Lasing with a Near-Confocal cavity in a high power FEL

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ABSTRACT

Lasing at high power in FELs has been achieved so far only with a near-concentric resonator [1]. Though this design can scale up to quite high power, it is ultimately limited by the mirror steering stability as the resonator design approaches concentricity. This constraint may be avoided by using a near-confocal resonator operated in a ring configuration. It is found that, if a small amount of gain focusing is present, the near-confocal resonator eigenmodes are modified such that the lowest order mode collapses around the electron beam and is large in the return (non-focusing) direction. This eigenmode is stable and is relatively insensitive to changes in the mirror radii of curvature and the strength of the electron beam focusing. This paper will present the theory of this new concept.

Keywords: accelerator, FEL, infrared, laser, superconducting, resonators

1. INTRODUCTION

The average saturation intensity in a high power FEL is orders of magnitude higher than one that can be tolerated on a mirror coating. In previous work [2] we have noted that optical resonators for high power FELs must provide a mode that is large on the mirrors but very small in the FEL wiggler. We define the magnification of the optical mode M as the ratio of the optical mode size at the nearest near-normal-incidence mirrors to the mode size at the waist. This quantity must be on the order of 100 or more to keep the mirrors from being destroyed. Others have proposed schemes using unstable confocal resonators. C. C. Shih proposed using two focusing corner cubes and a scraper mirror placed at the image point of the wiggler to achieve a large magnification without the penalty of tight alignment tolerances [3]. This idea has the advantages of very loose angular tolerances on the corner cube orientation, good suppression of aberrations, and no required transmissive elements. It has the disadvantage of requiring large angle p-plane reflections in the corner cubes, which is difficult at some wavelengths. Bhowmik proposed using two telescopes to expand the beam between the wiggler and the backleg mirrors [4]. This has the disadvantage of alignment difficulties and difficult-to-figure off-axis conic section mirrors. Both these schemes have a big disadvantage of the need for a scraper output coupler. This produces a narrow annulus that must then be modified using an axicon or similar device to produce a uniform output wavefront. It would be nice to have a stable resonator with transmissive output coupling to maintain a Gaussian output beam. Two stable resonator approaches are available to provide for such a large magnification.

The first approach is to use a nearly concentric stable resonator. The nearly concentric configuration is straightforward to align and decouples the length and mirror steering. These advantages are offset by a thermal runaway problem. When the mirrors heat up from the circulating laser light absorbed on the mirrors, they bulge out, increasing their radii of curvature. This increases the Rayleigh range and moves the cavity away from concentricity, thus reducing the mode size on the cavity mirrors. The high intensity from this arrangement causes the mirror to distort even more. This positive feedback can lead to very large distortion resulting in a runaway process. The nearly concentric cavity also has an extreme sensitivity to mirror steering and changes in the radii of curvature. The change in the mode angle θ due to a rotation in one mirror by an angle θ_M for a near-concentric resonator with magnification M is given by $\theta=(M/2)\theta_M$. Thus, for example, with a magnification of 100, a shift of ten μrad in one mirror will tilt the optical mode by 0.5 mrad. For a wavelength of 1 μm and a Rayleigh range of 50 cm this is already larger than the divergence of the optical mode. For high power systems one wants a magnification even larger than 100 and one quickly finds that active mirror stabilization is necessary for FEL operation. For high power systems, the limit of realizable mirror feedback systems is reached for power output around 100 kW. Similarly, the fractional change in the Rayleigh range for a given fractional change δ in the mirror radii of curvature is $\delta M/2$. This becomes problematic both for fabricating the mirrors and controlling the Rayleigh range during lasing. Making the high reflector a deformable mirror can mitigate both problems. If the sum of the radii of curvature can be maintained close to the design value, the shift in the waist position due to a difference in the ROCs is given by half the relative difference in the ROCs times

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the ROC. This is usually a negligible shift [5]. For large magnifications however, the control on the deformable mirror becomes almost impossible to achieve, requiring changes smaller than 0.1% in the radius of curvature.
The second approach is to use a stable ring confocal resonator. In a linear confocal resonator the mirror radii of curvature are both equal to the mirror separation. The resonator is metastable, sitting on a saddle point of the stability curve vs. the mirror radii of curvature. If the mirror ROCs are the same but slightly different from the resonator length, the cavity is stable. If the mirror ROCs are slightly different but their sum is equal to twice the resonator length, the resonator is unstable. From a practical standpoint this makes a confocal resonator quite difficult to build. A study of a confocal resonator by a group in Novosibirsk showed that lasing in both the stable and instable modes could be achieved in a storage ring FEL [6]. Changes from stable to unstable configurations were achieved by changing the mirrors.

Ozcan and Pantell pointed out that a confocal resonator is degenerate and can therefore support any optical mode. The mode can then shift to take advantage of changes in the gain medium. This result led them to propose the use of a confocal resonator with a tapered wiggler. The mode waist would move from an untapered section of the wiggler to tapered section as the laser saturated [7]. This behavior led us to the idea of using a near-confocal resonator for a high power system. The cavity can support a mode which is small in the gain region and large in the return leg. Since the wiggler gap is usually much smaller than the optical mode at the mirrors, one needs to return the optical beam outside the wiggler, necessitating a ring resonator configuration. When this is done one finds that the path length between the two focussing mirrors in either direction is not the same (though with a 6 mirror resonator the paths can be made the same). Such a resonator has different properties than a linear confocal resonator. This will be discussed in the next section. The geometries of near-concentric, near confocal and ring near confocal are shown in figure 1.

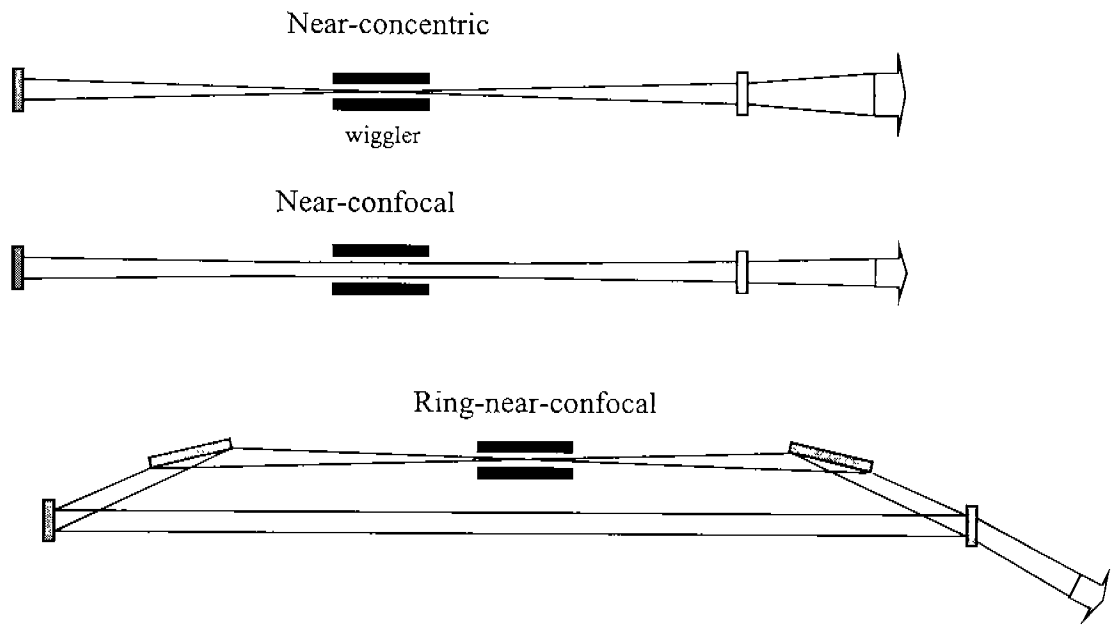


Figure 1. Geometries of near concentric, near confocal, and ring near confocal resonators. Angles in the ring confocal resonator are greatly exagerated. Note that the wiggler gap is larger for the near-confocal resonator to accomodate the larger optical mode.

The FEL gain medium provides a complex focussing term to the optical mode. Since the electron beam is usually smaller than the optical mode there is gain focussing in the wiggler. Since the FEL gain also shifts the phase of the optical mode, there is an effective refractive index as well, resulting in a net focussing of the optical mode [8]. In any saturated resonator, this focussing effect is not sufficient to lead to gain guiding as in a SASE FEL and, in a near concentric resonator, it has little effect on the resonator mode. Representing the gain as a complex Gaussian duct, it is possible to study the resonator using matrix methods. One finds that the focusing has a large effect on the nearly confocal ring resonator. The resulting mode is stable and not extremely sensitive to the miror ROCs or steering. This will be studied in the third section.

2. NEARLY CONFOCAL RING RESONATORS

If we assume the use of a 4 mirror resonator to allow the return beam to bypass the wiggler gap, and if we also assume that the ROCs of the mirrors are represented by $l/(1-\delta_i)$ where l is the distance between the two focussing mirrors through the wiggler and i is the mirror index, and that the return leg path length is $l(1-\delta_r)$ we find that the ABCD matric for the resonator starting in the wiggler center is:

$$\begin{pmatrix} \delta_1+\delta_2-1-2\delta_2(1-\delta_1)(1-\delta_r) & l\left(\delta_1\delta_2(1-\delta_r)+(\delta_1+\delta_2)/2\right) \\ -2(\delta_1+\delta_2+2\delta_r)/l & \delta_1+\delta_2-1-2\delta_1(1-\delta_2)(1-\delta_r) \end{pmatrix} \quad (1)$$

The stability condition for this resonator is approximately:

$$\delta_1\delta_2+\delta_r(\delta_1+\delta_2)/2>0 \quad (2)$$

The region of stability for δ_r=0.01 is shown in figure 2a.. The stable region is broadened into the second and fourth quadrants but a region of instability forms near the origin in the third quadrant. The Rayleigh range at the wiggler center is given by

$$z_R \cong \frac{\left[\delta_1\delta_2+\frac{1}{2}(\delta_1+\delta_2)\right]l}{2\sqrt{\delta_1\delta_2+\frac{1}{2}(\delta_1+\delta_2)\delta_r}} \cong \frac{l}{2}\sqrt{\frac{\delta}{\delta_r}} \quad (3)$$

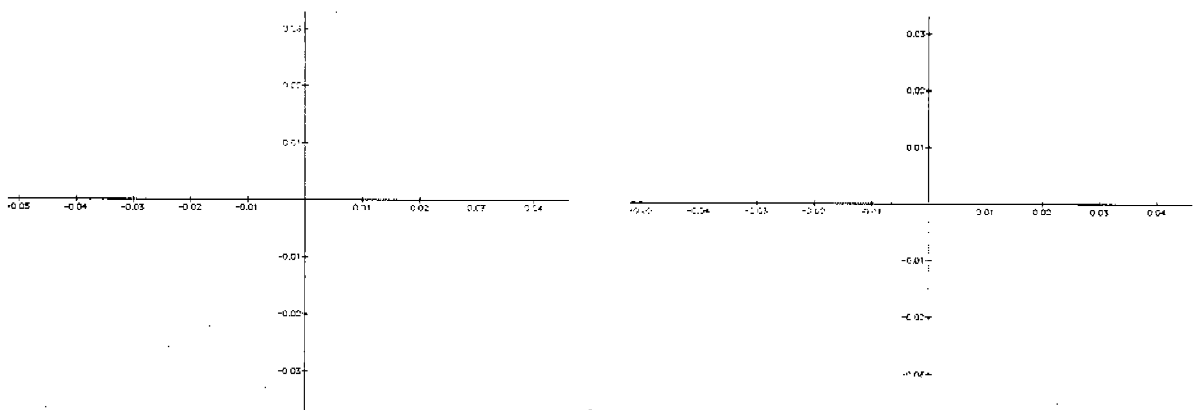
The latter approximation is for a symmetric resonator. For very small values of δ_1 and δ_2 one can make z_R quite small. For example, if $\delta_1=\delta_2=10^{-4}$ and δ_r=0.01 we find that the Rayleigh range is ~1/20 the cavity length and magnification is 100. This sounds good at first but one cannot adjust the ROC of the resonator mirrors with an accuracy of 10^{-4}. In fact one must maintain the ROC to much tighter tolerances than this so that the Rayleigh range is stable. The relative change in the Rayleigh range is the relative change in the ROC divided by δ. For $\delta=10^{-4}$ this is a severe problem.

How can we make the Rayleigh range less sensitive to the radii of curvature of the mirrors? One solution is to change the ABCD matrix so that the A and D terms are further away from unity. Increasing δ_r would help but in practice it is very hard to increase its value above a few percent. If we put a lens at the center of the resonator with focal length f we find that the new ABCD matrix, to lowest order in small quantities, is

$$\begin{pmatrix} \delta_1+\delta_2-1-\frac{L}{4f}(\delta_1+\delta_2) & \frac{l}{2}(\delta_1+\delta_2) \\ \frac{1}{f}-\frac{2}{l}(\delta_1+\delta_2+2\delta_r) & \delta_1+\delta_2-1-\frac{L}{4f}(\delta_1+\delta_2) \end{pmatrix}. \quad (4)$$

The stability criterion is changed greatly from the previous value. It is given by the equation

$$\delta_1\delta_2+\frac{1}{2}(\delta_1+\delta_2)\delta_r-\frac{l}{8f}(\delta_1+\delta_2)>0 \quad (5)$$



a. b.

Figure 2.Regions of stability for a ring-confocal resonator (a.) and for a ring confocal resonator with a lens at the resonator center (b). Grey regions are stable. The relations are represented in equations (2) and (5). In both cases the quantity δ_r was 0.01. The lens focal length was equal to the resonator length.

The stability regions near the g_1g_2 origin are shown in figure 2b. Note the stability diagram indicates that the system is only stable for $\delta<0$. The Rayleigh range is given by

$$z_R \cong \frac{l(\delta_1+\delta_2)}{4\sqrt{\delta_1\delta_2+\frac{1}{2}(\delta_1+\delta_2)\delta_r-\frac{l}{8f}(\delta_1+\delta_2)}} \cong \frac{l}{2}\sqrt{\frac{-\delta}{\frac{l}{4f}-\delta-\delta_r}}$$

The value of $l/4f$ can be nearly unity. This means that the value of δ may be much larger and still have a small Rayleigh range.

3. RING RESONATORS WITH FEL GUIDING

Since an FEL gain medium does focus the optical mode [6], one might expect that the resonator will still be stable but will have a small Rayleigh range in the wiggler. The gain medium in an FEL is not exactly the same as a simple thin lens however. It is more similar to a complex Gaussian duct [9]. The FEL gain medium provides both gain guiding and refractive guiding. The former leads to an imaginary component in the Gaussian duct ABCD matrix. The refractive guiding provides the real part of the Gaussian duct ABCD matrix. The gain and refractive guiding strengths are usually comparable. For this work we assume that they are equal. The imaginary part of the Gaussian duct provides an interesting characteristic to the resonator. As noted by Siegman [10]

> "It is possible to show quite generally that if an *arbitrarily small amount* of transversely increasing loss (or transversely decreasing gain) is added anywhere inside any purely real ABCD system, whether geometrically stable or unstable, then one of the eigenwaves for that system will always be modified so as to become both a confined and perturbation-stable Gaussian wave."

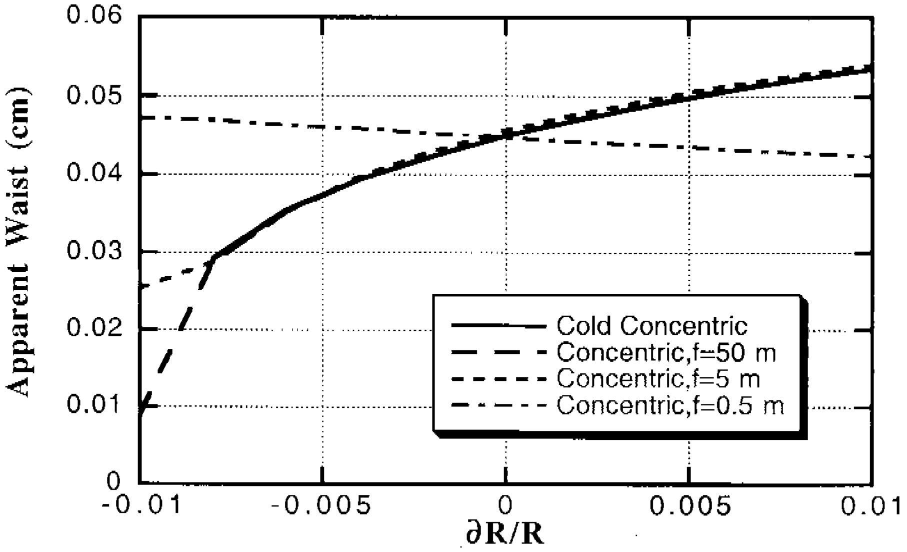


Figure 3. Here we show the apparent waist size vs. the fractional change in the radii of curvature in a symmetric, near-concentric resonator. The focal lengths correspond to 1%, 10%, and 100% gain in the FEL. The apparent waist size is calculated from the spot size and divergence outside the gain region.

Since the gain is transversely decreasing in the gain medium, we find that the resonator becomes universally stable assuming that the rest of the resonator has a real ABCD matrix. This is shown in figure 3 in which we show the apparent waist size at the wiggler exit vs. mirror radii of curvature for a near-concentric resonator. These curves were calculated using the program PARAXIA [11].

For a change of −1%, the resonator with no focusing is unstable. When a Gaussian duct is added in the region of the wiggler, the resonator becomes stable for this resonator. The resonator mode was calculated for a Gaussian duct one meter in length with a thick lens focal length of 50 meter, 5 meter, and 0.5 meters. These values correspond roughly to 1%, 10%, and 100% gain respectively. They therefore represent saturation in the high Q resonator, saturation in a low Q resonator, and the small signal regime of the laser. For low gain, the gain focusing has little effect except in regions where the cavity would otherwise be unstable. For high gain the waist size becomes nearly independent of the radii of curvature. The mode position does move around though so the optimum radii of curvature are still near the design values.

For the case of the ring-near-confocal resonator, the effect of guiding is much more striking. This is shown in figure 4. The slope of the curve is almost eliminated near the confocal point. This means that the Rayleigh range is relatively insensitive to the radii of curvature in the mirrors. The waist size is also not a strong function of the strength of the gain focusing. This is important when considering the turn-on of the laser. Finally, it should be noted that the slope of the waist size vs. radii of curvature is negative for the stronger focusing solutions. This means that the spot size on the mirrors will actually grow as the mirrors heat up, in comparison to the cold cavity solution, which shrinks rapidly as the radii of curvature grows.

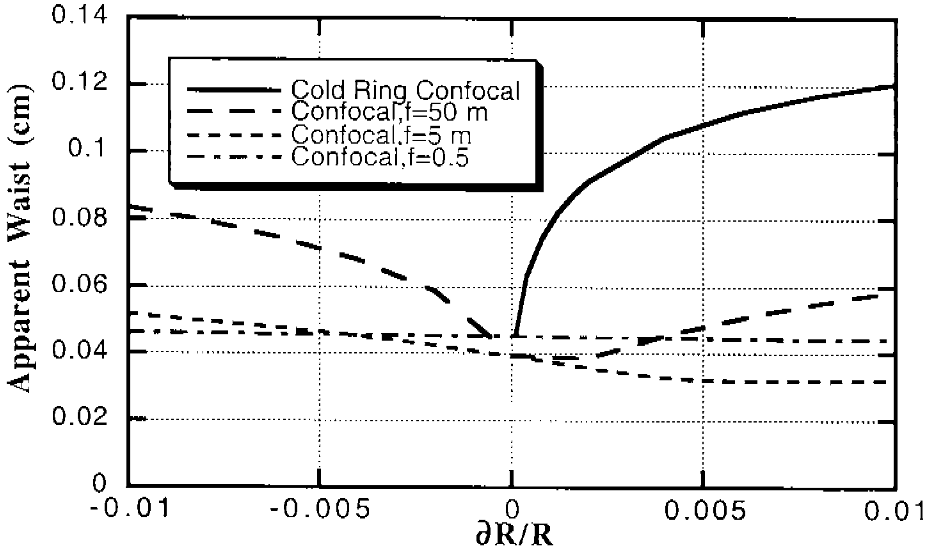


Figure 4. We show the apparent waist size vs. the fractional change in the radii of curvature of the mirrors for a ring-near-confocal resonator. The change in the radii of curvature is relative to the distance between the mirrors through the wiggler. The apparent waist size is calculated from the beam size and divergence at the exit of the wiggler.

A mode in a near-confocal resonator is relatively insensitive to mirror steering. The mode rotates by the same angle as the mirror. In fact, the limitation on mirror tilt is given by the shift in the waist position rather than the shift in the mode angle. For the ring-near-confocal resonator with gain focusing, the tilt sensitivity is increased over a simple near-concentric resonator but the overall limitations on the tilt are still dominated by the mode position shift rather than the mode rotation. This is evident from figure 5 in which we show the ratio of the mode rotation to the mirror tilt angle. Though the ratio does increase from the cold cavity value, it is still small enough that the mode does not rotate significantly. By comparison we show the same ratio for a near-concentric resonator with gain focusing in figure 6. In this case the already high sensitivity becomes even larger when focusing is added. The magnification in the ring-near-confocal resonator can be made very large without creating unacceptable mirror tilt sensitivity.

4. CONCLUSIONS

Though the Gaussian duct only approximates the gain medium of the FEL, the behavior with a more accurate model is expected to be similar to what we have found here. In future work we plan to model the resonator using a more realistic FEL gain model. The results of the simple gain model are encouraging. The insensitivity of the optical mode to either the radius of curvature or angle of the resonator mirror suggests that the resonator should be quite stable and insensitive to perturbations. The mode calculated here is the lowest order eigenmode of the resonator. Higher order modes will not couple strongly to the gain medium and so the mode selectivity should also be good. The output from the laser will simply be the wave transmitted through one of the mirrors. The mode quality of this beam can be quite high.

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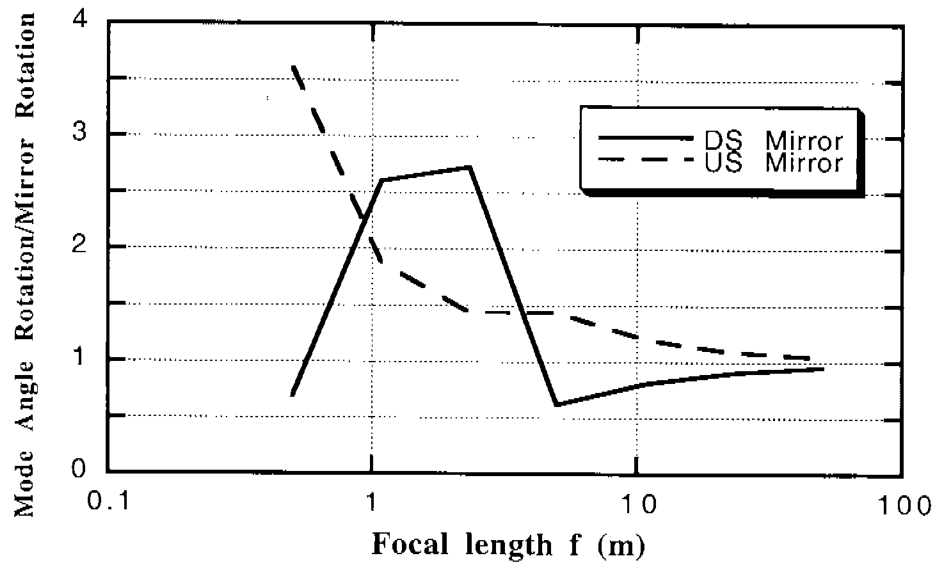

Figure 5. We plot the ratio of the rotation angle of the optical mode to the tilt of either the upstream or downstream mirror. The focal length typical of a saturated laser would be in the range of 2 to 5 meters.

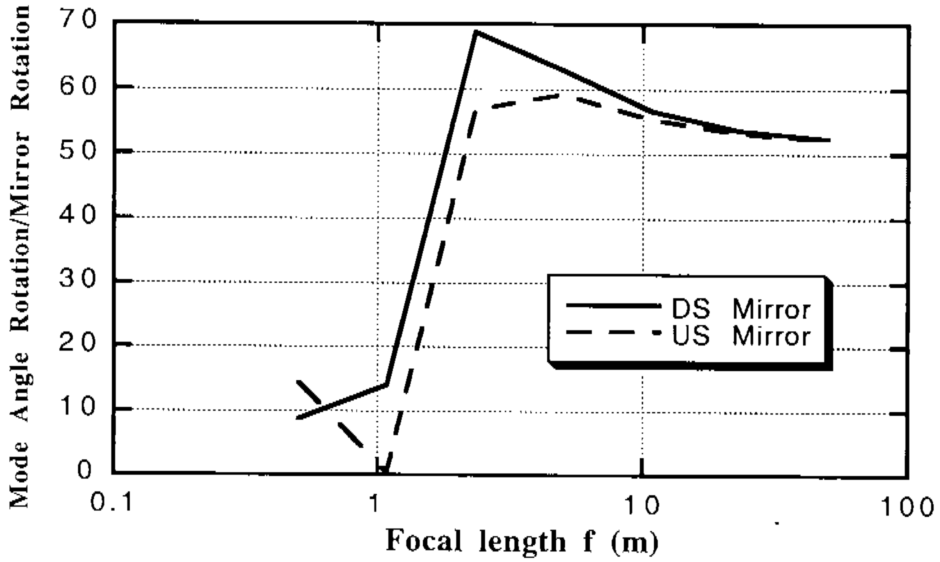

Figure 6. The ratio of the mode angle to the mirror rotation angle increases for moderate focusing. For very strong focusing it is reduced. The focal length for saturated lasing is in the range of 2 to 5 meters.

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