

Excited nucleon spectrum using a non-perturbatively improved clover fermion action

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We discuss the extraction of negative-parity baryon masses from lattice QCD calculations. The mass of the lowest-lying negative-parity $J = 1/2^-$ state is computed in quenched lattice QCD using an $\mathcal{O}(a)$ -improved clover fermion action, and a splitting found with the nucleon mass. The calculation is performed on two lattice volumes, and three lattice spacings enabling a study of both finite-volume and finite-lattice-spacing uncertainties. A measurement of the first excited radial excitation of the nucleon finds a mass comparable, or even somewhat larger than that of the negative-parity ground state, in accord with other lattice determinations but in disagreement with experiment. Results are also presented for the lightest negative-parity $I = 3/2$ state.

1. INTRODUCTION

The N^* spectrum exhibits many features that are emblematic of QCD; that they contain three valence quarks is a manifestation of SU(3) symmetry, while the linearly rising Regge trajectories are evidence for a flux-tube-like confining force. Finally, the fine and hyperfine structure reveals details of the interquark interactions. Thus the study of the spectrum of excited nucleon resonances has always been a vital component of the experimental hadronic physics program, as typified by the Hall B experimental effort at Jefferson Laboratory.

The motivation for studying the N^* spectrum

in many ways mirrors that for the hybrids, which also provide an arena in which to explore flux tubes. One distinction is that, in the case of nucleons, all spin-parity combinations are accessible to the quark model, and thus there are no nucleon “spin exotics”. However, both hybrids and excited nucleons share the feature that they are sensitive to the presence of excited glue.

The observed excited nucleon spectrum already poses many questions. There are “missing resonances”, predicted by the quark model but not yet observed experimentally. Most strikingly, there is the so-called Roper resonance, and the lightest negative-parity Λ state, the $\Lambda(1405)$ with anomalously light masses suggesting that

they may be “molecular” states, rather than true three-quark resonances. Lattice gauge theory calculations have a vital role to play in resolving these questions.

In an attempt to address these issues, there have been several recent calculations of the lowest-lying excited nucleon masses, emphasising in particular the extent to which the parity partners are accessible to lattice calculation. The first calculation employed the highly-improved D_{234} fermion action[1,2], whilst a second calculation employed domain-wall fermions[3,4]. Both calculations exhibited a clear splitting between the masses of the $N^{1/2+}(937)$ and $N^{1/2-}(1535)$ states, with an indication that the lowest-lying radial excitation of the nucleon has a mass considerably larger than that of the $N^*(1535)$, casting doubt on the nature of the Roper resonance.

In this talk, we present a calculation of the lowest lying negative parity nucleon mass using an $\mathcal{O}(a)$ -improved Sheikholeslami-Wohlert (SW), or clover, fermion action; preliminary results were presented in ref. [5], and a comprehensive analysis appeared in ref. [6]. By choosing the coefficient of the improvement term appropriately, all $\mathcal{O}(a)$ discretisation uncertainties are removed, ensuring that the continuum limit is approached with a rate proportional to a^2 . For a subset of our lattices, masses of the first radial excitation of the nucleon and of the parity partner of the Δ are presented.

The calculation of the excited nucleon spectrum places particularly heavy demands on lattice spectroscopy. The excited nucleon states are expected to be large; the size of a state is expected to double with each increase in orbital angular momentum. Thus a lattice study of the excited nucleon spectrum requires large lattice volumes, with correspondingly large computational requirements. Furthermore, the states are relatively massive, requiring a fine lattice spacing, at least in the temporal direction. These requirements could be satisfied with much greater economy using the clover fermion action than using the domain-wall or overlap formulation. Thus it is important to establish that the negative parity states are indeed accessible to calculations using the clover action. Finally, by comparing the

masses obtained using the clover action with a calculation, at a single quark mass, using the Wilson fermion action, we also gain insight into the nature of the interaction responsible for the splitting in the parity doublet.

The layout of the remainder of this talk is as follows. In the next section, I shall describe the construction of hadronic operators, and in particular the projection onto positive- and negative-parity states, and summarises other computational details. Section 3 contains the results for the masses of the lowest-lying positive- and negative-parity nucleon using the SW fermion action, and for a subset of the lattices the masses obtained for the Roper resonance. The talk concludes with a discussion, a comparison with other lattice and phenomenological calculations, and prospects for future studies.

2. SIMULATION DETAILS

2.1. Baryon operators

Historically, there has been relatively little study of the negative-parity baryon states, though there had been numerous computations of the nucleon radial excitations. It is therefore useful to review the rôle that parity plays in the construction of baryon operators, and how this rôle is manifest in the case of lattice calculations. We illustrate the discussion through consideration of the usual nucleon interpolation operators:

$$N_1^{1/2+} = \epsilon_{ijk}(u_i^T C \gamma_5 d_j) u_k, \quad (1)$$

$$N_2^{1/2+} = \epsilon_{ijk}(u_i^T C d_j) \gamma_5 u_k, \quad (2)$$

$$N_3^{1/2+} = \epsilon_{ijk}(u_i^T C \gamma_4 \gamma_5 d_j) u_k. \quad (3)$$

These operators have an overlap with particle states of both positive and negative parities. In order to construct operators of definite parity, we introduce the parity-projection operator:

$$P = (1 \pm \gamma_4), \quad (4)$$

and construct the correlators

$$C_{\pm}(t) = \sum_{\vec{x}} \langle 0 | T N(\vec{x}, t) (1 \pm \gamma_4) \bar{N}(0) | 0 \rangle, \quad (5)$$

where N is a baryon interpolating operator. On a periodic or anti-periodic lattice, the two time

orderings result in forward propagating states of positive (negative) parity, and backward propagating states which are anti-particles of negative (positive) parity states if the projection operator is chosen with positive (negative) sign. Thus the best delineation that may be constructed is between a forward propagating particle, and the backward propagating anti-particle of the parity partner. At large distances, where $t \gg 1$ and $N_t - t \gg 1$, the correlators assume the time behaviour

$$C_+(t) \rightarrow A^+ e^{-m^+ t} + A^- e^{-m^- (N_t - t)} \quad (6)$$

$$C_-(t) \rightarrow A^- e^{-m^- t} + A^+ e^{-m^+ (N_t - t)} \quad (7)$$

where m_i^+ and m_i^- are the lightest positive- and negative-parity masses respectively.

For the case of the nucleon ground state, the “diquark” part of both N_1 and N_3 couples upper (large) spinor components, while that in N_2 involves both an upper and lower spinor component and thus vanishes in the non-relativistic limit[1]. Thus our expectation is that the operators N_1 , N_3 should give a better overlap with the positive-parity ground state nucleon compared with operator N_2 , and this is confirmed in lattice simulations. For some of the lattices used in the calculation, the positive-parity nucleon mass is obtained using the non-relativistic quark operators [7], defined by

$$\psi \rightarrow \psi^{\text{NR}} = \frac{1}{2}(1 + \gamma_4)\psi, \quad \bar{\psi}^{\text{NR}} = \bar{\psi} \frac{1}{2}(1 + \gamma_4). \quad (8)$$

corresponding to the “large” components in the non-relativistic limit; this results in a factor of two reduction in computational effort at the expense of some loss of statistics.

2.2. Simulation Details

The calculation is performed in the quenched approximation to QCD. The standard Wilson gluon action is employed, whilst the quark propagators are computed using the Sheikholeslami-Wohlert clover fermion action, using the non-perturbative coefficient of the clover term determined in refs. [8] and [9]. Thus the hadron masses are free of all $\mathcal{O}(a^2)$ discretisation errors.

The propagators are computed using both local sources, and spatially extended “smeared”

sources, using either the fuzzed-source method (F) or Jacobi-smeared sources (J) as described in references [10] and [11] respectively. The simulation parameters are summarized in Tables 1. The errors on the fitted masses are computed using a bootstrap procedure. Different numbers of configurations are used at different quark masses at some of our β values, precluding the use of correlated fits in some of the chiral extrapolations. Thus a simple uncorrelated χ^2 fit is performed, with the uncertainties computed from the variation in the χ^2 . Calculations of the light hadron spectrum using these lattices were presented in references [15] and [16,17].

The clover term removes the leading chiral-symmetry-breaking effects from hadron masses at finite a . Since the lack of degeneracy between the positive- and negative-parity baryon states is a consequence of the spontaneous breaking of chiral symmetry, we also compare the measurements of the mass splitting at a single quark mass with those obtained using the Wilson fermion action.

3. RESULTS

The masses of the lowest-lying $N^{1/2+}$ and $N^{1/2-}$ states are obtained from a simultaneous, four-parameter fit to the positive- and negative-parity correlators $C_+(t)$ and $C_-(t)$ constructed from the “good” nucleon operator N_1 of eqn. 1. A clear signal for the mass of the lightest negative-parity state notably higher than that for the positive-parity state is observed, and is illustrated for one of our ensembles in Figure 1.

For the chiral extrapolation of the hadron masses, we adopt the ansatz

$$(am_X)^2 = (aM_X)^2 + b_2(am_\pi)^2, \quad (9)$$

where we use upper-case letters to denote masses obtained in the chiral limit and X is either $N^{1/2+}$ or $N^{1/2-}$. We include data at different volumes, but at the same β , in the chiral extrapolations, but treat the fuzzed and Jacobi-smeared data independently; they give slightly different masses, reflecting the different contributions of the excited states in the two cases. Note that the fit to $(am_X)^2$ is formally the same as one to am_X at small pion mass, but in practice provides a no-

Table 1

The parameters of the lattices used in the calculation. The labels J and F refer to use of Jacobi and “fuzzed” quark sources respectively.

β	c_{sw}	$L^3 \cdot T$	L [fm]	κ	Smearing
6.4	1.57	$32^3 \cdot 48$	1.6	0.1313, 0.1323, 0.1330, 0.1338, 0.1346, 0.1350	J
6.2	1.61	$24^3 \cdot 48$	1.6	0.1346, 0.1351, 0.1353	F
		$24^3 \cdot 48$	1.6	0.1333, 0.1339, 0.1344, 0.1349, 0.1352	J
		$32^3 \cdot 64$	2.1	0.1352, 0.1353, 0.13555	J
6.0	1.76	$16^3 \cdot 48$	1.5	0.13344, 0.13417, 0.13455	F
		$16^3 \cdot 32$	1.5	0.1324, 0.1333, 0.1338, 0.1342	J
		$24^3 \cdot 32$	2.2	0.1342, 0.1346, 0.1348	J

Lattice sizes in physical units are quoted using r_0 to set the scale [14].

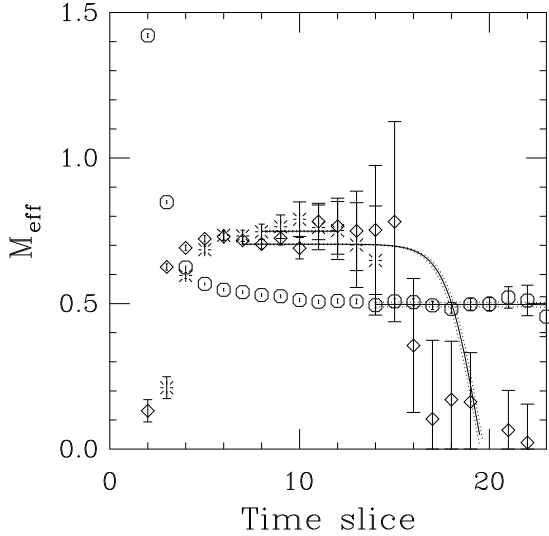


Figure 1. The effective masses for the $N^{1/2+}$ channel (circles) and the $N^{1/2-}$ channel (diamonds) at $\beta = 6.2$, with $\kappa = 0.1351$, using the nucleon interpolating operator of eqn 1. Also shown as the bursts is the effective mass of the positive-parity state determined using the “bad” nucleon interpolating operator N_2 .

tably better description of the data.

It is known that there are non-analytic terms contributing to the chiral extrapolation of physical quantities, and the contribution of the non-analytic terms arising from the pion-induced baryon self energies has been explored in reference [18]. In the quenched approximation, the leading, non-analytic terms can assume a different form. From quenched chiral perturbation theory, one expects leading non-analytic terms which are linear in m_π [19]; the coefficient of the leading term is predicted to be negative. We therefore also attempted to fit the data at $\beta = 6.4$ to the form

$$(am_X)^2 = (aM_X)^2 + b_1(am_\pi) + b_2(am_\pi)^2. \quad (10)$$

Fitting the $N^{1/2+}$ state we found a positive value of b_1 , though with a very large error that would still accommodate a negative value. Thus it is unclear whether eqn. 10 provides a reliable form with which to extrapolate data obtained with quark masses around that of the strange to the chiral limit, emphasising once again the need for data at the smaller quark masses at which the pion cloud emerges. The chirally extrapolated masses obtained using this procedure, with b_1 unconstrained, differ from those obtained using eqn. 9 by around 5%. In view of the difficulties discussed above, and because on some of our ensembles we have insufficient data points to perform a three-parameter fit, we quote our final results from fits to eqn. 9. An example of such a chiral extrapolation for the data at $\beta = 6.4$ is

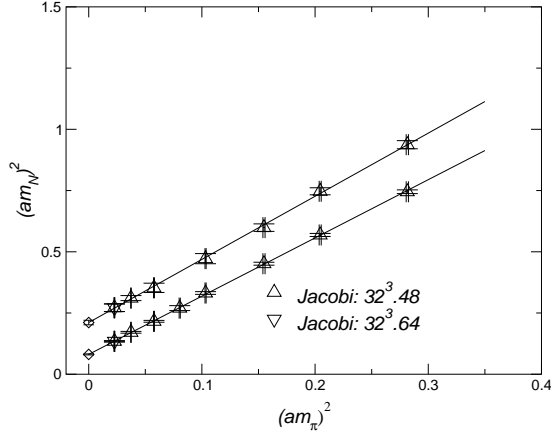


Figure 2. The masses in lattice units of the lowest-lying positive- and negative-parity nucleons at $\beta = 6.4$. The curves are from fits to m_X^2 using eqn. 9.

shown in Figure 2.

Ideally, the forms eqn. 9 and 10 should be applied to the baryons masses obtained in the infinite volume limit. Indeed, the talk by Stuart Wright at this workshop emphasised the importance of correctly accounting for finite-volume effects in chiral extrapolations [12]. Our data do not admit this procedure, but an investigation of the masses of the negative-parity state obtained on the smaller and larger of our lattices at $\beta = 6.0$ and 6.2 suggests that the masses of the negative-parity state on the larger lattices are smaller than those on the smaller lattices by around 5%.

In order to look at the discretisation uncertainties in our data, we show in Figure 3 the masses in units of r_0 against the a^2/r_0^2 , where $r_0 = 0.5$ fm is the hadronic scale [13,14].

On a subset of our lattices, we have looked at the ground-state mass obtained from the “bad” nucleon operator N_2 of eqn. 2. The effective mass in the positive-parity channel using this operator is shown in Figure 1; it suggests an effective ground state mass close to, and slightly higher than, that of the lightest negative-parity state. The signal degrades appreciably with decreasing

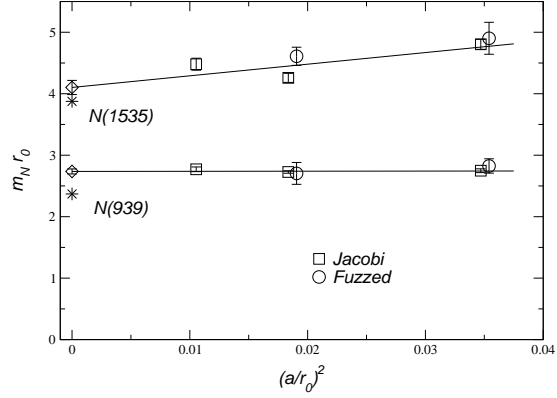


Figure 3. The masses of the lowest-lying positive- and negative-parity baryons in units of r_0^{-1} [13,14] against a^2 in units of r_0^2 . The lines are linear fits in a^2/r_0^2 to the positive- and negative-parity baryon masses. Also shown are the physical values.

pion mass, as can be seen in Figure 4 where we show the mass of the lightest positive-parity state from N_1 , the lightest negative-parity state from N_1 , and the mass of the lightest positive-parity state obtained with N_2 . A better determination of the first excited radial excitation would require the measurement of the cross-correlator between N_1 and N_2 , as done in reference [4]. However, the conclusion that the operator N_2 has a negligible overlap with the ground state nucleon at these values of the pseudoscalar mass seems inescapable.

The measurement of heavier excitations is inevitably noisier because of the reduced signal-to-noise ratio. On some of our lattices we have measured the correlators for the $I = 3/2$ Δ , using an interpolating operator

$$\Delta^{3/2,1/2} = \epsilon_{ijk}(u_i^T C \gamma_\mu u_j) u_k. \quad (11)$$

This has an overlap onto both spin-3/2 and spin-1/2 states, but these can be distinguished using a suitable spin projection, and the two parities delineated as described above. The masses of the ground state of both positive and negative parity are shown in Figure 5. Further success at deter-

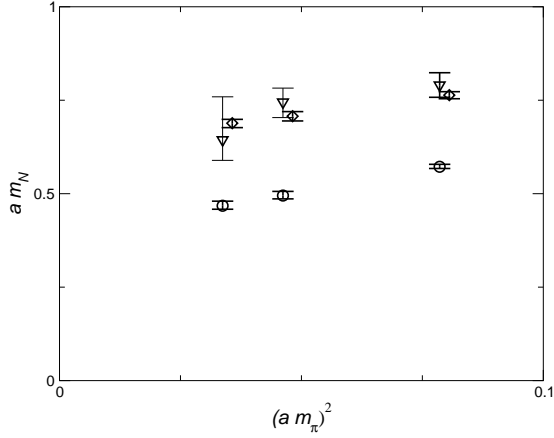


Figure 4. The masses (in lattice units) of the lowest-lying $N^{1/2+}$ (circles) and the $N^{1/2-}$ (diamonds) obtained using the “good” nucleon operator N_1 , and of the $N^{1/2+}$ using the “bad” operator N_2 , obtained on the $24^3 \times 48$ lattices at $\beta = 6.2$. The negative-parity points are displaced for clarity.

mining the higher spin states has been made in a recent determination of the $N^{3/2}$ masses [20].

4. CONCLUSIONS

We have seen that the use of the clover-improved fermion action enables the delineation of the positive- and negative-parity excited state masses, in accord with calculations using highly improved Wilson, and domain-wall fermion actions. On a subset of our lattices, we also computed propagators using the unimproved Wilson fermion action at a single quark mass, and found a parity mass splitting consistent with that of the clover fermion action. On reflection, this is hardly surprising; under $SU(6)$ spin-flavour symmetry, the low-lying positive-parity baryons can be assigned to an $l = 0$ multiplet, whilst the low-lying negative-parity baryons can be assigned to an $l = 1$ multiplet. Thus the mass-splitting can be likened to the $P - S$ splitting in the meson sector, which is well-known to be faithfully re-

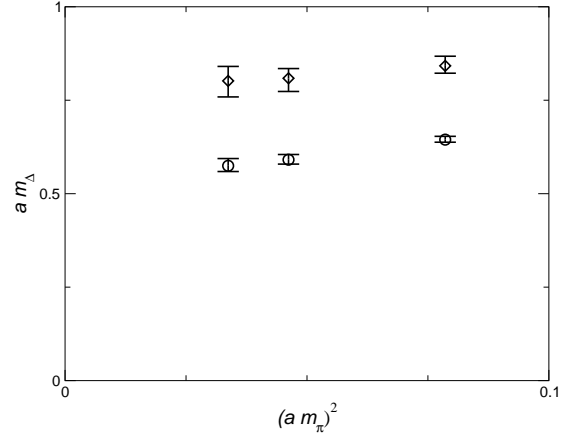


Figure 5. The masses in lattice units of the Δ (circles) and its parity partner (diamonds) against the pseudoscalar mass obtained on the $24^3 \times 48$ lattices at $\beta = 6.2$

produced using the Wilson fermion action.

The most striking feature common to all the lattice calculations is the apparent inversion of the ordering of mass of the lowest radial excitation of the nucleon, and the mass of the parity partner of the nucleon. In this calculation, we suggest a radial excitation comparable or a little heavier in mass than the negative-parity ground state; indeed the authors of reference [4] find a mass-splitting between the positive-parity radial excitation and the negative-parity state that is comparable to that between the negative-parity ground state and the nucleon. In contrast, the observed lightest radial excitation of the nucleon is the Roper resonance with a mass of 1440 MeV, whilst the lightest negative-parity state is the heavier $N^*(1535)$. In any case, we can see that lattice calculations of the spectrum are already posing interesting questions about the nature of the Roper resonance.

I have not discussed the issue of the nature of the $\Lambda(1405)$ in this talk. In the case of the positive-parity states, lattice calculations, such as that of ref. [15], suggest that the quark mass behaviour of the baryon masses, in the range over

which they are measured, is well described by terms corresponding to the “centres of gravity” of the quark masses within the hadron. If this observation continues to the negative-parity sector, it would be hard to imagine ground state Λ lower in mass than the $N^*(1535)$, perhaps favouring a molecular nature to the state. It should be noted, however, that a recent large N_C calculation has been able to accommodate a light $\Lambda(1405)$ [21].

We have seen that lattice calculations are already addressing some of the salient issues in baryon spectroscopy. The calculation of the radial excitations and the higher spin states will present further challenges; the need to overcome the reduced signal-to-noise ratio afflicting heavier excitations, the construction of suitable lattice operators to isolate higher excitations, and the exploitation of improved data analysis methods. As these issues are tackled, we will be able to tackle the more ambitious projects of transition form factors and hadronic decays.

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