Local Quark-Hadron Duality in Structure Functions*

W. Melnitchouk[a]

[a]Special Research Centre for the Subatomic Structure of Matter,
University of Adelaide, Adelaide 5005, Australia,
and Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

We examine the consequences of local duality for elastic scattering, and derive model-independent relations between structure functions at $x \sim 1$ and elastic electromagnetic form factors.

1. Introduction

The nucleon's deep-inelastic structure functions and elastic electromagnetic form factors parameterise fundamental information about its quark substructure. Both reflect dynamics of the internal quark wave functions describing the same physical ground state, albeit in different kinematic regions.

Exploration of the structure function—form factor connection is in fact as old as the first deep-inelastic scattering experiments themselves. In the early 1970s this connection was studied in the context of inclusive scattering in the resonance region and the onset of scaling behavior in deep-inelastic structure functions. In their pioneering paper, Bloom and Gilman [1] observed that the inclusive F_2 structure function at low W generally follows a global scaling curve which describes high W data, to which the resonance structure function averages. Furthermore, the equivalence of the averaged resonance and scaling structure functions appears to hold for each resonance, over localised regions in W, so that the resonance—scaling duality also exists locally.

Following Bloom and Gilman's empirical observations, de Rújula, Georgi and Politzer [2] pointed out that global duality can be understood from an operator product expansion of the QCD moments of structure functions. The weak Q^2 dependence of the low F_2 moments was interpreted as indicating that higher twist ($1/Q^2$ suppressed) contributions are either small or cancel. More recently, high precision data on the F_2 structure function from Jefferson Lab [3] have confirmed the original observations of Bloom and Gilman, demonstrating that local duality works remarkably well for each of the low-lying resonances, including the elastic, to rather low values of Q^2.

If the inclusive–exclusive connection via local duality is taken seriously, one can use measured structure functions in the resonance region to directly extract elastic form factors [2]. Conversely, empirical electromagnetic form factors at large Q^2 can be used to predict the $x \to 1$ behavior of deep-inelastic structure functions [1].

*Talk given at International Conference on Quark Nuclear Physics, Adelaide, Australia, February, 2000.

2. Elastic Structure Functions

The contributions to the inclusive structure functions of the nucleon from an elastic final state can be written explicitly in terms of the Sachs electric and magnetic form factors, G_E and G_M [4,5]:

$$F_1^{\rm el} = M\tau\, G_M^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2M}\right) \tag{1}$$

$$F_2^{\rm el} = \frac{2M\tau}{1+\tau}\left(G_E^2(Q^2) + \tau G_M^2(Q^2)\right)\, \delta\left(\nu - \frac{Q^2}{2M}\right), \tag{2}$$

for unpolarised scattering, and [4,5]:

$$g_1^{\rm el} = \frac{M\tau}{1+\tau} G_M(Q^2)\left(G_E(Q^2) + \tau G_M(Q^2)\right)\delta\left(\nu - \frac{Q^2}{2M}\right), \tag{3}$$

$$g_2^{\rm el} = \frac{M\tau^2}{1+\tau} G_M(Q^2)\left(G_E(Q^2) - G_M(Q^2)\right)\delta\left(\nu - \frac{Q^2}{2M}\right), \tag{4}$$

for the polarised case, where $\tau = Q^2/4M^2$.

Integrating the elastic structure functions over the Nachtmann variable ξ, where $\xi = 2x/(1+\sqrt{1+x^2/\tau})$, between the pion threshold ξ_{th} and $\xi = 1$, one finds "localised" moments of the structure functions:

$$\int_{\xi_{th}}^1 d\xi\, \xi^n\, F_1(\xi, Q^2) = \frac{\xi_0^{n+2}}{4-2\xi_0}\, G_M^2(Q^2), \tag{5}$$

$$\int_{\xi_{th}}^1 d\xi\, \xi^n\, F_2(\xi, Q^2) = \frac{\xi_0^{n+2}}{2-\xi_0}\, \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}, \tag{6}$$

$$\int_{\xi_{th}}^1 d\xi\, \xi^n\, g_1(\xi, Q^2) = \frac{\xi_0^{n+2}}{4-2\xi_0}\, \frac{G_M(Q^2)\left(G_E(Q^2) + \tau G_M(Q^2)\right)}{1+\tau}, \tag{7}$$

$$\int_{\xi_{th}}^1 d\xi\, \xi^n\, g_2(\xi, Q^2) = \frac{\xi_0^{n+2}}{4-2\xi_0}\, \frac{\tau G_M(Q^2)\left(G_E(Q^2) - G_M(Q^2)\right)}{1+\tau}, \tag{8}$$

where $\xi_0 = 2/(1+\sqrt{1+1/\tau})$ is the value of ξ at the nucleon pole ($x = 1$).

Differentiating Eqs.(5)–(8) with respect to Q^2 for $n = 0$ allows the inclusive structure functions near $x = 1$ to be extracted from the elastic form factors and their Q^2-derivatives:

$$F_1 \propto \frac{dG_M^2}{dQ^2}, \tag{9}$$

$$F_2 \propto \frac{G_M^2 - G_E^2}{4M^2(1+\tau)^2} + \frac{1}{1+\tau}\left(\frac{dG_E^2}{dQ^2} + \tau\frac{dG_M^2}{dQ^2}\right), \tag{10}$$

$$g_1 \propto \frac{G_M\left(G_M - G_E\right)}{4M^2(1+\tau)^2} + \frac{1}{1+\tau}\left(\frac{d(G_E G_M)}{dQ^2} + \tau\frac{dG_M^2}{dQ^2}\right), \tag{11}$$

$$g_2 \propto \frac{G_M\left(G_M - G_E\right)}{4M^2(1+\tau)^2} + \frac{\tau}{1+\tau}\left(\frac{d(G_E G_M)}{dQ^2} + \frac{dG_M^2}{dQ^2}\right). \tag{12}$$

Note that as $\tau \to \infty$ each of the structure functions F_1, F_2 and g_1 is determined by the slope of the square of the magnetic form factor, while g_2 (which in deep-inelastic scattering is associated with higher twists) is determined by a combination of G_E and G_M.

3. Neutron and Proton Structure Functions Below Threshold

Knowledge of structure functions at large x is vital for several reasons, not least of which is that it allows one to test mechanisms for the breaking of spin-flavor symmetry in the nucleon. There are a number of predictions for the $x \to 1$ behavior of the neutron to proton F_2 structure functions, ranging from 2/3 in the SU(6) symmetric quark model, to 1/4 in broken SU(6) through d quark suppression, to 3/7 in broken SU(6) via helicity flip suppression (see Refs.[6,7] and references therein). Although it is well established that the large-x F_2^n/F_2^p data deviate from the SU(6) expectation, they are at present inconclusive about the precise $x \to 1$ limit because of large nuclear corrections in the extraction of F_2^n from deuterium cross sections beyond $x \sim 0.6$ [6].

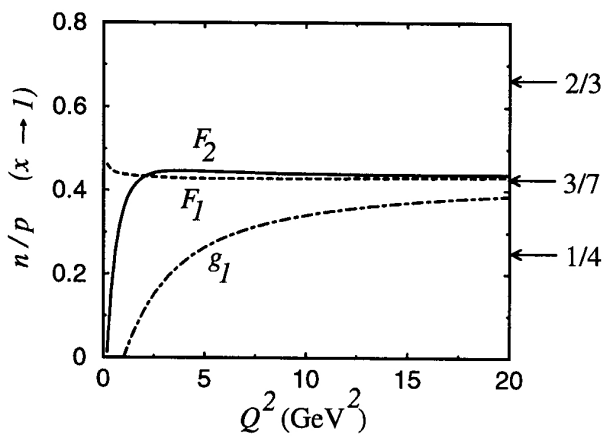

Figure 1. Neutron to proton ratio for F_1 (dashed), F_2 (solid) and g_1 (dot-dashed) structure functions in the limit $x \to 1$.

The ratios of the neutron to proton F_1, F_2 and g_1 structure functions are shown in Fig. 1 as a function of Q^2, using the parameterisation of the global form factor data in Ref.[8]. While the F_2 ratio varies quite rapidly at low Q^2, beyond $Q^2 \sim 3$ GeV^2 it remains almost Q^2 independent, approaching the asymptotic value $(dG_M^{n2}/dQ^2)/(dG_M^{p2}/dQ^2)$. Because the F_1^n/F_1^p ratio depends only on G_M, it remains flat over nearly the entire range of Q^2. At asymptotic Q^2 the model predictions for $F_1(x \to 1)$ coincide with those for F_2; at finite Q^2 the difference between F_1 and F_2 can be used to predict the $x \to 1$ behavior of the longitudinal structure function, or the $R = \sigma_L/\sigma_T$ ratio.

The pattern of SU(6) breaking for the spin-dependent structure function ratio g_1^n/g_1^p essentially follows that for F_2^n/F_2^p, namely 1/4 in the d quark suppression and 3/7 in the helicity flip suppression scenarios [6,7]. However, the g_1 structure function ratio approaches the asymptotic limit somewhat more slowly than F_1 or F_2, which may indicate a more important role played by higher twists in spin-dependent structure functions than in spin-averaged.

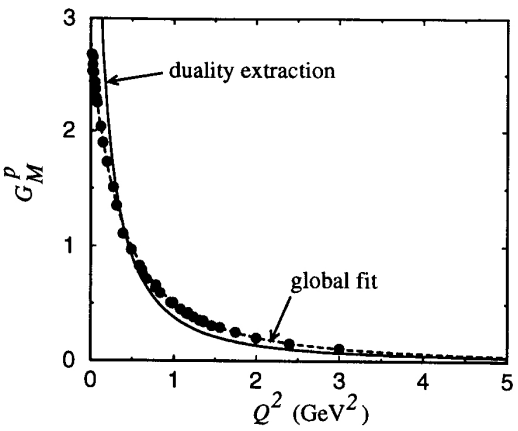

Figure 2. Proton magnetic form factor extracted from the inclusive structure function via Eq.(6).

It appears to be an interesting coincidence that the helicity retention model prediction of 3/7 is very close to the empirical ratio of the squares of the neutron and proton magnetic form factors, $\mu_n^2/\mu_p^2 \approx 4/9$. Indeed, if one approximates the Q^2 dependence of the proton and neutron form factors by dipoles, and takes $G_E^n \approx 0$, then the structure function ratios are all given by simple analytic expressions, $F_2^n/F_2^p \approx F_1^n/F_1^p \approx g_1^n/g_1^p \to \mu_n^2/\mu_p^2$ as $Q^2 \to \infty$. On the other hand, for the g_2 structure function, which depends on both G_E and G_M at large Q^2, one has a different asymptotic behavior, $g_2^n/g_2^p \to \mu_n^2/(\mu_p(1+\mu_p)) \approx 0.345$.

4. Extraction of Elastic Form Factors

If the resonance structure functions at large ξ are known, one can conversely extract the nucleon electromagnetic form factors from Eqs.(5)–(8). The G_M form factor of the nucleon can be extracted directly from the measured $F_1(\xi, Q^2)$ structure function in Eq.(5). Unfortunately, only the $F_2(\xi, Q^2)$ structure function of the proton has so far been measured in the resonance region. Nevertheless, to a good approximation one can assume that the ratio of electric to magnetic form factors is reasonably well known (see however Ref.[9]), and extract G_M from the F_2 structure function in the resonance region via Eq.(6).

Using the parameterisation of the recent $F_2(\xi, Q^2)$ data from Jefferson Lab [3], in Fig. 2 we show the extracted G_M^p compared with a compilation of elastic data. The agreement with date is quite remarkable over the entire of Q^2 between 0 and 3 GeV^2.

Another way to test the local duality relations is to combine Eqs.(5) and (6) and compare with the ratio of longitudinal to transverse cross sections, R:

$$R = \frac{\sigma_L}{\sigma_T} = \left(1 + \frac{1}{\tau}\right) \frac{F_2(x, Q^2)}{2F_1(x, Q^2)} - 1 . \tag{13}$$

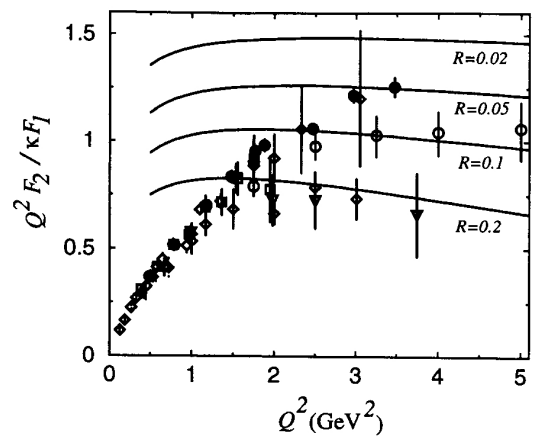

Figure 3. Local duality prediction for the ratio of Pauli to Dirac form factors.

In terms of R, the electric to magnetic form factor ratio is then:

$$\frac{G_E}{G_M} = \sqrt{\tau R}. \tag{14}$$

In terms of the Dirac and Pauli form factors (not to be confused with the inclusive $F_1(x, Q^2)$ and $F_2(x, Q^2)$ structure functions!):

$$F_1^{\text{Dirac}} = \frac{G_E + \tau G_M}{1+\tau}, \qquad F_2^{\text{Pauli}} = \frac{G_M - G_E}{1+\tau}, \tag{15}$$

one can equivalently write:

$$\frac{F_2^{\text{Pauli}}(Q^2)}{F_1^{\text{Dirac}}(Q^2)} = \frac{(1+R)\tau - (1+\tau)\sqrt{\tau R}}{(\tau - R)\tau}. \tag{16}$$

Empirically nothing is known about R at large x. In Fig. 3 we show the expected $Q^2 F_2^{\text{Pauli}}(Q^2)/F_1^{\text{Dirac}}(Q^2)$ ratio using several values of R, compared with the world data on the Pauli and Dirac form factors. Note that at large Q^2, the $F_2^{\text{Pauli}}(Q^2)$ form factor is expected from perturbative QCD to scale like $F_1^{\text{Dirac}}(Q^2)/Q^2$, so that asymptotically the ratio is expected to become flat. Measurement of these form factors at large Q^2 could therefore provide indirect information on the size of R at large x.

5. Discussion

The reliability of the duality predictions is of course only as good as the quality of the empirical data on the electromagnetic form factors and resonance structure functions. While the duality relations are expected to be progressively more accurate with increasing Q^2 [2], the difficulty in measuring form factors at large Q^2 also increases. Experimentally, the proton magnetic form factor G_M^p is relatively well constrained to $Q^2 \sim 30$ GeV^2, and the proton electric G_E^p to $Q^2 \sim 10$ GeV^2. The neutron magnetic form factor G_M^n has

been measured to $Q^2 \sim 5$ GeV^2, although the neutron G_E^n is not very well determined at large Q^2 (fortunately, however, this plays only a minor role in the duality relations, with the exception of the neutron to proton g_2 ratio, Eq.(12)). Future data on the $F_1(\xi, Q^2)$ structure function (via an L-T separation) in the resonance region will also be helpful in testing the local duality relations.

Along with the spin dependence, unraveling the flavor dependence of duality is also of fundamental importance. Although the local duality relations discussed here are empirical, a more elementary description of the quark–hadron transition requires understanding the transition from coherent to incoherent dynamics and the role of higher twists for individual quark flavors. This is as relevant for all the $N \to N^*$ transition form factors as for the elastic. The flavor dependence can be determined by either studying different hadrons, or tagging mesons in the final state of semi-inclusive scattering in the resonance region. Both the flavor and spin dependence of duality, and more generally the relationship between incoherent (single quark) and coherent (multi-quark) processes, will be addressed with an energy upgrade at Jefferson Lab, which should shed considerable light on the nature of the quark $\to$ hadron transition in QCD.

This work was supported by US DOE contract DE-AC05-84ER40150, and the Australian Research Council.

REFERENCES

1. E.D. Bloom and F.J. Gilman, Phys. Rev. Lett. 16 (1970) 1140.
2. A. De Rújula, H. Georgi and H.D. Politzer, Ann. Phys. 103 (1975) 315.
3. I. Niculescu, Ph.D. thesis, Hampton University, 1999; C. Keppel, talk presented at 7th International Workshop on Deep Inelastic Scattering and QCD (DIS 99), Zeuthen, Germany, Apr. 1999.
4. C.E. Carlson and N.C. Mukhopadhyay, Phys. Rev. D 58 (1998) 094029.
5. W. Melnitchouk, preprint ADP-00-13/T397, JLAB-THY-00-08.
6. W. Melnitchouk and A.W. Thomas, Phys. Lett. B 377 (1996) 11.
7. N. Isgur, Phys. Rev. D 59 (1999) 034013.
8. P. Mergell, U.-G. Meißner and D. Drechsel, Nucl. Phys. A596 (1996) 367.
9. M.K. Jones et al., Phys. Rev. Lett. 84 (2000) 1398.