

# Spin-dependent forces of quarks in baryon

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## Abstract

Nonperturbative spin-dependent forces of quarks in a baryon are calculated directly from the QCD Lagrangian in the framework of the Field Correlator Method both for heavy and light quarks. Resulting forces contain terms of 5 different structures, only one being known before in asymptotic form. Perturbative terms obtained by the same method are standard and have different signs and structures with respect to the corresponding nonperturbative ones, implying possible cancellations for some baryonic states.

## 1 Introduction

The spin structure of baryons presents a still unsolved problem, both on partonic and quark model level. For excited baryon spectrum the apparent small spin-orbit splitting of some baryonic states is a topic of vivid discussions [1]-[3]. Some baryonic states, like Roper resonance  $N(1440)$  or  $\Lambda(1405)$ , are not yet explained in the traditional framework of relativistic quark model (RQM) [2, 4]. More detailed information about the spin structure of baryons comes from the polarization experiments on electroproduction of excited resonances, [5, 6] which effectively measure the convolution of the baryon wave function, and is very sensitive to its structure.

Meanwhile the theoretical knowledge of the quark spin forces in the baryon is limited to the perturbative expressions calculated decades ago [7],

and nonperturbative spin-orbit Thomas term, written in the framework of RQM [8].

In applying these results to light baryons the notion of constituent quark masses is introduced in RQM, these appear in spin-dependent (SD) forces and play role of fitting parameters.

It is the purpose of the present paper to derive SD forces in a baryon in a most straightforward way from the QCD Lagrangian with the nonperturbative vacuum described by vacuum field correlators [9, 10]. Limiting to the lowest (Gaussian) field correlators one can express all terms of SD forces through the scalar functions  $D$  and  $D_1$  representing this Gaussian correlator [11]. High accuracy of such procedure is supported by recent lattice data [12] and contribution of higher correlators can be estimated to be of the order of few percent [13].

The functions  $D, D_1$  are themselves measured on the lattice [14] and also found in analytic approaches [15, 16].

An essential element of the present approach is that it is not connected to the heavy mass expansion, and can be applied also to light quarks in a baryon. In this case an effective Hamiltonian is constructed from first principles, which contains einbein (auxiliary) fields. It was shown previously, that stationary point of these einbein fields yields exactly the constituent quark masses which can be expressed unambiguously through the only parameter of this approach – the string tension  $\sigma$ .

The decisive check of this procedure is the calculation of baryon magnetic moments, since they are inversely proportional to the quark constituent masses. That was done in [17] and results agreed with all known experimental data within  $\sim 10\%$ .

The SD forces derived below in the paper are computed as a series in field correlators (cumulants) with growing powers of fields.

The lowest (Gaussian) term yields SD forces inversely proportional to the square of constituent quark masses. Having in mind the high accuracy of the Gaussian approximation [13] and baryon magnetic moments [17] one should expect that SD forces found below have the accuracy of the order of  $10\%$ .

Analogous expressions for heavy quarkonia [18, 19] and light mesons [20] have been reported earlier and estimated for realistic meson system respectively in [21] and [22].

The plan of the paper is as follows. In chapter 2 a general expression for the 3q baryon Green's function is introduced, and the Fock-Feynman-Schwinger (world-line) formalism is used to reveal the dependence on gauge

fields with spin operators explicitly written. Averaging over those with the help of the Field Correlator Method (FCM) one finally obtains an expression for the Green's function written in terms of field correlators.

In chapter 3 a special case of heavy quark masses is considered and all SD forces are obtained in closed form, expressed in terms of correlator functions  $D$  and  $D_1$ .

In chapter 4 the perturbative contribution to SD forces is written down. In chapter 5 a general case is considered when current quark masses can also be vanishingly small, and SD forces are written again in terms of integrals over functions  $D$  and  $D_1$  with constituent (dynamical) masses entering in the denominator.

Chapter 6 is devoted to the discussion of the relativistic structure of SD forces in the excited baryon spectrum. Comparison to other results in the literature is also made and possible extension of the method is suggested. Main points of the paper are summarized in Conclusions.

## 2 $3q$ Green's function with spin insertions

Following [9, 10, 23] we consider the  $3q$  Green's function, which can be written as

$$G_{3q}(x, y) = tr_L \left[ \Gamma_{out} \prod_{i=1}^3 (m_i - \hat{D}) \int D\mu_i D z^{(i)} e^{-K_i} \langle W_3 \exp g\sigma F \rangle \Gamma_{in} \right] \quad (1)$$

where  $tr_L$  is the trace over Dirac matrix indices,  $\Gamma_{out}(\Gamma_{in})$  are final (initial) state operators creating given  $J^{PC}$  assignment to the  $3q$  state and we have also denoted as in [17, 20]

$$K_i = \int_0^T dt \left[ \frac{m_i^2}{2\mu_i(t)} + \frac{\mu_i(t)}{2} (\dot{\mathbf{z}}^2(t) + 1) \right], \quad (2)$$

$$\langle W_3 \exp(g\sigma F) \rangle = tr_Y \exp \left[ \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int \langle \langle F(1) \dots F(n) \rangle \rangle d\rho(1) \dots d\rho(n) \right]. \quad (3)$$

Here and in what follows  $F(1)$  is always implied to be gauge-transported to one point  $x_0$ , namely  $F(1) \equiv F(z(1), x_0) = \Phi(x_0, z(1)) F(z(1)) \Phi(z(1), x_0)$  where  $\Phi(x, y)$  is defined in the Appendix, and finally,  $d\rho(n) = \sum_{i=1}^3 d\rho^{(i)}(n)$ , with

$$d\rho^{(i)}(n) \equiv ds_{\mu_n \nu_n}^{(i)}(u^{(n)}) + \frac{1}{i} \sigma_{\mu_n \nu_n}^{(i)} \frac{dt_n}{2\mu_i(t_n)}. \quad (4)$$

The integration in (3) extends over all 3 lobes of the minimal area surface ( $S_1 + S_2 + S_3$ ) inside the quark trajectories  $z^{(i)}(t)$  and the string-junction trajectory  $z^{(Y)}(t)$ . We have also denoted  $\sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$  and everywhere Euclidean space-time is used (till the last moment when the resulting Hamiltonian is obtained in Minkowski space-time) with  $\gamma$ -matrices

$$\gamma_4 = \gamma_0 \equiv \beta; \quad \gamma_i = -i\beta\alpha_i; \quad \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}.$$

Note also that notation  $tr_Y$  means

$$tr_Y P \equiv \frac{1}{6} e_{abc} e_{a'b'c'} P_{abc/a'b'c'}. \quad (5)$$

In the combination  $F(k)d\rho(k)$  in (3) one can write

$$F_{\mu\nu}\sigma_{\mu\nu}^{(i)} = \begin{pmatrix} \boldsymbol{\sigma}^{(i)}\mathbf{B}, & \boldsymbol{\sigma}^{(i)}\mathbf{E} \\ \boldsymbol{\sigma}^{(i)}\mathbf{E}, & \boldsymbol{\sigma}^{(i)}\mathbf{B} \end{pmatrix}, \quad i = 1, 2, 3. \quad (6)$$

As it was shown in [10] the spin-independent part of (3) which obtains neglecting the  $\Sigma$  term in (4), yields at large quark separations,  $|\mathbf{z}^{(i)} - \mathbf{z}^{(Y)}| \gg T_g$ ,  $i = 1, 2, 3$ , the familiar area-law asymptotics

$$\langle W_3 \rangle = \exp[-\sigma(S_1 + S_2 + S_3)] \quad (7)$$

implying linear confinement for each quark. In what follows we shall use the general expression (1) to derive the spin-dependent part of interaction both for heavy quarks (expansion in inverse powers of mass) and for light quarks.

### 3 Spin-dependent interaction in $1/m$ expansion

To illustrate the method we shall start with the derivation of spin-dependent (SD) forces via  $1/m$  expansion. Defining the SD potential as  $V_{SD}$ , one can write  $G_{3q} \sim e^{-TV_{SD}} \sim 1 - TV_{SD}$ , and for  $V_{SD}$  the following general form will be obtained below, similar (but not identical) to the corresponding form for heavy quarkonia [24, 18],

$$V_{SD}(\mathbf{R}^{(1)}, \mathbf{R}^{(2)}, \mathbf{R}^{(3)}) = \sum_{i=1}^3 \frac{\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}}{2m_i^2} \left( \frac{1}{R^{(i)}} \frac{dV_1}{dR^{(i)}} + \frac{1}{2R^{(i)}} \frac{d\varepsilon}{dR^{(i)}} \right)$$

$$\begin{aligned}
& + \frac{1}{N_c - 1} \sum_{i < j} \frac{(\boldsymbol{\sigma}^{(i)} \mathbf{L}^{(j)} + \boldsymbol{\sigma}^{(j)} \mathbf{L}^{(i)})}{2m_i m_j} \frac{1}{R^{(j)}} \frac{dV_2(R^{(i)}, R^{(j)})}{dR^{(j)}} + \\
& + \sum_{i < j} \left[ \frac{(\boldsymbol{\sigma}^{(i)} \boldsymbol{\sigma}^{(j)}) V_4(R_{ij})}{12m_i m_j (N_c - 1)} + \frac{3(\boldsymbol{\sigma}^{(i)} \mathbf{n})(\boldsymbol{\sigma}^{(j)} \mathbf{n}) - (\boldsymbol{\sigma}^{(i)} \boldsymbol{\sigma}^{(j)})}{12m_i m_j (N_c - 1)} V_3(R_{ij}) \right] + V_5, \quad (8)
\end{aligned}$$

where  $\mathbf{n} = \frac{\mathbf{R}_{ij}}{R_{ij}}$ ,  $\mathbf{R}_{ij} = \mathbf{R}_i - \mathbf{R}_j$ . We assume that current quark masses are large,  $m_i \gg \sqrt{\sigma}$ ,  $i = 1, 2, 3$ , and hence also  $\mu_i$  are large, since the latter are defined through  $m_i$  and  $\sigma$  in the stationary point analysis [10, 17] and always satisfy  $\mu_i = m_i + O(1/m_i)$ . Hence for simplicity we keep in the following  $\mu_i = m_i \gg \sqrt{\sigma}$  and expand in inverse powers of  $1/m_i$ .

As it was observed in [18, 19] the SD terms of the lowest order ( $1/m_i^2$ ,  $\frac{1}{m_i m_j}$ ) come from 3 different sources:

A) Diagonal terms in (6) are kept together with diagonal terms in  $\Lambda_i \equiv (m_i + \frac{1}{2}\gamma_\mu \dot{z}_\mu)$ , yielding one power of  $1/m_i$ . An additional power of  $\frac{1}{m_i}$  or  $\frac{1}{m_j}$  then comes from the expansion of  $\langle W_3 \rangle$ . This yields spin-orbit terms  $V_1'$ ,  $V_2'$  and  $V_5$ .

B) The off-diagonal terms are kept both in (6) and in  $\Lambda_i$ . This gives spin-orbit potential  $\frac{d\mathcal{E}}{dR}$ .

C) Diagonal terms from two matrices (6) with  $i \neq j$  are retained. This yields spin-spin potentials  $V_3$  and  $V_4$ . We now calculate the SD contributions from A) – C) point by point.

A) From (3), (4) one gets for  $i = 1$

$$\langle \text{tr}_Y [(1 + \frac{g}{2m_1} \sigma_k^{(1)} \int_0^T B_k(z^{(1)}, t^{(1)}) dt_1) W_3] \rangle \approx 1 - TV_{\text{SD}}^{(1)} \quad (9)$$

Using the relation  $igF_{\mu\nu}W = \frac{\delta}{\delta\sigma_{\mu\nu}(z)}W$  which obtains easily with nonabelian Stokes representation for  $W$ , one has

$$\langle \text{tr}_Y F_{\lambda\sigma}(x, z_0) W(C) \rangle = \text{tr}_Y \left\{ ig \int ds_{\mu\nu}(z) F_{\mu\nu}(z, z_0) F_{\lambda\sigma}(x, z_0) W_3(C) \right\} \quad (10)$$

one can rewrite the l.h.s. of (9) as

$$\text{tr}_Y W + \frac{ig^2}{2m_1} \sigma_k^{(1)} \int_0^T dt_1 \langle \text{tr}_Y B_k(z^{(1)}, t^{(1)}) \int_{S(C')} ds_{\mu\nu}(u) F_{\mu\nu}(u, x_0) W_3(C') \rangle. \quad (11)$$

In (10) and (11) the common reference point  $z_0$  is chosen to make both expressions gauge invariant; as it will be seen, this point will not appear in the final equations.

In (11) the contour  $C'$  is deformed due to orbital momentum of quarks as compared to the zeroth-order contour  $C_0$  consisting of straight lines. This is essential since otherwise the vacuum average  $\langle B_k W_3(C_0) \rangle$  vanishes since it is odd with respect to reflection  $z_i \rightarrow -z_i$   $i \neq k$ . Therefore all nonzero contribution in (11) is due to deflection of the quark path in  $C'$  from the straight line in  $C_0$ .

At this point we shall describe the quark trajectory  $z_\mu^{(i)}(t)$  and the corresponding string piece  $W_\mu^{(i)}$  from the quark position to the string junction (which we for simplicity take at the origin).

$$w_\mu^{(i)}(t, \beta) = z_\mu^{(i)}(t)\beta, \quad 1 \geq \beta \geq 0 \quad (12)$$

$$ds_{ik}^{(i)} = d\beta^{(i)} dt e_{ikm} \frac{\beta L_m^{(i)}}{im_i} \quad (13)$$

where  $L_m^{(i)}$  is the (Minkowskian) angular momentum of the  $i$ -th quark

$$L_s^{(i)} = im_i e_{skm} R_k^{(i)} \dot{z}_m^{(i)}, \quad \mathbf{R}^{(i)} = \mathbf{z}^{(i)} - \mathbf{z}^{(Y)} = \mathbf{z}^{(i)}. \quad (14)$$

Similarly  $d\sigma_{k4}^{(i)} = R_k^{(i)} d\beta^{(i)} du_4$ , and one arrives at the result

$$\begin{aligned} \langle B_k(z^{(i)}, t_i) W_3(C') \rangle &= ig \int d\beta^{(i)} du_4 \frac{\beta^{(i)} L_n^{(i)}}{im_i} \langle B_k B_n(u_4, \beta) W \rangle + \\ &+ ig \int d\beta^{(i)} du_4 R_l^{(i)}(u_4) \langle B_k E_l(u_4, \beta^{(i)}) W \rangle. \end{aligned} \quad (15)$$

Denoting

$$\langle B_k(z^{(1)}, t_1) E_i(\mathbf{u}, u_4) W_3 \rangle \equiv e_{kin}(u_n - z_n^{(1)}) \frac{\partial \Lambda_0}{\partial u_4} \quad (16)$$

one obtains

$$\begin{aligned} \sigma^{(i)} \mathbf{L}^{(i)} \left( \frac{1}{R} \frac{dV_1}{dR} \right)^{(i)} &= -g^2 \int_0^1 \beta d\beta \int_0^T du_4 \frac{\sigma_k^{(i)} L_n^{(i)}}{\langle W_3 \rangle} \langle B_k(z^{(i)}, t_i) B_n(\beta z^i, u_4) \\ &- \frac{\sigma^{(i)} \mathbf{L}^{(i)}}{\langle W_3 \rangle} \int_0^T du_4 (u_4 - t_1) d\beta \frac{\partial \Lambda_0}{\partial u_4} \end{aligned} \quad (17)$$

where we have used the relation  $u_n(u_4) - z_n^{(1)}(t_1) \cong \dot{z}_n^{(1)}(u_4)(u_4 - t_1)$ .

Until now we have not used the Gaussian dominance of the vacuum, i.e. the fact that  $\langle W_3 \rangle$  is saturated by the lowest cumulant  $\langle FF \rangle$ , which is found

to be an accurate approximation [13]. Using it one can write  $\langle FFW_3 \rangle \rightarrow \langle FF \rangle \langle W_3 \rangle$ , and introduce scalar functions  $D, D_1$  for tensor  $\langle FF \rangle$ , as it was done in [11].

Referring the reader to the Appendix for the corresponding relations, one finally obtains

$$\begin{aligned} \left( \frac{1}{R} \frac{dV_1}{dR} \right)^{(i)} &= - \int_0^R \frac{d\lambda}{R} \left( 1 - \frac{\lambda}{R} \right) \int_{-\infty}^{\infty} d\nu [D(\lambda, \nu) + D_1(\lambda, \nu) + \lambda^2 \frac{\partial D_1}{\partial \lambda^2}] - \\ &\quad - \int_{-\infty}^{\infty} \nu^2 d\nu \int_0^R \frac{d\lambda}{R} \frac{\partial D_1}{\partial \lambda^2}. \end{aligned} \quad (18)$$

Till now we have taken into account the interaction of the spin of the  $i$ -th quark with the surface  $S_i$ , which yields the term  $(\frac{1}{R} V_1')^{(i)}$  multiplied with  $(m_i^2)^{-1}$ . At this point we consider the interaction of the  $i$ -th quark spin with the (deformed) surface  $S_j$ , which will give the term  $V_2'$  in (8). For this one needs to consider a vacuum average of two  $F$ 's from two different surfaces  $S_i$  and  $S_j$ .

In general we have for two  $F$ 's transported to the same point  $x$  ( $\alpha, \dots, \eta$  are fundamental color indices)

$$\langle F(u, x)_{\alpha\xi} F(v, x)_{\gamma\eta} \rangle = \frac{\langle \text{tr} FF \rangle}{N_c^2 - 1} (\delta_{\alpha\eta} \delta_{\xi\gamma} - \frac{1}{N_c} \delta_{\alpha\xi} \delta_{\gamma\eta}). \quad (19)$$

Taking into account (5) and the relation

$$\text{tr}_Y \Phi_{\alpha\alpha'}(x, y) \Phi_{\beta\beta'}(x, y) \Phi_{\gamma\gamma'}(x, y) \equiv 1 \quad (20)$$

one obtains

$$\text{tr}_Y \langle F_{\alpha\alpha'}(u, x) F_{\beta\beta'}(v, x) \rangle = \frac{\langle \text{tr} F(u, x) F(v, x) \rangle}{N_c(N_c - 1)} \quad (21)$$

where we have also accounted for different orientation of plaquettes in  $S_i$  and  $S_j$ .

Now proceeding as in (15) one has

$$\begin{aligned} \langle B_k^{(i)} W_3(C_j') \rangle &= g \int_0^1 \beta^{(j)} d\beta^{(j)} du_4^{(j)} \frac{L_n^{(j)}}{m_j} \langle B_k^{(i)} B_n^{(j)}(u_4^{(j)}, \beta^{(j)} R^{(j)}) W_3 \rangle + \\ &\quad + ig \int d\beta^{(j)} du_4^{(j)} R_l^{(j)}(u_4^{(j)}) \langle B_k^{(i)} E_l^{(j)} W_3 \rangle. \end{aligned} \quad (22)$$

At this point one can use Gaussian dominance and relations (21) to obtain finally

$$\left(\frac{1}{R}V_2'(R)\right)_j = \int_0^1 d\beta^{(j)} \beta^{(j)} \int_{-\infty}^{\infty} d\nu [D(r^{(ij)}, \nu) + D_1 + ((r^{(ij)})^2 + \nu^2) \frac{\partial D_1}{\partial (r^{(ij)})^2}]. \quad (23)$$

In a similar way one obtains from the first term on the r.h.s. of (22), with the use of the last term on the r.h.s. of (A1).

$$V_5 = - \sum_{i>j} \int \frac{(\boldsymbol{\sigma}^{(i)} \mathbf{r}^{(ij)})(\mathbf{L}^{(j)} \mathbf{r}^{(ij)})}{2m_i m_j (N_c - 1)} \beta^{(j)} d\beta^{(j)} d\nu \frac{\partial D_1(r^{(ij)}, \nu)}{\partial (r^{(ij)})^2}. \quad (24)$$

Here we have defined  $\mathbf{r}^{(ij)} = \mathbf{R}^{(i)} - \beta^{(j)} \mathbf{R}^{(j)}$ , and one should take into account, that  $D, D_1$  depend on their arguments as

$$D(r, \nu) = D(\sqrt{r^2 + \nu^2}).$$

This concludes derivation of terms with the procedure A) and one goes over to the next point.

B) Following (9) one can write for the corresponding term of a given quark ( $i$ )

$$V_{SD}^{(\varepsilon)} T = - \frac{g}{(2m_i)^2} \left\langle \begin{pmatrix} m_i + \mu_i & -\boldsymbol{\sigma}^{(i)} \mathbf{p}^{(i)} \\ \boldsymbol{\sigma}^{(i)} \mathbf{p}^{(i)} & m_i - \mu_i \end{pmatrix} \begin{pmatrix} 0 & \boldsymbol{\sigma}^{(i)} \mathbf{E} \\ \boldsymbol{\sigma}^{(i)} \mathbf{E} & 0 \end{pmatrix} W_3(C) \right\rangle. \quad (25)$$

Now one can use relation

$$\langle E_k(z^{(i)}, t^{(i)}) W_3(C_0) \rangle = \frac{\delta \langle W_3(C_0) \rangle}{ig \delta \sigma_{k4}(z^{(i)}, t^{(i)})} = - \frac{1}{ig} \frac{\partial \varepsilon}{\partial R_k^{(i)}} \langle W_3(C_0) \rangle \quad (26)$$

where the following notation was introduced for the spin-independent potential  $\varepsilon(R^{(1)}, R^{(2)}, R^{(3)})$ .

$$\langle W_3(C_0) \rangle = \exp[-\varepsilon(R^{(1)}, R^{(2)}, R^{(3)}) T]. \quad (27)$$

Note that in (25-27) one can keep in  $W_3(C)$  the unperturbed (straight-line) contours for quark trajectories since the prefactor in (25) is already  $O(1/m^2)$ . Keeping in mind relation

$$\sigma_k^{(i)} E_k^{(i)} \sigma_l^{(i)} p_l^{(i)} = E_k^{(i)} p_k^{(i)} + i e_{kln} \sigma_n^{(i)} E_k^{(i)} p_l^{(i)} \quad (28)$$



one recovers the second term on the r.h.s. of (8).

C) Here one considers spin-spin interaction and the corresponding term looks like

$$1 - V_{\text{SD}}T = \text{tr}_Y \sum_{i>j} \langle (1 + \frac{g}{2m_i} \int \boldsymbol{\sigma}^{(i)} \mathbf{B}^{(i)}(z^{(i)}, t_i) dt_i) \times \\ \times (1 + \frac{g}{2m_j} \int \boldsymbol{\sigma}^{(j)} \mathbf{B}^{(j)}(z^{(j)}, t_j) dt_j) W_3 \rangle \quad (29)$$

Identifying spin-spin terms in (29) and using (21) and relations for  $\langle BBW_3 \rangle$  in Appendix one arrives at

$$V_{\text{SD}}^{(\sigma\sigma)} = \sum_{i<j} \int_{-\infty}^{\infty} \frac{d\nu \sigma_k^{(i)} \sigma_{k'}^{(j)}}{4m_i m_j (N_c - 1)} [\delta_{kk'} (D + D_1 + (\mathbf{u})^2 \frac{\partial D_1}{\partial (\mathbf{u})^2}) - \\ - u_k u_{k'} \frac{\partial D_1}{\partial (u)^2}] \quad (30)$$

where notations are used

$$\mathbf{u} = \mathbf{R}^{(i)} - \mathbf{R}^{(j)}, \quad \nu = t_i - t_j. \quad (31)$$

Rewriting (30) as

$$V_{\text{SD}}^{(\sigma\sigma)} = \sum_{i<j} \frac{\boldsymbol{\sigma}^{(i)} \boldsymbol{\sigma}^{(j)} V_4(u) + S_{ij} V_3(u)}{12m_i m_j (N_c - 1)} \quad (32)$$

one has

$$V_4(u) = \int_{-\infty}^{\infty} d\nu (3D(u, \nu) + 3D_1(u, \nu) + 2u^2 \frac{\partial D_1}{\partial u^2}) \quad (33)$$

$$V_3(u) = - \int_{-\infty}^{\infty} d\nu u^2 \frac{\partial D_1(u, \nu)}{\partial u^2}, \quad (34)$$

$$S_{ij} = 3(\boldsymbol{\sigma}^{(i)} \mathbf{n})(\boldsymbol{\sigma}^{(j)} \mathbf{n}) - \boldsymbol{\sigma}^{(i)} \boldsymbol{\sigma}^{(j)}, \quad \mathbf{n} = \frac{\mathbf{u}}{|\mathbf{u}|}. \quad (35)$$

This concludes definition of all NP spin-dependent terms in (8) to the order  $O(1/m^2)$  and in the approximation when only lowest,  $\langle FF \rangle$ , correlator is retained in the Wilson loop.

Now comparing our expressions for  $V'_1, V'_2, \varepsilon', V_3, V_4$  with the corresponding ones for heavy  $Q\bar{Q}$  case, given in [18, 19], one can see that they coincide exactly, the only difference being that one should sum up over all 3 quarks for  $V'$  and  $\varepsilon'$ , and take a double sum,  $i < j$ , for  $V'_2, V_3, V_4$ . In addition there is a term  $V_5$  which is of 3 body character and vanishes in two-body situation since in that situation  $\mathbf{L}^{(j)} \mathbf{r}^{(ij)} = \mathbf{L}^{(j)} \mathbf{r} \equiv 0$ .

## 4 Perturbative spin-dependent forces

We are now in position to consider also perturbative contributions to the SD potentials, which were calculated in [7]. The easiest way for us is to remember, that to the lowest order,  $O(\alpha_s)$ , all perturbative terms are pair-wise interaction of quarks, and they can be reconstructed from the expressions obtained above, Eqs. (18), (23), (24),(32)-(34), using  $O(\alpha_s)$  contribution to  $D_1(x)$ , while  $D(x)$  does not have contributions at this order, [25]

$$D(x) = D^{NP}(x), \quad D_1(x) = \frac{16\alpha_s}{3\pi x^4} + D_1^{NP}(x). \quad (36)$$

It is rewarding to realize that  $D_1$  does not enter into  $V_1'$  (terms containing  $D_1$  in (18) cancel exactly), so that perturbative contribution occurs only in the nondiagonal,  $i \neq j$ , terms in (8) and in  $\varepsilon'$ .

In the  $Q\bar{Q}$  case analogous calculations have been done and compared to standard ones in [18, 19].

We start with the  $\frac{d\varepsilon}{dR}$  term and rewrite it in the original form (26), not assuming that  $\varepsilon$  depends on  $R^{(i)}$  only, but also on  $\mathbf{R}^{(i)} - \mathbf{R}^{(j)}$ , as it is for the Coulomb term to be added to  $\varepsilon$ . In this way one obtains from (25), (26),(28)

$$V_{SD}^{(\varepsilon)} = \frac{e_{klm} p_l^{(i)} \sigma_n^{(i)}}{4m_i^2} \frac{\partial \varepsilon}{\partial R_k^{(i)}} \quad (37)$$

and for  $\varepsilon \rightarrow \varepsilon + V_{coul}$ ,  $V_{coul} = -\frac{2\alpha_s}{3} \sum_{i>j} \frac{1}{|\mathbf{R}^{(i)} - \mathbf{R}^{(j)}|}$  one obtains

$$V_{SD}^{(\varepsilon, pert)} = \frac{2\alpha_s}{3} \sum_{i>j} \left[ \frac{(\mathbf{R}^{(ij)} \times \mathbf{p}^{(i)}) \sigma^{(i)}}{4m_i^2 (R^{(ij)})^3} + \frac{(\mathbf{R}^{(ji)} \times \mathbf{p}^{(j)}) \sigma^{(j)}}{4m_j^2 (R^{(ij)})^3} \right]. \quad (38)$$

This expression coincides with the corresponding in [7]. Consider now nondiagonal spin-orbit term, equivalent to  $V_2'$ . Instead of using replacement (36) and doing integrations in (23), we start from more general expression (9) to derive

$$V_{SD}^{(2, pert)} = -\frac{2\alpha_s}{3(N_c - 1)} \sum_{i>j} \frac{[\sigma^{(i)}(\mathbf{R}^{(ij)} \times \mathbf{p}^{(j)}) + \sigma^{(j)}(\mathbf{R}^{(ji)} \times \mathbf{p}^{(i)})]}{m_i m_j (R^{(ij)})^3} \quad (39)$$

which is again in agreement with [7].

Next we consider the spin-spin interaction. Here it is straightforward to replace in  $V_4$  (33)  $D_1$  as in (36)  $D_1 \rightarrow D_1^{(pert)} = \frac{16\alpha_s}{3\pi x^4}$  to obtain

$$\begin{aligned} V_4^{(pert)}(r) &= \int_{-\infty}^{\infty} d\nu (3D_1^{(pert)}(r, \nu) + 2r^2 \frac{\partial D_1^{(pert)}}{\partial r^2}) = \\ &= -\frac{8\alpha_s}{3r} \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) \frac{1}{r} = \frac{32\pi\alpha_s}{3} \delta^{(3)}(\mathbf{r}). \end{aligned} \quad (40)$$

In a similar way one obtains for  $V_3^{(pert)}$ ,

$$V_3^{(pert)}(r) = \frac{4\alpha_s}{r^3}. \quad (41)$$

One can also persuade oneself that  $V_5$  has no perturbative counterpart since there  $\mathbf{L}^{(j)} \rightarrow \mathbf{L}^{(ij)}$  and it is orthogonal to  $\mathbf{r}^{(ij)}$  and therefore the total perturbative SD contribution to the order  $O(\alpha_s)$  can be written as

$$V_{SD}^{(pert)} = V_{SD}^{(\varepsilon, pert)} + V_{SD}^{(2, pert)} + \sum_{i < j} \frac{\sigma^{(i)} \sigma^{(j)} V_4^{(pert)}(R^{(ij)}) + S_{ij} V_3^{(pert)}(R^{(ij)})}{12m_i m_j (N_c - 1)} \quad (42)$$

where explicit form of 4 terms on the r.h.s. of (42) is given in (38), (39), (40) and (41).

Our results for  $V_{SD}^{(2, pert)}$ ,  $V_3^{(pert)}$ ,  $V_4^{(pert)}$  coincide with the corresponding expressions in [8], however our  $V_{SD}^{(\varepsilon, pert)}$  is 2 times smaller than the corresponding term in [8]. In the next sections we shall argue that for light quark this term gets indeed twice as big, since there one should replace  $m_i \rightarrow \mu_i$ , and for  $V_{SD}^{(\varepsilon)}$  the coefficient appears to be 2 times larger.

## 5 Spin-dependent forces for light quarks

In sections 3, 4 the SD forces have been obtained as an expansion in  $1/m_i$ ,  $1/m_j$  taking all 3 quark current masses large,  $m_i \gg \sqrt{\sigma}$ ,  $i = 1, 2, 3$ .

It was noticed before [20] however, that general expressions (1)-(4) for Green's functions written in FSR, with the einbein function  $\mu_i(t)$  introduced as in [10], allow to obtain expressions for SD forces also for light quarks without  $1/m$  expansion, and the corresponding terms for the meson case have been written before [10, 22]. Below we demonstrate in this section

that the same procedure works also for the  $3q$  case with light current quarks masses as well.

We start again with general form (3) and instead of expansion in  $1/m$  (or  $1/\mu$  which is equivalent for heavy quark) shall do the only approximation: keeping in the sum in the exponent (3) the lowest (Gaussian) cumulant  $\langle\langle F(1)F(2)\rangle\rangle$ . This approximation was recently supported by lattice data for Casimir scaling [12], while higher cumulants provide (for Wilson loop) less than 2% [13].

We start with the spin-spin interaction, which is easily obtained keeping in (3) the bilocal term and in (4) only the  $\sigma$ - dependent term. In (3) one should take into account that  $F(1)$  and  $F(2)$  belong to different lopes  $S_1, S_2$  of the  $S_{123}$  surface, hence  $n = 1$  for each of them; moreover one uses (19) and (21), the latter with opposite sign, since orientation of  $S(1, 3)$  and  $S(2, 3)$  is the same in our case. As a result one obtains

$$V_{\text{SD}}^{(\sigma\sigma)} = \sum_{i>j} \int_{-\infty}^{\infty} \frac{d(t_i - t_j)}{4\mu_i\mu_j(N_c - 1)} \sigma_{\mu_i\nu_i}^{(i)} \sigma_{\mu_j\nu_j}^{(j)} \langle F_{\mu_i\nu_i}(i) F_{\mu_j\nu_j}(j) \rangle. \quad (43)$$

The combination  $\sigma^{(i)}\sigma^{(j)}$  in (43) is a product of two  $4 \times 4$  matrices, which can be split into the product of Pauli spin matrices  $\sigma_i$  and chiral  $2 \times 2$  matrices  $\hat{1}$  and  $\hat{\rho}_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ . Thus one can rewrite (43) as

$$\begin{aligned} V_{\text{SD}}^{(\sigma\sigma)} = & \sum_{i>j} \int_{-\infty}^{\infty} \frac{g^2 d(t_i - t_j)}{4\mu_i\mu_j(N_c - 1)} \sigma_m^{(i)} \sigma_n^{(j)} [\langle \text{tr} B_m(i) B_n(j) \rangle (\hat{1} \times \hat{1}) + \\ & + \langle \text{tr} E_m(i) E_n(j) \rangle (\hat{\rho}_1 \times \hat{\rho}_1) + \langle \text{tr} B_m(i) E_n(j) \rangle (\hat{1} \times \hat{\rho}_1) + \\ & \langle \text{tr} E_m(i) B_n(j) \rangle (\hat{\rho}_1 \times \hat{1})] \equiv V_{\text{SD}}^{(\sigma\sigma)}(BB) + V_{\text{SD}}^{(\sigma\sigma)}(EE) + \\ & V_{\text{SD}}^{(\sigma\sigma)}(BE) + V_{\text{SD}}^{(\sigma\sigma)}(EB). \end{aligned} \quad (44)$$

Using formulas from Appendix for correlators of  $B, E$  one has

$$V_{\text{SD}}^{(\sigma\sigma)}(B_1 B) = \sum_{i>j} \frac{\sigma^{(i)} \sigma^{(j)} V_4(u) + S_{ij} V_3(u)}{12\mu_i\mu_j(N_c - 1)} (\hat{1} \times \hat{1}) \quad (45)$$

where  $\mathbf{u} \equiv \mathbf{R}^{(i)} - \mathbf{R}^{(j)}$ .

One can see that (45) coincides with (32-35) with substitution  $m_i, m_j \rightarrow \mu_i, \mu_j$ . For  $EE$  term one obtains

$$V_{\text{SD}}^{(\sigma\sigma)}(EE) = \sum_{i>j} \frac{\boldsymbol{\sigma}^{(i)} \boldsymbol{\sigma}^{(j)} \tilde{V}_4(u) + S_{ij} \tilde{V}_3(u)}{12\mu_i\mu_j(N_c - 1)} (\hat{\rho}_1 \times \hat{\rho}_1) \quad (46)$$

where we have defined

$$\tilde{V}_4(u) = \int_{-\infty}^{\infty} d\nu (3D(u, \nu) + 3D_1(u, \nu) + (3\nu^2 + u^2) \frac{\partial D_1(u, \nu)}{\partial \nu^2}) \quad (47)$$

$$\tilde{V}_3(u) = \int_{-\infty}^{\infty} d\nu u^2 \frac{\partial D_1(u, \nu)}{\partial u^2} = -V_3(u). \quad (48)$$

Finally for the last two terms in (44) one has

$$V_{\text{SD}}^{(\sigma\sigma)}(BE) = -V_{\text{SD}}^{(\sigma\sigma)}(EB) = \sum_{i>j} \frac{(\boldsymbol{\sigma}^{(i)} \times \boldsymbol{\sigma}^{(j)}) \frac{1}{2i} (\frac{\mathbf{p}^{(i)}}{\mu_i} + \frac{\mathbf{p}^{(j)}}{\mu_j})}{4\mu_i\mu_j(N_c - 1)} \int_{-\infty}^{\infty} \frac{\partial D_1(u, \nu)}{\partial u^2} \nu^2 d\nu. \quad (49)$$

This concludes calculation of spin-spin interaction.

We turn now to the calculation of spin-orbit terms. The corresponding expression in (3) can be written as

$$\begin{aligned} \langle W_3 \exp(g\sigma F) \rangle_{so} = \exp \left\{ \sum_{i=1}^3 i \int \frac{dt_i}{2\mu_i} \sigma_{\mu\nu}^{(i)} ds_{\rho\sigma}^{(i)}(u) D_{\mu\nu, \rho\sigma}(z, u) + \right. \\ \left. + \frac{i}{N_c - 1} \sum_{i \neq j} \int \frac{dt_i}{2\mu_i} ds_{\rho\sigma}^{(j)}(u) \sigma_{\mu\nu}^{(i)} D_{\mu\nu, \rho\sigma}(z, u) \right\} \quad (50) \end{aligned}$$

In (50) we have defined as in Appendix

$$\begin{aligned} D_{\mu\nu, \rho\sigma}(z, u) = \frac{g^2}{N_c} \langle \text{tr} F_{\mu\nu}(z(t_i)) F_{\rho\sigma}(u) \rangle = D(h) (\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\rho}) \\ + \frac{1}{2} [\partial_\mu h_\rho \delta_{\nu\sigma} + \text{perm.}] D_1(h) \end{aligned}$$

and  $h_\mu = z_\mu(t_i) - u_\mu$ .

The Dirac structure of the exponent in (50) is a sum, which can be written with notations from (44) as

$$V^{(so)} = \sum_{i=1}^3 V_i^{(so, \text{diag})} (\hat{\mathbf{1}}_i \times \hat{\mathbf{1}}_{jk}) + V_i^{(so, \text{nondiag})} (\hat{\rho}_1^{(i)} \times \hat{\mathbf{1}}_{jk}), \quad i \neq j, k. \quad (51)$$

In the  $4 \times 4$  matrix  $\sigma_{\mu\nu}^{(i)}$  we first consider the diagonal part. Repeating all the steps leading to (18) and (23), (24) one has the same expressions with the replacement  $m_i \rightarrow \mu_i$ ,  $i = 1, 2, 3$ , namely

$$V_{\text{SD}}^{(so \text{ diag})} = \left\{ \sum_{i=1}^3 \frac{\sigma^{(i)} \mathbf{L}^{(i)}}{2\mu_i^2} \frac{1}{R^{(i)}} \frac{dV_1}{dR^{(i)}} + \frac{1}{N_c - 1} \sum_{i < j} \frac{\sigma^{(i)} \mathbf{L}^{(i)} + \sigma^{(j)} \mathbf{L}^{(j)}}{2\mu_i \mu_j} \frac{1}{R^{(j)}} \frac{dV_2}{dR^{(j)}} \right\}. \quad (52)$$

Let us now consider the nondiagonal part in (51), which can be written as

$$\begin{aligned} \langle W_3 \exp(g\sigma F) \rangle_{(so, \text{nondiag})} &= \exp \left\{ i \sum_{i=1}^3 \int \frac{dt_i}{2\mu_i} \sigma_k^{(i)} (D_{k4,l4}^{(ii)}(z, u) ds_{l4}^{(i)}(u) \right. \\ &+ D_{k4,mn}(zu) ds_{nm}^{(i)}(u) + i \sum_{i \neq j} \int \frac{dt_i}{2\mu_i} \sigma_k^{(i)} (D_{k4,l4}^{(ij)}(z, u) ds_{l4}^{(j)}(u) + D_{k4nm}^{(ij)} ds_{nm}^{(j)}(u)) \left. \right\}_{ij} \end{aligned} \quad (53)$$

Now taking into account (13), (14) one has writing (53) in the form

$$\langle W_3 \exp(g\sigma F) \rangle_{(so, \text{nondiag})} = \exp \left[ -T \left\{ \sum_{i=1}^3 V_{so, \text{nondiag}}^{(ii)} + \sum_{i < j=1}^3 V_{(so, \text{nondiag})}^{(ij)} \right\} \right] \quad (54)$$

one finds from (53), replacing  $D_{\mu\nu, \rho\sigma}$  from (A1) and using (13), (14)

$$V_{(so, \text{nondiag})}^{(ii)} = \Delta_{EE}^{(ii)} + \tilde{\Delta}_{EE}^{(ii)} + \Delta_{EB}^{(ii)} \quad (55)$$

and similarly for  $V_{(so, \text{nondiag})}^{(ij)}$ . For terms on the r.h.s. of (55) one obtains

$$\begin{aligned} \Delta_{EE}^{(ii)} &= -i \frac{\sigma \mathbf{R}^{(i)}}{2\mu_i R^{(i)}} \Lambda^{(ii)}, \\ \Lambda^{(ii)} &= \int_0^{R^{(i)}} d\nu du (D(\nu, u) + D_1 + \nu^2 \frac{\partial D_1}{\partial \nu^2}) \\ \tilde{\Delta}_{EE}^{(ii)} &= -i \int (\sigma \mathbf{u})(\mathbf{R}^{(i)} \mathbf{u}) \frac{\partial D_1}{\partial u^2}(\nu, u) d\nu d\beta R^{(i)}, \quad \mathbf{u} = \mathbf{R}^{(i)} \beta; \\ \Delta_{EB}^{(ii)} &= \frac{i}{2\mu_i^3} \int_{-\infty}^{\infty} \nu^2 d\nu \frac{\partial D_1(\nu_1, h)}{\partial \nu^2} \int_0^1 \beta^{(i)} d\beta^{(i)} (\mathbf{L}^{(i)} \times \sigma^{(i)}) \mathbf{p}^{(i)}. \end{aligned} \quad (56)$$

Similarly for  $\Delta^{(ij)}$  one obtains from the second term on the r.h.s. of (53)

$$\Delta_{EE}^{(ii)} = -i \frac{\boldsymbol{\sigma} \mathbf{R}^{(j)}}{2\mu_i R^{(j)}} \frac{\Lambda^{(ij)}}{(N_c - 1)},$$

$$\Lambda^{(ij)} = R^{(i)} \int_0^1 d\beta^{(j)} \int_{-\infty}^{\infty} d\nu (D(\nu, r^{(ij)}) + D_1 + \nu^2 \frac{\partial D_1}{\partial \nu^2})$$

$$\tilde{\Delta}_{EE}^{(ii)} = -i \int \frac{(\boldsymbol{\sigma} \mathbf{r}^{(ij)})(\mathbf{R}^{(j)} \mathbf{r}^{(ij)})}{2\mu_i (N_c - 1)} d\nu \int_0^1 d\beta^{(j)} \frac{\partial D_1(\nu, r^{(ij)})}{\partial (r^{(ij)})^2}$$

$$\Delta_{EB}^{(ij)} = \frac{i}{2\mu_i \mu_j} \int_{-\infty}^{\infty} \nu^2 d\nu \frac{\partial D_1(\nu, r^{(ij)})}{\partial \nu^2} \int_0^1 \beta^{(j)} d\beta^{(j)} (\mathbf{L}^{(j)} \times \boldsymbol{\sigma}^{(j)}) \left( \frac{\mathbf{p}^{(i)}}{\mu_i} + \beta^{(j)} \frac{\mathbf{p}^{(j)}}{\mu_j} \right). \quad (57)$$

Here we have defined  $\mathbf{r}^{(ij)} = \mathbf{R}^{(i)} - \beta^{(j)} \mathbf{R}^{(j)}$ .

## 6 Discussion

Let us now discuss the results obtained in the paper. For the heavy-quark case the nonperturbative dependent potential is given in (8), and the perturbative part in (42), so that the total SD potential is

$$V_{\text{SD}}^{(total)} = V_{\text{SD}}^{(nonpert)} + V_{\text{SD}}^{(pert)}. \quad (58)$$

Perturbative part agrees with that obtained long ago in [7] and repeated in many subsequent papers. However in [8] the term  $V^{(s.pert)}$  is taken twice as big as (38) (or corresponding term in [7]). A possible modification for light quarks, which can produce this increase is discussed later.

The nonperturbative part  $V_{\text{SD}}^{(nonpert)}$  (8) consists of six terms, which were never fully written before.

Only asymptotics at large distances of the first term in (8) has been written before in [27] and later in [26], one can find it from (8)

$$V^{(nonpert)}(R^{(i)} \rightarrow \infty) = -\frac{\boldsymbol{\sigma}^{(i)} \mathbf{L}^{(i)} \boldsymbol{\sigma}}{4m_i^2 R^{(i)}}. \quad (59)$$

In [8] were instead postulated the pairwise nonperturbative spin-orbit forces, which contradict expressions derived in this paper and in [26]. All other

terms, proportional to  $V'_2, V_3, V_4$  and  $V_5$  have never been written for the 3q case, while for the  $q\bar{q}$  case the corresponding terms (except for  $V_5$ ) have been written in [18], [19]. The term  $V_5$  which has no counterpart in the  $q\bar{q}$  case, is completely new, and its physical implication is still unclear.

We now turn to be light quark case. Here the total SD "potential" is in general a sum of product of  $(4 \times 4)$  matrices, which can be written as

$$\hat{V}_{\text{SD}}^{(\text{light quarks})} = \sum_{i,j} (V_{\text{diag}}^{(ij)} \hat{1}_i \times \hat{1}_j + \hat{V}_{\text{nondiag}}^{(ij)}) \quad (60)$$

where  $\hat{V}_{\text{nondiag}}^{(ij)}$  contains terms like  $\hat{1}_i \times \hat{\rho}_{1j}, \hat{\rho}_{1i} \times \hat{1}_j, \hat{\rho}_{1i} \times \hat{\rho}_{1j}$ , and  $\rho_1 = \begin{pmatrix} 0 & \hat{1} \\ \hat{1} & 0 \end{pmatrix}$ , where each entry in  $\rho_1$ , is  $2 \times 2$  unit matrix.

Now for  $V_{\text{diag}}^{(ij)}$  one has

$$V_{\text{diag}}^{(ij)} = V_{\text{SD}}^{(\sigma\sigma)}(BB) + V_{\text{SD}}^{(\text{so.diag})} \quad (61)$$

where the first term on r.h.s. of (61) is given in (45) and the second in (52). One can see in these expressions for  $V_{\text{diag}}^{(ij)}$  the same terms as in (8) with exchange  $m_i \rightarrow \mu_i$  except for the spin-orbit term proportional to  $\frac{d\epsilon}{dR}$ . Before discussing two different strategies for obtaining this last term, let us look at the general structure of (60). It has the described above matrix form and depends on einbein fields  $\mu_i$ ,  $i = 1, 2, 3$ . The latter have been defined previously in [10, 17, 20, 23, 28] as scalars,  $2\mu_i = \frac{dz_i(\tau)}{d\tau}$ , and are assumed to be found from the stationary point equation in the path-integral form of the meson Green's function, or from the stationary point of the Hamiltonian. Now the spin-independent part of Hamiltonian is a unit matrix and hence can produce scalar stationary values for  $\mu_i$ . The situation changes however if one tries to incorporate also the SD part of Hamiltonian in the stationary point equation for  $\mu$ , since it would require  $\mu$  to have a matrix form similar to that of  $\hat{V}_{\text{SD}}$ .

This is possible in the generalized form of the FFFR, which is now under investigation, but in the present form the only possible way of treatment the SD part of Hamiltonian is to consider it as a perturbation. For light quarks it is not an expansion in  $1/\mu_i$ , and the whole expression (60) is obtained with the only and numerically good approximation – keeping the bilocal (Gaussian) correlator, neglecting all higher ones.

As it was shown in meson case [20, 28], this perturbation procedure works well even for lowest mesons, where SD corrections produce up to around



15% of the total mass (a similar situation holds true in earlier quark model calculations with fixed and prescribed constituent masses, see e.g. [8]). For heavier meson and baryon states the masses  $\mu_i$  grow rapidly with quantum numbers [20] and validity of perturbative treatment of SD terms becomes even better established. In what follows we describe a perturbative procedure of treating the nondiagonal terms.

To this end we must remember (as in point B of derivation in section 4) that nondiagonal terms are also present in the preexponential factor  $(m - \hat{D})$  in (1). Consider the largest nondiagonal term  $\Delta_{EE}^{(ii)}$  in (56), and take for simplicity its asymptotic form

$$V_{EE} = -i \sum_{i=1}^3 \frac{\boldsymbol{\alpha}^{(i)} \mathbf{n}^{(i)} \sigma}{2\mu_i}, \quad \mathbf{n}^{(i)} = \mathbf{R}^{(i)} / R^{(i)}. \quad (62)$$

One has (omitting index  $(i)$  for simplicity)

$$(m - \hat{D}) \exp(-V_{EE}T) \cong \begin{pmatrix} m + \mu, & -\boldsymbol{\sigma} \mathbf{p} \\ \boldsymbol{\sigma} \mathbf{p}, & m = \mu \end{pmatrix} \left(1 + i \frac{\sigma}{2\mu} \boldsymbol{\alpha} \mathbf{n} + \dots T\right). \quad (63)$$

Comparing with the leading term, given by the upper left corner, one normalizes  $(m - \hat{D})$  by extracting the factor  $(m + \mu)$  and thus obtains

$$Eq.(63) = (1 - V_{SD}^{(\varepsilon)}T + \dots), \quad V_{SD}^{(\varepsilon)} = \sum_{i=1}^3 \frac{\sigma \boldsymbol{\sigma}^{(i)} \mathbf{L}^{(i)}}{2\mu_i(m_i + \mu_i)R^{(i)}}. \quad (64)$$

In the heavy quark limit,  $\mu_i \approx m_i$  and (61) coincides with the term proportional to  $\frac{1}{R} \frac{d\varepsilon}{dR}$  in (8). For light quarks, when  $\mu_i \gg m_i$  however one has twice as large coefficient in(61) which coincides with the heavy-quark expansion with the light quark expression. Thus our total expression for SD potential treated as perturbation is a  $2 \times 2$  matrix

$$\tilde{V}_{SD}^{(light\ quarks)} = \sum_{i,j} V_{diag}^{(ij)} + V_{SD}^{(\varepsilon)} \quad (65)$$

where  $V_{diag}^{(ij)}$  is given in (61), (45), (52) and  $V_{SD}^{(\varepsilon)}$  is given in (64), with general form obtained by replacing  $\sigma \rightarrow \frac{\partial \varepsilon}{\partial R}$ . Now one can see from these expressions that we have a full correspondence between terms in (8) and in (65), where each term in (65) is obtained from the corresponding one in (8) by replacement  $m_i \rightarrow \mu_i$ , except for the term with  $\frac{d\varepsilon}{dR}$ , where one replaces  $2m_i^2 \rightarrow \mu_i(\mu_i + m_i)$ .

## 7 Concluding remarks

We have obtained all perturbative and nonperturbative spin-dependent terms in the 3q system in the approximation when lowest (bilocal) field correlator is retained in Wilson loop. The analogous procedure for mesons in [18, 19] yielded SD potentials satisfying Gromes relation [24], with correct asymptotics at large distances of Thomas precession type. For the 3q system we also get this asymptotics for spin-orbit terms in the form of a sum of one-body Thomas terms, in agreement with earlier results in [26]. All other nonperturbative terms and exact nonasymptotic form of Thomas terms are new. The signs of perturbative and nonperturbative spin-orbit terms are different and one may expect some cancellation, which should be checked in exact calculations of baryon spectra with spin splittings. All nonperturbative SD terms in (8) except for  $V_5$  have the structure similar to that of the  $Q\bar{Q}$  case, considered in [19], except that spin-orbit terms are of one-body rather than the two-body character. The new term  $V_5$  (24) does not have a  $Q\bar{Q}$  analog, and after averaging over coordinates has a structure similar to two-body spin-orbit force.

The large  $N_c$  structure of SD interaction can be clearly seen from explicit expressions and may be represented as leading ( $O(N_c^0)$ ) terms of one-body spin-orbit interaction, when both fields in the field correlator are on the same sheet of the 3-sheet surface, and suppressed ( $O(N_c^{-1})$ ) terms of spin-spin interactions and spin-orbit from two different sheets. Hence the 3q dynamics in the large  $N_c$  limit reduces to the uncorrelated motion of  $N_c$  quarks around a common center (string junction), which can be taken as infinitely heavy.

The general structure of the SD potential (8) at large  $N_c$  is in agreement with the classification done in [29] where the unsuppressed at large  $N_c$  terms are one-body spin-orbit potentials, while two-body spin-dependent terms are  $1/N_c$  suppressed. In addition in [29] appear also spin-flavour terms which can be associated with pion and kaon exchange forces. The latter were not considered in the present paper, but can be easily included in the same formalism, using the new chiral Lagrangian derived in [30]. It is shown there that in the  $q$ -string- $\bar{q}$  system pions are emitted by quarks with the known amplitude, so that the pion-exchange force can be predicted unambiguously and added to the those obtained in the present work. This would complete the overall picture of SD forces in baryon. The author is grateful to J. Goity for useful discussion, remarks and suggestions.

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## Appendix A Field correlators

From general definitions of  $g^2 \langle F_{\mu\nu} F_{\lambda\sigma} \rangle$  through  $D, D_1$  in [11] one gets

$$\begin{aligned} \frac{g^2}{N_c} \text{tr} \langle F_{\mu\nu}(x) \Phi(x, y) F_{\rho\sigma}(y) \Phi(y, x) \rangle &= (\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\rho}) D(z) + \\ &+ \frac{1}{2} [\partial_\mu z_\rho \delta_{\nu\rho} + \text{perm.}] D_1(z) \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \frac{g^2}{N_c} \text{tr} \langle B_i(x) \Phi(x, y) B_j(y) \Phi(y, x) \rangle &= (\delta_{ij} (D(z) + D_1(z) + z^2 \frac{\partial D_1}{\partial z^2}) - \\ &- z_i z_j \frac{\partial D_1}{\partial z^2}) \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \frac{g^2}{N_c} \text{tr} \langle E_i(x) \Phi(x, y) E_j(y) \Phi(y, x) \rangle &= (\delta_{ij} (D(z) + D_1(z) + z_4^2 \frac{\partial D_1}{\partial z^2}) + \\ &+ z_i z_j \frac{\partial D_1}{\partial z^2}) \end{aligned} \quad (\text{A.3})$$

$$\frac{g^2}{N_c} \text{tr} \langle B_i(x) \Phi(x, y) E_j(y) \Phi(y, x) \rangle = e_{ijk} z_4 z_k \frac{\partial D_1}{\partial z^2} \quad (\text{A.4})$$

where we have defined

$$z_\mu = x_\mu - y_\mu, \quad \mu = 1, 2, 3, 4, \quad \Phi(x, y) = P \exp ig \int_y^x A_\mu(u) du_\mu$$

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