.W. Filipone, in: Proceedings of 14th International Conference on Particles and Nuclei, Williamsburg, Va., May 1996, eds. C.E. Carlson and J.J. Domingo.
- [2] V.D. Burkert and Zhujun Li; Phys.Rev.D47,46(1993)
- [3] V.D. Burkert and B.L. Ioffe, Phys.Lett.B296,223(1992)
- [4] V. D. Burkert and B.L. Ioffe, J. Exp. Theo. Phys. 78,619(1994)
- [5] J. Soffer and O. Teryaev, Phys. Rev. Lett. 70, 3373(1993)
- [6] V. Burkert, D. Crabb, R. Minehart, et al., JLAB experiment E92-023
- [7] S. Kuhn, M. Taiuti, et al., JLAB experiment E93-009
- [8] Z.E. Mezziani, J.P. Chen, et al., JLAB experiment E94-010
- [9] W.X. Ma, D.H. Lu, A.W. Thomas, Z.P. Li, Nucl. Phys. A635, 497(1998)
- [10] O. Scholten and A. Yu Korchin, Eur.Phys.J.A6,211 (1999)
- [11] D. Drechsel, S.S. Kamalov, G. Krein, L. Tiator, Phys. Rev. D59:094021 (1999)
- [12] S.B. Gerasimov; Sov. J. Nucl. Phys. 2, 430 (1966)
- [13] S.D. Drell and A.C. Hearn, Phys. Rev. Lett.16 (1966) 908
- [14] V. Bernard, N. Kaiser, U. Meissner, Phys.Rev.D48,3062(1993)
- [15] X. Ji and J. Osborne, hep-ph/9905410
- [16] X. Ji, C.W. Kao, and J. Osborne, Phys.Lett.B472,1(2000)
- [17] K. Abe et al., Phys.Rev.D58:112003 (1998)
- [18] J.D. Bjorken, Phys. Rev. 179, 1547 (1969)
- [19] J. Ellis and R.L. Jaffe, Phys.Rev.D9(1974)1444
- [20] S.A. Larin and J.A.M. Vermaseren, Phys.Lett.B172, 109(1986); Particle Data Group, R.M. Barnett et al., Phys.Rev.D53,6100(1996)

Figure 1.

First moments $\Gamma_1(Q^2)$ of the polarized structure functions $g_{1p}(x, Q^2)$ and $g_{1n}(x, Q^2)$. The data points are from SLAC, circles - proton, squares - neutron. The lines at high Q^2 are pQCD evolutions of the asymptotic behavior to $O(\alpha_s^3)$. The curves labeled "proton" and "neutron" are predictions of the chiral expansion in next-to-leading order (see text). The arrow labeled "GDH" represents the slope of $\Gamma_1 p$ at $Q^2 = 0$ as predicted by the GDH sum rule for protons.

Figure 2.

First moment difference $\Gamma_1^p - \Gamma_1^n$. Data are from SLAC. Solid line labeled "Bjorken" represents the Bjorken sum rule, corrected to $O(\alpha_s^3)$ [20], the line labeled "ChPT" represents eqn.(7) and (8) for the proton-neutron difference of the chiral expansion, and the arrow represents the slope defined by the GDH SR at the photon point.



