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The origins of quark-hadron duality: How does the square of the sum become the sum of the squares?

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Abstract

Bloom-Gilman duality demonstrates empirically that the electroproduction of N^*'s at low momentum transfers averages smoothly around the scaling curve measured at large momentum transfers. The latter is proportional to the sum of the squares of the constituent charges whereas the former involves the coherent excitation of resonances and is driven by the square of summed constituent charges. We determine the minimal necessary conditions for this equality to be realised so that duality can occur and consider the implications for a range of processes that may be studied soon at CEBAF.

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When protons are probed by electron beams at energies of tens or hundreds of GeV, and at correspondingly large momentum transfers (Q^2), the scattering probability (summarised in the structure function $F_2(W^2, Q^2)$ where W is the mass of the hadronic system) is rather simple. It exhibits the well known property of scale invariance where $F_2 \sim F_2(W^2/Q^2)$, with small corrections that are well understood from perturbative QCD. Furthermore, the magnitude of $F_2(W^2/Q^2)$ is proportional to the sum of the squares of the (quark and antiquark) constituent charges.

Before the advent of QCD, Bloom and Gilman discovered [1] an empirical property of the data, namely that the electroproduction of N^*'s at lower energies and momentum transfers averages smoothly around the scaling curve measured at large momentum transfers. During the subsequent three decades, and especially following the advent of quantum chromodynamics, this enigma has received considerable theoretical attention [2]. The literature has primarily focussed on understanding how the Q^2 dependence of resonance excitation can conspire to mimic the $x' \sim \frac{Q^2}{W^2+Q^2}$ dependence of the deep inelastic data.

Recent high precision data from Jefferson Lab [3] have shown that this duality is observed for both proton and neutron targets, at least for spin averaged scattering, and that for the proton it occurs locally, *i.e.*, resonance by resonance. This has sparked a renewed interest in the origin of low energy duality, including a recent analysis in the context of a large-N_c-based relativistic quark model [4]. This work focussed on the dynamics required for a confined struck quark to behave as though it were free, but remarked in passing on the additional conditions on quark charges and dynamics required for duality to be realized locally. These data and the work of Ref. [4] have led us to focus in this paper on the latter issue, namely the circumstances whereby $F_2(x')$ - whose magnitude is in proportion to the sum of the squares of the (quark and antiquark) constituent charges - can in general match with the excitation of individual resonances which is driven by the coherently summed square of constituent charges.

In this note we draw attention to the necessary conditions for this duality to occur in general. We illustrate the essential physical principle in a simple pedagogical model of a system of two spinless electrically charged consitutents. This is then generalised to the more realistic case of three spinning quarks. Implications for both spin-dependent and spin-independent structure functions for proton and neutron targets will be displayed, local deviations from duality are predicted, and the possibility that there could be "precocious" factorisation in semi-inclusive hadroproduction is discussed. These ideas promise a lively program of experimental investigation for CEBAF.

A Simple Model

Consider a composite state made of two equal mass scalars, "quarks" q_1, q_2 with charges e_1, e_2 respectively at positions $\vec{r}_{1,2}$. The ground state wavefunction is $\psi_0(\vec{r})$, where $\vec{r}_{1,2} = \vec{R} \pm \vec{r}/2$ defines the centre of mass and internal spatial degrees of freedom. A photon of momentum $\vec{q}$ is absorbed with an amplitude proportional to $\Sigma_i e_i exp(i\vec{q}\cdot\vec{r}_i)$, which excites a "resonant" state with angular momentum L, described by the wavefunction $\psi_L(\vec{r})$.

Focussing on the internal coordinate $\vec{r}$, the transition amplitude is proportional to

$$(e_1+e_2)(exp(i\vec{q}\cdot\vec{r}/2)+exp(-i\vec{q}\cdot\vec{r}/2))+(e_1-e_2)(exp(i\vec{q}\cdot\vec{r}/2)-exp(-i\vec{q}\cdot\vec{r}/2))$$

The expansion, $exp(iqz/2) = \Sigma_L i^L P_L(cos\theta) j_L(qr/2)(2L+1)$, projects out the even and odd partial waves such that the amplitude is proportional to

$$M \sim \int drr^2 \psi_L^*(r)\psi_0(r) j_L(qr/2)[(e_1+e_2)\delta_{L=even} + (e_1-e_2)\delta_{L=odd}]. \qquad (1)$$

The resulting structure function, summed over resonance excitations, will have the form

$$F(q) \sim \sum_{n=0}^{\infty}[F_{2n}(q)(e_1+e_2)^2 + F_{2n+1}(q)(e_1-e_2)^2] \qquad (2)$$

which in general will be proportional to $e_1^2 + e_2^2$ only if the odd and even L states sum to equal strengths.

We now derive the circumstances under which this can occur. Consider the uu, $(ud \pm du)/\sqrt{2}$ and dd composite systems subject to the following process: $W^+ + (dd) \rightarrow (ud \pm du)/\sqrt{2} \rightarrow W^- + (uu)$. The overall Bose symmetry of the system will constrain the $ud + (-)du$ intermediate states to have $L = 2n$ ($L = 2n + 1$) respectively. Following analogous steps to those above, the amplitude will have the form

$$A\Big(W^+(q) + (dd) \rightarrow W^-(q) + (uu)\Big) \sim \sum_{n=0}^{\infty}[F_{2n}(q) - F_{2n+1}(q)] \tag{3}$$

In this case the $t-$channel involves transfer of exotic quantum numbers (charge 2, corresponding to constituents $uu\bar{d}\bar{d}$) and in the absence of such states, the amplitude must vanish according to the theory of hadronic duality [5], and as is seen consistently in hadronic data. This forces

$$\sum_{n=0}^{\infty} F_{2n}(q) \equiv \sum_{n=0}^{\infty} F_{2n+1}(q) \tag{4}$$

which with Eqn.(2) leads to

$$F(q) \sim \sum_{n=0}^{\infty} F_n(q)(e_1^2 + e_2^2) \tag{5}$$

whereby the square of the sum has become the sum of the squares.

This simple example exposes the physics rather clearly. The excitation amplitudes to resonance states contain both diagonal ($e_1^2 + e_2^2$) and higher twist terms ($\pm 2e_1e_2$) in the flavour basis. The former set add constructively for any L and the sum over the complete set of states can now logically give the deep inelastic curve [2]; the latter enter with opposite phases for even and odd L and destructively interfere. The critical feature that this exposes is that *at least one complete set of resonances of each symmetry-type has been summed over.*

In a non-relativistic SHO model it is possible to see how the above conspiracy arises. The contribution to $F(q)$ from the $n(\equiv L + 2k)$ set of degenerate levels (k being the radial and L the orbital quantum number) is $F_n(q) \sim (n!)^{-1}(q^2R^2)^n e^{-q^2R^2}$ from which one can immediately see that $\Sigma_n F_n(q) = 1$. It is interesting to note that any individual contribution, F_n, reaches its maximum value when $q^2R^2 = n$, at which point $F_n = F_{n-1}$. This coincidence is true for all juxtaposed partial waves at their peaks, which gives a rapid approach to the equality of F_{odd} and F_{even}. Analytically in the SHO one finds

$$F(q) \sim (e_1^2 + e_2^2) + 2e_1e_2e^{-4q^2R^2} \tag{6}$$

whereby the coherent (duality-violating) terms vanish like the fourth power of the elastic form factor.

SHO Quark Model

This pedagogical example contains all the essential physics needed to understand the more realistic case of the harmonic oscillator quark model. The example of spinless constituents above involved only electric multipoles, whereas introduction of spin involves both electric and magnetic multipole contributions. The symmetric and antisymmetric states generalise, respectively, to the **56** and **70** representations of $SU(6)_{flavor-spin}$. The destructive interference in the $s-$channel sum is as before and is now correlated with the overall Fermi antisymmetry of the $SU(6) \times SU(3)_{color} \times \psi(r)$ wavefunctions. The quark-parton model scaling curves proportional to $\Sigma_i e_i^2$ then obtain if $F_1(\mathbf{56}) \equiv F_1(\mathbf{70})$ and if the interaction of the photon with the magnetic moment of the quarks dominates.

This numerology was demonstrated long ago in Ref. [6], which exhibited the decomposition of the structure functions for $\gamma + N \rightarrow$ hadrons when the γ and N spins are antiparallel ($\sigma_{1/2}$) or parallel ($\sigma_{3/2}$) (i.e., when the net spin projection along the initial photon direction is $1/2; 3/2$). The relative strengths of the supermultiplets for the case where magnetic interactions dominate are listed in Table 1 from which $g_1^{p,n} \sim \sigma_{1/2} - \sigma_{3/2}$ and $F_1^{p,n} \sim \sigma_{1/2} + \sigma_{3/2}$ can be constructed. The familiar leading twist ratios are obtained immediately from an equal weighting of the **56** and **70** contributions: $F^n/F^p = 2/3$; $A^p \equiv g_1^p/F_1^p = 5/9$; $A^n = 0$.

Table 1: Relative Photoproduction Strengths in the Quark Model

$SU(6):$	$\sigma^p_{1/2}$	$\sigma^p_{3/2}$	$\sigma^n_{1/2}$	$\sigma^n_{3/2}$
56	11	6	6	6
70	10	0	3	3

These results imply that for $F_1^{p,n}$ and g_1^p duality will not be realised unless the **56** and **70** states ($N, \Delta(1236)$ and the negative parity N^*, Δ^* with $W \leq 1.8$ GeV) have been integrated over. However, for g_1^n there is the tantalising possibility that duality may be more localised as the **56** alone, consisting of $N, \Delta(1236)$ already satisfies $\int dW g_1^n(W,q) = 0$ in this idealised model, and the dominance of magnetic interactions is also realised for the neutron target. In reality SU(6) is broken, in particular by the differing masses of the N and Δ due to higher twist one-gluon-exchange hyperfine effects in QCD. Hence it will be interesting to investigate these general ideas in explicit quark models to predict the detailed approach to duality in a world where SU(6) is broken. Some qualitative ideas on the $x \to 1$ dependence of F^n/F^p and $A^{n,p}$ already exist in the literature[7, 8]; with the above insights into how the leading twist results relate to the symmetries of the excited hadronic states, explicit calculations summing over the above resonances and incorporating SU(6) breaking in QCD can now be developed and their implications for CEBAF in particular explored.

When we break Table 1 into its $SU(3) \times SU(2)$ content, we see that for the proton duality may be satisfied by $W \leq 1.6$ GeV. This is because the $\mathbf{70};{}^{\mathbf{4}}\mathbf{8}$ and $\mathbf{70};{}^{\mathbf{2}}\mathbf{10}$, which are at ~ 1.7 GeV, make negligible contributions (Table 2). For neutron targets, by contrast, one must include the $\mathbf{70};{}^{\mathbf{4}}\mathbf{8}$, which necessitates integrating up to 1.8 GeV. This region above 1.7 GeV also contains **56** at $N = 2$ in the harmonic oscillator. Thus we anticipate systematic deviations from local duality, in accord with the new data from Ref. [3]. These data showed that the $S_{11}(1530)$ region ($\mathbf{70};{}^{\mathbf{2}}\mathbf{8}$) and the $F_{15}(1680)$ ($\mathbf{56};{}^{\mathbf{2}}\mathbf{8}$) are enhanced relative to the deep inelastic scaling curve for proton targets. As Table 2 shows, the **70** contribution for a proton target is concentrated in the $\mathbf{70}^{\mathbf{2}}\mathbf{8}$, hence the enhancement seen in Ref. [3].

Table 2: Relative Photoproduction Strengths of $\mathbf{56}, \mathbf{0}^+$ and $\mathbf{70}, \mathbf{1}^-$ Multiplets

$SU(6):$	$[\mathbf{56}, \mathbf{0}^+]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{56}, \mathbf{0}^+]^{\mathbf{4}}\mathbf{10}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{4}}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

In contrast to the proton case, this table predicts that for neutron targets, the $S_{11}(1530)$ region ($[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{8}$) will fall **below** the scaling curve. The third resonance region, containing $[\mathbf{70}, \mathbf{1}^-]^{\mathbf{4}}\mathbf{8}$ as well as $[\mathbf{56}, \mathbf{2}^+]^{\mathbf{2}}\mathbf{8}$ and $[\mathbf{56}, \mathbf{2}^+]^{\mathbf{4}}\mathbf{10}$, is expected to be locally enhanced over the scaling curve for both proton and neutron targets. Note that to order q^2 the $[\mathbf{56}, \mathbf{0}^+]$ and $[\mathbf{70}, \mathbf{1}^-]$ multiplets are sufficient to realise duality. Formally the analyis can be extended to higher q^2 by including correspondingly higher multiplets; however, the credibility of the non-relativistic harmonic oscillator may become questionable. These predictions will be interesting tests of our analysis.

Inclusion of both magnetic and electric interactions shows that the duality is non-trivial. Inasmuch as the magnetic terms dominate at large Q^2 in the quark model, duality can be realised for the dominantly transverse scattering of the deep inelastic region. For the longitudinal structure function, F_L, duality is again realised, with the breakdown into **56** and **70** as in Table 3:

Table 3: Relative Longitudinal Production Strengths, as in Table 2

$SU(6):$	$[\mathbf{56}, \mathbf{0}^+]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{56}, \mathbf{0}^+]^{\mathbf{4}}\mathbf{10}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{4}}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^{\mathbf{2}}\mathbf{10}$	*total*
F_L^p	1	0	1	0	1	3
F_L^n	0	0	1	0	1	2

However, for $F_1(Q^2 \to 0)$ both electric and magnetic multipoles contribute and interfere with phases determined by the J^P and the spin-L_z correlations in the various **56** and **70** states. This causes dramatic Q^2 dependence in polarisation asymmetries [6, 9] and enables the connection to the Drell-Hearn-Gerasimov sum rule at $Q^2 = 0$. Thus we predict that *Bloom-Gilman duality must fail at Q^2 where the electric and magnetic multipoles have comparable strengths.* Calculations in simplistic models successfully predicted that this would be at $Q^2 \sim 0.5$ GeV^2[9]; these results now merit, and are receiving, more detailed examination [10]. The data from Ref. [3] show that even as low as $Q^2 = 0.5$ (GeV/c)^2 the integrated strengths of spectra at fixed Q^2 are within 10% of the corresponding integrals over the scaling curve. It will be interesting to verify the predicted breakdown at lower Q^2 and also to test if duality in the **magnetic** multipoles holds all the way to $Q^2 = 0$.

A Possible Extension to Fragmentation Functions

Our conclusion that the destructive interference between hadronic states of different symmetries is a critical feature of duality, can be applied to semi-inclusive hadroproduction, such as $\gamma(q)N \to \pi + X$. In the quark parton model in the ideal valence quark region when $u(x) = 2d(x)$ and at large z, where $D_u^{\pi^-}/D_u^{\pi^+} \to 0$, it is trivial to obtain relations such as $F(\gamma p \to \pi^+ + X)/F(\gamma p \to \pi^- + X) = 8$ (= 1/2 for neutron targets) [11]. The coherent picture described above can be applied to these processes. We find that destructive interference leads to factorisation and to duality, with for example

$$F(\gamma p \to \pi^+ + X) \equiv \sum_{(W' = \mathbf{56}, \mathbf{70})} F(\gamma p \to \pi^+ W') \tag{7}$$

where

$$F(\gamma N \to \pi^+ W') = \sum_{(N^*, N'^*)} F_{(\gamma N \to N^*)} D_{(N^* \to N'^* \pi)} \sim \sum_q e_q^2 q(x) D_{q \to \pi}(z) \tag{8}$$

where $D_{q\to\pi}$ is the quark $\to \pi$ fragmentation function, $F_{(\gamma N \to N^*)}$ is the $\gamma N \to N^*$ transition form factor, and $D_{(N^* \to N'^* \pi)}$ is a function representing the decay $N^* \to N'^* \pi$ where W' is the invariant mass of the final state N'^*. The breakdown of $F(\gamma N \to \pi W')$ into the individual states in the supermultiplets for the final W' states is shown in Table 4:

Table 4: $SU(6)$ and $SU(3) \times SU(2)$ Multiplet Contributions to Inclusive $\pi^{\pm}$ Photproduction

W'	$\gamma p \to \pi^+ + W'$	$\gamma p \to \pi^- + W'$	$\gamma n \to \pi^+ + W'$	$\gamma n \to \pi^- + W'$
$\mathbf{56}; \mathbf{8}$	100	0	0	25
$\mathbf{56}; \mathbf{10}$	32	24	96	8
$\mathbf{70};^{\mathbf{2}}\mathbf{8}$	64	0	0	16
$\mathbf{70};^{\mathbf{4}}\mathbf{8}$	16	0	0	4
$\mathbf{70}; \mathbf{10}$	4	3	12	1
$TOTAL$	216	27	108	54

Note the self-consistency of the results: $F(\gamma p \to (\pi^+ + \pi^-))/F(\gamma n \to (\pi^+ + \pi^-))$ recovers the 3/2 ratio familiar for the "total" F^p/F^n. We see also that duality may be obtained at large Q^2, W^2 when W' is integrated over the range up to 1.7 GeV ($\mathbf{56} + \mathbf{70}$). We see also that to a good approximation it may ensue for the $\mathbf{56}$ alone when Born terms and the Δ are included. To the extent that this is true, one may expect factorisation and approximate duality at small $Q^2, W'^2 \leq 3\text{GeV}^2$. A possible example of this was noticed long ago in Ref [12] for $\gamma p(n) \to \pi\Delta$ when $0.2 \leq |t| \leq 1\text{GeV}^2$.

In $\gamma N \to \pi + W'$, the above Table strictly applies only to the imaginary part of the amplitudes. One should in principle also consider $u-$channel diagrams, where the π is emitted prior to the photoabsorbtion. These diagrams would give an inversion of the ratios for π^+/π^-. Ref. [12] has shown, via fixed t dispersion relations, that at least for small Q^2, the s and u-channel resonances tend to cancel in the real part, whereby the charge ratios are preserved. There is some indication from data on photoproduction that this is the case empirically [13, 12], but the dependence of these processes on Q^2 needs to be checked in explicit models and the precision of these ratios tested with data from CEBAF. We hope to return to these questions elsewhere.

Finally, we comment on a characteristic feature of one particle inclusive data: the production of jets. In this resonance duality approach, for any individual partial wave L there is a specified angular distribution. Upon summing over L, the jet characteristics of the quark parton model arise as a result of the constructive interference of spherical harmonics in the near forward and destructive interference in the backward hemisphere [14]. As may be anticipated from the uncertainty principle, after summing over L one expects an angular spread of the jet given by $\Delta\phi \sim h/L_{max}$.

In this letter we have outlined the general features of a programme to understand not only duality, but also to define the "semi-local" averaging procedures that must be employed to see duality at low energies. We shall report elsewhere on detailed predictions that may be tested in the forthcoming experimental programs for CEBAF at Jefferson Laboratory. If we are successful in defining these averaging procedures, it would have a great impact on our ability to measure structure functions in kinematic regions that hitherto have been believed to be inaccessible.

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