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Vector Meson Dominance Model for Radiative Decays Involving Light Scalar Mesons

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We study a vector dominance model which predicts a fairly large number of currently interesting decay amplitudes of the types $S \to \gamma\gamma$, $V \to S\gamma$ and $S \to V\gamma$, where S and V denote scalar and vector mesons, in terms of three parameters. As an application we first use our approach to obtain the usual puzzling result $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0^0\gamma) \lesssim 1$ rather than the observed value of about 3. The model makes it easy to test in detail a recent proposal to solve this puzzle by including the isospin violating a_0^0-f_0 mixing and we find that it seems successful.

There is increasing interest in a possible nonet of light scalar mesons (all of mass $< 1\,\text{GeV}$). In addition to the well established $f_0(980)$ and $a_0(980)$ evidence of both experimental and theoretical nature for a very broad σ ($\simeq 560$) and a very broad κ ($\simeq 900$) has been presented [1]. The latter two resonances are difficult to identify cleanly because they appear to be of non Breit-Wigner type, signaling strong interference with the non-resonant background.

Such a nonet would most likely represent meson states more complicated than quark-anti quark type and hence would be of great importance for a full understanding of QCD in its non-perturbative low energy regime.

Clearly it is important to study the properties of the $f_0(980)$ and $a_0(980)$ from the point of view of how they fit into a putative nonet family. In particular, the reactions $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ have recently been observed [2] with good accuracy and are considered as useful probes of scalar properties. The theoretical analysis was initiated by Achasov and Ivanchenko [3] and followed up by many others [4]. The models employed are essentially variants of the single K meson loop diagram to which a ϕ-type vector meson, a photon and two pseudoscalars or a scalar are attached.

In the present note we introduce a complementary approach which emphasizes the "family" or symmetry aspects of the analysis. This enables us to study the correlations among a fairly large number of related radiative amplitudes in terms of a few parameters, without making a commitment to a particular quark structure for the scalars.

Our framework is that of a standard non-linear chiral Lagrangian containing, in addition to the pseudoscalar nonet matrix field ϕ, the vector meson nonet matrix ρ_μ and a scalar nonet matrix field denoted N. Under chiral unitary transformations of the three light quarks; $q_{\rm L,R} \to U_{\rm L,R} \cdot q_{\rm L,R}$, the chiral matrix $U = \exp(2i\phi/F_\pi)$, where $F_\pi \simeq 0.131\,\text{GeV}$, transforms as $U \to U_{\rm L} \cdot U \cdot U_{\rm R}^\dagger$. The convenient matrix $K(U_{\rm L}, U_{\rm R}, \phi)$ [5] is defined by the following transformation property of ξ ($U = \xi^2$): $\xi \to U_{\rm L} \cdot \xi \cdot K^\dagger = K \cdot \xi \cdot U_{\rm R}^\dagger$, and specifies the transformations of "constituent-type" objects. The fields we need transform as

$$
\begin{aligned}
& N \to K \cdot N \cdot K^\dagger \,, \\
& \rho_\mu \to K \cdot \rho_\mu \cdot K^\dagger + \frac{i}{\tilde{g}} K \cdot \partial_\mu K^\dagger \,, \\
& F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}\,[\rho_\mu\,,\,\rho_\nu] \to K \cdot F_{\mu\nu} \cdot K^\dagger
\end{aligned}
\tag{1}
$$

where the coupling constant $\tilde{g}$ is about 4.04. One may refer to Ref. [6] for our treatment of the pseudoscalar-vector Lagrangian and to Ref. [7] for the scalar addition. The entire Lagrangian is chiral invariant (modulo the quark mass term induced symmetry breaking pieces) and, when electromagnetism is added, gauge invariant.

It should be remarked that the effect of adding vectors to the chiral Lagrangian of pseudoscalars only is to replace the photon coupling to the charged pseudoscalars as,

$$
\begin{aligned}
& ie\mathcal{A}_\mu \,{\rm Tr}\left(Q\phi \overleftrightarrow{\partial_\mu} \phi\right) \to \\
& \quad e\mathcal{A}_\mu \Big[k\tilde{g}F_\pi^2 {\rm Tr}\,(Q\rho_\mu) \\
& \qquad + i\left(1 - \frac{k}{2}\right) {\rm Tr}\left(Q\phi \overleftrightarrow{\partial_\mu} \phi\right)\Big] + \cdots \,,
\end{aligned}
\tag{2}
$$

where $\mathcal{A}_\mu$ is the photon field, $Q = {\rm diag}(2/3, -1/3, -1/3)$ and $k = \left(\frac{m_v}{\tilde{g}F_\pi}\right)^2$ with $m_v \simeq 0.76\,\text{GeV}$. The ellipses stand for symmetry breaking corrections. We see that in this model, Sakurai's vector meson dominance [8] simply amounts to the statement that $k = 2$ (the KSRF relation [9]). This is a reasonable numerical approximation which is essentially stable to the addition of symmetry breakers [6, 10] and we employ it here by neglecting the last term in Eq. (2). Although vector meson dominance must be somewhat modified in cases where the axial anomaly plays a role [11], it generally works quite well for processes such as those we study here.

The new feature of the present work is the inclusion of strong trilinear scalar-vector-vector terms in the effective Lagrangian:

$$\begin{aligned}\mathcal{L}_{SVV} = {} & \beta_A\, \epsilon_{abc}\epsilon^{a'b'c'}\, [F_{\mu\nu}(\rho)]^a_{a'}\, [F_{\mu\nu}(\rho)]^b_{b'}\, N^c_{c'} \\ & + \beta_B\, \mathrm{Tr}\,[N]\, \mathrm{Tr}\,[F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)] \\ & + \beta_C\, \mathrm{Tr}\,[N F_{\mu\nu}(\rho)]\, \mathrm{Tr}\,[F_{\mu\nu}(\rho)] \\ & + \beta_D\, \mathrm{Tr}\,[N]\, \mathrm{Tr}\,[F_{\mu\nu}(\rho)]\, \mathrm{Tr}\,[F_{\mu\nu}(\rho)]\ . \qquad (3)\end{aligned}$$

Chiral invariance is evident from (1) and the four flavor-invariants are needed for generality. (A term $\sim \mathrm{Tr}(FFN)$ is linearly dependent on the four shown). Actually the β_D term will not contribute in our model so there are only three relevant parameters β_A, β_B and β_C. Equation (3) is analogous to the PVV interaction which was originally introduced as a $\pi\rho\omega$ coupling a long time ago [12]. With (2) one can now compute the amplitudes for $S \to \gamma\gamma$ and $V \to S\gamma$ according to the diagrams of Fig. 1.

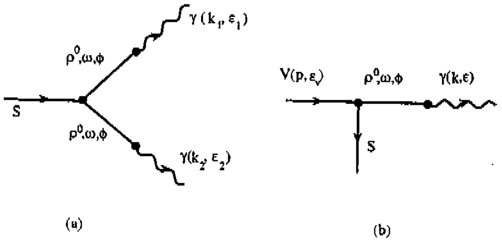

FIG. 1: Feynman diagrams for (a) $S \to \gamma\gamma$ and (b) $V \to S\gamma$.

The decay matrix element for $S \to \gamma\gamma$ is written as $(e^2/\tilde{g}^2) X_S \times (k_1 \cdot k_2\, \epsilon_1 \cdot \epsilon_2 - k_1 \cdot \epsilon_2\, k_2 \cdot \epsilon_1)$ where ϵ_μ stands for the photon polarization vector. It is related to the width by

$$\Gamma\,(S \to \gamma\gamma) = \alpha^2 \frac{\pi}{4} m_S^3 \left| \frac{X_S}{\tilde{g}^2} \right|^2 , \qquad (4)$$

and X_S takes on the specific forms:

$$\begin{aligned} X_\sigma &= \frac{4}{9}\beta_A \left(\sqrt{2}s - 4c\right) + \frac{8}{3}\beta_B \left(c - \sqrt{2}s\right) , \\ X_{f_0} &= -\frac{4}{9}\beta_A \left(\sqrt{2}c + 4s\right) + \frac{8}{3}\beta_B \left(\sqrt{2}c + s\right) , \\ X_{a_0} &= \frac{4\sqrt{2}}{3}\beta_A\ . \qquad (5)\end{aligned}$$

Here $\alpha = e^2/(4\pi)$, $s = \sin\theta_S$ and $c = \cos\theta_S$ where the scalar mixing angle, θ_S is defined from

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} N_3^3 \\ (N_1^1 + N_2^2)/\sqrt{2} \end{pmatrix} . \qquad (6)$$

Furthermore ideal mixing for the vectors, with $\rho^0 = (\rho_1^1 - \rho_2^2)/\sqrt{2}$, $\omega = (\rho_1^1 + \rho_2^2)/\sqrt{2}$, $\phi = \rho_3^3$, was assumed for simplicity.

Similarly, the decay matrix element for $V \to S\gamma$ is written as $(e/\tilde{g}) C_V^S \times [p \cdot k \epsilon_V \cdot \epsilon - p \cdot \epsilon k \cdot \epsilon_V]$. It is related to the width by

$$\Gamma(V \to S\gamma) = \frac{\alpha}{3} \left|k_V^S\right|^3 \left| \frac{C_V^S}{\tilde{g}} \right|^2 , \qquad (7)$$

where $k_V^S = (m_V^2 - m_S^2)/(2m_V)$ is the photon momentum in the V rest frame. For the energetically allowed $V \to S\gamma$ processes we have

$$\begin{aligned} C_\phi^{f_0} &= \frac{2\sqrt{2}}{3}\beta_A c - \frac{4}{3}\beta_B \left(\sqrt{2}c + s\right) \\ &\quad + \frac{\sqrt{2}}{3}\beta_C \left(c - \sqrt{2}s\right) , \\ C_\phi^\sigma &= -\frac{2\sqrt{2}}{3}\beta_A s - \frac{4}{3}\beta_B \left(c - \sqrt{2}s\right) \\ &\quad - \frac{2}{3}\beta_C \left(c + \frac{1}{\sqrt{2}}s\right) , \\ C_\phi^{a_0} &= \sqrt{2}\,(\beta_C - 2\beta_A) , \\ C_\omega^\sigma &= \frac{2\sqrt{2}}{3}\beta_A \left(c + \sqrt{2}s\right) + \frac{2\sqrt{2}}{3}\beta_B \left(c - \sqrt{2}s\right) \\ &\quad - \frac{2}{3}\beta_C \left(\sqrt{2}c + s\right) , \\ C_{\rho^0}^\sigma &= -2\sqrt{2}\beta_A c + 2\sqrt{2}\beta_B \left(c - \sqrt{2}s\right) . \qquad (8)\end{aligned}$$

In addition, the same model predicts amplitudes for the energetically allowed $S \to V\gamma$ processes: $f_0 \to \omega\gamma$, $f_0 \to \rho^0\gamma$, $a_0^0 \to \omega\gamma$, $a_0^0 \to \rho^0\gamma$ and, if κ^0 is sufficiently heavy $\kappa^0 \to K^{*0}\gamma$. The corresponding width is

$$\Gamma(S \to V\gamma) = \alpha \left|k_S^V\right|^3 \left| \frac{D_S^V}{\tilde{g}} \right|^2 , \qquad (9)$$

where $k_S^V = (m_S^2 - m_V^2)/(2m_S)$ and

$$\begin{aligned} D_{f_0}^\omega &= \frac{2}{3}\beta_A \left(-2c + \sqrt{2}s\right) + \frac{2}{3}\beta_B \left(2c + \sqrt{2}s\right) \\ &\quad + \frac{2}{3}\beta_C \left(c - \sqrt{2}s\right) , \\ D_{f_0}^{\rho^0} &= -2\sqrt{2}\beta_A s + 2\beta_B \left(2c + \sqrt{2}s\right) , \\ D_{a_0}^\omega &= 2\beta_C , \\ D_{a_0}^{\rho^0} &= \frac{4}{3}\beta_A , \\ D_{\kappa^0}^{K^{*0}} &= -\frac{8}{3}\beta_A\ . \qquad (10)\end{aligned}$$

All the different decay amplitudes are described by the parameters β_A, β_B and β_C. The reason β_D does not appear at all and β_C does not appear for $S \to \gamma\gamma$ is that, noting Eq. (2), the $\mathrm{Tr}(F_{\mu\nu})$ factor is seen to give zero when coupled to an external photon line. Because the σ and κ are so broad, the simple two body final state approximation in decays like ω, $\phi \to \sigma\gamma \to \pi^0\pi^0\gamma$ is not accurate. It is better to consider these decays as having three body final states with the terms in Eq. (3) giving the vertices and to take into account large width corrections in the scalar propagators as well as non resonant background.

These formulas can be used for different choices of the quark structure of the scalar nonet N_a^b (e.g. the usual $q_a\bar{q}^b$ scenario or the "dual" scenario $Q_a\bar{Q}^b$ where $Q_a \sim \epsilon_{abc}\bar{q}^b\bar{q}^c$). The characteristic mixing angle θ_S is expected to differ, depending on the scheme. In the literature, besides conventional $q\bar{q}$ models, $qq\bar{q}\bar{q}$ models [13], meson-meson "molecule" models [14] and unitarized meson-meson [15] models have been investigated. Recently models featuring mixing between a $qq\bar{q}\bar{q}$ nonet and a heavier $q\bar{q}$ nonet have been proposed [16]; in this case two sets of interactions like Eq. (3) should be included.

Now we shall illustrate the procedure for the model of a single putative scalar nonet [7] with a mixing angle, $\theta_S \simeq -20°$ (characteristic of $qq\bar{q}\bar{q}$ type scalars). At first we shall neglect the iso-spin violating a_0-f_0 mixing, which will play an important role later.

The parameters β_A and β_B may be estimated from the $S \to \gamma\gamma$ processes. Substituting $\Gamma_{\rm exp}(a_0 \to \gamma\gamma) = (0.28 \pm 0.09)\,{\rm keV}$ (obtained using [17] $B(a_0 \to K\bar{K})/B(a_0 \to \eta\pi) = 0.177 \pm 0.024$) into Eqs. (4) and (5) yields $\beta_A = (0.72 \pm 0.12)\,{\rm GeV}$ (assumed positive in sign). Of course, this value is independent of the value of θ_S. Then, $\Gamma_{\rm exp}(f_0 \to \gamma\gamma) = 0.39 \pm 0.13\,{\rm keV}$ yields either $\beta_B = (0.61 \pm 0.10)\,{\rm GeV}^{-1}$ or $\beta_B = (-0.62 \pm 0.10)\,{\rm GeV}^{-1}$. In turn we formally predict $\Gamma(\sigma \to \gamma\gamma)$ to be either $(0.024 \pm 0.023)\,{\rm keV}$ or $(0.38 \pm 0.09)\,{\rm keV}$ respectively.

Next consider the ϕ radiative decays. Assuming $\phi \to \eta\pi^0\gamma$ is dominated by $\phi \to a_0\gamma$, $\Gamma_{\rm exp}(\phi \to a_0\gamma) = (0.47 \pm 0.07)\,{\rm keV}$ and Eq. (8) determines β_C as either $(7.7 \pm 0.5)\,{\rm GeV}^{-1}$ or $(-4.8 \pm 0.5)\,{\rm GeV}^{-1}$. Note that $|\beta_A|$ and $|\beta_B|$ are almost an order of magnitude smaller than $|\beta_C|$. Thus, the ϕ radiative decay rates are mainly determined by $|\beta_C|$. Knowing β_A, β_B and β_C we can predict $\Gamma(\phi \to f_0\gamma)$ using Eq. (8). There are four possibilities due to the two possibilities each for β_B and β_C. The largest number, $\Gamma(\phi \to f_0\gamma) = (0.21 \pm 0.03)\,{\rm keV}$ corresponds to the choice $\beta_B = (-0.62 \pm 0.10)\,{\rm GeV}^{-1}$ and $\beta_C = (7.7 \pm 0.5)\,{\rm GeV}^{-1}$.

Unfortunately this is still considerably smaller than the listed value [17]: $\Gamma_{\rm exp}(\phi \to f_0\gamma) = (1.51 \pm 0.41)\,{\rm keV}$. [18] Actually, we have just reproduced in our framework an emerging puzzle [2] which may be stated as: Why does experiment yield $B(\phi \to f_0\gamma)/B(\phi \to a_0\gamma) = 3.2^{+1.0}_{-0.6}$ rather than about unity (0.46 ± 0.09 in our case [20]) as expected from theoretical estimates ?

Recently Close and Kirk [19] proposed that this puzzle may be solved by considering the effects of the isospin violating $a_0^0(980)$-$f_0(980)$ mixing. We will now, using a very different method of calculation, provide supporting evidence for the correctness of this proposal. Analogous mixings in the pseudoscalar and vector sectors are very well known – the $\eta\pi^0$ mixing provides the dominant interaction for the $\eta \to 3\pi$ decay and the $\omega\rho^0$ mixing noticeably modifies the shape of $e^+e^- \to 2\pi$ cross section near the ρ, ω resonance region. The a_0^0-f_0 mixing might be expected to be even more significant since not only are the two masses nearly equal but their widths are also likely to be nearly equal. This important feature was not considered in [19]. One may simply introduce the mixing by a term in the effective Lagrangian: $\mathcal{L}_{af} = A_{af}a_0^0 f_0$. A recent calculation [21] for the purpose of finding the effect of the scalar mesons in the $\eta \to 3\pi$ process obtained the value $A_{af} = -4.66 \times 10^{-3}\,{\rm GeV}^2$. It is convenient to treat this term as a perturbation. Then the amplitude for $\phi \to f_0\gamma$ includes a correction term consisting of the $\phi \to a_0^0\gamma$ amplitude given in Eq. (8) multiplied by A_{af} and by the a_0 propagator (in which the usual prescription for each scalar meson mass, $m_S \to m_S - i\Gamma_S/2$ [22] is made). The $\phi \to a_0^0\gamma$ amplitude has a similar correction. Then we may simply illustrate the improvement in the $\phi \to f_0\gamma/\phi \to a_0^0\gamma$ ratio by assuming the reasonable β_C dominance noted above; this yields

$$\frac{{\rm amp}(\phi \to f_0\gamma)}{{\rm amp}(\phi \to a_0^0\gamma)} = \frac{\frac{1}{3}\left(c - \sqrt{2}s\right) - z}{1 + \frac{z}{3}\left(c - \sqrt{2}s\right)} \,,$$

$$z = \frac{A_{af}}{2m_{f_0}} \frac{1}{m_{f_0} - m_{a_0} - \frac{i}{2}\left(\Gamma_{f_0} - \Gamma_{a_0}\right)} \,. \qquad (11)$$

This ratio is 0.47 when the correction z is neglected. Finding the exact value of z is difficult; according to the Review of Particle Physics [17] $m_{a_0} = (984.7 \pm 1.3)\,{\rm MeV}$, $\Gamma_{a_0} = 50$–$100\,{\rm MeV}$, $m_{f_0} = 980 \pm 10\,{\rm MeV}$ and $\Gamma_{f_0} = 40$–$100\,{\rm MeV}$. Clearly one must rely on theoretical models to get at least a rough answer. From column 1 of Table II in Ref. [23] we read $m_{f_0} = 987\,{\rm MeV}$ and $\Gamma_{f_0} = 65\,{\rm MeV}$ while in Eq. (4.2) of Ref. [24] we read $\Gamma_{a_0} = 70\,{\rm MeV}$. With the central value of m_{a_0} we get $z = -0.47 + 0.51i$. This results in an enhanced ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0^0\gamma) \simeq 1.4$, which is much closer to the experimental one !

We next recalculate β_A, β_B and β_C including the effects of the a_0-f_0 mixing in all of the $S \to \gamma\gamma$, $V \to S\gamma$ and $S \to V\gamma$ processes. A scan shows that the values of $(m_{f_0} - m_{a_0})$ and $|\Gamma_{f_0} - \Gamma_{a_0}|$ taken above are in fact optimum for maximum enhancement of the $\phi \to f_0\gamma/\phi \to a_0\gamma$ width ratio. The two (out of 4) parameter sets which yield the highest values of the f_0/a_0 ratio as well as other predictions are listed in Table I. Note that the f_0/a_0 ratio can be boosted to 2.5 ± 0.5 by increasing A_{af} by 20 per cent [21]. It will be interesting to see if future experiments confirm the pattern of predicted widths.

Elsewhere, we will study flavor symmetry breaking effects, treatment of the $S\gamma$ final states as $PP\gamma$, corrections beyond first order in z and the case of mixed $q\bar{q}$ and $qq\bar{q}\bar{q}$ scalar nonets.

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β_A	0.86 ± 0.10	0.13 ± 0.14
β_B	0.23 ± 0.11	-0.65 ± 0.08
β_C	9.8 ± 0.6	8.3 ± 0.7
f_0/a_0 ratio	1.9 ± 0.4	1.8 ± 0.4
$\Gamma(\sigma \to \gamma\gamma)$	0.016 ± 0.019	0.20 ± 0.06
$\Gamma(\phi \to \sigma\gamma)$	150 ± 23	44 ± 12
$\Gamma(\omega \to \sigma\gamma)$	35 ± 5	41 ± 6
$\Gamma(\rho \to \sigma\gamma)$	1.7 ± 1.2	7.8 ± 2.6
$\Gamma(f_0 \to \omega\gamma)$	$(1.1 \pm 0.2) \times 10^3$	800 ± 138
$\Gamma(f_0 \to \rho\gamma)$	14.1 ± 4.6	8.5 ± 3.3
$\Gamma(a_0 \to \omega\gamma)$	713 ± 90	501 ± 81
$\Gamma(a_0 \to \rho\gamma)$	2.5 ± 0.6	5.5 ± 1.4

TABLE I: Fitted values of β_A, β_B and β_C together with the predicted values of the ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ and the decay widths of $V \to S + \gamma$ and $S \to V + \gamma$. Units of β_A, β_B and β_C are GeV^{-1} and those of the decay widths are keV.

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