

# Weak Transitions in $A=6$ and 7 Nuclei

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## Abstract

The  ${}^6\text{He}$  beta decay and  ${}^7\text{Be}$  electron capture processes are studied using variational Monte Carlo wave functions, derived from a realistic Hamiltonian consisting of the Argonne  $v_{18}$  two-nucleon and Urbana-IX three-nucleon interactions. The model for the nuclear weak axial current includes one- and two-body operators with the strength of the leading two-body term—associated with  $\Delta$ -isobar excitation of the nucleon—adjusted to reproduce the Gamow-Teller matrix element in tritium  $\beta$ -decay. The measured half-life of  ${}^6\text{He}$  is under-predicted by theory by  $\simeq 8\%$ , while that of  ${}^7\text{Be}$  for decay into the ground and first excited states of  ${}^7\text{Li}$  is over-predicted by  $\simeq 9\%$ . However, the experimentally known branching ratio for these latter processes is in good agreement with the calculated value. Two-body axial current contributions lead to a  $\simeq 1.7\%$  (4.4%) increase in the value of the Gamow-Teller matrix element of  ${}^6\text{He}$  ( ${}^7\text{Be}$ ), obtained with one-body currents only, and slightly worsen (appreciably improve) the agreement between the calculated and measured half-life. Corrections due to retardation effects associated with the finite lepton momentum transfers involved in the decays, as well as contributions of suppressed transitions induced by the weak vector charge and axial current operators, have also been calculated and found to be negligible. The approximate character of the variational wave functions employed here is presumably at the origin of the present unsatisfactory situation between theory and experiment.

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## I. INTRODUCTION

The present study deals with weak transitions in the  $A=6$  and  $7$  systems within the context of a fully microscopic approach to nuclear structure and dynamics, in which nucleons interact among themselves via realistic two- and three-body potentials, and with external electro-weak probes via realistic currents consisting of one- and many-body components. To the best of our knowledge, calculations of the super-allowed  ${}^6\text{He}$  beta- ( $\beta^-$ ) decay and  ${}^7\text{Be}$  electron- ( $\epsilon^-$ ) capture processes have relied in the past on relatively simple shell-model or two- and three-cluster wave functions. The shell model calculations have typically failed to reproduce the measured Gamow-Teller matrix elements governing these weak transitions, unless use was made of an effective one-body Gamow-Teller operator, in which the nucleon's axial coupling constant  $g_A$  is quenched with respect to its free value—for a recent summary of a shell-model analysis of  $\beta$ -decay rates in  $A \leq 18$  nuclei, see Ref. [1].

More phenomenologically successful models have been based on  $\alpha NN$  (for  $A=6$ ) or  $\alpha-t$  and  $\alpha-\tau$  (for  $A=7$ ) clusterization, and have used either Faddeev techniques with a separable representation of the  $NN$  and  $\alpha N$  potentials [2] or the resonating-group method [3]. However, while these studies do provide useful insights into the structure of the  $A=6$  and  $7$  nuclei, their connection with the underlying two- (and three-) nucleon dynamics is rather tenuous. For example, it is unclear whether the required quenching of  $g_A$  in the shell-model calculations reflects deficiencies in the associated wave functions—a lack of correlations or limitations in the model space—and/or in the model for the axial current operator—which typically ignores many-body terms—or, rather, a true modification of the nucleon axial coupling in medium.

In this work, we use variational Monte Carlo (VMC) wave functions [4–7], derived from a realistic Hamiltonian consisting of the Argonne  $v_{18}$  two-nucleon [8] and Urbana-IX three-nucleon [9] interactions. The VMC wave functions provide a reasonable description of the energy spectra of low-lying states in  $A=6$ – $8$  nuclei [5,10], and of elastic and inelastic electromagnetic form factors and radiative widths of  ${}^6\text{Li}$  states [11].

The model for the nuclear weak vector and axial-vector currents is that developed in Refs. [12,13], consisting of one- and two-body terms. The weak vector charge and current operators are constructed from their isovector electromagnetic counter-parts [14,15], in accordance with the conserved-vector-current (CVC) hypothesis. The leading two-body term in the axial current is due to  $\Delta$ -isobar excitation, while the leading two-body axial charge operator is associated with the long-range pion-exchange term [16], required by low-energy theorems and the partially-conserved-axial-current relation. The largest model dependence is in the weak axial current. To minimize it, the  $N \rightarrow \Delta$  transition axial coupling constant has been adjusted to reproduce the experimental value of the Gamow-Teller matrix element in tritium  $\beta$ -decay in an essentially exact calculation, using correlated-hyperspherical-harmonics wave functions, derived from the same Hamiltonian adopted here [17].

This manuscript falls into seven sections. In Sec. II explicit expressions for the  $\beta$ -decay and  $\epsilon$ -capture rates are derived in terms of reduced matrix elements of multipole operators of the nuclear weak current, whose model is succinctly described in Sec. IV. The  $A=6$  and  $7$  nuclei VMC wave functions are reviewed in Sec. III, while predictions for the  ${}^6\text{He}$   $\beta$ -decay and  ${}^7\text{Be}$   $\epsilon$ -capture rates are presented and discussed in Sec. VI. Our conclusions are summarized in Sec. VII.

## II. TRANSITION RATES

Nuclear  $\beta$ -decay is induced by the weak-interaction Hamiltonian [18]

$$H_W = \frac{G_V}{\sqrt{2}} \int d\mathbf{x} e^{-i(\mathbf{p}_e + \mathbf{p}_\nu) \cdot \mathbf{x}} l_\sigma j^\sigma(\mathbf{x}), \quad (2.1)$$

where  $G_V$  is the Fermi coupling constant ( $G_V = 1.14939 \cdot 10^{-5} \text{ GeV}^{-2}$  [19]),  $l_\sigma$  is the leptonic weak current

$$l_\sigma = \bar{u}_e \gamma_\sigma (1 - \gamma_5) v_\nu, \quad (2.2)$$

and  $j^\sigma(\mathbf{x})$  is the hadronic weak current density. The electron and (electron) anti-neutrino momenta and spinors are denoted, respectively, by  $\mathbf{p}_e$  and  $\mathbf{p}_\nu$ , and  $u_e$  and  $v_\nu$  ( $e^-$  emission is being discussed here). The Bjorken and Drell [20] conventions are used for the metric tensor  $g^{\sigma\tau}$  and  $\gamma$ -matrices. However, the spinors are normalized as  $u_e^\dagger u_e = v_\nu^\dagger v_\nu = 1$ . The amplitude for the process  ${}^A Z \rightarrow {}^A(Z+1)e^- \bar{\nu}_e$  is then given by

$$\langle f | H_W | i \rangle = \frac{G_V}{\sqrt{2}} l^\sigma \langle -\mathbf{q}; {}^A(Z+1), J_f M_f | j_\sigma^\dagger(\mathbf{q}) | {}^A Z, J_i M_i \rangle, \quad (2.3)$$

where  $\mathbf{q} = \mathbf{p}_e + \mathbf{p}_\nu$ ,  $|{}^A Z, J_i M_i\rangle$  and  $|{}^A(Z+1), J_f M_f\rangle$  represent the initial and final nuclear states, the latter recoiling with momentum  $-\mathbf{q}$ , with spins  $J_i$  and  $J_f$  and spin projections  $M_i$  and  $M_f$ , respectively, and

$$j^\sigma(\mathbf{q}) = \int d\mathbf{x} e^{i\mathbf{q} \cdot \mathbf{x}} j^\sigma(\mathbf{x}) \equiv (\rho(\mathbf{q}), \mathbf{j}(\mathbf{q})). \quad (2.4)$$

Standard techniques [13,18] are now used to carry out the multipole expansion of the weak charge ( $\rho(\mathbf{q})$ ) and current ( $\mathbf{j}(\mathbf{q})$ ) operators in a reference frame in which the  $\hat{z}$ -axis defines the spin-quantization axis, and the direction  $\hat{\mathbf{q}}$  is specified by the angles  $\theta$  and  $\phi$ :

$$\langle J_f M_f | \rho^\dagger(\mathbf{q}) | J_i M_i \rangle = \sqrt{4\pi} \sum_{l \geq 0} X_0^l C_l(q), \quad (2.5)$$

$$\langle J_f M_f | \hat{\mathbf{e}}_{q0}^* \cdot \mathbf{j}^\dagger(\mathbf{q}) | J_i M_i \rangle = \sqrt{4\pi} \sum_{l \geq 0} X_0^l L_l(q), \quad (2.6)$$

$$\langle J_f M_f | \hat{\mathbf{e}}_{q\pm 1}^* \cdot \mathbf{j}^\dagger(\mathbf{q}) | J_i M_i \rangle = -\sqrt{2\pi} \sum_{l \geq 1} X_{\mp 1}^l [\pm M_l(q) + E_l(q)], \quad (2.7)$$

where the dependence on the direction  $\hat{\mathbf{q}}$ , and on the initial and final spins and spin projections is contained in the functions  $X_\lambda^l$ , with  $\lambda=0, \pm 1$ , defined as

$$X_\lambda^l(\hat{\mathbf{q}}; J_f M_f, J_i M_i) \equiv (-i)^l \left( \frac{2l+1}{2J_f+1} \right)^{1/2} D_{l_z, \lambda}^l(-\phi, -\theta, 0) \langle J_i M_i, l l_z | J_f M_f \rangle, \quad (2.8)$$

with  $l_z = M_f - M_i$ . In Eqs. (2.5)-(2.7) the  $q$ -dependent coefficients are the reduced matrix elements (RME's) of the Coulomb ( $C$ ), longitudinal ( $L$ ), transverse electric ( $E$ ) and transverse magnetic ( $M$ ) multipole operators, as given in Refs. [13,18], and the vectors  $\hat{\mathbf{e}}_\lambda$  denote the linear combinations

$$\hat{\mathbf{e}}_{q0} \equiv \hat{\mathbf{e}}_{q3} , \quad (2.9)$$

$$\hat{\mathbf{e}}_{q\pm 1} \equiv \mp \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_{q1} \pm i\hat{\mathbf{e}}_{q2}) , \quad (2.10)$$

with  $\hat{\mathbf{e}}_{q3} = \hat{\mathbf{q}}$ ,  $\hat{\mathbf{e}}_{q2} = \hat{\mathbf{z}} \times \mathbf{q}/|\hat{\mathbf{z}} \times \mathbf{q}|$ ,  $\hat{\mathbf{e}}_{q1} = \hat{\mathbf{e}}_{q2} \times \hat{\mathbf{e}}_{q3}$ . Since the weak charge and current operators have scalar/polar-vector ( $V$ ) and pseudo-scalar/axial-vector ( $A$ ) components, each multipole consists of the sum of  $V$  and  $A$  terms:

$$T_{l_z} = T_{l_z}(V) + T_{l_z}(A) , \quad (2.11)$$

where  $T=C, L, E$ , and  $M$ , and the parity of  $l$ th-pole  $V$ -operators is opposite of that of  $l$ th-pole  $A$ -operators. The parity of Coulomb, longitudinal, and electric  $l$ th-pole  $V$ -operators is  $(-)^l$ , while that of magnetic  $l$ th-pole  $V$ -operators is  $(-)^{l+1}$ . Finally, in Eq. (2.8) the  $D_{l_z, l_z'}^l$  are rotation matrices in the standard notation of Ref. [21].

The rate for nuclear beta decay is then obtained from

$$d\Gamma_{fi}^\beta = 2\pi \delta(E_i - E_f) \frac{1}{2J_i + 1} \sum_{M_i M_f} \sum_{s_e s_\nu} |\langle f | H_W | i \rangle|^2 \frac{d\mathbf{p}_e}{(2\pi)^3} \frac{d\mathbf{p}_\nu}{(2\pi)^3} , \quad (2.12)$$

where  $E_i = m_i$  is the rest mass of the initial nucleus  ${}^A Z$ ,  $E_f = p_\nu + \sqrt{p_e^2 + m_e^2} + \sqrt{q^2 + m_f^2}$  is the energy of the final state,  $m_e$  and  $m_f$  being the rest masses of the electron and final nucleus  ${}^A(Z+1)$ . Carrying out the spin sums leads to [13,18]

$$\begin{aligned} \frac{1}{2J_i + 1} \sum_{M_i M_f} \sum_{s_e s_\nu} |\langle f | H_W | i \rangle|^2 &= G_V^2 \frac{4\pi}{2J_i + 1} \left[ (1 + \mathbf{v}_e \cdot \hat{\mathbf{v}}_\nu) \sum_{l \geq 0} |C_l(q)|^2 \right. \\ &\quad + (1 - \mathbf{v}_e \cdot \hat{\mathbf{v}}_\nu + 2\mathbf{v}_e \cdot \hat{\mathbf{q}} \mathbf{v}_\nu \cdot \hat{\mathbf{q}}) \sum_{l \geq 0} |L_l(q)|^2 \\ &\quad - 2\hat{\mathbf{q}} \cdot (\mathbf{v}_e + \hat{\mathbf{v}}_\nu) \sum_{l \geq 0} \text{Re} [C_l(q) L_l^*(q)] \\ &\quad + (1 - \mathbf{v}_e \cdot \hat{\mathbf{q}} \mathbf{v}_\nu \cdot \hat{\mathbf{q}}) \sum_{l \geq 1} [ |M_l(q)|^2 + |E_l(q)|^2 ] \\ &\quad \left. - 2\hat{\mathbf{q}} \cdot (\mathbf{v}_e - \hat{\mathbf{v}}_\nu) \sum_{l \geq 1} \text{Re} [M_l(q) E_l^*(q)] \right] , \quad (2.13) \end{aligned}$$

where  $\mathbf{v}_e = \mathbf{p}_e / \sqrt{p_e^2 + m_e^2}$  and  $\hat{\mathbf{v}}_\nu = \hat{\mathbf{p}}_\nu$  are the velocities of the electron and neutrino, respectively. Note that angular momentum and parity selection rules restrict the number of non-vanishing  $l$ -multipoles contributing to the transition.

A few comments are now in order. Firstly, the expression above is valid for  $e^-$  emission, however, the rate for  $e^+$  emission corresponding to a transition  ${}^A Z \rightarrow {}^A(Z-1)e^+\nu_e$  has precisely the same form, but for the sign in the last term of Eq. (2.13),  $+2\hat{\mathbf{q}} \cdot (\mathbf{v}_e - \hat{\mathbf{v}}_\nu) \dots$ . Secondly, for allowed transitions with  $|J_i - J_f| = \pm 1, 0$  and  $\pi_i \pi_f = 1$  the above rate is easily shown to reduce to the familiar expression in terms of Fermi [ $F \propto C_0(q=0; V)$ ] and Gamow-Teller [ $GT \propto E_1(q=0; A) = \sqrt{2} L_1(q=0; A)$ ] matrix elements, see Sec. V and Ref. [18]. Finally, a more realistic treatment—like that outlined in Ref. [22], for instance—takes into account the distortion of the outgoing  $e^\pm$  wave function in the Coulomb field of

the residual nuclear system. The simple approximation consisting in multiplying the right-hand-side of Eq. (2.13) by the ratio of the charged lepton density at the nuclear radius to the density at infinity has been deemed, however, to suffice for our purposes here (see Sec. V). One of the objectives of the present study is to estimate, in an allowed decay such as the  ${}^6\text{He} \rightarrow {}^6\text{Li} e^- \bar{\nu}_e$  under consideration [with  $(J_i^{\pi_i}, T_i) = (0^+, 1)$  and  $(J_f^{\pi_f}, T_f) = (1^+, 0)$ ], the size of corrections associated with i) retardation effects due to the finite lepton momentum transfer in the decay, and ii) transitions other than those of F [ $C_0(q; V)$ ] and/or GT [ $L_1(q; A)$  and  $E_1(q; A)$ ] type. For example, a naive analysis of the 6-body decay above would ignore the momentum transfer dependence—it is in the range  $0 \leq q \leq 4$  MeV/c—as well as the contributions arising from transitions induced by the axial charge and vector current operators via  $C_1(q; A)$  and  $M_1(q; V)$  RME's, respectively. Of course, available tabulations of F and GT matrix elements extracted from experiment [23] do attempt to estimate these corrections (as well as those due to electron screening, the finite extent of the nuclear charge distribution, etc.). The latter are typically obtained, however, within the context of a shell-model description of the nuclear wave functions. Thus, it is interesting to re-examine the issue above within the present framework using more realistic wave functions.

The  $e^-$  capture (so-called  $\epsilon$ -capture) process is governed by the same weak-interaction Hamiltonian in Eq. (2.1). However, the lepton weak-current density is now given by

$$l_\sigma(\mathbf{x}) = e^{-i\mathbf{p}_\nu \cdot \mathbf{x}} \bar{u}_\nu \gamma_\sigma (1 - \gamma_5) \langle \text{Atom}_f | \psi_e(\mathbf{x}) | \text{Atom}_i \rangle, \quad (2.14)$$

where  $\psi_e(\mathbf{x})$  is the electron field operator, and  $|\text{Atom}_i\rangle$  and  $|\text{Atom}_f\rangle$  are the initial and final atomic states, respectively. A realistic description of nuclear electron capture requires, therefore, a careful treatment of the atomic physics aspects of the process [24], such as those relating to atomic wave-function overlaps, exchange contributions, and electron-correlation effects. We defer to Sec. V for a discussion of some of these issues in the context of the  ${}^7\text{Be}$   $\epsilon$ -capture of interest here. In this section, however, we simply approximate

$$\langle \text{Atom}_f | \psi_e(\mathbf{x}) | \text{Atom}_i \rangle \simeq \frac{R_{1s}(x)}{\sqrt{4\pi}} \chi(s_e) \equiv \frac{R_{1s}(x)}{\sqrt{4\pi}} u(\mathbf{p}_e, s_e) \quad p_e \rightarrow 0, \quad (2.15)$$

ignoring atomic many-body effects and relativistic corrections—this latter approximation is reasonably justified for low  $Z$ , since the  $e^-$  velocity is  $\simeq Z\alpha \ll 1$ . In Eq. (2.15),  $R_{1s}(x)$  is the 1s radial solution of the Schrödinger equation, and the two-component spin state  $\chi(s_e)$  has been conveniently replaced by the four-component spinor  $u(\mathbf{p}_e, s_e)$  in the limit  $p_e \rightarrow 0$ , which allows us to use standard techniques to carry out the spin sum over  $s_e$  at a later stage. The transition amplitude reads

$$\langle f | H_W | i \rangle = \frac{G_V}{\sqrt{8\pi}} R_{1s}(0) \tilde{l}^\sigma \langle -\mathbf{p}_\nu, {}^A(Z-1), J_f M_f | j_\sigma^\dagger(\mathbf{p}_\nu) | {}^A Z, J_i M_i \rangle, \quad (2.16)$$

where  $\tilde{l}_\sigma \equiv \bar{u}_\nu \gamma_\sigma (1 - \gamma_5) u_e$ , and the  $e^-$  radial wave function has been factored out from the matrix element of  $j^\sigma(\mathbf{p}_\nu)$  by approximating it with its value at the origin. The resulting differential rate is then written as [17,18]

$$d\Gamma_{fi}^\epsilon = 2\pi \delta(E_i - E_f) \frac{1}{2J_i + 1} \sum_{M_i M_f} \sum_{s_e s_\nu} |\langle f | H_W | i \rangle|^2 \frac{d\mathbf{p}_\nu}{(2\pi)^3}, \quad (2.17)$$

where the initial and final energies are now given by  $E_i = m_i + m_e$  and  $E_f = p_\nu + \sqrt{p_\nu^2 + m_f^2}$ , respectively. Note that atomic binding energy contributions have been neglected, since they are of the order  $(Z\alpha)^2 m_e/2$ . The square of the amplitude summed over the spins can be obtained, *mutatis mutandis*, from that corresponding to  $e^+$  emission discussed above in the limit  $p_e = 0$  ( $\mathbf{q} = \mathbf{p}_\nu$ ):

$$\frac{1}{(2J_i + 1)} \sum_{M_i M_f} \sum_{s_e s_\nu} |\langle f | H_W | i \rangle|^2 = G_V^2 \frac{|R_{1s}(0)|^2}{4\pi} \frac{4\pi}{(2J_i + 1)} \left[ \sum_{l \geq 0} |C_l(p_\nu) - L_l(p_\nu)|^2 + \sum_{l \geq 1} |M_l(p_\nu) - E_l(p_\nu)|^2 \right]. \quad (2.18)$$

### III. WAVE FUNCTIONS

The VMC wave function,  $\Psi_T(J^\pi; T)$ , for a given nucleus, is constructed from products of two- and three-body correlation operators acting on an antisymmetric single-particle state with the appropriate quantum numbers. The correlation operators are designed to reflect the influence of the interactions at short distances, while appropriate boundary conditions are imposed at long range [4-7]. The  $\Psi_T(J^\pi; T)$  has embedded variational parameters that are adjusted to minimize the expectation value

$$E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq E_0, \quad (3.1)$$

which is evaluated by Metropolis Monte Carlo integration.

A good variational trial function has the form

$$|\Psi_V\rangle = \left[ 1 + \sum_{i < j < k} \tilde{U}_{ijk}^{TNI} \right] \left[ \mathcal{S} \prod_{i < j} (1 + U_{ij}) \right] |\Psi_J\rangle. \quad (3.2)$$

The Jastrow wave function,  $\Psi_J$ , is fully antisymmetric and has the  $(J^\pi; T)$  quantum numbers of the state of interest. For the  $s$ -shell nuclei we use the simple form

$$|\Psi_J\rangle = \left[ \prod_{i < j < k} f_{ijk}^c \right] \left[ \prod_{i < j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle. \quad (3.3)$$

Here  $f_c(r_{ij})$  and  $f_{ijk}^c$  are central two- and three-body correlation functions and  $\Phi_A$  is a Slater determinant in spin-isospin space, e.g.,

$$|\Phi_4(0000)\rangle = \mathcal{A} |p \uparrow p \downarrow n \uparrow n \downarrow\rangle. \quad (3.4)$$

The  $U_{ij}$  and  $\tilde{U}_{ijk}^{TNI}$  are noncommuting two- and three-nucleon correlation operators, and  $\mathcal{S}$  indicates a symmetric sum over all possible orders. The  $U_{ij}$  includes spin, isospin, and tensor terms:

$$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O_{ij}^p, \quad (3.5)$$

where the  $O_{ij}^p$  are the same operators that appear in the AV18  $NN$  potential. The  $f_c(r)$  and  $u_p(r)$  functions are generated by the solution of a set of coupled differential equations which contain a number of variational parameters [4]. The  $\tilde{U}_{ijk}^{TNI}$  has the spin-isospin structure of the dominant parts of the  $NNN$  interaction as suggested by perturbation theory.

The optimal  $U_{ij}$  and  $\tilde{U}_{ij;k}^{TNI}$  do not change significantly from nucleus to nucleus, but  $\Psi_J$  does. For the  $p$ -shell nuclei,  $\Psi_J$  includes a one-body part that consists of 4 nucleons in an  $\alpha$ -like core and  $(A - 4)$  nucleons in  $p$ -shell orbitals. We use  $LS$  coupling to obtain the desired  $JM$  value, as suggested by standard shell-model studies [25]. We also need to sum over different spatial symmetries  $[n]$  of the angular momentum coupling of the  $p$ -shell nucleons [26]. The one-body parts are multiplied by products of central pair and triplet correlation functions, which depend upon the shells ( $s$  or  $p$ ) occupied by the particles and on the  $LS[n]$  coupling:

$$|\Psi_J\rangle = \mathcal{A} \left\{ \prod_{i < j < k} f_{ijk}^c \prod_{i < j \leq 4} f_{ss}(r_{ij}) \prod_{k \leq 4 < l \leq A} f_{sp}(r_{kl}) \sum_{LS[n]} \left( \beta_{LS[n]} \prod_{4 < l < m \leq A} f_{pp}^{LS[n]}(r_{lm}) \right) |\Phi_A(LS[n]JMTT_3)_{1234:5\dots A}\rangle \right\}. \quad (3.6)$$

The operator  $\mathcal{A}$  indicates an antisymmetric sum over all possible partitions into 4  $s$ -shell and  $(A - 4)$   $p$ -shell particles. The pair correlation for both particles within the  $s$ -shell,  $f_{ss}$ , is set to the  $f_c$  of the  $\alpha$ -particle. The  $f_{sp}$  is similar to  $f_{ss}$  at short range, but with a long-range tail going to a constant  $\approx 1$ , while the  $f_{pp}^{LS[n]}$  is allowed to depend on the particular single-particle channel.

The  $LS[n]$  components of the single-particle wave function are given by:

$$|\Phi_A(LS[n]JMTT_3)_{1234:5\dots A}\rangle = |\Phi_4(0000)_{1234}\rangle \prod_{4 < l \leq A} \phi_p^{LS[n]}(R_{\alpha l}) \quad (3.7)$$

$$\left\{ \left[ \prod_{4 < l \leq A} Y_{1m_l}(\Omega_{\alpha l}) \right]_{LM_L[n]} \times \left[ \prod_{4 < l \leq A} \chi_l(\frac{1}{2}m_s) \right]_{SM_S} \right\}_{JM} \times \left[ \prod_{4 < l \leq A} \nu_l(\frac{1}{2}t_3) \right]_{TT_3}.$$

The  $\phi_p^{LS}(R_{\alpha l})$  are  $p$ -wave solutions of a particle in an effective  $\alpha$ - $N$  potential that has Woods-Saxon and Coulomb parts. They are functions of the distance between the center of mass of the  $\alpha$  core and nucleon  $l$ , and may vary with  $LS[n]$ . The wave function is translationally invariant, so there is no spurious center of mass motion.

Two different types of  $\Psi_J$  have been constructed in recent VMC calculations of light  $p$ -shell nuclei: an original shell-model kind of trial function [5] which we will call Type I, and a cluster-cluster kind of trial function [6,7] which we will call Type II. In Type I trial functions, the  $\phi_p^{LS}(r)$  has an exponential decay at long range, with the depth, range, and surface thickness of the Woods-Saxon potential serving as variational parameters. The  $f_{sp}$  goes to a constant somewhat less than unity, while  $f_{pp}^{LS[n]}$  is similar to  $f_{ss}$  at short-range with an added long-range tail that is larger for states of lesser spatial symmetry  $[n]$ . Details for these  $A = 6, 7$  trial functions are given in Ref. [5].

In Type II trial functions,  $\phi_p^{LS}(r)$  is again the solution of a  $p$ -wave differential equation with a potential containing Woods-Saxon and Coulomb terms, but with an added Lagrange multiplier that turns on at long range. This Lagrange multiplier imposes the boundary condition

$$[\phi_p^{LS[n]}(r \rightarrow \infty)]^n \propto W_{km}(2\gamma r)/r, \quad (3.8)$$

where  $W_{km}(2\gamma r)$  is the Whittaker function for bound-state wave functions in a Coulomb potential and  $n$  is the number of  $p$ -shell nucleons. The  $\gamma$  is related to the cluster separation energy which is taken from experiment. The accompanying  $f_{sp}$  goes exactly to unity (more rapidly than in the Type I trial function) and the  $f_{pp}^{LS[n]}$  are taken from the exact deuteron solution in the case of  ${}^6\text{Li}$ , or the VMC triton ( ${}^3\text{He}$ ) solution in the case of  ${}^7\text{Li}$  ( ${}^7\text{Be}$ ). Consequently, the Type II trial function factorizes at large cluster separations as

$$\Psi_T \rightarrow \psi_\alpha \psi_\tau W_{km}(2\gamma r_{\alpha\tau})/r_{\alpha\tau}. \quad (3.9)$$

where  $\psi_\alpha$  and  $\psi_\tau$  are the wave functions of the clusters and  $r_{\alpha\tau}$  is the separation between them. More details on these wave functions are given in Refs. [6,7]. In the case of  ${}^6\text{He}$ , which does not have an asymptotic two-cluster threshold, we generate a  $f_{pp}^{LS[n]}$  correlation assuming a weakly bound  ${}^1S_0$   $nn$  pair.

For either type of trial function, a diagonalization is carried out in the one-body basis to find the optimal values of the  $\beta_{LS[n]}$  for a given  $(J^\pi; T)$  state. The trial function, Eq.(3.2), has the advantage of being efficient to evaluate while including the bulk of the correlation effects. A more sophisticated variational function can be constructed by including two-body spin-orbit correlations and additional three-body correlations [4,27], but the time to compute these extra terms is significant, while the gain in the variational energy is relatively small.

The wave function at a given spatial configuration  $\mathbf{R} = \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A$  can be represented by a vector of  $2^A \times I(A, T)$  complex coefficients in spin and isospin space [5]. For the nuclei considered here, this gives vectors of 320 in  ${}^6\text{Li}$ , 576 in  ${}^6\text{He}$ , and 1,792 in  ${}^7\text{Li}$  and  ${}^7\text{Be}$ . The spin, isospin, and tensor operators  $O_{ij}^{p=2,6}$  contained in the Hamiltonian and other operators of interest are sparse matrices in this basis.

#### IV. NUCLEAR WEAK CURRENT

The model for the nuclear weak current has been most recently and exhaustively described in Ref. [13]. Here we summarize only its main features.

The nuclear weak current consists of vector and axial-vector parts,

$$\rho_\pm(\mathbf{q}) = \rho_\pm(\mathbf{q}; V) + \rho_\pm(\mathbf{q}; A), \quad (4.1)$$

$$\mathbf{j}_\pm(\mathbf{q}) = \mathbf{j}_\pm(\mathbf{q}; V) + \mathbf{j}_\pm(\mathbf{q}; A), \quad (4.2)$$

with corresponding one- and two-body components. The weak vector current is constructed from the isovector part of the electromagnetic current, in accordance with the conserved-vector-current (CVC) hypothesis

$$\mathbf{j}_\pm(\mathbf{q}; V) = [T_\pm, \mathbf{j}_z(\mathbf{q}; \gamma)], \quad (4.3)$$

where  $\mathbf{j}_\pm(\mathbf{q}; V)$  is the charge-lowering (-) or charge-raising (+) weak vector current,  $\mathbf{j}_z(\mathbf{q}; \gamma)$  is the isovector part of the electromagnetic current, and  $T_\pm$  is the (total) isospin-lowering or isospin-raising operator. A similar relation holds between the electromagnetic charge operator and its weak vector counterpart. For reference, we list only the expressions for the one-body terms in  $\rho(\mathbf{q}; V)$  and  $\mathbf{j}(\mathbf{q}; V)$ , in the notation of Ref. [13]:



$$\rho_i^{(1)}(\mathbf{q}; V) = \rho_{i,\text{NR}}^{(1)}(\mathbf{q}; V) + \rho_{i,\text{RC}}^{(1)}(\mathbf{q}; V) , \quad (4.4)$$

with

$$\rho_{i,\text{NR}}^{(1)}(\mathbf{q}; V) = \tau_{i,\pm} e^{i\mathbf{q}\cdot\mathbf{r}_i} , \quad (4.5)$$

$$\rho_{i,\text{RC}}^{(1)}(\mathbf{q}; V) = -i \frac{(2\mu^v - 1)}{4m^2} \tau_{i,\pm} \mathbf{q} \cdot (\boldsymbol{\sigma}_i \times \mathbf{p}_i) e^{i\mathbf{q}\cdot\mathbf{r}_i} , \quad (4.6)$$

and

$$\mathbf{j}_i^{(1)}(\mathbf{q}; V) = \frac{1}{2m} \tau_{i,\pm} \left[ \mathbf{p}_i , e^{i\mathbf{q}\cdot\mathbf{r}_i} \right]_+ - i \frac{\mu^v}{2m} \tau_{i,\pm} \mathbf{q} \times \boldsymbol{\sigma}_i e^{i\mathbf{q}\cdot\mathbf{r}_i} , \quad (4.7)$$

where  $[\dots, \dots]_+$  denotes the anticommutator,  $\mathbf{p}$ ,  $\boldsymbol{\sigma}$ , and  $\boldsymbol{\tau}$  are the nucleon's momentum, Pauli spin and isospin operators, respectively, and  $\mu^v$  is the isovector nucleon magnetic moment ( $\mu^v = 4.709$  n.m.). Finally, the isospin raising and lowering operators are defined as

$$\tau_{i,\pm} \equiv (\tau_{i,x} \pm i\tau_{i,y})/2 . \quad (4.8)$$

The one-body terms in the axial charge and current operators have the standard expressions [13] obtained from the non-relativistic reduction of the covariant single-nucleon current, and retain terms proportional to  $1/m^2$ ,  $m$  being the nucleon mass:

$$\rho_i^{(1)}(\mathbf{q}; A) = -\frac{g_A}{2m} \tau_{i,\pm} \boldsymbol{\sigma}_i \cdot \left[ \mathbf{p}_i , e^{i\mathbf{q}\cdot\mathbf{r}_i} \right]_+ , \quad (4.9)$$

and

$$\mathbf{j}_i^{(1)}(\mathbf{q}; A) = \mathbf{j}_{i,\text{NR}}^{(1)}(\mathbf{q}; A) + \mathbf{j}_{i,\text{RC}}^{(1)}(\mathbf{q}; A) , \quad (4.10)$$

with

$$\mathbf{j}_{i,\text{NR}}^{(1)}(\mathbf{q}; A) = -g_A \tau_{i,\pm} \boldsymbol{\sigma}_i e^{i\mathbf{q}\cdot\mathbf{r}_i} , \quad (4.11)$$

$$\begin{aligned} \mathbf{j}_{i,\text{RC}}^{(1)}(\mathbf{q}; A) = & \frac{g_A}{4m^2} \tau_{i,\pm} \left( \boldsymbol{\sigma}_i \left[ \mathbf{p}_i^2 , e^{i\mathbf{q}\cdot\mathbf{r}_i} \right]_+ - \left[ \boldsymbol{\sigma}_i \cdot \mathbf{p}_i \mathbf{p}_i , e^{i\mathbf{q}\cdot\mathbf{r}_i} \right]_+ - \frac{1}{2} \boldsymbol{\sigma}_i \cdot \mathbf{q} \left[ \mathbf{p}_i , e^{i\mathbf{q}\cdot\mathbf{r}_i} \right]_+ \right. \\ & \left. - \frac{1}{2} \mathbf{q} \left[ \boldsymbol{\sigma}_i \cdot \mathbf{p}_i , e^{i\mathbf{q}\cdot\mathbf{r}_i} \right]_+ + i \mathbf{q} \times \mathbf{p}_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \right) - \frac{g_P}{2m m_\mu} \tau_{i,\pm} \mathbf{q} \boldsymbol{\sigma}_i \cdot \mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}_i} . \end{aligned} \quad (4.12)$$

The axial-vector coupling constant  $g_A$  is taken to be [28]  $1.2654 \pm 0.0042$ , by averaging values obtained from the beta asymmetry in the decay of polarized neutrons and the half-lives of the neutron and super-allowed  $0^+ \rightarrow 0^+$  transitions. The last term in Eq. (4.12) is the induced pseudo-scalar contribution ( $m_\mu$  is the muon mass), for which the coupling constant  $g_P$  is taken as [29]  $g_P = -6.78 g_A$ .

Some of the two-body axial-current operators are derived from  $\pi$ - and  $\rho$ -meson exchanges and the  $\rho\pi$ -transition mechanism. These mesonic operators, first obtained in a systematic way in Ref. [30], have been found to give rather small contributions in weak transitions involving few-nucleon systems [12,13,17]. The two-body weak axial-charge operator includes a pion-range term, which follows from soft-pion theorem and current algebra arguments [16,31], and short-range terms, associated with scalar- and vector-meson exchanges. The latter are obtained consistently with the two-nucleon interaction model, following a procedure [32] similar to that used to derive the corresponding weak vector-current operators [13].

The dominant two-body axial current operator, however, is that due to  $\Delta$ -isobar excitation [12,13]. Since the  $N\Delta$  transition axial-vector coupling constant  $g_A^*$  is not known experimentally, the associated contribution suffers from a large model dependence. To reduce it [12,13], the coupling constant  $g_A^*$  has been adjusted to reproduce the experimental value of the Gamow-Teller (GT) matrix element in tritium  $\beta$  decay,  $0.957 \pm 0.003$  [12]. The value used for  $g_A^*$  in the present work is  $g_A^*=1.17 g_A$  [17].

The  $\Delta$ -excitation operator used here is that derived, in the static  $\Delta$  approximation, using first-order perturbation theory. This approach is considerably simpler than that adopted in Ref. [13], where the  $\Delta$  degrees of freedom were treated non-perturbatively, by retaining them explicitly in the nuclear wave functions [33]. However, it is important to emphasize [13] that results obtained within the perturbative and non-perturbative schemes are within a % of each other typically, once  $g_A^*$  is fixed, independently in the perturbative and non-perturbative calculations, to reproduce the experimentally known GT matrix element, see Table XV in Ref. [13].

## V. CALCULATION

The calculation of the  $\beta$ -decay and  $\epsilon$ -capture rates proceeds in two steps: firstly, the Monte Carlo evaluation of the weak charge and current operator matrix elements, and the subsequent decomposition of these in terms of reduced matrix elements (RME's); secondly, the evaluation of the rate by carrying out the integrations in Eqs. (2.12) and (2.17).

The RME's are obtained from Eqs. (2.5)–(2.7) by choosing appropriately the  $\hat{q}$ -direction. For example, in the  $\beta$ -decay of  ${}^6\text{He}$  the  $C_1(q; A)$  and  $L_1(q; A)$  RME's are determined from

$$C_1(q; A) = \frac{i}{\sqrt{4\pi}} \langle {}^6\text{Li}, 10 | \rho_+^\dagger(q\hat{z}; A) | {}^6\text{He}, 00 \rangle, \quad (5.1)$$

$$L_1(q; A) = \frac{i}{\sqrt{4\pi}} \langle {}^6\text{Li}, 10 | \hat{z} \cdot \mathbf{j}_+^\dagger(q\hat{z}; A) | {}^6\text{He}, 00 \rangle, \quad (5.2)$$

while the  $E_1(q; A)$  and  $M_1(q; V)$  RME's from

$$E_1(q; A) = -\frac{i}{\sqrt{2\pi}} \langle {}^6\text{Li}, 10 | \hat{z} \cdot \mathbf{j}_+^\dagger(q\hat{x}; A) | {}^6\text{He}, 00 \rangle, \quad (5.3)$$

$$M_1(q; V) = -\frac{1}{\sqrt{2\pi}} \langle {}^6\text{Li}, 10 | \hat{y} \cdot \mathbf{j}_+^\dagger(q\hat{x}; V) | {}^6\text{He}, 00 \rangle, \quad (5.4)$$

where  $J_i, M_i=0,0$  and  $J_f, M_f=1,0$  and the spin-quantization axis is along  $\hat{z}$ . The matrix elements above are computed, without any approximation, by Monte Carlo integration.

The wave functions are written as vectors in the spin-isospin space of the  $A$  nucleons ( $A=6$  or  $7$  here) for any given spatial configuration  $\mathbf{R} = (\mathbf{r}_1, \dots, \mathbf{r}_A)$ . For the given  $\mathbf{R}$ , the state vector  $O^\dagger(\mathbf{R})\Psi_i(\mathbf{R})$  is calculated with techniques similar to those developed in Ref. [14]— $O(\mathbf{R})$  is any of the operators  $\rho(\mathbf{q}; V)$ ,  $\mathbf{j}(\mathbf{q}; V)$ , etc., and  $\Psi_i(\mathbf{R})$  is the wave function of the initial nucleus. The spatial integrations are carried out with the Monte Carlo method by sampling  $\mathbf{R}$  configurations according to the Metropolis algorithm, using a probability density proportional to  $\langle \Psi_f^\dagger(\mathbf{R})\Psi_f(\mathbf{R}) \rangle$ , where  $\Psi_f(\mathbf{R})$  is the wave function of the final nucleus and the notation  $\langle \dots \rangle$  implies sums over the spin-isospin states. Typically 20,000 configurations are enough to achieve a relative error  $\leq 1\%$  on the matrix elements.

Once the RME's have been obtained, the calculation of the total transition rate is reduced to performing the integrations over the outgoing momenta in Eqs. (2.12) and (2.17). A glance at Eqs. (2.12) and (2.13) shows that the differential rate depends on the magnitude of the electron momentum  $p_e$  and the variable  $x_{e\nu} = \hat{\mathbf{p}}_e \cdot \hat{\mathbf{p}}_\nu$ , since the magnitude of the neutrino momentum is fixed by the energy-conserving  $\delta$ -function. The total rate can therefore be written as

$$\Gamma_{fi}^\beta = \frac{G_V^2 m_e^5}{2\pi^3} \gamma_{fi}^\beta, \quad (5.5)$$

where the constant  $\gamma_{fi}^\beta$  is simply given by

$$\gamma_{fi}^\beta \equiv \frac{2\pi}{2J_i + 1} \int_0^{\bar{p}_e^*} d\bar{p}_e \bar{p}_e^2 F(Z_f, \bar{p}_e) \int_{-1}^1 dx_{e\nu} \bar{p}_\nu^2 f_{\text{rec}}^{-1} [\dots]. \quad (5.6)$$

Here all momenta and rest masses have been expressed in units of the electron mass ( $\bar{p}_e = p_e/m_e$ ,  $\bar{m}_i = m_i/m_e$ ,  $\bar{m}_e = 1$ , etc.),  $[\dots]$  denotes the content of the large square brackets on the right-hand-side of Eq. (2.13),  $f_{\text{rec}}^{-1}$  is the recoil factor resulting from integrating out the  $\delta$ -function,

$$f_{\text{rec}} = \left| 1 + \frac{\bar{p}_e x_{e\nu}}{\bar{m}_f} + \frac{\bar{p}_\nu}{\bar{m}_f} \right|, \quad (5.7)$$

the neutrino momentum  $\bar{p}_\nu$  is

$$\bar{p}_\nu = \frac{2\bar{\Delta}}{1 + \bar{p}_e x_{e\nu}/\bar{m}_f + \sqrt{(1 + \bar{p}_e x_{e\nu}/\bar{m}_f)^2 + 2\bar{\Delta}/\bar{m}_f}}, \quad (5.8)$$

where  $\bar{\Delta} = \bar{m}_i - \bar{m}_f - \sqrt{\bar{p}_e^2 + 1 - \bar{p}_e^2/(2\bar{m}_f)}$ , and lastly the function  $F(Z_f, \bar{p}_e)$  accounts approximately for wave-function distortion effects of the outgoing electron in the Coulomb field of the final nucleus with atomic number  $Z_f$  and radius  $\bar{R}_f$  (expressed in units of the Compton wavelength of the electron) [22]

$$F(Z, \bar{p}_e) = 2(1 + \gamma_0) (2\bar{p}_e \bar{R})^{2(\gamma_0 - 1)} \frac{|\Gamma(\gamma_0 + i\nu)|^2}{|\Gamma(2\gamma_0 + 1)|^2} e^{\pi\nu}, \quad (5.9)$$

with  $\gamma_0 \equiv \sqrt{1 - (\alpha Z)^2}$  and  $\nu \equiv \alpha Z/v_e$ . The maximum allowed electron momentum is denoted with  $\bar{p}_e^*$ , the upper integration limit in Eq. (5.6), and is given by

$$\bar{p}_e^* = \sqrt{\left[ \sqrt{\bar{m}_f^2 + 2\bar{m}_f(\bar{m}_i - \bar{m}_f) + 1} - \bar{m}_f \right]^2 - 1} . \quad (5.10)$$

Neglecting the recoil of the final nucleus leads to:  $f_{\text{rec}}=1$ ,  $\bar{p}_\nu = \bar{m}_i - \bar{m}_f - \sqrt{\bar{p}_e^2 + 1}$ , and  $\bar{p}_e^* = \sqrt{(\bar{m}_i - \bar{m}_f)^2 - 1}$ .

Note that there is an implicit dependence on  $p_e$  and  $x_{e\nu}$  in the RME's via the momentum transfer  $q$ . It is convenient to make this dependence explicit by expanding the RME's as

$$T_l(q) = q^m \sum_{n \geq 0} t_{l,2n} q^{2n} , \quad (5.11)$$

consistently with the known expansions of the multipole operators in powers of  $q$  [13]. Here  $m = l, l \pm 1$ , depending on the RME considered. For example, in the  ${}^6\text{He}$   $\beta$ -decay one has  $C_1(q; A) = q(c_{1,0} + c_{1,2}q^2 + \dots)$ ,  $L_1(q; A) = l_{1,0} + l_{1,2}q^2 + \dots$ , etc. . Given the low momentum transfers involved in all transitions under consideration,  $q \leq 4$  MeV/c, the leading and next-to-leading order terms  $t_{l,0}$  and  $t_{l,2}$  are sufficient to reproduce accurately  $T_l(q)$ . Incidentally, the long-wavelength-approximation corresponds, typically, to retaining only the  $t_{l,0}$  term. Finally, standard numerical techniques—gaussian quadratures—are used to carry out the integrations in Eq. (5.6).

It is useful to consider the case of allowed transitions for which  $|J_i - J_f| = \pm 1, 0$  and  $\pi_i \pi_f = 1$ , such as the  ${}^6\text{He}$   $\beta$ -decay of interest in the present study. Ignoring retardation corrections due to the finite momentum transfer involved in the decay, one finds that the only surviving RME's in the limit  $q \rightarrow 0$  are  $C_0(V)$ ,  $L_1(A)$ , and  $E_1(A)$ —of course,  $C_0(V)$  vanishes unless  $J_i = J_f$ . The one-body terms in the associated multipole operators read in this limit [13,18]:

$$C_{00}(q=0; V) = \frac{1}{\sqrt{4\pi}} \sum_i \tau_{i,\pm} , \quad (5.12)$$

$$E_{11z}(q=0; A) = \sqrt{2} L_{11z}(q=0; A) = -\frac{i}{\sqrt{6\pi}} g_A \sum_i \tau_{i,\pm} \sigma_{i,11z} , \quad (5.13)$$

where  $\sigma_{i,11z}$  denote the spherical components of  $\sigma_i$ . The constant  $\gamma_{fi}^\beta$  is then obtained as, neglecting nuclear recoil corrections,

$$\gamma_{fi}^\beta = \frac{|F|^2 + g_A^2 |GT|^2}{2J_i + 1} f \quad (5.14)$$

$$f \equiv \int_1^{\bar{m}_i - \bar{m}_f} d\bar{E}_e \bar{E}_e (\bar{E}_e^2 - 1)^{1/2} (\bar{m}_i - \bar{m}_f - \bar{E}_e)^2 F(Z_f, \bar{E}_e) , \quad (5.15)$$

since the integration over  $x_{e\nu}$  can now be performed trivially. The familiar definitions of the Fermi and Gamow-Teller (reduced) matrix elements,

$$F \equiv \langle J_f || \sum_i \tau_{i,\pm} || J_i \rangle , \quad (5.16)$$

$$GT \equiv \langle J_f || \sum_i \tau_{i,\pm} \sigma_i || J_i \rangle , \quad (5.17)$$

have been introduced, as well as  $\bar{E}_e = \sqrt{\bar{p}_e^2 + 1}$ . Combining Eqs. (5.5) and (5.14) leads to the standard expression of the decay rate for allowed transitions as, for example, in Ref. [23].

Finally, the total rate for  $\epsilon$ -capture easily follows from Eqs. (2.17) and (2.18):

$$\Gamma_{fi}^\epsilon = \frac{G_V^2 m_e^5}{4\pi^2} \bar{E}_\nu |\bar{R}_{1s}(0)|^2 f_{\text{rec}}^{-1} \frac{4\pi}{2J_i + 1} [\dots], \quad (5.18)$$

where now  $[\dots]$  denotes the content of the large square brackets on the right-hand-side of Eq. (2.18), and the recoil factor is given by  $f_{\text{rec}}^{-1} = |1 - \bar{E}_\nu / (\bar{m}_i + 1)| \simeq 1$  with  $\bar{E}_\nu = [(\bar{m}_i + 1)^2 - \bar{m}_f^2] / [2(\bar{m}_i + 1)] \simeq \bar{m}_i + 1 - \bar{m}_f$ . Again, we have expressed masses and energies in units of  $m_e$ , and  $R_{1s}(0) = m_e^{3/2} \bar{R}_{1s}(0)$ . In particular, for allowed transitions and ignoring recoil effects leads to the familiar result [24]

$$\Gamma_{fi}^\epsilon = \frac{G_V^2 m_e^5}{4\pi^2} \bar{E}_\nu |\bar{R}_{1s}(0)|^2 \frac{|F|^2 + g_A^2 |GT|^2}{2J_i + 1}. \quad (5.19)$$

As already pointed out in Sec. II, however, a more accurate treatment of the atomic physics aspects is warranted for a meaningful comparison with experiment. In the case of the  ${}^7\text{Be}$   $\epsilon$ -capture, of interest here, such a program has indeed been carried out by Chen and Crasemann [34]. They use multi-configurational Hartree-Fock (MCHF) wave functions to represent the initial ground state of the  ${}^4\text{Be}$  atom as

$$\Psi_i = C_1 \Phi(1s^2 2s^2) + C_2 \Phi(1s^2 2p^2), \quad (5.20)$$

and the final  $1s$ - or  $2s$ -hole states, after  $K$  or  $L_I$  captures in standard nomenclature [24], as respectively

$$\Psi_K = C'_1 \Phi'(1s 2s^2) + C'_2 \Phi'(1s 2p^2), \quad (5.21)$$

$$\Psi_L = \Phi'(1s^2 2s), \quad (5.22)$$

and then proceed to evaluate the matrix elements  $\langle \Psi_K | \psi_e(\mathbf{x}=0) | \Psi_i \rangle$  and similarly for  $\Psi_L$ . The MCHF approach goes beyond the independent-particle approximation, commonly adopted in the analysis of electron capture, by retaining the effects of Coulomb correlations among the electrons. The end-result is that the rate, including both  $K$  and  $L_I$  captures, can be conveniently written as in Eq. (5.18), but for the replacement  $|\bar{R}_{1s}(0)|^2 \rightarrow B \times |\bar{R}_{1s}(0)|^2$ , where

$$B = B_K + \left| \frac{\bar{R}_{2s}(0)}{\bar{R}_{1s}(0)} \right|^2 B_L \quad (5.23)$$

in the notation of Ref. [34], with  $B_\alpha = |\langle \Psi_\alpha | \psi_e(\mathbf{x}=0) | \Psi_i \rangle|^2$  and  $\alpha = K$  or  $L$ . Using the values listed in Table III of Ref. [34] for  $B_K$ ,  $B_L$ , and the ratio of (radial) wave functions at the origin, as well as the value for  $\bar{R}_{1s}(0)$  from Table IX of Ref. [24], we find the relevant combination  $B \times |\bar{R}_{1s}(0)|^2$  to be equal to  $7.2403 \times 10^{-5}$  for the case of  ${}^7\text{Be}$ .

## VI. RESULTS

In this section we report results for the  $\beta$ -decay of  ${}^6\text{He}$  and the  $\epsilon$ -capture in  ${}^7\text{Be}$ . The variational Monte Carlo (VMC) wave functions of the  $A=6$  and  $7$  nuclei have been obtained from a realistic Hamiltonian consisting of the Argonne  $v_{18}$  two-nucleon [8] and Urbana-IX three-nucleon [9] interactions, the AV18/UIX model. The binding energies and radii predicted by the VMC wave functions of type I and II discussed in Sec. III are listed for reference in Table I.

The contributions of the different components of weak vector and axial-vector charge and current operators to the reduced matrix elements (RME's) contributing to the  $\beta$ -decay of  ${}^6\text{He}$  and the  $\epsilon$ -capture in  ${}^7\text{Be}$  are reported in Tables II and VII. In these tables the column labeled "1-body" lists the contributions associated with the one-body terms of the charge and current operators, including relativistic corrections proportional to  $1/m^2$ . These are the operators given in Eqs. (4.4), (4.7), (4.9), and (4.10) of Sec. IV. The column labeled "Mesonic" lists the contributions from two-body vector and axial-vector charge and current operators, associated with pion and vector-meson exchanges, namely those of Eqs. (4.16)–(4.17), (4.30)–(4.31), (4.32)–(4.34), and (4.35)–(4.37) of Ref. [13]. Lastly, the column labeled " $\Delta$ " lists the contributions arising from  $\Delta$  excitation, obtained in perturbation theory and in the static  $\Delta$  approximation, as in Eqs. (4.44), (4.48), (4.50) and (4.52) of Ref. [13]. We reiterate that the coupling constant  $g_A^*$  in the  $N\Delta$  axial two-body current has been set equal to the value  $1.17 g_A$ , required to reproduce the tritium Gamow-Teller (GT) matrix element in a calculation based on the same treatment of  $\Delta$  degrees of freedom, and using correlated-hyperspherical-harmonics trinucleon wave functions corresponding to the AV18/UIX Hamiltonian model [17] (see discussion in Sec. IV).

The contributions of the different components of the weak axial current to the GT matrix elements occurring in the  ${}^6\text{He}$   $\beta$ -decay and  ${}^7\text{Be}$   $\epsilon$ -capture are reported in Tables III, V, and VI. The notation in these tables is similar to that just discussed above, with the only difference that it is now in reference to the axial current only. Furthermore, the one-body contributions are separated into the contributions associated with the leading and next-to-leading terms in the non-relativistic expansion of the covariant single-nucleon axial current, Eqs. (4.11) and (4.12).

Having clarified the notation in (most of) the tables, we now proceed to discuss the results for the  ${}^6\text{He}$   $\beta$ -decay and  ${}^7\text{Be}$   $\epsilon$ -capture separately in the next two subsections.

### A. The ${}^6\text{He}$ $\beta$ -decay

The  ${}^6\text{He}$   $\beta$ -decay is induced by the weak axial-vector charge and current, and weak vector current operators via the multipoles  $C_1(q; A)$ ,  $L_1(q; A)$ ,  $E_1(q; A)$ , and  $M_1(q; V)$ . The values for the associated RME's, obtained with type I VMC wave functions, are reported in Table II at a value  $0.015 \text{ fm}^{-1}$  of the lepton momentum transfer  $q$ . Note that in the decay  $q$  varies, ignoring tiny recoil corrections, between 0 and  $\simeq 0.020 \text{ fm}^{-1}$ , corresponding to an end-point energy  $m({}^6\text{He}) - m({}^6\text{Li}) = 4.013 \text{ MeV}/c^2$ .

The largest (in magnitude) RME's are the  $L_1(A)$  and  $E_1(A)$  due to transitions induced by the weak axial current  $\mathbf{j}(\mathbf{q}; A)$ . This is to be expected since the decay  ${}^6\text{He} \rightarrow {}^6\text{Li} e^- \bar{\nu}_e$  is a superallowed one, having  $(J_i^{\pi_i}, T_i) = (0^+, 1)$  and  $(J_f^{\pi_f}, T_f) = (1^+, 0)$ . The two-body "Mesonic" con-

tributions to  $\mathbf{j}(q; A)$  are individually very small, moreover they interfere destructively and nearly cancel out. The two-body axial current contributions due to  $\Delta$  excitation are at the level of  $\simeq 1.5\%$  of the leading one-body contributions.

The transitions induced by the axial charge and vector current are inhibited, since the first non-vanishing terms in the long-wavelength expansions of the associated multipole operators are linear in  $q$ , and hence their contributions  $C_1(A)$  and  $M_1(V)$  suppressed by one power of  $qR$  ( $R$  is the nuclear radius) with respect to  $L_1(A)$  and  $E_1(A)$ . The two-body contributions to  $M_1(V)$  and  $C_1(A)$  are relatively large, and increase (in magnitude) the one-body results by 9% and 25%, respectively. Among the ‘‘Mesonic’’ terms, the  $\pi$  axial charge and vector current operators are dominant, while the vector-meson exchange as well as  $\Delta$  operators play a minor role. It is important to stress the model-independent character of the  $\pi$ -exchange two-body operators, whose vector and axial-vector structures are dictated, respectively, by gauge invariance [13] and current algebra [16] arguments.

The  $M_1(V)$  RME is approximately related by CVC to the RME of the electromagnetic multipole operator  $M_1(\gamma)$  connecting the  ${}^6\text{Li}^*(3.56 \text{ MeV})$  excited state with  $(J^\pi, T)$  assignments  $(0^+, 1)$  to the  ${}^6\text{Li}(\text{g.s.})$  ground state  $(J^\pi, T)=(1^+, 0)$  via

$$\begin{aligned} M_1(q; V) &= -\frac{1}{\sqrt{2\pi}} \langle {}^6\text{Li}, 10 | [T_\pm, \hat{\mathbf{y}} \cdot \mathbf{j}_z^\dagger(q\hat{\mathbf{x}}; \gamma)] | {}^6\text{He}, 00 \rangle \\ &\simeq -\sqrt{2} M_1(q; \gamma), \end{aligned} \quad (6.1)$$

where we have made use of Eqs. (4.3) and (5.4), and have assumed that the  ${}^6\text{Li}(\text{g.s.})$  as well as the  ${}^6\text{He}(\text{g.s.})$  and  ${}^6\text{Li}^*(3.56 \text{ MeV})$  states are members, respectively, of an iso-singlet and an iso-triplet. Of course, electromagnetic terms and isospin-symmetry-breaking components in the strong-interaction sector, both of which are present in the AV18/UIX Hamiltonian model, will spoil the relation above to some extent. However, the present VMC wave functions of type I for  ${}^6\text{Li}(\text{g.s.})$ ,  ${}^6\text{Li}^*(3.56 \text{ MeV})$ , and  ${}^6\text{He}(\text{g.s.})$  are pure  $T = 0$  and  $TM_T=10$  and 1–1, respectively. Similar (type I) wave functions have been recently employed in Ref. [11] to carry out a calculation of the elastic and transition form factors of the  $A=6$  systems and, in particular, of the radiative width of the  ${}^6\text{Li}^*(3.56 \text{ MeV})$  state. The values for the  $M_1(\gamma)$  RME were found to be  $-i 2.81 \times 10^{-3}$  and  $-i 3.09 \times 10^{-3}$  including one-body only and both one- and two-body terms in the electromagnetic current operator [11] in agreement, on the basis of Eq. (6.1), with those reported in Table II. Incidentally, the radiative width of the  ${}^6\text{Li}^*(3.56 \text{ MeV})$  was predicted to be [11] 7.49 eV and 9.06 eV with one- and (one+two)-body currents. The experimental value is  $(8.19 \pm 0.17) \text{ eV}$ .

In Table III we list results for the GT RME, related to the  $L_1(q=0; A)$  and  $E_1(q=0; A)$  RME’s via

$$E_1(q=0; A) = \sqrt{2} L_1(q=0; A) = -\frac{i}{\sqrt{6\pi}} g_A GT, \quad (6.2)$$

with  $g_A=1.2654$ . A few comments are in order. Firstly, the predicted  ${}^6\text{He}$  GT RME is about 5% larger than that derived from Eqs. (5.5) and (5.14), 2.173, using the most recent tabulation of the  $\log(ft)$  value for the  ${}^6\text{He}$  decay,  $2.910 \pm 0.002$ , reported in Ref. [35]. This over-prediction is already present at the level of the one-body contributions, those associated with two-body operators further increase the discrepancy from about 3% to 5%. Secondly, the difference between the results obtained with type I and II wave functions are very small.

The type II wave functions, in contrast to those of type I, incorporate long-range Coulomb correlation effects and the correct two-body clustering behavior in the asymptotic region, as discussed in Sec. III. However, these asymptotically improved wave functions have only a marginal impact on the value of the GT RME, by reducing it by only  $\simeq 0.2\%$ . Thirdly, the relativistic corrections to  $\mathbf{j}(\mathbf{q}; A)$  (proportional to  $1/m^2$ ) are relatively large, comparable to the leading two-body contributions associated with  $\Delta$  excitation, and have been neglected in all previous studies we are aware of. Lastly, the  $E_1(q=0; A)$  RME, derived from Eq. (6.2), is  $-i0.6657$  (total), which should be compared to the value  $-i0.6654$  (total, type I) obtained at  $q=0.015 \text{ fm}^{-1}$ . Thus retardation corrections are tiny, as expected.

Finally, in Table IV we list the values for the half-life of  ${}^6\text{He}$  derived from Eqs. (5.5) and (5.6) under different approximation schemes for purpose of illustration (note that  $\tau_{1/2}=\ln 2/\Gamma$ ). The first row in Table IV is obtained by retaining only the  $L_1(A)$  and  $E_1(A)$  RME's evaluated at  $q=0$ , namely neglecting retardation corrections as well as the contributions from transitions induced by the vector current and axial charge. It is equivalent to using Eqs. (5.14) and (5.15), apart from negligible recoiling corrections. The second row again includes only the  $L_1(A)$  and  $E_1(A)$  RME's, but now keeps their full momentum transfer dependence. The third row includes all contributing RME's with their intrinsic  $q$ -dependence. The last row reports the measured half-life from Ref. [35].

It is important to stress that in the present calculation the effects of Coulomb distortion of the outgoing electron wave function are considered within the approximate scheme discussed in Sec. V. In existing tabulations, such as those in Ref. [23], these effects are treated more realistically, by solving the Dirac equation for the electron in the field generated by the (extended) charge distribution of the daughter nucleus. Obviously, this approach complicates considerably the formulae derived in Sec. II using plane waves. However, the "error" made in our present treatment should not be large, as can be inferred from the following argument. If one takes the experimental value for the GT matrix element (2.173) to compute back the experimental half-life, but using Eqs. (5.5) and (5.6), one obtains a value of 820.4 ms, which should be compared to the "true" experimental half-life of 806.7 ms. This 1.6% difference is presumably due to our present approximate scheme for dealing with Coulomb distortions of the electron waves. Lastly, if these were to be altogether ignored [by setting  $F(Z, \bar{p}_e)=1$  in Eq. (5.6)], the resulting calculated values for  $\tau_{1/2}$ , in the same approximation as in the third row of Table IV, would be 823.9 ms and 803.1 ms with 1- and (1+2)-body operators, respectively.

## B. The ${}^7\text{Be}$ $\epsilon$ -capture

The  ${}^7\text{Be}$  nucleus decays by electron capture to the ground state of  ${}^7\text{Li}$  and to its first-excited state at 0.478 MeV. The  $(J^\pi, T)$  assignments of  ${}^7\text{Be}(\text{g.s.})$ ,  ${}^7\text{Li}(\text{g.s.})$ , and  ${}^7\text{Li}^*(0.48 \text{ MeV})$  are  $(3^-/2, 1/2)$ ,  $(3^-/2, 1/2)$ , and  $(1^-/2, 1/2)$ , respectively.

The calculated GT matrix elements for the transitions to the ground and first excited states of  ${}^7\text{Li}$  are given in Tables V and VI. The first and second rows in Table V (VI) list the results obtained with VMC wave functions of type I, and based on random walks consisting of 20,000 (15,500) and 10,000 (10,000) configurations, respectively. Note that the statistical Monte Carlo errors are at the 1% level for the decay to  ${}^7\text{Li}$ , and almost an order of magnitude smaller for the decay to  ${}^7\text{Li}^*$ . Indeed, the central values computed in the longer and shorter



random walks are fully consistent (within errors) in the ground- to excited-state transition, but barely so in the ground- to ground-state transition. The third row in Tables V and VI lists the results obtained with VMC wave functions of type II and based on 10,000 point random walks. As for the  $A=6$  case, it appears that the asymptotically improved wave functions of type II lead to values for the GT matrix elements not statistically different from those of type I. The two-body contributions increase by roughly 3% the one-body matrix elements, and as discussed below, bring theory into better agreement with experiment.

The  $\epsilon$ -capture to  ${}^7\text{Li}(\text{g.s.})$  also proceeds through a Fermi-type transition. The Fermi matrix element, defined as in Eq. (5.16), is  $F = -\sqrt{2} J_f + 1 = -2$  with wave functions of type I, in which isospin-symmetry-breaking components are ignored. However, the type II wave functions do include long-range Coulomb correlations, and therefore break the isospin symmetry of the iso-doublet  ${}^7\text{Be}(\text{g.s.})$ - ${}^7\text{Li}(\text{g.s.})$ . The Fermi matrix element is calculated to be, in this case,  $-1.999$ .

In Table VII we report the contributions from the individual components of the weak vector and axial charge and current operators to the dominant RME's occurring in the transition to the first-excited state of  ${}^7\text{Li}$  (the corresponding neutrino energy is  $E_\nu=0.384$  MeV or  $\bar{E}_\nu=0.752$  in units of the electron mass). The  $M_1(V)$  and  $C_1(A)$  transition strengths are down by 3 and 4 order of magnitudes with respect to the leading  $E_1(A)$  and  $L_1(A)$ . There are in principle additional contributions from order 2 multipoles, such as  $C_2(V)$  and  $M_2(A)$ , for example, however, these are expected to be even more suppressed than those due to  $M_1(V)$  and  $C_1(A)$ . No attempt has been made to calculate them. One should note that the retardation corrections in the  $E_1(q; A)$  and  $L_1(q; A)$  RME's are negligible. Indeed,  $E_1(q=0; A) = \sqrt{2} L_1(q=0; A) = 6.456 \times 10^{-1}$  in the (1+2)-body calculation, which should be compared with  $6.427 \times 10^{-1}$  from Table VII. Lastly, the  $M_1(V)$  RME can be expressed via CVC, again ignoring isospin-symmetry-breaking effects, as

$$M_1(q; V) \simeq -i \frac{\sqrt{2}}{3} \frac{q}{2m} \left[ \langle {}^7\text{Be}^* \parallel \mu_1(\gamma) \parallel {}^7\text{Be} \rangle - \langle {}^7\text{Li}^* \parallel \mu_1(\gamma) \parallel {}^7\text{Li} \rangle \right], \quad (6.3)$$

where  ${}^7\text{Be}^*$  is the first excited state of  ${}^7\text{Be}$  at 0.429 MeV with  $(J^\pi, T) = (1^-, 1/2)$ , and  $\mu_1(\gamma)$  is the magnetic moment operator. From the experimentally known radiative widths of the  ${}^7\text{Be}^*$  and  ${}^7\text{Li}^*$  states [35], the isovector combination above of transition magnetic moments is found to be  $(5.87 \pm 0.14)$  n.m., and the resulting  $M_1(q; V)$  is  $-i 5.66 \times 10^{-4}$ , which should be compared to the predicted values of  $-i 5.010 \times 10^{-4}$  and  $-i 5.904 \times 10^{-4}$  with one- and (one+two)-body currents from Table VII, respectively. Incidentally, in the case of the transition to the ground state of  ${}^7\text{Li}$ , one finds, using the experimental values for the  ${}^7\text{Be}$  and  ${}^7\text{Li}$  magnetic moments (respectively,  $-1.398(15)$  n.m. and  $3.256424(2)$  n.m. from Ref. [35]), that the isovector combination similar to that in Eq. (6.3) (with  ${}^7\text{Be}^* \rightarrow {}^7\text{Be}$  and  ${}^7\text{Li}^* \rightarrow {}^7\text{Li}$ ) is  $i(0.1007 \pm 0.0003) \times 10^{-2}$ , while the calculated values are  $i 0.111 \times 10^{-2}$  and  $i 0.132 \times 10^{-2}$  with one- and (one+two)-body currents.

Finally, in Table VIII we report the predicted half-life and branching ratio of  ${}^7\text{Be}$  to the ground and first-excited states of  ${}^7\text{Li}$ . We have ignored the tiny corrections due to retardation effects and transitions induced by the vector current and axial charge operators, i.e. Eq. (5.19) has been used for  $\Gamma^\epsilon$ , but for the replacement  $|\bar{R}_{1s}(0)|^2 \rightarrow B \times |\bar{R}_{1s}(0)|^2 = 7.2403 \times 10^{-5}$ . The half-life is over-predicted by about 9%, while the branching ratio is under-predicted by 1%. Two-body contributions reduce significantly the discrepancy between the calculated and measured quantities.

## VII. CONCLUSIONS

In the present study we have reported on calculations of the  ${}^6\text{He}$   $\beta$ -decay and  ${}^7\text{Be}$   $\epsilon$ -capture rates, using variational Monte Carlo (VMC) wave functions derived from a realistic Hamiltonian, and a realistic model for the nuclear weak current and charge operators, consisting of one- and two-body terms. Both processes are super-allowed, and are therefore driven almost entirely by the axial current (and, additionally, by the vector charge in the case of the  ${}^7\text{Be}$  decay to the ground state of  ${}^7\text{Li}$ ). The two-body part in the axial current operator has been adjusted to reproduce the experimentally known Gamow-Teller (GT) matrix element in  ${}^3\text{H}$   $\beta$ -decay.

The GT matrix element in  ${}^6\text{He}$  is over-predicted by about 5%, while those in  ${}^7\text{Be}$  connecting to the ground- and first-excited states of  ${}^7\text{Li}$  are under-predicted by 5%, when compared to the experimental values. However, the observed branching ratio in the  ${}^7\text{Be}$   $\epsilon$ -capture is reasonably well reproduced by theory. We have verified explicitly that the (relatively small) discrepancies between the measured and calculated GT matrix elements are not explained by the inclusion of retardation effects or by shift of strength to suppressed transitions induced by the weak vector current and axial charge operators.

One- and two-body axial current contributions interfere constructively, leading to a 1.7% (4.4% on average) increase in the one-body prediction for the GT matrix element in  ${}^6\text{He}$  ( ${}^7\text{Be}$ ). As a result, the inclusion of two-body operators has the effect of slightly worsening (significantly improving) the agreement between theory and experiment in  ${}^6\text{He}$  ( ${}^7\text{Be}$ ) systems. It is important to stress that the same model for the nuclear weak current adopted here, has been recently shown to provide an excellent description of the process  ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$  [17], in which two-body (vector and axial-vector) operators contribute 12% of the total rate.

The origin of the current unsatisfactory situation between theory and experiment is more likely to be in the approximate character of the VMC wave functions used here. We have explored the sensitivity of the results to alternative VMC wave functions, denoted as type II in Sec. III, which incorporate a better treatment of the asymptotic behavior, in particular the clustering properties into 2+4 or 3+4 sub-systems. No significant changes in the calculated values have been found. Thus, the next logical step in our quest for a quantitative understanding of weak transitions in the  $A=6$  and 7 (as well as  $A=8$ ) systems, is to repeat the present calculations with the more accurate Green's function Monte Carlo wave functions [9,10,36], and also to investigate the numerical implications of models for the nuclear weak current derived from effective field theory approaches, such as those in Refs. [37,38]. Most of the computational techniques and computer codes developed here can be carried over to this planned next stage.

Finally, it is important to establish whether the present approach, based on realistic interactions and currents, leads to a consistent description of the available experimental data on weak transitions in light nuclei, *beyond* the  $A=3$  systems for which its validity has already been ascertained. One important implication of such a program should be made clear: it would serve to corroborate the robustness of our recent predictions for the cross sections of the proton weak captures on  ${}^1\text{H}$  [12,37] and  ${}^3\text{He}$  [13,38].

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TABLES

TABLE I. Binding energies (MeV) and point radii (fm) of  $A=6$  and  $7$  nuclei obtained with VMC wave functions of type I and II for the AV18/UIX Hamiltonian model. Also listed are the corresponding GFMC and experimental values.

Nucleus	Wave function	$B$	$\langle r_p^2 \rangle^{1/2}$	$\langle r_n^2 \rangle^{1/2}$
${}^6\text{He}(\text{g.s.})$	Type I	23.99(7)	1.97(1)	2.88(2)
	Type II	23.78(7)	2.28(3)	3.23(5)
	GFMC	28.1(1)	1.97(1)	2.94(1)
	Expt	29.27		
${}^6\text{Li}(\text{g.s.})$	Type I	27.09(7)	2.49(1)	2.49(1)
	Type II	27.34(7)	2.50(1)	2.50(1)
	GFMC	31.1(1)	2.57(1)	2.57(1)
	Expt	31.99	2.43	
${}^7\text{Be}(\text{g.s.})$	Type I	30.49(9)	2.44(1)	2.30(1)
	Type II	30.00(10)	2.44(1)	2.33(1)
	GFMC	36.2(1)	2.52(1)	2.33(1)
	Expt	37.60		
${}^7\text{Li}(\text{g.s.})$	Type I	32.09(9)	2.30(1)	2.44(1)
	Type II	31.87(9)	2.32(1)	2.43(1)
	GFMC	37.8(1)	2.33(1)	2.52(1)
	Expt	39.24	2.27	
${}^7\text{Li}^*(0.48 \text{ MeV})$	Type I	31.97(9)	2.29(1)	2.44(1)
	Type II	31.43(10)	2.31(1)	2.42(1)
	GFMC	37.5(2)		
	Expt	38.76		

TABLE II. Contributions to the (purely imaginary) reduced matrix elements  $E_1(A)$ ,  $L_1(A)$ ,  $M_1(V)$ , and  $C_1(A)$  at  $q=0.015 \text{ fm}^{-1}$  in  ${}^6\text{He}$   $\beta$ -decay. See text for notation.

RME	1-body	Mesonic	$\Delta$	(1+2)-body
$10^1 \times E_1(A)$	-6.540(15)	-0.004(3)	-0.110(3)	-6.654(16)
$10^1 \times L_1(A)$	-4.623(11)	-0.003(2)	-0.078(2)	-4.704(11)
$10^3 \times M_1(V)$	3.922(9)	0.300(2)	0.057(1)	4.279(10)
$10^4 \times C_1(A)$	-4.584(32)	-0.977(10)	-0.171(5)	-5.733(34)

TABLE III. Contributions to the Gamow-Teller matrix element in  ${}^6\text{He}$   $\beta$ -decay. See text for notation.

Wave function	1-body NR	1-body RC	Mesonic	$\Delta$	(1+2)-body
Type I	2.254(5)	-0.0094(3)	0.0014(9)	0.0376(10)	2.284(5)
Type II	2.246(10)	-0.0100(3)	0.0011(10)	0.0418(11)	2.278(10)

TABLE IV. Values in ms for the  ${}^6\text{He}$  half-life obtained in a number of approximation schemes. See text for an explanation. The measured half-life is also listed.

	1-body	(1+2)-body
$E_1(0; A)$ and $L_1(0; A)$	762.9	743.2
$E_1(q; A)$ and $L_1(q; A)$	763.5	744.0
All	764.4	745.1
Expt		$806.7 \pm 1.5$

TABLE V. Contributions to the Gamow-Teller matrix element in the  ${}^7\text{Be}$   $\epsilon$ -capture to the  ${}^7\text{Li}$  ground state. See text for notation.

Wave function	1-body NR	1-body RC	Mesonic	$\Delta$	(1+2)-body
Type I(20k)	2.366(29)	-0.038(2)	0.0039(18)	0.110(3)	2.441(29)
Type I(10k)	2.288(42)	-0.034(3)	-0.0029(26)	0.110(5)	2.361(42)
Type II(10k)	2.321(41)	-0.041(2)	-0.0008(9)	0.108(5)	2.387(41)

TABLE VI. Contributions to the Gamow-Teller matrix element in the  ${}^7\text{Be}$   $\epsilon$ -capture to the  ${}^7\text{Li}$  first excited state. See text for notation.

Wave function	1-body NR	1-body RC	Mesonic	$\Delta$	(1+2)-body
Type I(15.5k)	2.157(6)	-0.028(1)	0.0064(8)	0.080(1)	2.215(6)
Type I(10k)	2.156(7)	-0.028(1)	0.0063(10)	0.080(2)	2.215(7)
Type II(10k)	2.154(7)	-0.034(1)	0.0049(11)	0.093(2)	2.218(8)

TABLE VII. Contributions to the (purely imaginary) reduced matrix elements  $E_1(A)$ ,  $L_1(A)$ ,  $M_1(V)$ , and  $C_1(A)$  in the  ${}^7\text{Be}$   $\epsilon$ -capture to the  ${}^7\text{Li}$  first excited state at  $\bar{E}_\nu=0.7515$  (in units of the electron mass). See text for notation.

RME	1-body	Mesonic	$\Delta$	(1+2)-body
$10^1 \times E_1(A)$	6.192(18)	0.016(3)	0.220(4)	6.427(19)
$10^1 \times L_1(A)$	4.405(13)	0.017(2)	0.163(3)	4.584(13)
$10^4 \times M_1(V)$	-5.010(13)	-0.746(6)	-0.148(2)	-5.904(14)
$10^5 \times C_1(A)$	5.328(90)	0.456(20)	0.163(17)	5.948(93)

TABLE VIII. The half-life and branching ratio of  ${}^7\text{Be}$  to the ground and first excited states of  ${}^7\text{Li}$ , predicted with one- and (one+two)-body currents, are compared with the experimental values. The VMC wave functions of type I are used.

	1-body	(1+2)-body	Expt
$\tau_{1/2}(\text{days})$	62.29	58.24	$53.22 \pm 0.06$
$\xi$ (%)	10.20	10.33	$10.44 \pm 0.04$