

Extracting forward strong amplitudes from elastic differential cross sections

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(July 17, 2001)*

Abstract

The feasibility of a model-independent extraction of the forward strong amplitude from elastic nuclear cross section data in the Coulomb-nuclear interference region is assessed for π and K^+ scattering at intermediate energies. Theoretically-generated “data” are analyzed to provide criteria for optimally designing experiments to measure these amplitudes, whose energy dependence (particularly that of the real parts) is needed for disentangling various sources of medium modifications of the projectile-nucleon interaction. The issues considered include determining the angular region over which to make the measurements, the role of the most forward angles measured, and the effects of statistical and systematic errors. We find that there is a region near the forward direction where Coulomb-nuclear interference allows reliable extraction of the strong forward amplitude for both pions and the K^+ from .3 to 1 GeV/c.

25.80Dj, 25.80Nv, 24.10Jv, 13.75Gx

I. INTRODUCTION

In the scattering of a charged particle from a nucleus, the measured elastic differential cross section is the square of a scattering amplitude, $F_{el}(\theta)$, which is the sum of a Coulomb amplitude $f_c(\theta)$ and a strong amplitude $F_N(\theta)$,

$$F_{el}(\theta) = f_c(\theta) + F_N(\theta) , \quad (1)$$

Many outstanding issues of strong interaction physics may be addressed if one can determine $F_N(\theta)$ from the experimental data. For example, for pion and kaon beams, medium modifications of the underlying projectile-nucleon scattering amplitude are of great current interest [1,2], and efforts are being made to extract this information from scattering data. Typically, modern analyses utilize sophisticated optical model codes [2,3] based on microscopic models of the underlying dynamics, in which direct comparisons are made between the differential cross sections calculated by the codes and measured differential cross sections.

We wish to emphasize that for pion-nucleus scattering, especially in the GeV range of energies and for kaon-nucleus scattering at energies above several hundred MeV, it is extremely valuable for the theorists to have precise information on the *amplitude* $F_N(\theta)$ for $\theta \approx 0^\circ$. Measurements of $d\sigma_{el}/d\Omega(\theta)$ at small θ contain this information through the interference of the Coulomb and strong interactions in Eq. 1. Determinations of $\text{Re}F_N(0)$ and $\text{Im}F_N(0)$ are strong constraints on the underlying models. We make the case in this paper that the considerable inconsistency existing in currently available data sets [4] can be remedied at existing facilities with appropriate measurement strategies. We hope that our results will facilitate precise measurements of $d\sigma_{el}/d\Omega(\theta)$ at small θ at these laboratories.

The issues involved in designing measurement strategies may be examined through application of optical models such as those mentioned above. However, a much simpler approach is possible due to the fact that [4] for π -nucleus scattering at energies above the Δ_{33} resonance ($T_\pi \geq 300$ MeV) and K^+ -nucleus scattering at all energies, the region over which the Coulomb amplitude is an appreciable fraction of the strong amplitude extends from a few degrees out to angles generally beyond ninety degrees. This is an ideal situation for utilizing Coulomb-nuclear interference in a model independent way to extract the strong amplitude from elastic differential cross section measurements.

To accomplish this, elastic differential cross section data needs to be taken in the near forward direction. Such measurements do not require a large amount of beam time; the measurements are all made at angles where the differential cross section is quite large. A set of data spanning a large energy region at reasonably spaced energy intervals is thus possible. Data taken with P_{lab} at intervals of 25 MeV/c from 300 MeV/c to 1 GeV/c would take less time than is required for the measurement of a single differential cross section out to a reasonable angle. This is because the differential cross section is diffractive and thus falls exponentially with angle. Points with decent statistics at large angles where the cross section has typically fallen by four orders of magnitude require more beam time than do a large number of measurements at small angles.

The primary purpose of the present work is to assess the feasibility of measuring the real and imaginary parts of the forward scattering amplitude with sufficient accuracy to clarify the issues alluded to above. Our secondary purpose is to present enough of the details of the argument and analysis so that the experimentalists can optimally design such an experiment.

In Sect. II, we review a simple procedure for extracting the strong amplitude from the elastic differential cross section and provide a summary of the results of our analysis (details are given in the Appendices) based on this method. We utilize the model of Refs. [2,3] to generate theoretical elastic differential cross sections at a set of angles. These results constitute model data to which we add statistical or systematic errors of various magnitudes. The momentum-space model of [2,3] produces results which are very similar to the measured elastic differential cross sections for π [5,6] and for K^+ [6,7]. We thus believe that the analysis done here on these model data sets is directly applicable to real data. In Section III we discuss underlying physics issues involved. The final section summarizes the paper and presents our overall conclusions.

II. PROCEDURE AND RESULTS

To use Coulomb-nuclear interference to extract the forward strong amplitude from the elastic differential cross section, we envision making an expansion of $F_N(\theta)$ and fitting the coefficients of the various terms to data. In order to determine this amplitude most effectively, certain questions need to be addressed. These would include the following: what is the optimal angular range over which measurements should be made; to what order should one expand the strong amplitude; how important is it to measure to the most forward-possible angle; how do statistical and systematic errors affect the extracted values of the strong forward amplitude; and, most importantly, how stable and reliable is the method for extracting the forward strong amplitude? We define the procedure we use for answering these questions in Sect. II.A and present our results in Sect. II.B, below.

A. Procedure

The nuclear amplitude $F_N(\theta)$ that we will use for our analysis is defined in terms of the *point* Coulomb scattering amplitude $f_{c,pt}(\theta)$,

$$F_{el}(\theta) = f_{c,pt}(\theta) + F_N(\theta) . \quad (2)$$

Alternatively, we could define the nuclear amplitude relative to the extended Coulomb amplitude or introduce the strong amplitude related to $F_N(\theta)$ through the Bethe phase [8–10]. All such definitions are mathematically equivalent, and we choose to work with the definition (the one corresponding to Eq. 2) that simplifies the empirical analysis. Since we anticipate the use of this amplitude as a constraint on the optical model, all three nuclear amplitudes are equally suitable for this purpose.

The simplest procedure for obtaining $F_N(0)$ is based on a Taylor series expansion of $F_N(\theta)$ in powers of $\sin^2 \theta/2$. Truncating the series after three terms, we then have six real parameters, A_R , B_R , C_R , A_I , B_I , and C_I defined by:

$$F_N(\theta) = A_R \left(1 - B_R \sin^2 \theta/2 - C_R \sin^4 \theta/2 \right) + i A_I \left(1 - B_I \sin^2 \theta/2 - C_I \sin^4 \theta/2 \right) . \quad (3)$$

The experimental differential cross section minus the point Coulomb differential cross section is given by $d\sigma_N/d\theta$

$$\begin{aligned} \frac{d\sigma_N}{d\theta}(\theta) &\equiv \frac{d\sigma_{el}}{d\theta}(\theta) - \frac{d\sigma_{c,pt}}{d\theta}(\theta) \\ &= |F_N(\theta)|^2 + 2 \operatorname{Re} [f_{c,pt}(\theta) F_N^*(\theta)] , \end{aligned} \quad (4)$$

where $d\sigma_{el}/d\theta$ is the experimental elastic differential cross section and $d\sigma_{c,pt}/d\theta$ is the point Coulomb differential cross section. The various expansion parameters of $F_N(\theta)$ given in Eq. 3 may then be determined by a fit of Eq. 4 to the forward angle experimentally measured differential cross sections, $d\sigma_{el}/d\theta$.

This work assesses the above procedure for extracting the forward strong amplitude based on theoretically generated model data sets. We utilize the the momentum-space theory of Ref. [2,3] to generate the model data. To study the effects of statistical errors, Gaussian distributed errors are added to the theoretically generated model data. To understand the effects of systematic errors, these data sets are renormalized both upward and downward. We do the analysis for both π^- and K^+ at $P_{lab} = 500$ MeV/c and 1 GeV/c. This study demonstrates that the extraction of the forward strong amplitude, both the real and imaginary part, is feasible, and we provide the information needed to optimally design an experiment.

The strong interaction amplitude extrapolated to zero degrees is then given by $F_N(0) = A_R + i A_I$. As we have stated, once $F_N(0)$ is obtained, it may be used in conjunction with optical model descriptions such as those of Ref. [2,3] to constrain the underlying microscopic models. If desired, the real and imaginary parts of the strong amplitude may be extracted directly [8–10] from A_R and A_I using a Bethe-phase analysis [4]. Note that the real part of the forward strong amplitude can also be extracted from transmission experiments [11], a technique that has been applied [12] to resonance energy π -nucleus scattering.

B. Results

The details of the assessment of the above procedure for obtaining $F_N(0)$ are provided in the Appendices. In order to determine the feasibility of an experiment, we examine the angular range over which data should be taken (Appendix 1), the effects of statistical (Appendix 2) and systematic (Appendix 3) errors, and the importance of taking very forward angle points (Appendix 4). These details should prove valuable for the design of an experiment. Here we summarize our results. Our goal is to extract the forward amplitude, i.e. A_I and A_R . In the Appendices we also provide some guidance on the possibility of extracting the next term in the expansion, B_I and B_R .

We find in Appendix 1 that there is a substantial angular region over which the strong and Coulomb amplitudes are reasonably comparable. We also find that there is an angular region where the strong amplitude is linear in $\sin^2 \theta/2$. We explored going beyond linear order in the expansion by extending the angular region of the measurements. This extension did *not* improve the ability to extract the forward amplitude. The smallest angle of this region determined by requiring the magnitude of the Coulomb amplitude to be twice the size of the strong amplitude. This implies, for both pions and K^+ , that $\theta_{min} = 4^\circ$ for 500 MeV/c and $\theta_{min} = 2^\circ$ for 1 GeV/c. The maximum angle is determined by the conditions that the linear approximation hold and that the extrapolation yield an accuracy to, say, better than 1%. This would require a measurement out to about 16° .

In Appendix 2 we find that the magnitude of the error in the extracted forward amplitude is proportional to the statistical errors in the data. For π^- the percent error in A_I is approximately equal to the percent error in the data. The error in A_R is of the same magnitude, but, because the value of A_R is much smaller, it corresponds to larger percent error in this quantity. The situation is different for K^+ . The percent error in A_R is approximately three times the error in the data at both energies examined, while for A_I it is seven times at 500 MeV/c and three times at 1 GeV/c.

Because the cross sections are large throughout the angular region for these experiments, it is easy to have the statistical errors much smaller than the systematic errors. For systematic errors above about 10%, the analysis becomes nonlinear and is not reliable, as discussed in Appendix 3. Systematic errors of 5% or less do yield reliable results. For π^- scattering at 500 MeV/c, each 1% systematic error yields only 0.3% error in the extrapolation for A_R and 0.5% for A_I . At 1 GeV/c, the behavior of A_I is nonlinear, but stable. There is never more than 0.7% error in the extracted value for each 1% systematic error. The error in A_R is comparable in magnitude to the error in A_I , which, again, corresponds to a much larger percent error in A_R at 1 GeV/c. For K^+ each 1% systematic error produces at 500 MeV/c an error of 5.7% for A_I , 1.5% for A_R , and at 1 GeV/c an error of 1.4% for A_I and 3% for A_R . Systematic errors below 5% would very significantly constrain theories for both pions and K^+ .

Since the normalization of the incident beam can be a large source of systematic error, a special setup that would allow for measurements into the far forward direction would be desirable. Measurements at the very forward angles where the Coulomb scattering becomes dominant would allow the normalization of the beam to be determined by comparing the measurements to the known Coulomb amplitude.

Finally, the forward-most points are the most difficult to take and yet we would expect them to be the most important in controlling the extrapolation to $\theta = 0^\circ$. We examine in Appendix 4 the consequences of increasing the errors on the first two experimental points at θ_{min} and $\theta_{min} + 0.5^\circ$, or eliminating them altogether. For statistical errors of 1% or 2%, the removal of these data points roughly doubles the magnitude of the errors in the extracted quantities, with the exception of A_R for the K^+ where at 500 MeV/c the extracted value is little affected and at 1 GeV/c it increases by a factor of three. The forward points taken with good statistics and with systematic errors no worse than the remaining points can, in general, reduce errors by a factor of two or more.

III. DISCUSSION

The strong amplitude at zero degrees as a function of energy contains information which is complimentary to that obtained by measuring exclusive cross sections as a function of angle. The imaginary part of the forward strong amplitude is related through unitarity to the total cross section. Its extraction from differential cross section measurements is an experimental check on the consistency of two independent experimental measurements, elastic scattering and transmission experiments. Because it is independent of the total cross section, the real part of the forward strong amplitude provides new information to help decide among competing models. Moreover, the real part of the strong amplitude at zero degrees is a quantity that is not driven by small corrections to the theory, such as is the exact

depth of the minima in the diffraction pattern. It is a qualitative feature of the reaction, which like the total cross section, puts constraints on the theory at the qualitative level.

There is a growing body of evidence [1] that the kaon interaction with a nucleon is enhanced in the nuclear medium compared to that in free space. Several phenomenological analyses [13] of this data have been performed, but a consistent picture of the underlying physics has not yet emerged. Measurement of the strong forward scattering amplitude for kaon-nucleus elastic scattering as a function of energy would provide an independent check on the total cross section measurements. Since the real part of the forward amplitude contains new information, it would presumably help determine the underlying physics behind the in-medium increase in the interaction.

The situation is somewhat different for the pion. The π -nucleon interaction is dominated by a number of overlapping resonances. Evidence from photo-reactions indicate the existence of an in-medium modification [14] of the resonances (their mass, width, and coupling constants). In Ref. [2], modifications of the properties of the excited hadrons were taken from the photo-reaction [14] and the effects on π -nucleus total reaction cross sections were predicted and compared with data from [15]. From that work (see also [16]) one would conclude that there is a medium enhanced two-body cross section for the π similar to that found for the K^+ . However, because of the inconsistencies referred to above [4], this result is not as convincing as one would like. Data of the type that we discuss here, namely precise π -nucleus elastic scattering in the forward direction from 300 MeV/c to 1 GeV/c, could resolve the existing discrepancy and provide a critical quantitative characterization of the medium effect for pions.

The data would provide, at the same time, the real part of the forward strong amplitude as a new and important clue to the underlying dynamics. For the π^- , models suggest that $F_N(0)$ may pass through zero just below 1 GeV/c [4]. This situation presents an interesting physics opportunity arising from the fact that this zero is strongly associated with a zero in the real part of a two-body amplitude dominated by numerous resonances centered at various energies throughout the GeV region. The signs and magnitudes of the real parts of the corresponding partial waves contributing to the amplitude occur in such a way that their sum vanishes close to 1 GeV/c. The precise energy at which this amplitude crosses zero for a nuclear target would thus be a sensitive measure of medium-induced mass shifts for this set of resonances.

IV. SUMMARY AND CONCLUSIONS

We find that for intermediate energy π and K^+ elastic scattering there is a region near the forward direction where Coulomb-nuclear interference allows reliable extraction of the strong forward amplitude. We find that statistical and systematic errors in the data are reflected in an understandable and predictable way in the errors in the extracted value of $F_N(0)$, as long as the errors are kept at less than about 5%. We provide guidance for the angular region over which to take the data. The accuracy of the extrapolation to zero degrees is sensitive to how far forward one can take data, as is demonstrated in Tables II and III of Appendix 2. In addition, taking data to even smaller angles where the known Coulomb interaction dominates could be a way to control the systematic error associated with beam normalization.

New data making use of such an analysis could resolve limitations of and outstanding disagreements among various data sets that exist. For example, in Ref. [4] the elastic differential cross section data [5] for π^- scattering from ^{12}C at $P_{lab} = 610, 710, 790,$ and 895 MeV/c was used to extract the zero-degree elastic scattering amplitude. The total cross sections determined from the imaginary part of the zero-degree amplitude were consistently lower than those [15] measured by transmission experiments. Additionally, the real part of the zero degree scattering amplitude was found to have an energy dependence that is not present in theoretical calculations [3]. There exist other elastic differential cross section measurements for pions [17] but the quality of these data do not allow a stable extraction of the forward strong scattering amplitude. The data [6,7] on elastic K^+ -nucleus scattering is even more limited. Although the results produced total cross sections that were consistent with transmission measurements [18], in order to extract the forward strong amplitude from the data of [7], model assumptions had to be made.

We have provided criteria that will enable the optimal design of an experiment. Since the imaginary part of $F_N(0)$ is related by unitarity to the total cross section, this quantity will provide an independent check on its measured values. The real part of $F_N(0)$ is a quantity which would offer considerable constraint on theories. It would provide information on the important question of the medium modification of the properties of both the target nucleons and, in the case of π scattering, the produced excited nucleons. For π scattering, the energy at which the real part of the amplitude passes through zero would be a good measure of medium-induced mass shifts of the excited hadrons. In this case data for both π^+ and π^- would be valuable; a consistent theoretical treatment of these two cases would indicate that all of the Coulomb effects had been adequately accounted for.

A convincing understanding of the underlying physics that is determining intermediate energy meson-nucleus reactions will require a consistent interpretation of a number of reactions. In addition to elastic scattering and total cross sections that are the subject of this work, this would include quasielastic scattering of kaons [19] and pions, both with [20] and without [21] charge exchange. The measurement of $\text{Re } F_N(0)$ would be an important ingredient in this broader program.

ACKNOWLEDGMENTS

The work of DJE is supported, in part, by the U.S. Department of Energy under grant DE-FG02-96ER40975. The Southeast Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility under DOE contract DE-AC05-84ER40150. The work of MBJ was supported, in part, by the US Department of Energy under contract No. W-7405-ENG-36. The work of CMC was supported, in part, by the ROC National Science Council under grant NSC 86-2112-M-129-001. CMC and DJE would like to thank the Los Alamos National Laboratory for their hospitality during part of this work.

APPENDIX:

1. Angular range

The first question is to determine the angular region over which $f_{c,pt}(\theta)$ and $F_N(\theta)$ are comparable in size. In Figs. 1-4 we plot the theoretical values for the real and imaginary parts of $f_{c,pt}(\theta)$ and $F_N(\theta)$ versus $k^2 \sin^2 \theta/2$ for elastic scattering of π^- and K^+ from ^{12}C at $P_{lab} = 500$ MeV/c and 1 GeV/c. First, we see that in general the amplitudes are non-zero and of comparable size for at least a part of the angular region depicted. The exception to this is $\text{Re } F_N(\theta)$ for π^- at 1 GeV/c. For the pion, $\text{Re } F_N(0)$ passes through zero at a momentum just below 1 GeV/c and is thus small and not of a typical size at 1 GeV/c. One also notices that $\text{Im } f_{c,pt}(\theta)$ goes to zero for increasing θ as expected. From Eq. 4 we see that the measured cross section is still dependent on both the real and imaginary parts of $F_N(\theta)$ and thus our methodology is still valid even though the Coulomb amplitude has become nearly real. We will find that it is the convergence of the Taylor series that determines the maximum angle θ_{max} out to which data need be taken. We also see that there is a significant range over which the strong amplitude is nearly linear in $\sin^2 \theta/2$. If the goal is to extract A_R and A_I , then a linear expansion which includes only A 's and B 's should suffice.

For the minimum angle θ_{min} at which to take data, we adopt the criteria that the point Coulomb amplitude $f_{c,pt}(\theta_{min})$ be approximately one half of the strong amplitude $F_N(\theta_{min})$. Points where the Coulomb amplitude is dominant will carry little information about the strong amplitude and thus should not be included in the analysis. From Figs. 1-4, we find that for both pions and K^+ this implies $\theta_{min} = 4^\circ$ at 500 MeV/c and $\theta_{min} = 2^\circ$ at 1 GeV/c. Because the physical size of a spectrometer can limit how far forward measurements can be made, we revisit in Appendix 4 the question of how important are the small angle data points for determining $F_N(0)$.

Given θ_{min} , what is the optimal value for the maximum angle θ_{max} at which to take data? The answer to this question will depend on the accuracy to which one wishes to work. Our point of view will be that we wish to learn both the real and the imaginary part of $F_N(0)$, i.e. A_I and A_R . To find θ_{max} , we generate theoretical values of $d\sigma_N/d\theta$ at a discrete set of points θ_i . We then fit [22] the polynomial expansion for $F_N(\theta_i)$ to determine the expansion coefficients. As we increase the number of points, and hence the value of θ_{max} , the values of A_R and A_I from the fit will increasingly differ from the exact values. The results of these calculations are given in Figs. 5-8 for π^- elastic scattering from ^{12}C at 500 MeV/c and 1 GeV/c, and similarly in Figs. 9-12 for K^+ . The curves were generated with θ_i starting at θ_{min} and then taking evenly spaced points at intervals given by $\delta\theta = 0.5^\circ$. In each figure the solid line represents the results of using two terms, i.e. the A 's and B 's, in the Taylor series; the dashed lines use three terms. Given a desired accuracy for A_R or A_I , the value of θ_{max} can be determined from these graphs. For example, we have included on the graphs as dotted lines the exact value of $A \pm 1\%$. Using two terms in the expansion and insuring that the value of A_R is valid to better than 1%, Fig. 5 suggests that at 500 MeV/c one must use fewer than 23 points. Here 23 points corresponds to $\theta_{max} = \theta_{min} + 23 \times \delta\theta = 4^\circ + 23 \times 0.5^\circ = 15.5^\circ$. Utilizing a three-term expansion, this becomes 48 points ($\theta_{max} = 28^\circ$). Looking at Fig. 6 one finds that for A_I at 500 MeV/c, a 1% error gives 22 points ($\theta_{max} = 16^\circ$) for a two term expansion and 45 points ($\theta_{max} = 26.5^\circ$) for the three-term expansion.

We have found that in fitting to the model data, even in the case of no statistical errors, there are multiple local minima in χ^2 as a function of the expansion coefficients. However,

for the cases we examined in this work, the absolute minimum always corresponded to parameters which go continuously to the exact answer as the model data was improved. Thus for the level of errors we have investigated the technique is stable.

2. Effect of statistical errors

To understand the role statistical errors in the data have in generating errors in the extracted value of $F_N(0)$, we generate model data by adding Gaussian distributed random errors to each of the values of the cross section $d\sigma/d\theta(\sigma_i)$. We do this ten times to generate ten model data sets. We utilize a two term expansion and thus extract A_R , A_I , B_R , and B_I for each of these data sets. From the ten sets we can get the average of each extracted parameter and its standard deviation. We do this entire process three times, setting the errors in the model data to produce standard deviations of 1%, 2%, and 5%.

The results are presented in Table I, where the results are given for π^- and K^+ elastic scattering from ^{12}C at $P_{lab} = 500$ MeV/c and 1 GeV/c. From this table, some general guidelines can be determined concerning the errors in the extracted expansion coefficients as a function of the errors in the data. For π^- the percent error in A_I is approximately equal to the percent error in the data. This is not surprising as A_I is the dominant term in the expansion. The error for A_R is roughly equal in absolute magnitude to the error for A_I . Since A_R is smaller than A_I the percent error in A_R is larger. One might also wish to reliably extract the B coefficients. The percent error in the B_I coefficient is approximately five times larger than the percent error in the data. The B_R coefficient is poorly determined; it would take an exceptionally precise experiment to learn anything about it. Adding in the C coefficients to the expansion and including data to larger angles was found not to improve the situation. We do not recommend working with a three-term expansion.

For K^+ elastic scattering the situation is somewhat different. The values of A_R and A_I are comparable. The percent error in A_R is approximately three times the percent error in the data; the percent error in A_I is approximately seven times the percent error in the data at 500 MeV/c and three times at 1 GeV/c. As was the case for π^- , it does not appear possible to extract a reliable value of B_R . At $P_{lab} = 500$ MeV/c, the value of B_I can be determined only at the level of its magnitude and sign, while at 1 GeV/c it can be determined at the level of a percent error that is ten times the percent error of the data.

3. Effect of systematic errors

Since the data is to be taken in the forward direction where count rates are high, it should be possible to have the statistical errors smaller than the systematic errors, particularly if good quality data can be taken at θ_{min} . To study the effects of systematic errors, we take the model data and increase it uniformly by 5% and by 10%. We have also decreased the model data uniformly by 5% and 10% and find results that generally indicate a reasonably linear effect.

In Table IV we present results for π^- and K^+ elastic scattering from ^{12}C at $P_{lab} = 500$ MeV/c and 1 GeV/c when we increase and decrease the model data by 5% and 10%. The first thing to note is that if the systematic errors are too large the error in the extracted

coefficients can become a nonlinear function of the size of the systematic error. This occurs here for +10% systematic error and π^- and K^+ scattering at 500 MeV/c as can be seen in the behavior of A_I . Experiments should thus keep the systematic errors at 5% or less to avoid this possibility.

For π^- scattering at 500 MeV/c, the value of A is particularly stable; for every 1% systematic error in the measured cross section, there is only 0.3% error in the extracted value of A_I and 0.5% error in the extracted value of A_R . The errors in the extracted values of A_I for π^- at 1 GeV/c are not linear. However, the sign of the nonlinearity is such that it leads to a stable value for A_I . For each 1% systematic error there is never more than a 0.7% error in the extracted value of A_I . As noted earlier for π^- at 1 GeV/c, the value of A_R is atypically small thus giving errors which on a percentage basis are large. There is a 14% error in the extracted value of A_R for each 1% systematic error. If one takes a typical value for A_R over this energy region of 3.0 fm, then one finds that the error in A_R is 2% for every 1% systematic error.

For K^+ scattering at 500 MeV/c we find that every 1% of systematic error produces 5.7% error in the extracted value of A_I and 1.5% in the extracted error of A_R . At 1 GeV/c these become 1.4% for A_I and 3% for A_R . For K^+ the value of A_R can more accurately be determined at 500 MeV/c and this gradually changes as the momentum increases so that A_I is more accurately determined at 1 GeV/c.

4. Importance of the most forward data points

Since the data is being extrapolated to zero degrees, the first few data points at the smallest angles might hold a special significance. At the same time, the most forward data points can be the most poorly determined as they often involve a larger background caused, for example, by the beam scattering off the spectrometer. To better understand this, we have repeated the above analysis with the first two data points having larger errors than the rest. In Table II we show results where the errors for the data are set at 1% with the exception of the first two points which have errors of 2% or 5%.

Increased errors on the first two data points increases the errors on the extracted values of the A 's noticeably. Roughly, the errors in the extracted values of the A 's is doubled by having an error of 2% or 5% on the first two data points. This result led us to examine what happens if these first two data points were removed from the data set. Does it have a positive effect to include very forward data points knowing they are inferior or would it be a better strategy to include only the most reliable data points? The results with the two most forward points removed are given in Table III.

For π^- the removal of the first two data points roughly doubles the error for the extracted value of A_I when the errors in the data are 1% or 2%. For A_R this is also true at $P_{lab} = 500$ MeV/c but at 1 GeV/c the increase in the error for A_R is closer to a factor of three to four. As noted earlier, $\text{Re } F_N(0)$ is atypically small for this case and thus the errors in its determination are of a typical size but are a large fraction of the actual value. For K^+ a similar result holds. The error in the value of A_I is increased by somewhat more than a factor of two. The error in A_R at 500 MeV/c, however, is little affected, while at 1 GeV/c the increase is roughly a factor of three. If the data has a 5% error, the errors on the

extracted values of the parameters are sufficiently large that it is difficult to see a pattern in the change caused by the removal of the first two points.

In general, poor quality data for the first several points is slightly better than not including the points. In both cases, the first several points taken with good statistics improves significantly the quality of the parameters extracted from the data reducing the errors by a factor of two or more.

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FIGURES

FIG. 1. The two pieces of the scattering amplitude, $f_{c,pt}(\theta)$ and $F_N(\theta)$, versus $k^2 \sin^2 \theta/2$ for elastic scattering of π^- from ^{12}C at $P_{lab} = 500$ MeV/c as calculated in the model of Ref. [3]. The solid lines are $\text{Re } f_{c,pt}(\theta)$ and $\text{Re } F_N(\theta)$, the dashed lines are $\text{Im } f_{c,pt}(\theta)$ and $\text{Im } F_N(\theta)$. In both cases the Coulomb amplitude is the amplitude which is singular at the origin. The x-axis corresponds to an angular range of 0° to 15° .

FIG. 2. The same as Fig. 1 except the pion momentum is 1 GeV/c and the x-axis corresponds to an angular range of 0° to 7.5° .

FIG. 3. The same as Fig. 1 except the reaction is elastic scattering of K^+ from ^{12}C at $P_{lab} = 500$ MeV/c. The x-axis correspond to an angular range of 0° to 30°

FIG. 4. The same as Fig. 3 except the K^+ momentum is 1 GeV/c and the x-axis corresponds to an angular range of 0° to 15° .

FIG. 5. The value of A_R found from fitting $d\sigma_N/d\theta$ utilizing the expansion of $F_N(\theta)$, Eq. 3, versus the number N of model data points that were fitted. The reaction is π^- elastic scattering from ^{12}C at $P_{lab} = 500$ MeV/c. The data points begin at $\theta_{min} = 4^\circ$ and are evenly spaced with $\delta\theta = 0.5^\circ$. The solid curve is for the case where two terms are kept in the Taylor series; the dashed curved is for three terms. The dotted lines represent the exact value of $A_R \pm 1\%$.

FIG. 6. The same as Fig. 5 except A_I is presented.

FIG. 7. The same as Fig. 5 except the reaction is π^- incident on ^{12}C at $P_{lab} = 1$ GeV/c and $\theta_{min} = 2^\circ$.

FIG. 8. The same as Fig. 7 except A_I is presented.

FIG. 9. The same as Fig. 5 except the reaction is K^+ incident on ^{12}C at $P_{lab} = 500$ MeV/c.

FIG. 10. The same as Fig. 9 except A_I is presented.

FIG. 11. The same as Fig. 7 except the reaction is K^+ incident on ^{12}C at $P_{lab} = 1$ GeV/c.

FIG. 12. The same as Fig. 11 except A_I is presented.

TABLES

TABLE I. The values of the expansion coefficients A_R , A_I , B_R , and B_I extracted from the model data for π^- and K^+ scattering from ^{12}C at $P_{lab} = 500$ and 1000 MeV/c. The results for model data with statistical errors that have a standard deviation of 1%, 2%, and 5% are presented.

meson	P_{lab} (MeV/c)	θ_{min}	θ_{max}	error	A_R (fm)	A_I (fm)	B_R	B_I
π^-	500	4°	15°	exact	-2.75	6.37	-5.28	-30.5
				1%	-2.77 ± 0.03	6.39 ± 0.03	-6.13 ± 0.99	-31.6 ± 1.0
				2%	-2.74 ± 0.09	6.33 ± 0.06	-2.33 ± 3.73	-28.5 ± 3.1
				5%	-2.78 ± 0.20	6.41 ± 0.20	-4.85 ± 8.51	-32.9 ± 8.3
π^-	1000	2°	7.5°	exact	0.53	11.0	-31.0	-40.8
				1%	0.52 ± 0.06	10.9 ± 0.2	-26.5 ± 9.5	-40.3 ± 1.9
				2%	0.48 ± 0.13	10.9 ± 0.2	-19.8 ± 18.3	-40.7 ± 3.1
				5%	0.40 ± 0.29	10.9 ± 0.4	-22.1 ± 20.5	-42.1 ± 5.7
K^+	500	4°	12.5°	exact	-2.28	2.20	7.60	-5.46
				1%	-2.35 ± 0.07	2.54 ± 0.23	16.5 ± 8.0	-1.63 ± 4.50
				2%	-2.30 ± 0.09	2.25 ± 0.19	10.0 ± 6.7	-4.07 ± 3.90
				5%	-2.04 ± 0.32	2.37 ± 1.87	-1.5 ± 7.4	-29.9 ± 31.7
K^+	1000	2°	7.5°	exact	-2.44	6.15	22.6	-13.7
				1%	-2.42 ± 0.06	6.07 ± 0.22	18.1 ± 8.6	-15.5 ± 1.8
				2%	-2.44 ± 0.09	5.99 ± 0.44	16.9 ± 12.5	-16.0 ± 2.5
				5%	-2.46 ± 0.31	6.44 ± 0.86	33.9 ± 20.6	-17.4 ± 7.1

TABLE II. The values of the expansion coefficients A_R , A_I , B_R , and B_I extracted from the model data for π^- and K^+ scattering from ^{12}C at $P_{lab} = 500$ and 1000 MeV/c. The error on the data is 1% except for the first two data points where the errors are 2% or 5% as listed under the column labeled "error".

meson	P_{lab} (MeV/c)	θ_{min}	θ_{max}	error	A_R (fm)	A_I (fm)	B_R	B_I
π^-	500	4°	15°	exact	-2.75	6.37	-5.28	-30.5
				2%	-2.76 ± 0.10	6.38 ± 0.07	-5.50 ± 2.62	-31.0 ± 3.3
				5%	-2.71 ± 0.06	6.34 ± 0.08	-3.40 ± 4.86	-29.0 ± 3.8
π^-	1000	2°	7.5°	exact	0.53	11.0	-31.0	-40.8
				2%	0.49 ± 0.18	10.9 ± 0.3	-26.9 ± 12.1	-40.8 ± 3.9
				5%	0.42 ± 0.18	11.1 ± 0.3	-30.8 ± 13.0	-43.2 ± 4.6
K^+	500	4°	12.5°	exact	-2.28	2.20	7.60	-5.46
				2%	-2.34 ± 0.07	2.46 ± 0.40	14.3 ± 10.3	-3.89 ± 3.24
				5%	-2.33 ± 0.08	2.49 ± 0.33	14.6 ± 9.7	-3.26 ± 4.5
K^+	1000	2°	7.5°	exact	-2.44	6.15	22.6	-13.7
				2%	-2.42 ± 0.11	6.15 ± 0.36	21.5 ± 9.9	-15.1 ± 2.5
				5%	-2.41 ± 0.13	6.22 ± 0.43	23.7 ± 9.0	-15.0 ± 2.3

TABLE III. The same as Table I except the forward most two data points have been removed from the analysis.

meson	P_{lab} (MeV/c)	θ_{min}	θ_{max}	error	A_R (fm)	A_I (fm)	B_R	B_I
π^-	500	5°	15°	exact	-2.75	6.37	-5.28	-30.5
				1%	-2.72 ± 0.07	6.34 ± 0.06	-4.02 ± 2.98	-29.2 ± 2.9
				2%	-2.80 ± 0.13	6.37 ± 0.11	-4.92 ± 5.09	-31.3 ± 5.2
				5%	-2.59 ± 0.25	6.27 ± 0.18	2.3 ± 11.8	-26.7 ± 8.8
π^-	1000	3°	7.5°	exact	0.53	11.0	-31.0	-40.8
				1%	0.35 ± 0.28	11.1 ± 0.4	-27.8 ± 15.6	-43.5 ± 7.1
				2%	0.29 ± 0.61	11.2 ± 0.6	-33.9 ± 17.9	-47.1 ± 11.6
				5%	-0.11 ± 1.91	10.9 ± 0.9	6.3 ± 43.8	-44.9 ± 16.3
K^+	500	5°	12.5°	exact	-2.28	2.20	7.60	-5.46
				1%	-2.32 ± 0.07	2.40 ± 0.47	12.9 ± 10.8	-4.30 ± 3.21
				2%	-2.33 ± 0.11	2.58 ± 0.45	16.3 ± 12.7	-3.58 ± 5.38
				5%	-2.19 ± 0.23	2.54 ± 0.84	6.0 ± 5.9	-14.4 ± 19.4
K^+	1000	3°	7.5°	exact	-2.44	6.15	22.6	-13.7
				1%	-2.36 ± 0.15	6.34 ± 0.66	23.9 ± 15.2	-17.8 ± 13.9
				2%	-2.25 ± 0.34	6.61 ± 1.09	27.4 ± 20.6	-22.1 ± 9.2
				5%	-1.82 ± 1.01	7.23 ± 2.10	32.9 ± 27.1	-34.7 ± 26.3

TABLE IV. The values of the expansion coefficients A_R , A_I , B_R , and B_I extracted from the model data for π^- and K^+ scattering from ^{12}C at $P_{lab} = 500$ and 1000 MeV/c. In order to simulate systematic errors, the model data are increased/decreased by 5% and 10% as indicated.

meson	P_{lab} (MeV/c)	θ_{min}	θ_{max}	sys err	A_R (fm)	A_I (fm)	B_R	B_I
π^-	500	4°	15°	-10%	-2.91	6.17	-6.95	-34.5
				-5%	-2.85	6.28	-6.22	-32.7
				exact	-2.75	6.37	-5.28	-30.5
				+5%	-2.68	6.46	-3.70	-28.4
				+10%	-2.64	6.39	18.7	-18.8
π^-	1000	2°	7.5°	-10%	-0.24	11.0	-38.2	-49.3
				-5%	0.15	11.0	-35.3	-44.8
				exact	0.53	11.0	-31.0	-40.8
				+5%	0.79	10.6	16.1	-42.2
				+10%	1.21	10.7	15.1	-37.5
K^+	500	4°	12.5°	-10%	-1.93	3.66	31.5	-6.91
				-5%	-2.17	3.11	24.3	-20.4
				exact	-2.28	2.20	7.60	-5.46
				+5%	-2.47	1.74	6.04	-4.66
				+10%	-2.64	1.21	5.25	-2.84
K^+	1000	2°	7.5°	-10%	-1.66	7.07	37.7	-23.7
				-5%	-2.07	6.66	31.9	-17.8
				exact	-2.44	6.15	22.6	-13.7
				+5%	-2.75	5.58	7.88	-15.8
				+10%	-3.10	5.33	8.33	-14.1























