<arttitle> Record High Power Terahertz Radiation from Relativistic Electrons.

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<abs> Calculations and measurements confirm the production of coherent broadband THz radiation from relativistic electrons with an average power of nearly 20 watts. The radiation has qualities closely analogous to the THz radiation produced by ultrafast laser techniques (spatially coherent, short duration pulses with transform-limited spectral content). But in contrast to conventional THz radiation, the intensity is many orders of magnitude greater due to a relativistic enhancement. The absorptive and dispersive properties of materials in this spectral range provide contrast for a unique type of imaging[1,2]. The striking improvement in power reported here could revolutionize this application by allowing full-field, real-time image capture. High peak and average power THz sources are also critical in driving new non-linear phenomena with excellent signal to noise, and for pump-probe studies of dynamical properties of novel materials, both of which are central to future high-speed electronic devices[3,4]. It should also be useful for studies of molecular vibrations and rotations, low frequency protein motions, phonons, superconductor bandgaps, electronic scattering and collective electronic excitations (e.g., charge density waves).

<p> The THz region, (1 THz = 33 cm^{-1} or 4 meV), lies in the far infrared spectral range where conventional thermal sources are very weak. For example, a 2000 K blackbody source provides less than 1 microwatt/cm^{-1} of power in a typical spectroscopy application. While narrowband sources have been available using FEL technology[5], a significant advancement in broadband THz sources has occurred over the past decade with the advent of coherent THz radiation emission from photocarriers in a biased semiconductor[6]. In a typical setup, a sub-picosecond laser photoexcites electrons (and holes) in GaAs. These mobile charges are accelerated under intense, pulsed, electric fields (produced between metal electrodes on the GaAs surface) to generate electromagnetic radiation. The entire process occurs in about 1 ps, resulting in a pulsed output having spectral content extending up into the THz spectral range. Since the photocarriers accelerate in unison and the source dimensions are smaller than the wavelength, the radiating fields of individual carriers add in phase (coherent or super-radiant emission) to produce an intensity that scales as N^2, where N is the number of charge carriers. An energy per pulse of about 1 μJ has been achieved, implying MW peak powers, but at repetition rates of 1 kHz such that average power levels are only 1 mW[6].

<p> The present work describes a different process for producing coherent THz radiation by accelerated electrons. Like the method described above, the process begins with pulsed laser excitation in GaAs, but makes use of photoemission to produce bunches of free electrons in space. Using the energy recovery linac (ERL) at the Jefferson Lab Free Electron Laser (JLab FEL)[7], very short electron bunches (~ 500 fs) are brought to relativistic energies (~ 40 MeV) in a linac and then transversely accelerated by a magnetic field to produce the desired THz emission as synchrotron

radiation. This accelerator is capable of a running with a relatively high average beam current (up to 5 mA) that is unique in the world. Like the THz emitter described above, the electrons experience a common acceleration. If the electron bunch dimensions are small (in particular, the bunch length is less than the wavelength of observation), one again obtains a multiparticle coherent enhancement[8,9]. Such coherent synchrotron radiation has been observed from electrons accelerated in linacs[10-13], and more recently it has been discussed and observed from electron bunches in storage rings[14-17], but not at THz frequencies or in a form stable enough for use as a light source. Active programs to study THz radiation from linacs or storage rings are underway at BESSY II and DESY in Germany, and at Brookhaven National Laboratory and Lawrence Berkeley National Laboratory in the USA. Some linacs can create very short bunches (< 1ps) and produce coherent radiation up to a few THz, but most are limited to repetition rates of a few Hz, so the average power is quite low. The repetition rate for storage rings is on the order of 100 MHz, but the electron bunches are significantly longer (~ 100 ps) due to longitudinal damping through synchrotron radiation emission. Thus the emission is limited to the very low frequency regime (far-IR), or arises from instabilities that momentarily modify the bunch shape.

<p> The JLab ERL accelerator system overcomes some of the limitations of conventional linacs and storage rings. Electron bunches as short as ~ 500 fs are produced by the standard technique of energy modulation (chirping) followed by compression in the dispersive region of a magnetic chicane[18]. Since it operates as a linac, it is capable of producing and maintaining very short bunches. The time for an electron bunch to pass through the accelerator is less than 1 μs, thus longitudinal damping is negligible. But unlike most linacs, it operates at a very high repetition rate

(up to 75 MHz) by using superconducting RF cavities and recovering the energy of the spent electron bunches[7] so that the average current is orders of magnitude higher than conventional linacs.

<p> Conceptually it is easy to understand the many orders of magnitude gain realized in these experiments in comparison to the more conventional (non-relativistic) THz source. The diagram in Fig. 1 shows the two processes for comparison. In both cases a short light pulse from a mode-locked laser strikes a GaAs wafer, generating charge carriers. Thus the number of radiating charges should be comparable (same order of magnitude) for both cases. We can therefore compare the power produced per electron, and use Larmor's formula[19] for the radiated power. In CGS units it takes the form

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$$Power = \frac{2e^2a^2}{3c^3}\gamma^4 \qquad (1)$$

<p-ni>where e is the charge, a is the acceleration, c the speed of light and γ is the ratio of the mass of the electron to its rest mass. In case A (the conventional THz emitter), these carriers immediately experience a force from the bias field (~100 volts across a 100 micron gap) of ~10^6 V/m, which results in an acceleration of 10^{17} m/s^2. The entire process is completed in less than 1 ps, resulting in spectral content up to a few THz. In case B, the same number of charge carriers are brought to a relativistic energy of >10 MeV in a linac, after which a magnetic field bends their path into a circle of radius ρ = 1m resulting in an acceleration $c^2/\rho = 10^{17}$ m/sec^2, the same as for case A. An observer for case B also detects a brief pulse of electromagnetic radiation as an electron bunch passes by. But in this case, two factors control the pulse duration; one is the

bunch length and the other is the time for the relativistically compressed acceleration field from each electron to sweep past. The latter is given approximately by[19]

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$$\delta t = \frac{4\rho}{3\gamma^{3}c} \qquad (2)$$

<p-ni> and determines the spectral range emitted by each electron. The bunch length determines the spectral range over which the coherent enhancement occurs. For an electron energy of 10 MeV ($\gamma = 21$), and with $\rho = 1$ m, we obtain a δt of about 500 fs, which is comparable to the bunch length. The resulting spectral content extends up to about 1 THz, the same spectral range as for case A. With all factors except γ the same, we see from eq. 1 that the power radiated by a relativistic electron exceeds that from a conventional THz emitter by a factor of $\gamma^4 = 21^4 = 2 \times 10^5$. In practice, the electron energy can be significantly larger, but this simply adds intensity at higher frequencies and leaves the low frequency (THz) intensity essentially unchanged. This higher frequency emission would be relatively weak (incoherent) unless the bunch lengths could be shortened. A ten-fold reduction in bunch length combined with a factor of ~2 increase in electron energy would extend the spectral range to 10 THz. The relativistic enhancement at this frequency would be close to 2 million.

<p> We mention again that both cases benefit from multiparticle coherent emission, since many radiating charges are contained physically within one period of the emitted THz light. Theoretically, the more general expression for the power emitted by an electron *bunch*, as a function of frequency (ω) and solid angle (Ω), is derived by extending the classical electrodynamics[19] theory for a single electron, to a system of N electrons, thus[8,9]

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$$\frac{d^2 I}{d\omega \; d\Omega} = \left[N[1-f(\omega)] + N^2 f(\omega) \right] \times \underbrace{\frac{e^2\omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \hat{n} \times \left(\vec{\beta} \times \hat{n} \right) \; e^{i\omega\left(t - \frac{\hat{n} \cdot \vec{r}(t)}{c} \right)} dt \right|^2}_{\zeta} \tag{3}$$

<p-ni> where e is the charge on an electron, β is the ratio of the velocity of the particle bunch to the velocity of light, $\vec{n}$ is a unit vector along the direction of propagation (to the observer), $\vec{r}(t)$ is the location of the electron bunch center, and N is the number of particles in the bunch. The term $N^2 f(\omega)$ represents the coherent enhancement, and includes the form factor $f(\omega)$ which is the Fourier transform of the normalized longitudinal particle distribution within the bunch, i.e.

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$$f(\omega) = \left| \int_{\infty}^{\infty} e^{i\omega \hat{n} \cdot \ddot{z} / c} S(z) dz \right|^2 \tag{4}$$

<p-ni> where $S(z)$ is the distribution function for particles in the bunch, measured relative to the bunch center.

<p> A practical solution to the term labeled ζ has been presented by Hulbert and Williams[20]. For the present calculations we assume 40 MeV electron bunches each carrying 100 pC of charge, passing through a 1 m radius bend at a 37.4 MHz repetition rate. For simplicity, we assume each bunch has a Gaussian particle distribution of width σ, yielding a Gaussian form factor

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$$f(\omega) = e^{-\left(\frac{\omega\sigma}{c}\right)^2} = e^{-4\pi^2\sigma^2\left(\frac{1}{\lambda}\right)^2} \tag{5}$$

<p-ni> where λ is the wavelength of the light at frequency ω. The bunches are not strictly Gaussian in practice, but this approximation is useful for estimating the overall power and spectral content. Since the number of particles is large, the coherent emission readily dominates over any incoherent emission (as would occur for wavelengths smaller than the bunch length). Because the transverse source dimensions are smaller than the diffraction limit, the emitted light also has an extremely high degree of transverse spatial coherence. This allows for interferometry by a simple form of wavefront division[21].

<p> Fig. 2 shows calculation results in units of (average) W/cm^{-1} over the range 1-1,000 cm^{-1}, or 0.03 to 33 THz. The calculation assumes 60 mrad vertical × 60 mrad horizontal, based on the extraction optics described in the next section. In the same figure we compare two other broadband sources for this region, namely a 2000 K thermal source and a synchrotron radiation source. The superiority of the JLab ERL in the THz range is clear. We also note that the calculation assumes that the ERL has an average current of 3.8 mA compared with 800 mA at the NSLS, Brookhaven, so that in the incoherent regime the NSLS has 800/3.8 = 211 times the power per cm^{-1}. However, in the THz spectral range (where the JLab ERL emits coherently and the NSLS does not) the ERL has 3.9×10^6 times the power of the NSLS.

<p> In our experiments using the ERL THz source, the electrons were generated using the frequency doubled output of a Coherent Antares model Nd:YLF laser operating at, or a sub-multiple of, 74.8 MHz, and with an average power of a few watts. Light of wavelength 530 nm was incident on a negative electron affinity Cs coated GaAs cathode. The resulting photoelectrons were accelerated using a DC voltage of 300 kV into a superconducting linac and accelerated to an energy of 40 MeV. Although

the electrons are initially emitted from the cathode with a ~40 ps FWHM pulse length they become tightly bunched in the accelerator to pulse lengths less than 1 ps. After transiting the accelerator system, the electrons are decelerated in the same linac to 10 MeV energy before reaching the beam dump, thus recovering most of the beam energy. This energy recovery allows average currents of up to 5 mA and electron bunches containing up to 135 pC using an rf system nominally capable of accelerating only 1.1 mA beam current.

<p> The ERL THz radiation was extracted from a dipole magnet of 1 m bending radius immediately prior to the free-electron laser cavity, the latter being unimportant for these experiments. For the total power measurements the light exited the accelerator vacuum chamber through a 10 mm aperture diamond window subtending an angle of 20 × 20 mrad relative to the source point. The emerging beam was focused onto a calibrated Molectron J25 $LiTaO_3$ pyroelectric detector, calibrated with equipment traceable to NIST. This detector had a nearly flat response of out to THz wavelengths due to a black organic coating, and a responsivity of 8.83 Volts/Joule (±2%). Measurements of the total power confirmed the predictions of the calculations for this aperture, charge per bunch and repetition.

<p> The spectral content of the JLab ERL THz light was analyzed using a Nicolet Nexus 670 rapid-scan Michelson interferometer with a silicon beamsplitter. The light was detected using a 4.2K Infrared Laboratories bolometer with a 2 mm × 2 mm boron-doped Si composite element, fed from a 12 mm diameter f/4 Winston cone. It was fitted with a black polyethylene filter to ensure no light above 600 cm^{-1} was detected. The diamond window on the accelerator was replaced by a larger crystal quartz window to increase the light collection to 60 × 60 mrad. An 80 cm focal length

spherical mirror produced a 48 mm diameter collimated beam compatible with the Nicolet 670 interferometer optics. A switching mirror allowed a remote choice of source, namely the THz light from the accelerator, or a T=1300 K thermal reference source.

<p> For the spectroscopy experiments, the analysis and detection system did not have sufficient dynamic range to cover the 7 decades in power difference between the 2 sources. However, as mentioned earlier, the ERL THz source could be run at a precisely defined lower repetition rate. In this way we could reduce the average power without changing the spectral content. We chose to make measurements at 584 kHz, instead of 37.4 MHz, and at a charge per bunch of 34 pC instead of 100 pC, thereby reducing the ERL THz power by a factor of $\{(37 \times 10^6) / (584 \times 10^3)\} \times (100/34)^2$, or approximately 550.

<p> We have another reference point for determining the absolute power since we were able to switch sources from the ERL THz emission port to a 1300 K thermal source (the spectrometer's standard globar source). This allowed us to measure the relative power using the same spectrometer and detection system. At a frequency of 12 cm^{-1} we obtained a ratio of intensity from the ERL THz source to that of the globar of 2×10^4. To compare with the calculation, we multiply the THz source results by the reduction factor of 550, as discussed earlier. This implies a measured advantage of the JLab ERL THz source over the globar of 10^7. The calculation predicts an enhancement of $0.6 / (6 \times 10^{-8}) = 10^7$. The level of agreement is somewhat surprising since our simple arguments have ignored diffraction and other effects on the detection efficiency of both sources. However, the result does indeed affirm the large ERL THz power.

<p> We show the results of the measurements in Fig. 3. In the same figure we show a curve calculated for a bunch length of 500 fs, which is reasonable for the machine operating under the conditions of the experiment. We have scaled the data to fit the theory on the basis of the absolute power measurements. The spectral onset of the super-radiant enhancement on the high frequency side is clearly seen, with the shape matching the theoretical prediction. Note that there is a severe discrepancy on the lower frequency side due to diffraction effects. This can be understood in the following way. At 10 cm^{-1} and with an f/17 beam, the diffraction-limited source size is 17 mm, almost the same as the extraction optics. At 1 cm^{-1}, the diffraction-limited source size would, at 170 mm, be more than 3 times larger than the vacuum chamber containing the electron beam!

<p> One additional property of super-radiant emission from electrons is the dependence of the intensity on the square of the number of particles per bunch from Eq. 3. In Fig. 4 we plot the integrated intensity as a function of bunch charge, which shows good agreement with the theoretical N^2 curve. We note in passing that in our experiments we were able to observe considerable changes in spectral weight depending on operating conditions, and we were able to enhance certain spectral regions in a controlled way via the machine parameters. Thus the electron bunch distributions were not purely Gaussian. Since we measured the intensity, not field, of the emitted light, we were unable to determine uniquely the electron density distributions, but this might be possible in future experiments using coherent detection.

<p> Finally we measured the polarization of the emitted THz light. The intensity ratio for the horizontal to vertical polarization components is 3 for synchrotron radiation in the long-wavelength limit. This assumes full collection of the emitted

radiation. We note that the dominant intensity is near 30 cm^{-1}, which has a natural opening angle of 86 millirad. Since the emission pattern is "clipped" by the 60 mrad collection optics, the calculated ratio is expected to be higher, approaching a value of 6. Using a wire-grid polarizer placed between the Michelson modulator and the detector, we measured a ratio of 5 and consider this to be good agreement.

<p> We have demonstrated that the short bunches which circulate in energy recovery electron circulating rings with sub-picosecond electron bunch lengths yield broadband high brightness THz radiation with close to 1 W/cm^{-1} of average power into the diffraction limit and peak powers about 10^4 times higher than this. The JLab FEL is the first of a new generation of such machines, and already this accelerator is being upgraded to double the average current. The THz light will be extracted immediately before the FEL undulator where the bunches are at their shortest. Eventually machines such as these may yield average currents of 100 mA, making them highly competitive with storage rings. However, one caveat remains, this being the question of stability. Since energy recovery linac light sources are being considered for x-ray applications, it is important to measure the IR light, at least as an indicator of sources of noise. We made preliminary measurements using a frequency analyzer and the results will be published in due course.

<bibcit> 1. Chen, Q. and Zhang, X.-C., Electro-Optic THz Imaging. *Ultrafast Laser:Technology and Applications*, Fermann, Galvanauskas and Sucha (eds), Marcel Dekker, Inc (2001).

<bibcit> 2. Zhang, X.-C. Generation and Detection of Terahertz Electromagnetic Pulsed Radiation from Semiconductor Crystals with Femtosecond Optics. *Compound Optoelectronic Materials and Devices*, ISBN 0-646-21807-7, 69, (1995).

<bibcit> 3. Cole, B. E. ; Williams, J. B. ; King, B. T. Coherent manipulation of semiconductor quantum bits with terahertz radiation. *Nature* **410** 60-63 2001

<bibcit> 4. Huber, R., Tauser, F., Brodschelm, A., Bichler, M., Abstreiter, G., and Leitenstorfer, A., How many-particle interactions develop after ultrafast excitation of an electron-hole plasma. *Nature* **414** 286-289 (2001).

<bibcit> 5. Winnerl, S., Schomburg, E., Brandl, S., Kus, O., Renk, K.F., Wanke, M.C., Allen, S.J., Ignatov, A.A., Ustinov, V., Zhukov, A. and Kop'ev, P.S., Frequency doubling and tripling of terahertz radiation in a GaAs/AlAs superlattice due to frequency modulation of Bloch oscillations *Applied Physics Letters* **77**, 1259-1261 (2000).

<bibcit> 6. You, D., Jones, R. R., Bucksbaum, P. H. and Dykaar, D. R., Generation of high-power sub-single-cycle 500-fs electromagnetic pulses. *Optics Letters* **18**, 290-292 (1993).

<bibcit> 7. Neil, G.R., Bohn, C.L., Benson, S.V., Biallas, G., Douglas, D., Dylla, H.F., Evans, R., Fugitt, J., Grippo, A., Gubeli, J., Hill, R., Jordan, K., Krafft, G., Li, R., Merminga, L., Piot, P., Preble, J., Shinn, M., Siggins, T., Walker, R. and Yunn, B., Sustained Kilowatt Lasing in a Free-Electron Laser with Same Cell Energy Recovery. *Phys. Rev. Lett.* **84**, 662-665 (2000).

<bibcit> 8. Nodvick, S., Saxon, D.S., Suppression of coherent radiation by electrons in a synchrotron. *Phys. Rev.* **96**, 180-184 (1954).

<bibcit> 9. Hirschmugl, C. J., Sagurton, M. and Williams, G. P. Multiparticle Coherence Calculations for Synchrotron Radiation Emission. *Phys. Rev.* A**44**, 1316-1320 (1991).

<bibcit> 10. Nakazato, T., Oyamada, M., Niimura, N., Urasawa, S., Konno, O., Kagaya, A., Kato, R., Kamiyama, T., Torizuka, Y., Nanba, T., Kondo, Y., Shibata, Y., Ishi, K., Ohsaka, T. and Ikezawa, M. Observation of coherent synchrotron radiation. *Phys. Rev. Lett.* **63**, 1245-1248 (1989).

<bibcit> 11. Happek, U., Blum, E.B. and Sievers, A.J. Observation of coherent transition radiation. *Phys. Rev. Lett.* **67**, 2962-2965 (1991).

<bibcit> 12. Wang, D. X., Krafft, G. A. and Sinclair, C. K. Measurement of femtosecond electron bunches using a rf zero-phasing method. *Phys. Rev.* **E57**, 2283-2286 (1998).

<bibcit> 13. Lihn, Hung-chi, Kung, P., Settakorn, C. and Wiedemann, H. Observation of stimulated transition radiation. *Phys. Rev. Letts*. **76**, 4163-4166 (1996).

<bibcit> 14. Tamada, H., Tsutsui, H., Shimoda, K., and Mima, K. Features of the compact photon storage ring. *Nuclear Instr. & Meth.* **A331**, 566 (1993).

<bibcit> 15. Andersson, A., Johnson M. S. and Nelander, B. Coherent synchrotron radiation in the far infrared from a 1-mm electron bunch. *Optical Engineering,* **39**, 3099-3105 (2000)

<bibcit> 16. Arp, U., Fraser, G.T., Hight Walker, A.R., Lucatorto, T.B., Lehmann, K.K., Harkay, K., Sereno N. and Kim, K.-J. Spontaneous coherent microwave emission and the sawtooth instability in a compat storage ring. *Phys. Rev. Special Topics, Accelerators and Beams* **4**, 54401-1-54401-9 (2001)

<bibcit> 17. Carr, G.L., Kramer, S.L, Murphy, J.B., Lobo, R.P.S.M. and Tanner, D.B. Observation of coherent synchrotron radiation from the NSLS VUV ring. *Nucl. Instr. & Meth.* **A463**, 387-392 (2001).

<bibcit> 18. Yu, L.H., Johnson, E., Li, D. and Umstadter. D. Femtosecond free-electron laser by chirped pulse amplification. *Phys. Rev.* **E49**, 4480-4486 (1994).

<bibcit> 19. Jackson, J.D. *Classical Electrodynamics*, Wiley, New York 1975.

<bibcit> 20. Hulbert S.L. and Williams, G.P. Synchrotron Radiation Sources. Handbook of Optics: Classical, Vision, and X-Ray Optics, 2nd ed., vol. III. Bass, Michael, Enoch, Jay M., Van Stryland, Eric W. and Wolfe William L. (eds.). New York: McGraw-Hill, 32.1-32.20 (2001).

<bibcit> 21. Moeller, K. D., Siddons, D. P., Hirschmugl, C. J., Scardino, D., Petrone, P., Carlson, D. and Williams, G. P. *Applied Optics* **30**, 4297-4301 (1991).

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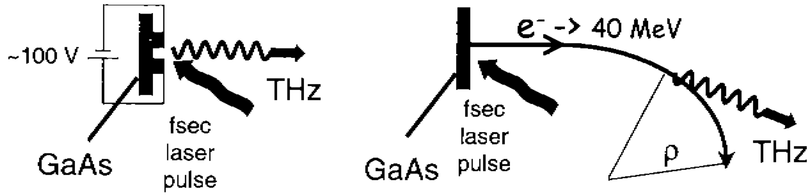
<LEGEND> Figure 1. Comparison between coherent THz radiation generated by a conventional laser-driven THz source (A) and the relativistic source described here (B).

<LEGEND> Figure. 2. Calculations of the average power emitted by a 10 mm^2 2000 K thermal source (dashed line), the NSLS VUV ring at Brookhaven National Laboratory (dotted line), and the JLab ERL (solid line). The NSLS was calculated for a stored current of 800 mA and 90 mrad (vertical) by 90 mrad (horizontal) collection. The JLab ERL was calculated for 500 fs FWHM, 100 pC bunches at a repetition rate of 75 MHz (for an average current of 5 mA) and 60 mrad (vertical) by 60 mrad (horizontal) collection.

<LEGEND> Figure. 3. Comparison between measured (solid line) and calculated (dashed line) THz spectral intensity.

<LEGEND> Figure. 4. Measured THz intensity as a function of beam current (square symbols), showing the quadratic dependence expected for coherent emission (solid line).



$$E = \frac{100V}{10^{-4}m} = 10^6 \,{}^{V}\!/_{m}$$

$$a = \frac{F}{m} = \frac{10^6 V \cdot e/m}{0.5MeV/c^2} = \frac{10^6(3\times10^8)^2}{0.5\times10^6} \cong 10^{17}\,{}^{m}\!/_{sec^2}$$

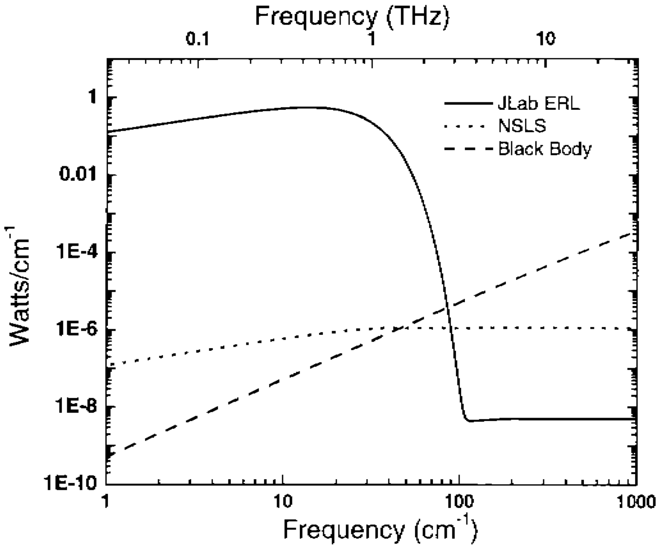
A

$$a = \frac{c^2}{\rho} = \frac{(3\times10^8)^2}{1} \cong 10^{17}\,{}^{m}\!/_{sec^2}$$

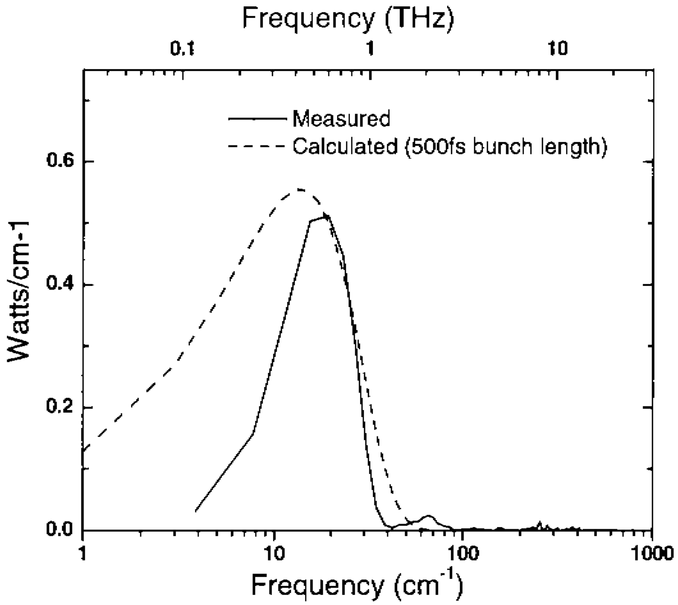
$$if\ \rho = 1\ m$$

B

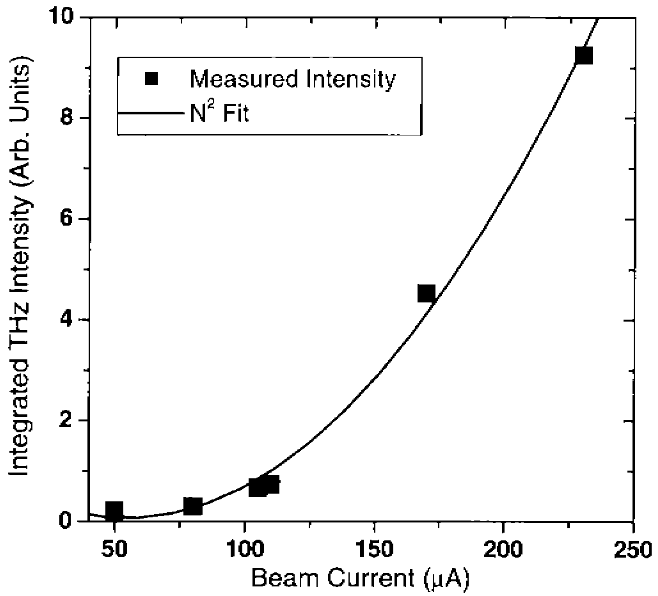
Carr, Martin, McKinney, Jordan, Neil & Williams Fig. 1



Carr, Martin, McKinney, Jordan, Neil & Williams Fig. 2



Carr, Martin, McKinney, Jordan, Neil & Williams Fig. 3



Carr, Martin, McKinney, Jordan, Neil & Williams Fig. 4