A Pair Polarimeter for Linearly Polarized High Energy Photons

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Abstract

Electron-positron pair production, the main process for high energy photon interaction with matter, has a large analyzing power for linear polarized photons. Our calculations show that in the GeV energy range a compact polarimeter based on silicon micro-strip detectors is feasible. We calculated the cross section as a function of the angle between the polarization plane and the line between positron and electron positions in the detector. For a thin radiator and equal energies of the positron and electron the ratio $\sigma_{||}/\sigma_{\perp}$ is about 1.6. At a beam intensity of 10^6 photons per second a polarization measurement with a 1% statistical accuracy will require less than one hour of data taking.

Key words: Polarimeter for γ-rays, Silicon micro-strip

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1 Introduction

In 1950 C. N. Young [1] and Berlin&Madansky [2] pointed out that in photoproduction of an electron-positron pair the production plane is correlated with the plane of the photon polarization. A full QED analysis of polarization effects in pair production was done by H. Olsen and L. C. Maximon [3]. They provided a convenient formula for the pair production cross section at different polarizations of the photon. Maximon&Olsen also showed [4] a very rapid change of the azimuthal asymmetry vs coplanarity angle of the positron-photon and the electron-photon planes when this angle is near π. Since 1960, polarimeters based on pair production in amorphous converters have been built for photon energies up to 1 GeV. They were used in experiments to study nucleon and resonance physics [5–8]. The scheme of such polarimeters was based on magnetic separation of the pair components and a measurement of the effect of the photon polarization on the distribution of the tracks along the direction of the magnetic field.

Recently, an idea for a polarimeter scheme based on silicon micro-strip detectors was reported [9]. In principle the full geometry of pair production as well as its energies can be measured. The event rate vs azimuth of the vector which connects the crossings of the detector plane by a positron and an electron is the most easily measurable parameter, and, as it was found by numerical integration [10], provides a much larger analyzing power than the previously

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In the present communication we present the design considerations of the polarimeter and detailed calculations of the asymmetry for the proposed scheme.

2 Kinematics of the reaction

The coordinate system and kinematics are presented in Figure 1. The photon momentum $\mathbf{k}$ is directed along the Z axis. The photon polarization vector $\mathbf{e}$ is parallel to the X axis. The directions of the positron (electron) momentum $\mathbf{p}_{+(-)}$ are described by the plane angles $\theta_{+(-)}$ and $\phi_{+(-)}$. The points P and N represent the positions, where the particles cross the detector plane. The angle between planes $\mathbf{k},\mathbf{p}_+$ and $\mathbf{k},\mathbf{p}_-$ is $\phi_\pm$, the so-called coplanarity angle of the reaction. The detector plane is also shown. Here a new angle $\omega_\pm$ is defined by the vector $\vec{PN}$ and the X axis.

3 Cross section of the reaction

We have used the differential cross section of e^+e^- pair production by polarized photons given in [3]:

$$\frac{\partial^3\sigma}{\partial E_-\partial\Omega_-\partial\Omega_+} = \frac{\bar{\Phi}}{8\pi^2}\cdot\frac{[1-F(q)]^2}{\delta\cdot q^4}\cdot\frac{4}{k\delta}\cdot(X_{unp}+P_l\cdot X_{pol}) \tag{1}$$

$$X_{unp} = (E_+^2+E_-^2)\frac{u_+^2+u_-^2+2u_+u_-\cos(\phi_+-\phi_-)}{\Delta_+\Delta_-}+2E_+E_-(\Delta_+^{-1}-\Delta_-^{-1})^2 \tag{2}$$

$$X_{pol} = -2E_+E_-\left[\frac{u_+^2}{\Delta_+^2}\cos 2\phi_+ + \frac{u_-^2}{\Delta_-^2}\cos 2\phi_- + \frac{u_+}{\Delta_+}\frac{u_-}{\Delta_-}\cos(\phi_+ + \phi_-)\right] \tag{3}$$

where k is the photon energy; E_+ and E_- are the positron and electron energies; $\partial\Omega_+$ and $\partial\Omega_-$ are the elements of solid angles for the directions of the positron and electron; $\bar{\Phi} = \alpha r_0^2 Z^2$; Z is the atomic number of the atom in the converter; α is the fine structure constant; r_0 is the electron classical radius; q is the momentum of the recoil nuclei; $F(q)$ is an atomic form factor, $F(q) = 1/[1 + (111qZ^{-\frac{1}{3}})^2]$; δ is the minimum value of the momentum transfer q, with $\delta = k/(2E_+E_-)$; the combinations $\Delta_{+(-)}$ are $1 + u^2_{+(-)}$; and the reduced plane angles are defined as $u_{+(-)} = E_{+(-)}\theta_{+(-)}$. The rest mass of the electron is used as a unit of energy and momentum in these formulas. X_{unpol} provides the cross section for unpolarized photons and X_{polar} provides the polarization dependent part of the cross section; P_l is the degree of photon linear polarization. It is useful to express the momentum transfer as:

$$q^2 = u_+^2 + u_-^2 + 2u_+u_- \cos(\phi_+ - \phi_-) + \frac{1}{2}\left(\frac{\Delta_+}{E_+} + \frac{\Delta_-}{E_-}\right)^2 \tag{4}$$

from where it is easy to see that q^2 has a minimum and the cross section a maximum at $\phi_+ - \phi_- = \phi_\pm \sim \pi$.

3.1 General considerations of the azimuthal asymmetry

The azimuthal dependence of the cross section for any reaction with linearly polarized photons and unpolarized nuclei has a term proportional to $(\cos\phi)^2$, where ϕ is the azimuthal angle between the photon polarization vector $\mathbf{e}$ and a vector constructed from other available vectors in the process [11].

The general form of the cross section is $1 + \Sigma \cdot \cos 2\phi$, where in the case of pair photoproduction $\Sigma = (X_{unp} - X_{pol})/(X_{unp} + X_{pol})$. The $\cos 2\phi = 2(\mathbf{e} \cdot \mathbf{l})^2 - 1$, where $\mathbf{e}$ is the photon polarization vector, and $\mathbf{l}$ is an available vector in the process.

When one integrates the cross section over the momentum transfer to the nucleus and the momentum of one of the produced particles, two remaining vectors are the photon momentum $\mathbf{k}$ and the momentum of the second produced particle. In this case the $\mathbf{l}$ vector is $\mathbf{n}_{single}$ a normal to the plane based on the momenta of the photon and the detected particle, e.g. for the positron $\mathbf{n}_{single} = \mathbf{k} \times \mathbf{p}_+ / |\mathbf{k} \times \mathbf{p}_+|$. A complete analysis of the asymmetry for this case of single particle detection, its dependence on the coplanarity angle and the energy of the detected particle is given in [4].

If both produced particles are detected, a new vector $\mathbf{n}_{pair}$ can be constructed from their momenta. A vector normal to the pair-production plane can be made as a vector product of $\mathbf{p}_+$ and $\mathbf{p}_-$. The unit vector directed along the projection of the mentioned vector product onto the plane perpendicular to the photon momentum is $\mathbf{n}_{pair}$:

$$\mathbf{n}_{pair} = \frac{\mathbf{k} \times [\mathbf{k} \times [\mathbf{p}_+ \times \mathbf{p}_-]]}{|\mathbf{k} \times [\mathbf{k} \times [\mathbf{p}_+ \times \mathbf{p}_-]]|} \tag{5}$$

Such an approach was analyzed by S. R. Kelner in [12], where the analytic integration was performed in a logarithmic approximation. An average asymmetry of 0.143 was found after integration over directions and energies of the positron and electron.

The momenta of the pair particles can be used to construct another vector:

$$\mathbf{n}_\omega = \frac{(\mathbf{p}_+ - \mathbf{k}\cdot(\mathbf{p}_+\cdot\mathbf{k})/k^2) - (\mathbf{p}_- - \mathbf{k}\cdot(\mathbf{p}_-\cdot\mathbf{k})/k^2)}{|(\mathbf{p}_+ - \mathbf{k}\cdot(\mathbf{p}_+\cdot\mathbf{k})/k^2) - (\mathbf{p}_- - \mathbf{k}\cdot(\mathbf{p}_-\cdot\mathbf{k})/k^2)|} \tag{6}$$

This vector is almost the same as $\mathbf{n}_{pair}$. An advantage of the $\mathbf{n}_\omega$ vector is that it can be directly measured in an experiment, where the positron and electron tracks are detected [10]. As it is shown in Figure 1, the vector $\mathbf{n}_\omega$ is parallel to the vector $\vec{PN}$, which connects the crossings of the positron and electron trajectories through the detector plane.

3.2 Calculation of the asymmetry

The cross section for pair photoproduction has a narrow maximum at low momentum transfer values, so accurate integration requires considerable CPU time. We performed an integration over the directions of the momenta of the positron and electron using the NAG library package [13]. The integration was done over the polar angles θ_+ and θ_+, and the coplanarity angle $\phi_\pm$. To calculate the asymmetry the cross section according to the formula 1 was integrated with two different weighting factors, one for for the total cross section and $2 \cdot \cos 2a$ for the polarization-dependent part. When we calculated the single particle asymmetry, $a = \phi_+$ while for the proposed polarimeter scheme $a = \omega_\pm$.

The asymmetry trends to be little bit higher for a low-Z converter. We did our calculation for a carbon converter.

Multiple scattering in the converter introduces random fluctuations of the directions of the positron and electron momenta. The resulting fluctuations of the positions of the positron and electron trajectories in the detector plane were represented by two additional dimensions in the integration. The effect of multiple scattering was calculated for the case of equal energies of the pair particles.

3.3 Asymmetry as a function of ratio E_+/E_γ

The asymmetry has a large variation vs energy ratio between the positron and electron, as shown in Figure 2, which is in agreement with [4]. At $E_+ = E_-$ there is a wide smooth maximum for the asymmetry. Integrated over directions of the positron and electron the asymmetry has a maximum of 0.24. The asymmetry averaged over the positron energy spectra is 0.14. Because of the slow variation of the asymmetry vs E_+/E_γ near its maximum, the energy analysis can be an effective method for increasing the asymmetry without introducing too large a systematic error.

3.4 Asymmetry as a function of the photon energy

The asymmetry has a very slow dependence on the photon energy. This is shown in Figure 3 for equal energies of the positron and electron.

A slightly larger dependence of the asymmetry vs the photon energy was observed when the events with a fixed but small distance between pair components are removed. This is required in an experiment due to the limited two track resolution of the detector. At low energy the asymmetry rises because the maximum distance between the positron and electron is limited by the finite size of the detector.

3.5 Asymmetry as a function of the positron-electron distance

The pair production cross section and Σ asymmetry are shown in Figure 4 as functions of r_{+-}, the distance between the positron and electron tracks in the detector. The cross section is proportional to r_{+-} at small distances because of the growing phase space and drops quickly at large distances. In our example the distance 0.8 mm corresponds to an angle between pair components of $2/E_\gamma$. The asymmetry is also low at small values of r_{+-}. It has a maximum at r_{+-} of 1 mm. In an experiment the average asymmetry will be measured. If $r_{max} > 2$ mm the sensitivity of the average asymmetry to the value of r_{max} is less than 2% for a 1 mm change in r_{max}. Another curve demonstrates the sensitivity of Σ to the value of r_{min}. In analysis of the experiment the events with r_{+-} less than of 0.2-0.3 mm will be excluded because of the limited two-track resolution of the detector. It is easy to see that uncertainty in such a cut due to detector resolution should not introduce significant systematics in the asymmetry measurement.

3.6 *Asymmetry as a function of the converter thickness*

The positron and electron are produced inside the converter and have multiple scattering on their way out. The average multiple-scattering angle is $\theta_{ms,+(-)} = \frac{14}{E_{+(-)}} \cdot \sqrt{\frac{2}{t}}$, where t is the converter thickness in units of radiation length. The asymmetry drops for larger converter thicknesses, as shown in Figure 5. The calculation was done for $E_+ = E_-$ and different cuts on the minimal distance between the positron and electron. The effect of multiple scattering is less than one expects from a comparison of the multiple scattering angle and typical angle between pair components, because in the region of small distances between the positron and electron the asymmetry is low even for a very thin converter, as shown in Figure 4.

4 Concept of the polarimeter

Our choice of polarimeter scheme is based on the following considerations:

First, the use of micro-strip detectors (MSD) allows detection of two tracks at small relative distance and an accurate determination of the coordinates for each of the pair components without the use of a magnetic field to separate the particles. As a result, the properties of the original asymmetry in the pair-production process are preserved and can be used to maximize the effectiveness of the polarimeter.

Second, the detection of both pair components allows determination of the direction of the vector $\mathbf{n}_\omega$ introduced above.

Third, the integration over a wide range of emission angles, as well as coplanarity angle, allows to minimize sensitivity to the cuts in these parameters and reduce the systematic uncertainties of the asymmetry measurement.

Fourth, the effect of scattering of the pair components in the radiator reduces the value of the asymmetry, which limits the useful thickness of the converter. For example, the use of MSD as a converter will be attractive if its thickness can be reduced to 100 μm for silicon.

Fifth, the particle path from the converter to MSD must be shielded from stray magnetic field to a level below 0.1 Gauss.

5 Layout of the polarimeter

The layout of the pair polarimeter is shown in Figure 6. The polarimeter consists of a veto detector (VD), a converter (CO), a pair detector (PD), a vacuum straight section (VS), a set of micro-strip detectors (MSD), and trigger counters (TC) with or without analyzing magnet (AM).

The veto detector as well as the pair detector can be made of a thin plastic scintillator or a gas multi wire proportional chamber (MWPC). In the last case, because of the very low amount of matter in the MWPC, the appropriate thickness converter should be placed in front of the pair detector.

The path of the pair components from production in the converter through the vacuum straight section should be shielded from any stray magnetic field. For the polarimeter under discussion, which has a length of 100 cm, a magnetic field on the level of 0.1 G on the path of the pair can lead to a systematic error in polarization measurement of 1%.

The MSD are the central part of the polarimeter, providing accurate determination of the coordinates for the positron and electron. With a typical photon beam size of 1 cm, the MSD should cover an area of 2×2 cm^2, to avoid systematics related to acceptance variation. A pitch of 50 μm on the MSD should be used. This allows at least 15 μm position resolution and better than 100 μm two-track resolution.

For reliable event reconstruction, in the presence of the stray hits, four coordinate planes are needed. Because stray hits are generated by soft photons and partly by e^+e^- production in the MSD, the number of them is proportional to MSD thickness. To reduce the number of stray hits, double-sided MSD are preferable. Double-sided MSD have the additional advantage of short distance between coordinate planes and as a result reduce the parallax effect in track analysis. For a low-energy photon beam the space needed for the polarimeter is not very large and a scintillator fiber hodoscope can be used instead of MSD.

Behind the MSD the trigger counters (TC) are located. There are two possible arrangements of the trigger. In the simplest case the trigger can be organized by a pair of plastic scintillator counters, each of which detects both produced particles, the positron and electron. The second option is based on the pair spectrometer, which measures the energy of each pair component. In the last case a larger analyzing power will be achieved.

Summary

Here, we describe a proposal and calculations on a polarimeter for linearly polarized high-energy photons. The proposed detection of the electron-positron pairs, produced from a thin amorphous converter, by silicon micro-strip detectors allows to build a compact polarimeter for photon energies of several GeV. The calculation of the production cross section as a function of the angle between the polarization plane and the vector connecting the positron and electron crossings through the detector plane shows an asymmetry which is larger than those of previously known schemes of pair polarimeters by a factor of 1.4. The analyzing power is about 0.24 for the thin converter and equal energies of an electron and a positron. In addition to higher analyzing power our polarimeter will measure the full azimuthal distribution continuously, which reduces the sensitivity to many possible sources of systematics. The polarimeter scheme, which uses a magnetic field for separation of a positron and an electron, in combination with the unique spatial resolution of the MSD in an active converter and the pair detector, can be extended to photon energies up to tens of GeV.

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Figure 1. The kinematics of the e^+e^- pair photoproduction.

Figure 2. The asymmetry and cross section of pair production.

Figure 3. The asymmetry as a function of photon energy.

Figure 4. The cross section and asymmetry as a function of pair distance.

Figure 5. The asymmetry as a function of converter thickness.

Figure 6. The layout of the photon polarimeter.

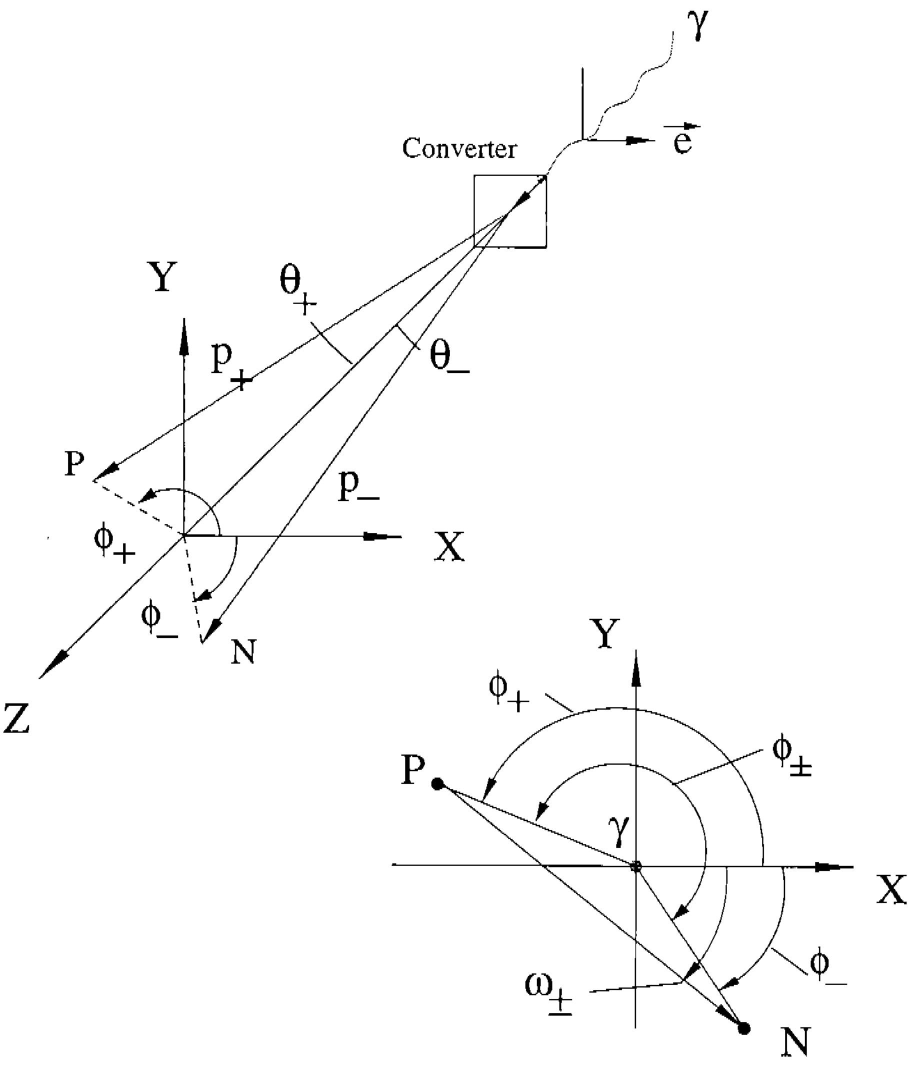

Fig. 1. The kinematics of the e^+e^- pair photoproduction (upper picture) and the azimuth angles in the detector plane. The photon momentum is directed along the Z axis. The photon polarization vector $\vec{e}$ is parallel to the X axis. The angle $\phi_+(\phi_-)$ is between the photon polarization plane and the plane constructed by the momentum of the photon and momentum of a positron (an electron). The angle $\phi_\pm$ is the coplanarity angle. The labels P and N indicate the positions of the crossings of the detector plane by a positron and an electron. The angle $\omega_\pm$ between the X axis and the vector $\vec{PN}$ is a directly measurable parameter in an experiment.

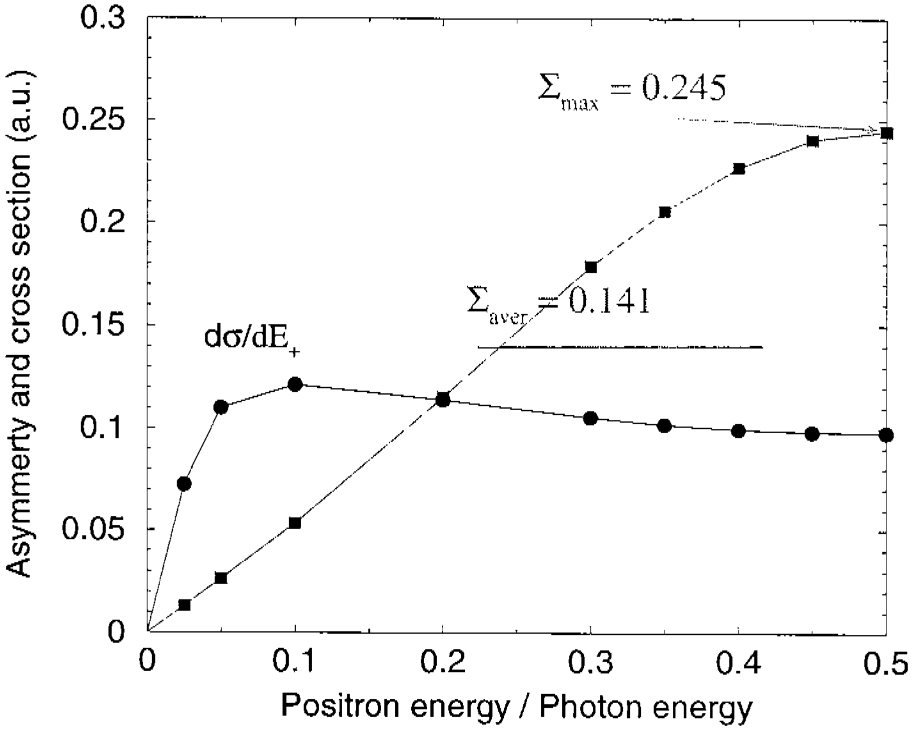

Fig. 2. The asymmetry and cross section of pair production by 2 GeV 100% linearly polarized photons as a function of positron energy. Σ_{aver} shows the asymmetry averaged over the full range of the positron energy.

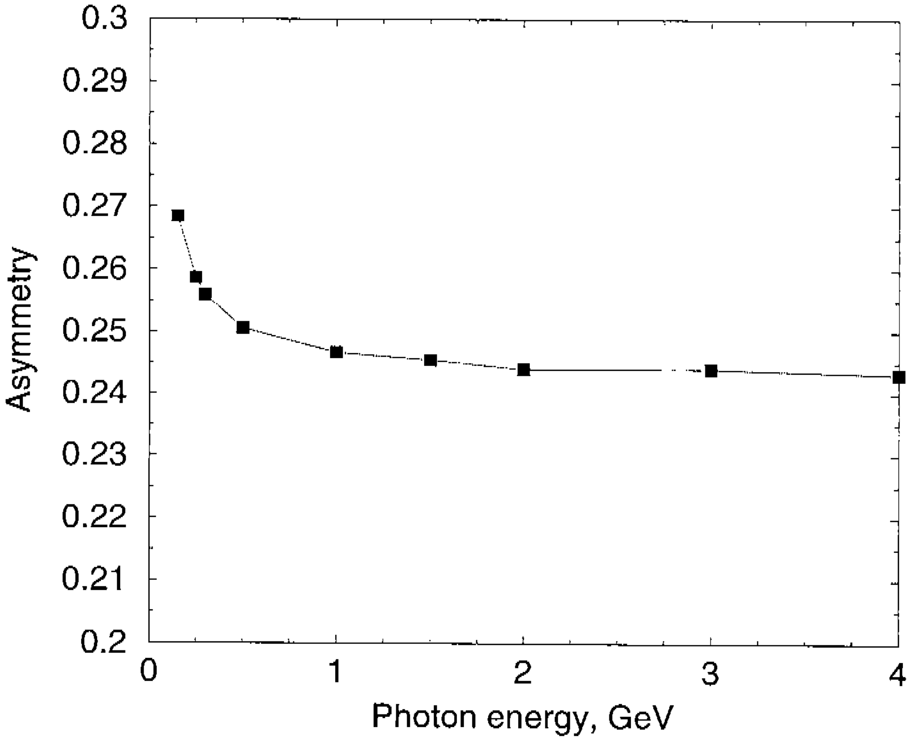

Fig. 3. The asymmetry of pair production by 100% linearly polarized photons in a very thin carbon converter as a function of photon energy for $E_+ = E_-$, integrated over directions of the pair.

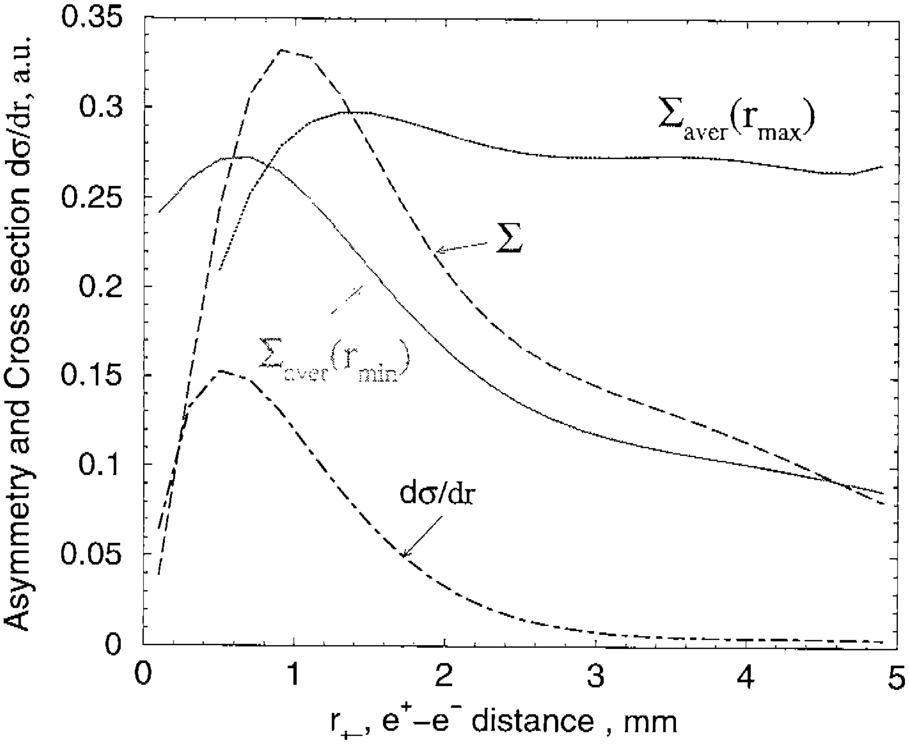

Fig. 4. The cross section of pair production $d\sigma/dr$ (dot-dash), the Σ asymmetry (long dash), the Σ asymmetry averaged over the interval of r with fixed $r_{max} = 5$ mm, and Σ asymmetry averaged over the interval of r with fixed $r_{min} = 0.5$ mm. The photon energy is 2 GeV, $E_+ = E_-$, the distance between the converter and the detector plane is 800 mm.

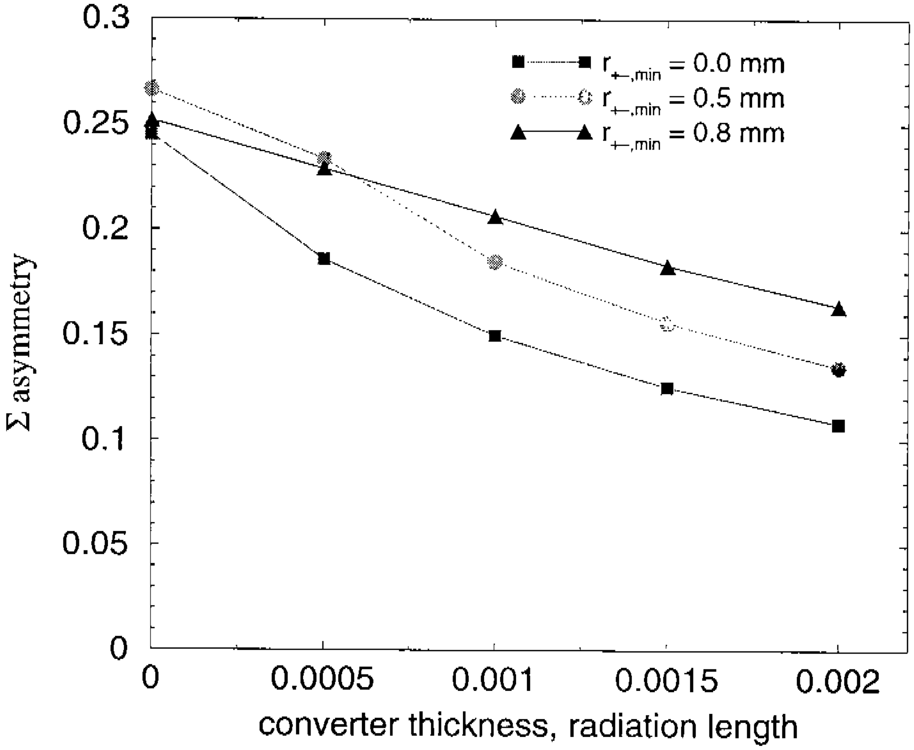

Fig. 5. The asymmetry of pair production by 2 GeV 100% linearly polarized photons as a function of converter thickness for equal energies of the positron and electron. The effect of the cut on minimal distance between a positron and an electron is presented. A 20 by 20 mm detector was located at a distance 800 mm from the converter.

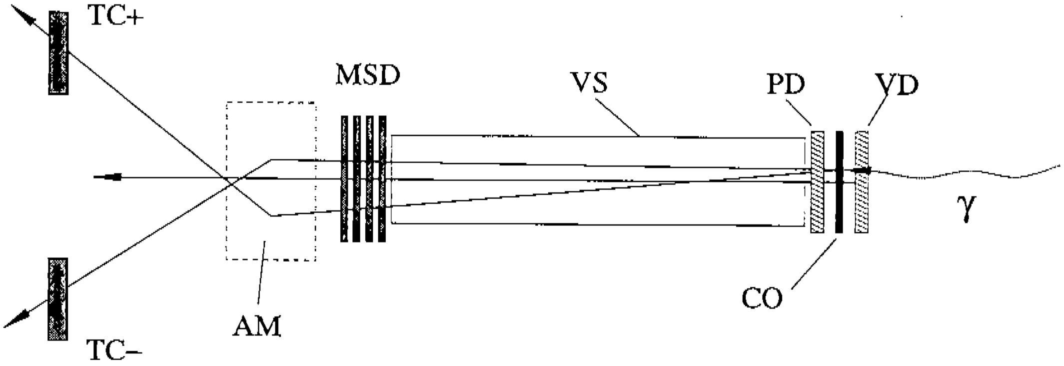

Fig. 6. The layout of the photon polarimeter. The photon strikes from the right side. The veto detector is labeled as VD, the converter as CO, the pair detector as PD, the vacuum straight section as VS, the set of micro-strip detectors as MSD, the trigger counters as TC+ and TC-, and the analyzing magnet as AM.