NUCLEON COMPTON SCATTERING WITH TWO SPACE–LIKE PHOTONS*

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We calculated two–photon exchange effects for elastic electron–proton scattering at high momentum transfers. The corresponding nucleon Compton amplitude is defined by two space–like virtual photons that appear to have significant virtualities. We make predictions for a) a single–spin beam asymmetry, and b) a single–spin target asymmetry or recoil proton polarization caused by an unpolarized electron beam.

1. Introduction

The two-photon exchange mechanism in elastic electron-nucleon scattering can be observed experimentally by a) measuring the C-odd difference between electron–proton and positron–proton scattering cross sections; b) analyzing deviations from the Rosenbluth formula and c) studying T-odd parity–conserving single–spin observables. This paper concentrates on the latter.

Early measurements of the parity–conserving single–spin observables include induced polarization of the recoil proton in elastic ep-scattering[1]

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and the target asymmetry[2]. These experiments were able to set only upper bounds that appeared to be at one per cent level. Corresponding theoretical calculations were given in Refs.[3] and Ref.[4], with deep–inelastic intermediate states considered in the latter.

The transverse beam asymmetry of spin-$\frac{1}{2}$ particle scattering on a nuclear target was first calculated by N.F. Mott[5] in 1932, providing, for example, an operating principle for low–energy (about 1 MeV) electron beam polarimeters [6]. The measurement of this asymmetry at higher energies of several hundred MeV was reported recently by SAMPLE Collaboration [7]. The observed magnitude of this effects is about 10^{-5} and it appears to be the only nonzero parity-conserving single–spin effect measured so far for elastic ep–scattering. Here we present the first (to the best of our knowledge) theoretical evaluation of this asymmetry that takes nucleon structure into consideration. We also present results of our calculations of parity–conserving single–spin effects due to initial or final proton polarization in elastic *ep*–scattering. Since we are dealing with large transferred momenta, we describe excitation of intermediate hadronic states (Fig.1b) in terms of deep–inelastic structure functions of the non–forward Compton amplitude with two space–like photons.

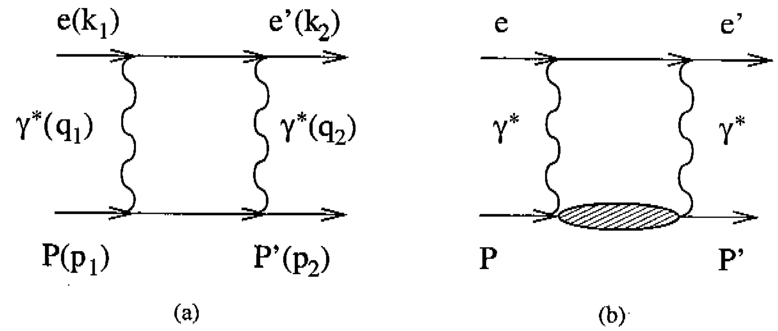

Figure 1. Two-photon exchange mechanism responsible for single–spin asymmetries in elastic *ep*–scattering. a) Elastic intermediate state. b) Inelastic intermediate states.

2. Formalism

In QED, beam and target parity–conserving single–spin asymmetries and polarizations are caused by the two–photon exchange mechanism (Fig.1). In the leading order of electromagnetic coupling constant, the imaginary (absorptive) part of the two–photon exchange amplitude interferes with a (real) amplitude of the lowest–order one–photon exchange to produce a single–spin effect due to transverse (namely, normal to the scattering plane) polarization of either the electron or the proton. As was noticed by De Rujula and collaborators over 30 years ago[4], the quantity which governs transverse polarization effects is ***the absorptive part of the non–forward Compton amplitude for off–shell photons scattering from nucleons.***

Using parity- and time-reversal invariance, one can demonstrate that a) the beam asymmetry is zero in the ultrarelativistic beam energies, b) beam and target asymmetries are independent observables and c) the target asymmetry and recoil proton polarization are equal.

There are two basic contributions to the two–photon exchange mechanism shown in Fig.1 which differ by the intermediate hadronic state. In the first case (Fig.1a) the intermediate state is purely elastic, containing only a proton and electron. In the second case (Fig.1b) the target proton is excited producing continuum of particles in the intermediate state.

A general formula for the transverse single–spin asymmetries includes integration over the loop 4–momentum k. It can be written as[4]

$$A^{el,in}_{l,p} = \frac{8\alpha}{\pi^2}\frac{Q^2}{D(Q^2)}\int dW^2 \frac{S+M^2-W^2}{S+M^2}\frac{dQ_1^2}{Q_1^2}\frac{dQ_2^2}{Q_2^2}\frac{1}{\sqrt{K}}\,B^{el,in}_{l,p}, \qquad (1)$$

where $S = 2k_1p_1$, $Q^2_{1,2} = -q^2_{1,2}$, M is the proton mass and notation for particle momenta is shown in Fig.1. For the elastic intermediate state, the integration is two dimensional because of two Dirac deltas that put the intermediate lepton and proton on the mass shell. It does not apply to the proton in the inelastic case, so the additional integration over W^2 has to be done (from M^2 to $S+M^2$), resulting in a triple integral. The quantity $D(Q^2)$ comes from Born (*i.e.*, one–photon exchange) contribution. The formulae for the relevant quantities read

$$\begin{aligned} D(Q^2) &= 8(Q^4(F_1+F_2)^2 + 2S_m(F_1^2+\tau F_2^2), \\ S_m &= S^2 - SQ^2 - M^2Q^2, \tau = Q^2/4M^2, \\ d\Omega &= \frac{2dQ_1^2 dQ_2^2}{\sqrt{K}}, \\ K &= \sqrt{1 - z_1^2 - z^2 - z_2^2 + 2zz_1z_2}. \end{aligned} \qquad (2)$$

Here z, z_1 and z_2 are cosines of the scattering angles in c.m.s. They are related to Q^2, Q_1^2 and Q_2^2 as follows:

$$Q^2 = \frac{S^2}{2(S+M^2)}(1-z), \quad Q^2_{1,2} = \frac{S(S-W^2+M^2)}{2(S+M^2)}(1-z_{1,2}). \qquad (3)$$

The above formula (3) also sets the limits of the integration region for Q_1^2 and Q_2^2, which is shown in Fig. 2 for the representative electron beam energy $E_b = 5$ GeV. It can be seen from Fig. 2 that the virtualities of the exchanged photons, albeit limited, can become significantly larger than the overall transferred momentum Q^2. Experimentally, by selecting the electron scattering angles and beam energies, one can control the limits of photon virtualities contributing to the single–spin asymmetries.

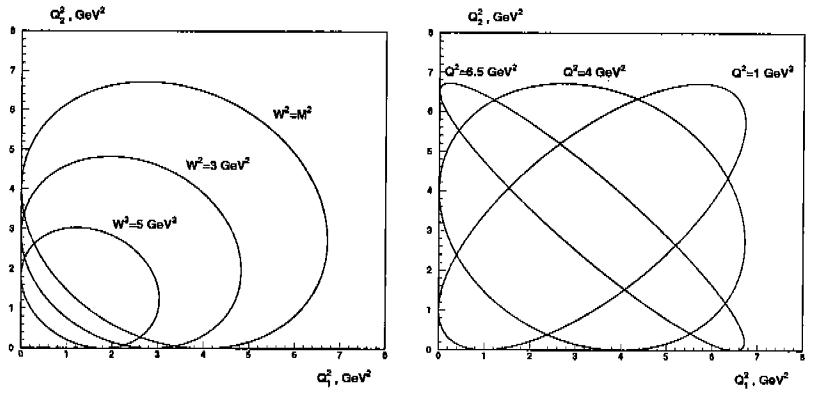

Figure 2. Integration region over Q_1^2 and Q_2^2 in Eq.(2) for elastic ($W^2 = M^2$) and inelastic contributions. The latter (left) is given for Q^2=4 GeV^2 and two values of W^2, which is an integration variable in this case. The elastic case is shown on the right as a function of external Q^2. The electron beam energy is E_b= 5 GeV.

The quantities B in Eq.(1) result from contraction of leptonic and hadronic tensors. Explicitly, they read

$$B_l^{el,in} = \frac{i}{4} Tr[(\hat{k}_2 + m)\gamma_\alpha(\hat{k} + m)\gamma_\beta\gamma_5\hat{\eta}_l(\hat{k}_1 + m)\gamma_\mu] \times$$
$$\frac{1}{4} Tr[(\hat{p}_2 + M)W^{el,in}_{\alpha\beta}(\hat{p}_1 + M)\Gamma^*_\mu] \qquad (4)$$
$$B_p^{el,in} = \frac{1}{4} Tr[\hat{k}_2\gamma_\alpha\hat{k}\gamma_\beta\hat{k}_1\gamma_\mu]\frac{i}{4} Tr[(\hat{p}_2 + M)\gamma_5\hat{\eta}_p W^{el,in}_{\alpha\beta}(\hat{p}_1 + M)\Gamma^*_\mu(q^2)],$$

where $k(p)$ is the 4–momentum of the intermediate electron (proton), m is the electron mass, $\eta_{l,p}$ is the electron (proton) polarization vector and the nucleon electromagnetic current is parameterized in the form

$$\Gamma_\mu(q^2) = (F_1(q^2) + F_2(q^2))\gamma_\mu - F_2(q^2)\frac{(p_1 + p_2)_\mu}{2M}. \qquad (5)$$

The only quantities than remain to be defined are elastic and inelastic non–forward tensors $W_{\alpha\beta}$[a]. For the elastic case, it is known exactly:

$$W^{el}_{\alpha\beta} = 2\pi\delta(W^2 - M^2)\Gamma_\alpha(q_2^2)(\hat{p} + \hat{q}_1 + M)\Gamma_\beta(q_1^2). \quad (6)$$

To compute the non–forward inelastic hadronic tensor, we need model assumptions. We use an expression for it inspired by Blumlein and Robaschik calculation [9],

$$W^{in}_{\alpha\beta} = 2\pi\left(-g_{\alpha\beta} + \frac{p^b_\alpha q^b_\beta + p^b_\beta q^b_\alpha}{q^b p^b} W_1 + p^b_\alpha p^b_\beta \frac{W_2}{M^2}\right)\frac{\hat{q}^b}{q^b p^b} \quad (7)$$

where $q^b = (q_1 - q_2)/2$ and $p^b = (p_1 + p_2)/2$.

These tensors are normalized such that in the forward direction they reproduce conventional relations between inclusive structure functions and elastic form factors,

$$W_1 = 2M\tau(F_1 + F_2)^2\delta(W^2 - M^2), \; W_2 = 2M(F_1^2 + \tau F_2^2)\delta(W^2 - M^2). \quad (8)$$

For the elastic intermediate state, we use unitarity to obtain model–independent analytic expressions for the target and beam asymmetries B^{el},

$$B^{el}_p = \frac{\sqrt{S_m}}{4M\sqrt{Q^2}}[Q_1^2Q_2^2(S_qF_1(2\mathcal{F}_{22} + \mathcal{F}_+) - \frac{1}{2}(4F_1M^2 - F_2Q^2)(2\mathcal{F}_{11} + \mathcal{F}_+))$$
$$-SS_qQ^2(2F_2\mathcal{F}_{11} - F_1\mathcal{F}_+ - F_1\mathcal{F}_-) + \frac{(Q^2 - Q_+^2)S}{8S_m}\sum_{ij} C^p_{ij}F_i\mathcal{F}_j)] \quad (9)$$
$$B^{el}_l = \frac{m\sqrt{S_m}}{4\sqrt{Q^2}}[(-2Q_1^2Q_2^2(F_1 + F_2)(2\mathcal{F}_{11} + \mathcal{F}_+) + \frac{Q^2 - Q_+^2}{4M^2S_m}\sum_{ij} C^l_{ij}F_i\mathcal{F}_j)]$$

where $Q_+^2 = Q_1^2 + Q_2^2$ and expressions for the coefficients C_{ij} are given in the Appendix.

The index $i = 1, 2$ corresponds to form factors $F_{1,2} = F_{1,2}(Q^2)$, and the index j takes values $11, 22, +, -$ where $\mathcal{F}_j$ are quadratic combinations of elastic form factors with the arguments $Q^2_{1,2}$:

$$\begin{aligned} \mathcal{F}_{11} &= F_1(Q_1^2)F_1(Q_2^2), \; \mathcal{F}_{22} = F_2(Q_1^2)F_2(Q_2^2), \\ \mathcal{F}_+ &= F_1(Q_1^2)F_2(Q_2^2) + F_1(Q_2^2)F_2(Q_1^2), \\ \mathcal{F}_- &= \frac{Q_1^2 - Q_2^2}{Q^2}(F_1(Q_1^2)F_2(Q_2^2) - F_1(Q_2^2)F_2(Q_1^2)). \end{aligned} \quad (10)$$

The asymmetries $B^{el}_{l,p}$ are given in the symmetric form with respect to the transformation $Q_1^2 \leftrightarrow Q_2^2$. It can be seen also that

$$B^{el}_{l,p}(Q^2 = Q_1^2) = B^{el}_{l,p}(Q^2 = Q_2^2) = 0$$

[a]Strictly speaking, we deal only with imaginary parts of these tensors

so there are no infrared singularities in Eq.(1).

An inelastic contribution to the asymmetries reads

$$
\begin{aligned}
B_l^{in} &= \frac{mQ^2W_1}{16\nu_b N_s}\left[R_1 + 2R_2 + 8R_3\right] + \frac{mQ^2W_2}{32M^2\nu_b N_s}R_4 \\
B_p^{in} &= \frac{(Q_+^2 - Q^2)(SF_2 - 2M^2F_1) + 2F_2S_qw_m}{128\nu_b^2 N_s M^3}(4M^2W_1T_1 + \\
&\quad \nu_b W_2 T_2)Q^2
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
R_1 &= (2\nu_b(2S - 2\nu_b - Q^2)F_1 + (F_1 + F_2)Q_+^2Q^2)(w_mQ^2 - 2\nu_bS + \\
&\quad 2M^2(Q^2 - Q_+^2)), \\
R_2 &= (w_mQ^2 - (Q^2 - Q_+)S)\nu_bQ^2(F_1 + F_2), \\
R_3 &= (S^2 - M^2Q^2 - Q^2S)\nu_b^2F_1, \\
R_4 &= (\nu_b(4F_1M^2 - F_1Q^2 - 2F_2Q^2) + 2(F_1 + F_2)Q^2S_q)(w_mQ^2 - \\
&\quad 2\nu_bS + 2M^2(Q^2 - Q_+^2)), \\
T_1 &= 2Q_+^2Sw_m + 2Q^2(S - w_m)^2 - 2Q^2Q_+^2(S + M^2) - \\
&\quad Q^2Q_+^2(S - w_m) + Q_+^4S, \\
T_2 &= 4M^2(Q^2Q_+^2 - Q^2S_w - Q_+^2S) - 4Q_+^2S^2 + Q^2S_w + \\
&\quad 3Q^2Q_+^2S + 8(s - Q^2)SS_w, \\
w_m &= W^2 - M^2,\ \nu_b = w_m + \frac{Q_+^2 - Q^2}{2}, \\
N_s &= \frac{1}{2}\sqrt{Q^2(S^2 - M^2Q^2 - SQ^2)},\ S_w = S + M^2 - W^2.
\end{aligned} \tag{12}
$$

In general, the structure functions $W_{1,2}$ are functions of four invariant variables, Q^2, $Q_{1,2}^2$ and W^2. These structure functions were neither measured nor calculated theoretically. A possibility to construct a model for them was discussed in Ref.[8], where upper bounds were obtained from the positivity conditions. Following Ref.[8], we can write

$$
W_{1,2}(W^2, Q_1^2, Q_2^2, Q^2) = \left[W_{1,2}^{DIS}(W^2, Q_1^2)W_{1,2}^{DIS}(W^2, Q_2^2)\right]^{1/2} F(Q^2), \tag{13}
$$

where we assumed additional form factor–like dependence $F(Q^2)$ on the overall 4–momentum transfer. If the deep–inelastic conditions ($W > 2$ GeV, $Q^2 > 1$ GeV^2) take place, the non–forward Compton form factor $F(Q^2)$ may be related to the integral of Generalized Parton Distributions (GPD)[10] at large transverse momenta t (with the Mandelstam variable t equal to $-Q^2$ in our case). Note that since the *absorptive* part of the non–forward Compton amplitude contributes, then $x = \xi$ part of GPDs is selected similar to the single–spin asymmetry arising from interference of the Bethe–Heitler process with virtual Compton scattering[11]. The computed quantities are then described by the zeroth moments of nucleon GPDs and it is therefore natural to assume for further estimates that the introduced form factor $F(Q^2)$ depends on Q^2 like the nucleon form factor described, with a good accuracy, by the dipole formula $F(Q^2) = (1 + \frac{Q^2}{0.71GeV^2})^{-2}$.

This model choice for the non–forward Compton amplitude has three main properties. It is symmetric with respect to the transformation $Q_1^2 \leftrightarrow Q_2^2$, has a correct forward ($Q^2 = 0$) limit and assumes form factor–like suppression with respect to the overall transferred momentum Q^2. The latter was not considered in the early papers[4,8], leading to dramatic overestimates of the two–photon–exchange effects for elastic ep–scattering.

3. Numerical results and conclusions

Our results for the single–spin asymmetries are presented in Fig.3. The asymmetries are kinematically suppressed in the forward and backward directions by a factor $\sin(\Theta_{c.m.})$. The target asymmetry increases with increasing beam energies, while the beam asymmetry decreases due to the additional supression factor m/E_b. The target asymmetry (= recoil polarization) is evaluated at the per cent level. Below the pion threshold, only the nucleon intermediate state is allowed and the calculation becomes model–independent, based only on unitarity and known values of proton form factors. At higher energies, the contribution from excited intermediate hadronic states exceeds the elastic contribution. As can be seen from Fig.4, the integration region where both exchanged photons are highly virtual plays an important role. Measurements of the single–spin asymmetries due to two–photon exchange provide information about the absorptive part of the virtual Compton amplitude with two space–like photons at large values of the Mandelstam variable t.

To our knowledge, this is the first published calculation of the single–spin beam asymmetry in elastic ep–scattering for the kinematics where nucleon structure effects become important. Our results appear to be in reasonable agreement with recent SAMPLE data[7].

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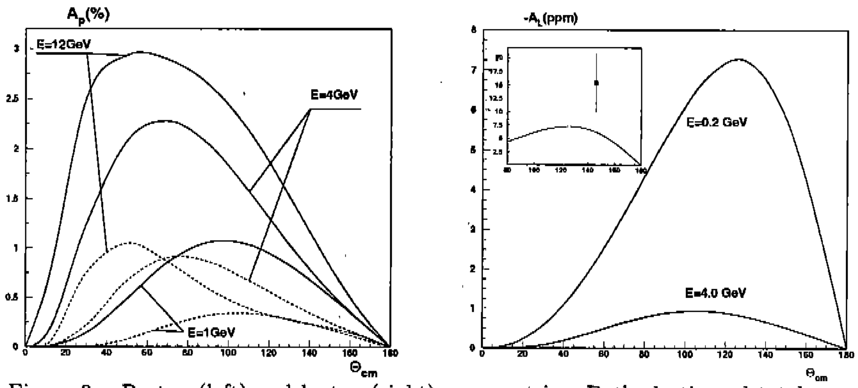

Figure 3. Proton (left) and lepton (right) asymmetries. Both elastic and total contribution are shown for proton asymmetry and only elastic one for lepton asymmetry. The plot in the insert gives comparison with SAMPLE data[7] at E_B= 0.2 GeV.

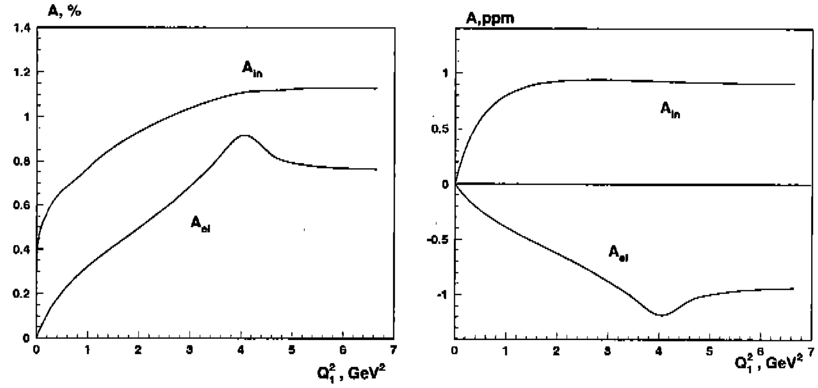

Figure 4. Elastic and inelastic contributions to the target (left) and beam (right) asymmetries versus the upper integration limit over the virtuality of one of the two exchanged photons. Importance of high virtualities is evident.

Appendix

Expressions for the coefficients $C_{i,j}$ in Eq.(9) are as follows

$$C^p_{1,11} = 4M^2 Q_+^2 (Q^2 + 2S)$$
$$C^p_{1,22} = 2(-(Q^2 - Q_+^2)(2S^2 - M^2 Q^2) + Q^4 S)$$
$$C^p_{1,+} = (4Q^2 S_q (S + M^2) + Q_+^2 (4M^2 Q^2 + 4M^2 S - 2SS_q))$$
$$C^p_{1,-} = -Q^2 (2M^2 Q^2 - 2SS_q)$$
$$C^p_{2,11} = -4Q^2 (2S_q (S + M^2) + SQ_+^2)$$
$$C^p_{2,22} = -Q^2 (-(Q^2 - Q_+^2)(S - M^2) - 2S^2 + 2Q^2 S) S/M^2$$

$$
\begin{aligned}
C^p_{2,+} &= -3Q^2SQ^2_+ \\
C^p_{2,-} &= SQ^4 \\
C^l_{1,11} &= 2M^2(2Q^2_+M^2Q^2 + Q^2_+Q^2S + 2Q^2_+S^2 + 8M^4Q^2 + 12M^2Q^2S) \\
C^l_{1,22} &= Q^2((Q^2 - Q^2_+)M^2(S + 2M^2) - S^2(Q^2_+ + 4M^2)) \\
C^l_{1,+} &= M^2(4Q^2_+M^2Q^2 + 3Q^2_+Q^2S + 2Q^2_+S^2 + 2M^2Q^4 + Q^4S - 6Q^2S^2) \\
C^l_{1,-} &= -2M^2Q^4(M^2 + S) \\
C^l_{2,11} &= -4M^2(M^2Q^4 + Q^2S^2 + Q^4S - Q^2_+S^2) \\
C^l_{2,22} &= -S^2Q^2(Q^2_+ - Q^2) \\
C^l_{2,+} &= S^2(2Q^2_+M^2 - Q^2_+Q^2/2 - 2M^2Q^2 + Q^4) \\
C^l_{2,-} &= S^2Q^4/2
\end{aligned}
$$

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