

# Nonperturbative quark dynamics in baryon

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## Abstract

Field-correlator method is used to calculate nonperturbative dynamics of quarks in a baryon. General expression for the 3q Green's function is obtained using Fock-Feynman-Schwinger (world-line) path integral formalism, where all dynamics is contained in the 3q Wilson loop with spin-field insertions. Using lowest cumulant contribution for Wilson loop one obtains an Y-shaped string with a deep hole at the string junction position. Using einbein formalism for the quark kinetic terms one automatically obtains constituent quark masses, calculable through the string tension. Resulting effective action for 3q plus Y-shaped strings is quantized in the path-integral formalism to produce two versions of Hamiltonian, in the c.m. and another in the light-cone system, incorporating confining strings with string junction and quarks with dynamically produced constituent masses. Hyperspherical formalism is used to solve for masses and wave functions. Simple estimates in lowest approximation yield baryon masses in good agreement with experiment without fitting parameters.

## 1 Introduction

Baryons are for a long time an object of an intensive theoretical study [1]-[11]. Both the perturbative dynamics and confinement interaction were considered decades ago [1]-[5] and a series of papers of Isgur and collaborators [7, 8] has enlightened the structure of the baryon spectrum in good general agreement

with experiment. In those works dynamics was considered as a QCD motivated and relativistic effects in kinematics have been accounted for. Recently a more phenomenological approach based on large  $N_c$  expansion for baryons [12]-[15] was applied to baryon spectra [16, 17] and clearly demonstrated the most important operators forming the spectrum of 70-plet.

Summing up the information from the quark-based model one has a picture of baryon spectra with basically oscillator-type spectrum, modified by presence of spin-dependent forces and other corrections. E.g. the orbital excitation with  $\Delta L = 1$  "costs" around 0.5 GeV, while the radial one (actually two types) amounts to around 0.8 GeV. Moreover, hyperfine splitting, which in experiment is large (for  $\Delta - N$  system it is 0.3 GeV) is underestimated using perturbative forces with  $\alpha_s \approx 0.4$  and spin-orbit splitting typically small in experiment, needs some special cancellations in theory [18].

Moreover, some states cannot be well explained in the standard quark models. A good example is  $N^*(1440)$ , which is too low to be simply a radial excitation and moreover its experimental electroproduction amplitudes [19] are in evident conflict with theory [20].

This example is probably not unique, and one can notice an interesting pattern in "radial" excitations of  $N, \Delta, \Lambda$  and  $\Sigma$ : in all cases three lowest states  $M_1, M_2, M_3$  have intervals  $\Delta_1 \equiv M_2 - M_1 \sim 400 \div 500$  MeV,  $\Delta_2 \equiv M_3 - M_1 \sim 600 - 700$  MeV.

Namely, in

$N(939)$	$\Delta(1232)$	$\Lambda(1116)$	$\Sigma(1193)$
$N(1440)$	$\Delta(1600)$	$\Lambda(1600)$	$\Sigma(1660)$
$N(1710)$	$\Delta(1920)$	$\Lambda(1810)$	$\Sigma(1770)$
			$\Sigma(1880)$
$\Delta_1 = 500,$ $\Delta_2 = 770;$	$\Delta_1 = 370,$ $\Delta_2 = 690;$	$\Delta_1 = 484,$ $\Delta_2 = 700;$	$\Delta_1 = 467,$ $\delta_2 = 600$

one can see that  $\Delta_2$  corresponds to usual radial excitation, while the energy interval  $\Delta_1$  cannot be explained in a simple way in standard quark models.

One can say more about difficulties with the interpretation of the  $\Lambda(1405)$  state, quantitative descriptions of  $\Delta N$  transitions etc. [19, 20].

In this situation it sounds reasonable to apply new dynamical approaches, which are directly connected to the basic QCD Lagrangian and where all approximations can be checked both theoretically and numerically on the lattice.

Here belong the field correlator method (FCM) started in [21, 22] (for a review see [23] and for more dynamical applications in [24]). It is aimed at expressing all observables in terms of gauge-invariant field correlators. Its use is largely facilitated by recent observation on the lattice [25, 26], that the lowest bilocal correlator gives dominant contribution to the quark-antiquark forces, while higher correlators contribute around 1%. The use of FCM for meson spectra [27, 28] has shown, that gross features of spectra can be calculated through only string tension, while fine and hyperfine structure require the knowledge of another characteristics of bilocal correlator – the gluon correlation length [29] which is known from lattice data [30] and analytic calculations [31, 32].

In dynamical applications of the FCM to baryon spectra two different schemes are used presently; the relativistic Hamiltonian method (RHM) and the method of Dirac orbitals. The first was suggested in [33, 34] and used for baryon Regge trajectories in [35, 36] and for magnetic moments in [37]. The second method was suggested in [38] and exploited to calculate baryon magnetic moments in [39].

Recently an important element was added to the RHM for baryons, namely all spin-dependent forces between quarks have been calculated in the same Gaussian approximations [40]. To finalize the RHM for baryons one still needs to construct the full baryonic Hamiltonian taking into account the energy of string motion, nonperturbative self-energy corrections [41] etc. The present paper is aimed at the fulfilling this task. It contains the detailed derivation of the full baryonic Hamiltonian both in the c.m. and in the light-cone system of coordinates, simple estimates of spectra for spin-averaged masses and a preparatory discussion of future explicit detailed calculations.

The paper is organized as follows. In section 2 the  $3q$  Green's function is written down using the Fock-Feynman-Schwinger representation [42], in section 3 the c.m. and relative coordinates are introduced and the einbein field  $\mu(t)$  is introduced which will give rise to the quark constituent mass. In section 4 the resulting effective action is quantized and the full c.m. Hamiltonian is explicitly written down.

Section 5 is devoted to the derivation of the light-cone Hamiltonian, its

physical interpretation and correspondence to the partonic model.

In section 6 the construction of the baryon wave function is discussed, expansion of its coordinate part into a sum of hyperspherical harmonics and analytic estimates of spin-averaged spectra.

Spin-dependent forces are discussed in section 7 and section 8 is devoted to conclusions.

## 2 Baryon Green's function

One can define initial and final states of baryon as a superposition of 3q states

$$\Psi_{in,out}(x^{(1)}, x^{(2)}, x^{(3)}) = \Gamma_{in,out} e_{a_1 a_2 a_3} \prod \psi_{\gamma_i}^{a_i}(x^{(i)}, x^{(0)}) \quad (1)$$

where  $a_i$  are color indices, while  $\gamma_i$  contain both flavour and Dirac indices, and a sum over appropriate combinations of these last indices is assumed with  $\Gamma$  as coefficients.

The 3q Green's function can be written as

$$G_{3q}(x^{(i)}|y^{(k)}) = \langle tr_Y \Gamma_{out} \prod_{i=1}^3 S_{a_i b_i}^{(i)}(x^{(i)}, y^{(i)}) \Gamma_{in} \rangle \quad (2)$$

where we have neglected the quark determinant and defined

$$tr_Y = \frac{1}{6} \sum_{a_i, b_i} e_{a_1 a_2 a_3} e_{b_1 b_2 b_3} \quad (3)$$

and  $S^{(i)}(x^{(i)}, y^{(i)})$  is the quark Green's function in the external gluonic fields (vacuum and perturbative gluon exchanges). For the latter one can use the exact Feynman-Schwinger (FS) form [22, 24, 42]

$$S(x, y) = (m - \hat{D}) \int_0^\infty ds (Dz)_{xy} e^{-K} W_z(x, y) \exp g \int_0^s \sigma_{\mu\nu} F_{\mu\nu}(z(\tau)) d\tau \quad (4)$$

where  $W_z$  is the phase factor along the contour  $C_z(x, y)$  starting at  $y$  and finishing at  $x$ , which goes along trajectory which is integrated in  $(Dz)_{xy}$ ,  $K = m^2 s + \frac{1}{4} \int_0^s \dot{z}^2 d\tau$

$$W_z(x, y) = P \exp ig \int_y^x A_\mu dz_\mu \quad (5)$$

and  $P$  is the ordering operator, while

$$\sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \sigma\mathbf{B}, & \sigma\mathbf{E} \\ \sigma\mathbf{E}, & \sigma\mathbf{B} \end{pmatrix} \quad (6)$$

The average over gluon fields, implied in (2) by angular brackets, is convenient to perform after the nonabelian Stokes theorem is applied to the product of  $W_z$ .

Consider to this end the gauge-invariant quantity

$$W_3(x, y) \equiv \text{tr}_Y \prod_{i=1}^3 W_{z_i}(x, y) \quad (7)$$

We can now write the nonabelian Stokes theorem expressing  $A_\mu$  in  $W_{z_i}$  (e.g. using general contour gauge) through  $F_{\mu\nu}(u, x^{(0)}) = \phi(x^{(0)}, u)F_{\mu\nu}(u)\phi(u, x^{(0)})$  where  $\phi(x, y)$  is a parallel transporter, and  $x^{(0)}$  convenient to choose at the common point  $x = x^{(1)} = x^{(2)} = x^{(3)}$ .

Making final points also coincident,  $y = y^{(1)} = y^{(2)} = y^{(3)}$ , one can use the identity

$$\text{tr}_Y \phi_{a_1 b_1}(x, y) \phi_{a_2 b_2}(x, y) \phi_{a_3 b_3}(x, y) = 1 \quad (8)$$

and rewrite  $W_3$  as

$$\langle W_3(x, y) \rangle = \text{tr}_Y \exp \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int_{\sum s_i} \langle \langle F(1) \dots F(n) \rangle \rangle d\sigma(1) \dots d\sigma(n) \quad (9)$$

Note that integration in (9) is over all three lobes, made of contours  $C_{z_i}(x, y)$  and the string junction trajectory  $z_Y(s)$ , with  $z_Y(0) = y, z_Y(1) = x$ . The actual form of  $z_Y(s)$  is defined by the minimal action principle and not necessarily coincides with the trajectory of the center of mass of  $3q$  system. An important specification is needed at this point.

For this case of 3-lobe loop as well as for several separate loops one can use the following gauge-invariant averaging formula, where both field correlators are transported to one point  $x$  and  $a, b, c$  are fundamental color indices

$$\langle F(u, x)_{ab} F(v, x)_{cd} \rangle = \frac{\langle \text{tr}(F(u, x)F(v, x)) \rangle}{N_c^2 - 1} (\delta_{ad}\delta_{bc} - \frac{1}{N_c} \delta_{ab}\delta_{cd}). \quad (10)$$

Now whenever  $F(u, x), F(v, x)$  are on the same lobe, then indices  $b$  and  $c$  coincide and one obtains

$$\text{tr}_Y \langle F(u, x)_{ab} F(v, x)_{bd} \rangle = \frac{\langle \text{tr} F(u, x)F(v, x) \rangle}{N_c} \quad (11)$$

where (8) was used. For  $u, v$  on different lobes, one instead have

$$tr_Y \langle F(u, x)_{ab} F(v, x)_{cd} \rangle = \frac{\langle tr(F(u, x)F(v, x)) \rangle}{N_c(N_c^2 - 1)}. \quad (12)$$

As the last step in this chapter, one can include the quark spin operator  $\sigma_{\mu\nu} F_{\mu\nu}$  into the cluster expansion (9), with the help of relation

$$\begin{aligned} & \langle F_{\mu\nu}(u, x) \exp ig \int_S F_{\lambda\sigma}(v, x) ds_{\lambda\sigma}(v) \rangle = \\ & = \frac{1}{ig} \frac{\delta}{\delta s_{\mu\nu}(v)} \langle \exp ig \int_S F_{\lambda\sigma}(v, x) ds_{\lambda\sigma}(v) \rangle. \end{aligned} \quad (13)$$

Exponentiating the operator  $F_{\mu\nu}$  one arrives at the shift operator  $\exp \frac{1}{i}(s_{\mu\nu} \frac{\delta}{\delta s_{\mu\nu}(u)})$  and finally gets

$$\begin{aligned} & \langle W_3 \exp g \sum_{i=1}^3 \sigma_{\mu\nu}^{(i)} \int_0^{s_i} F_{\mu\nu}(z^{(i)}(\tau_1^{(i)})) d\tau^{(i)} \rangle \equiv \langle W_3 \exp(g\sigma F) \rangle = \\ & tr_Y \exp \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int_{\sum S_i} \langle \langle F(1) \dots F(n) \rangle \rangle d\rho(1) \dots d\rho(n) \end{aligned} \quad (14)$$

where we have defined  $d\rho(n) = \sum_{i=1}^3 d\rho^{(i)}(n)$

$$d\rho^{(i)}(n) = ds_{\mu_n \nu_n}^{(i)}(u^{(n)}) + \frac{1}{i} \sigma_{\mu_n \nu_n}^{(i)} d\tau^{(i)}(n). \quad (15)$$

Here index  $i = 1, 2, 3$  refers to three lobes of the total surface  $S_3$ , and it is understood that whenever  $F(N)$  under the cumulant sign  $\langle \langle \dots \rangle \rangle$  is multiplied by  $d\tau^{(i)}$ , it is taken at the point  $z^{(i)}(\tau_n^{(i)})$ , lying on the quark trajectory  $z^{(i)}(\tau)$  which forms the boundary of the lobe ( $i$ ).

Inserting (4), (14) into (2) one obtains

$$G_{3q}(x, y) = tr_L [\Gamma_{out} \prod_{i=1}^3 (m_i - \hat{D}^{(i)})_R \int_0^\infty ds_i (Dz^{(i)})_{xy} e^{-K_i} \langle W_3 \exp(g\sigma F) \rangle \Gamma_{in}]. \quad (16)$$

Here  $tr_L$  is the trace over Lorentz indices, and  $(m_i - \hat{D}^{(i)})_R$  is the value of operator  $(m_i - \hat{D}^{(i)})$  when acting on the path integral, which was found in [43] to be

$$(m_i - \hat{D}^{(i)})_R = m_i - i\hat{p}^{(i)}. \quad (17)$$

Eqs. (16), (14) give an exact and most general expression for the  $3q$  Green's function, which is however intractable if all FC are retained there.

To simplify we shall use the observation from lattice calculations [25, 26] that lowest (Gaussian) correlator gives the dominant contribution (more than 95%) to the static  $Q\bar{Q}$  quark potential. Assuming that situation is similar for  $3Q$  case and also for light baryons, we now keep in (14) and (9) only lowest cumulant  $\langle\langle FF \rangle\rangle$  and express it in terms of scalar function  $D, D_1$  as in [21]

$$\frac{g^2}{N_c} \langle tr F_{\mu\nu}(u, x) F_{\rho\lambda}(v, x) \rangle = (\delta_{\mu\rho}\delta_{\nu\lambda} - \delta_{\mu\lambda}\delta_{\nu\rho}) D(u-v) + \frac{1}{2} \left[ \frac{\partial}{\partial u_\mu} (u-v)_\rho \delta_{\nu\lambda} + perm. \right] D_1(u-v). \quad (18)$$

Here we have replaced parallel transporters  $\Phi(u, x)\Phi(x, v)$  by the straight-line transporter  $\Phi(u, v)$ , since for the generic situation with  $|u-v| \sim T_g$ ,  $|u-x| \sim |v-x| \sim R$ ,  $R \gg T_g$ , the former and the later are equal up to the terms  $O((T_g/R)^2)$ . Now in view of (12), (13) one can write in Gaussian approximation

$$\begin{aligned} \langle W_3 \exp(g\sigma F) \rangle &= \exp\left[-\frac{g^2}{2N_c} \sum_{i=1}^3 \int \langle tr F(u) F(v) \rangle d\rho^{(i)}(u) d\rho^{(i)}(v) - \right. \\ &\quad \left. - \frac{g^2}{N_c(N_c-1)} \sum_{i \neq j} \int \langle tr F(u) F(v) \rangle d\rho^{(i)}(u) d\rho^{(j)}(v) \right] \end{aligned} \quad (19)$$

where  $d\rho^{(i)}$  is defined in (15). Here  $\langle tr FF \rangle$  can be expressed in terms of  $D, D_1$  and one has a closed expression for the term  $\langle W_3 \exp(g\sigma F) \rangle$ , which acts as a dynamical kernel in the path integral (16).

Now for large sizes of Wilson loop  $W_3$ , such that  $R^{(i)} \gg T_g$ , one can discard  $D_1$  and retain  $D$  in (18), since only the latter ensures area law (and moreover, lattice data [30] show that  $D_1 \ll D$ ). Then the diagonal terms in the sum of the exponent in (19) can be written as (neglecting spin-dependent part for the moment)

$$\langle W_3 \rangle_{diag} = \exp(-\sigma(S_1 + S_2 + S_3)) \quad (20)$$

where  $S_i$  is the area of the minimal surface between trajectory of quark ( $i$ ) and trajectory of string junction (Y-trajectory), and we have used the relation [21, 22]

$$\sigma = \frac{1}{2} \int d^2x D(x). \quad (21)$$

Let us turn now to nondiagonal terms in (19). Since  $D(x), D_1(x)$  are dying exponentially fast for  $x > T_g$  [?], only region of width  $T_g$  around  $Y$  trajectory contributes to this term, which one can write as

$$V_{nondiag.T} = \sum_{i \neq j} V_{nondiag.}^{(ij)} = \frac{g^2}{N_c(N_c - 1)} \sum_{i \neq j} \int \langle \text{tr} F_{\mu\nu}(u) F_{\rho\lambda}(v) \rangle ds_{\mu\nu}^{(i)}(u) ds_{\rho\lambda}^{(j)}(v). \quad (22)$$

Separating out time components,  $u = (u_4, \mathbf{u})$ ,  $v = (v_4, \mathbf{v})$ , one can write for  $D$  contribution

$$V_{nondiag.}^{(D)} = \frac{1}{N_c - 1} \sum_{i \neq j} \int D(u^{(i)} - v^{(j)}) d(u_{\parallel}^{(i)} - v_{\parallel}^{(j)}) d\left(\frac{u_{\perp}^{(i)} + v_{\perp}^{(j)}}{2}\right) d(u_4^{(i)} - v_4^{(j)}) \quad (23)$$

where we have introduced for  $\mathbf{u}, \mathbf{v}$  parallel and transverse components.

Since  $|u^{(i)} - v^{(j)}| = \sqrt{(u_4^{(i)} - v_4^{(j)})^2 + (u_{\parallel}^{(i)} - v_{\parallel}^{(j)})^2 + (u_{\perp}^{(i)} - v_{\perp}^{(j)})^2}$  is growing fast with  $|u_{\perp} - v_{\perp}|$  one can estimate (20), (23) as

$$V_{nondiag.}^{(D)} \sim \sigma T_g, \quad V_{diag.}^{(D)} = \sigma(R^{(1)} + R^{(2)} + R^{(3)}). \quad (24)$$

Being always smaller than  $V_{diag.}^{(D)}$  for large  $R^{(i)}$ , nevertheless  $V_{nondiag.}^{(D)}$  brings about an interesting cancellation for small  $R^{(i)}$ . Estimating integrals in (23) for small  $R^{(i)}$ ,  $R^{(i)} \lesssim T_g$ , and using for  $D(x)$  the Gaussian form,  $D(x) = D(0) \exp(-\frac{x^2}{4T_g^2})$ , one has

$$V^{(3)} = V_{diag.} + V_{nondiag.} = \frac{\sigma}{2\sqrt{\pi}T_g} (\sum_i \mathbf{R}^{(i)})^2. \quad (25)$$

It is clear that for a symmetric configuration  $R^{(1)} = R^{(2)} = R^{(3)}$  one has  $V^{(3)} = 0$ . To study further this cancellation let us take into account that if the triangle made of quarks has all angles less than  $120^\circ$  (since string junction is at Torricelli point). In this case one can write

$$R^2 \equiv (\sum_i \mathbf{R}^{(i)})^2 = (R^{(3)} - R^{(1)})^2 + (R^{(2)} - R^{(1)})^2 - (R^{(3)} - R^{(1)})(R^{(2)} - R^{(1)}) \quad (26)$$

i.e.  $V^{(3)}$  vanishes quadratically in differences of quark distances from the string junction. Practically this brings about a strong effective cancellation in  $V^{(3)}$  for  $3q$  system with equal masses at approximately equal distances.



Numerically and analytically this fact was discovered first in [44] for static  $3Q$  potential. It was argued there, that the cancellation brings about smaller slope of  $V(3Q)$  at small to intermediate distances, as was indeed found on the lattice [45]. The reference to a triangular configuration in [45] is irrelevant, since the latter is impossible to construct in a gauge-invariant way [46]. Explicit expressions for  $V(3)$  in general case are given in Appendix of [46].

### 3 Gaussian representation for the effective action of quarks and string

Consider now the exponent of the FSR for the  $3q$  Green's function (16), (19) in the simplified case when i) spin interaction is neglected and ii) large distances  $|\mathbf{R}_i| \gg T_g$  are taken into account.

In this situation one can use the form (20) instead of (19), and writing the exponential term in (16) as

$$G_{3q}(x, y) = tr_L[\Gamma_{out} \prod (m_i - i\hat{p}_i) \int_0^\infty ds_i (Dz_4^{(i)})_{xy} \Gamma_{in}] e^{-A} \quad (27)$$

where  $A$  plays the role of effective action,

$$A = \sum_{i=1}^3 (K_i + \sigma S_i). \quad (28)$$

Our purpose is finally to construct the effective Hamiltonian, considering  $A$  as an effective action for 3 quarks and the composite string with the string junction. To achieve this goal one must i) go over to the real time corresponding to the chosen hypersurface in the  $4d$  Euclidean time (later on to be transformed into Minkowskian time), ii) to transform the Nambu-Goto form at  $S_i$  into a quadratic form, as it is necessarily done in string theory (since otherwise the path integral (27) is not properly defined). Both operations are the same as in the  $q\bar{q}$  case, considered in [34] and we shall follow closely that procedure.

The resulting Hamiltonian depends on the choice of the hypersurface, and for the  $q\bar{q}$  system both the c.m. [34] and light-cone [47, 48] cases were considered.

Below the c.m. Hamiltonian will be derived, and to this end we choose the hyperplane intersecting all 3 quark trajectories and  $Y$  trajectory at one

common time  $t$ , to be considered in the interval  $0 \leq t \leq T$ , so that quark coordinates are  $z^{(i)} = (t, \mathbf{z}^{(i)})$ , and string junction coordinate  $z_Y = (t, \mathbf{z}_y)$ .

Now one can make a change of variables, introducing the einbein variable [34, 49]  $\mu(t)$  for a given trajectory  $z^{(i)}(\tau^{(i)})$ ,  $0 \leq \tau \leq s$ , one defines

$$d\tau^{(i)} \equiv \frac{dz_4^{(i)}(\tau^{(i)})}{\frac{dz_4^{(i)}(\tau^{(i)})}{d\tau^{(i)}}} = \frac{dz_4^{(i)}(\tau^{(i)})}{2\mu_i(z_4^{(i)})} \quad (29)$$

so that kinetic term  $K_i$  becomes

$$K_i = \int_0^T dt \left[ \frac{m_i^2}{2\mu_i(t)} + \frac{\mu_i(t)}{2} (\dot{\mathbf{z}}^2(t) + 1) \right]. \quad (30)$$

The transition from the integral over  $ds_i dz_4^{(i)}$  to the integral over  $D\mu^{(i)}(t)$  is known to have a nonsingular Jacobian (see and Appendix A of [34] for more details and explanations)

$$D\mu^2(t) \sim \exp\left[-i \frac{\text{const}}{\varepsilon} \int_0^T \sqrt{\mu^2(t)} dt\right] ds D z_4(t) \quad (31)$$

where  $\varepsilon \sim 1/\Lambda$ , and  $\Lambda$  is an ultraviolet cut-off parameter.

Hence the integrals in (27) can be rewritten as

$$\prod_i ds_i D^4 z^{(i)} \rightarrow \prod D\mu_i D^3 z^{(i)} \quad (32)$$

where the integration measure for  $D\mu_i$  can be specified further to be [34]:  $D\mu(t) \sim \prod_{n=1}^N \frac{d\mu(t_n)}{\mu^{3/2}(t_n)}$ . As a next step one introduces the c.m. and relative coordinates

$$R = \frac{1}{\mu_+(t)} \sum \mu_i(t) \mathbf{z}^{(i)}(t) \\ \dot{\boldsymbol{\xi}} = \sqrt{\frac{3}{2}} \left( \frac{\mu_1 \dot{\mathbf{z}}_1 + \mu_2 \dot{\mathbf{z}}_2}{2} - \mu_3 \dot{\mathbf{z}}_3 \right) \frac{1}{\mu_+}, \quad \dot{\boldsymbol{\eta}} = \frac{\mu_1 \dot{\mathbf{z}}_1 - \mu_2 \dot{\mathbf{z}}_2}{\mu_+ \sqrt{2}}. \quad (33)$$

From our discussion above it is clear that the time  $t$  coincides with the fourth component of the c.m. coordinate,  $t = R_4$ , and the whole quark kinetic term in (28):

$$\sum_{i=1}^3 K_i = \int_0^T dt \left[ \sum \left( \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \frac{1}{2} \mu_+(t) \mathbf{R}^2 + \frac{1}{2} \mu_\eta \dot{\boldsymbol{\eta}}^2 + \frac{1}{2} \mu_\xi \dot{\boldsymbol{\xi}}^2 \right]. \quad (34)$$

The area-law term in (28) can be written as follows

$$\sum_{i=1}^3 \sigma S_i = \sigma \sum_{i=1}^3 \int_0^T dt \int_0^1 d\beta_i \sqrt{(\dot{w}_\mu^{(i)})^2 (w'^{(i)})^2 - (\dot{w}_\mu^{(i)} w_\mu'^{(i)})^2} \quad (35)$$

where  $w_\mu^{(i)}(t, \beta)$  is the  $i$ -th string position at the time  $t$  and coordinate  $\beta$  along the string. In the spirit of our approach one should take the world sheets of the strings corresponding to the minimal area of the sum of surfaces between quark trajectories and a  $Y$ -trajectory of the string junction. At this point we make a simplifying approximation [33, 34] that strings at any moment  $t$  can be represented by pieces of straight lines. In this way one disregards string excitations (hybrids) and mixing between these excitations and ground state baryons. This can be done for ground states since the mass gap for string excitations is around 1 GeV [24].

For higher excited states the mixing should be taken into account analogously to what was done in meson sector [50].

Thus one writes

$$w_\mu^{(i)}(t, \beta) = z_\mu^{(i)}(t)\beta + z_\mu^{(Y)}(t)(1 - \beta) \quad (36)$$

and time derivatives of  $w_\mu^{(i)}$  in (32) can be replaced using (33) by time derivatives of  $z_\mu^{(i)}, z_\mu^{(Y)}$ . In this way the string does not possess the dynamical degrees of freedom of its own (in this straight-line approximation). To recover the latter one can use background perturbation theory and consider the states with  $3q$  and additional valence gluon(s). The latter describes gluonic excitation of baryon and has its own dynamical degree of freedom. Note that this way of systematic description of string excitation is different from the *ad hoc* assumption that string is described by Nambu-Goto action with all dynamical string degrees of freedom included, which does not follow from the QCD Lagrangian.

Consider now string-junction trajectory. In line with the whole approach one requires that at any given moment  $z_\mu^{(Y)}(t)$  occupies the position which gives the minimal string energy, i.e.  $z_\mu^{(Y)}(t)$  should coincide with the Torricelli point, giving the minimum of the sum of lengths of 3 strings:

$$L = \sum_{i=1}^3 |\mathbf{z}^{(i)}(t) - \mathbf{z}^{(Y)}(t)|, \quad \frac{\partial L}{\partial z_k^{(Y)}(t)} = 0. \quad (37)$$

Therefore  $\mathbf{z}^{(Y)}(t)$  is not an independent dynamical d.o.f. and  $\mathbf{z}^{(Y)}$  is expressed in terms of  $\mathbf{z}^{(i)}$ ,  $i = 1, 2, 3$ .

Now one can introduce (as is usual in string theory [51]) the auxiliary fields (einbein fields [49]) to replace the untractable square-root terms in (35) by quadratic expressions. In this way one writes

$$S_i = \frac{1}{2\tilde{\nu}_i} [(\dot{w}^{(i)})^2 + (\sigma\tilde{\nu}_i)^2(r^{(i)})^2 - 2\eta_i(\dot{w}_k^{(i)}r_k^{(i)}) + (\eta_i)^2(r^{(i)})^2]. \quad (38)$$

Here  $\tilde{\nu}_i(t, \beta) \geq 0$  and  $\eta_i(t, \beta)$  are two einbein fields (which are integrated out to yield back the form (35)), and  $\mathbf{r}^{(i)} = \mathbf{z}^{(i)} - \mathbf{z}^{(Y)}$ . As a result one has for the  $3q$  Green's function

$$G_{3q}(x, y) = \int DRD\xi D\eta \prod_{i=1}^3 D\mu_i D\tilde{\nu}_i D\eta_i \text{tr}(\Gamma_{out}(m_i - i\hat{p}_i)\Gamma_{in}) e^{-A}. \quad (39)$$

## 4 Quantization of the strings and derivation of the $3q$ -string Hamiltonian

The action (28) using (36), (38) and can be written as

$$\begin{aligned} A = \int_0^t dt \sum_{i=1}^3 \left[ \frac{m_i^2}{2\mu_i} + \frac{\mu_i \dot{\mathbf{z}}_i^2}{2} + \frac{\mu_i}{2} + \int_0^1 d\beta_i \frac{\sigma^2 r_i^2}{2\nu_i} + \right. \\ \left. + \frac{1}{2} \int_0^1 d\beta_i \nu_i (\dot{\mathbf{r}}_i \beta_i + \dot{\mathbf{r}}_Y)^2 + \frac{1}{2} \int_0^1 d\beta_i \nu_i \eta_i^2 \mathbf{r}_i^2 - \right. \\ \left. - \int_0^1 d\beta_i \nu_i \eta_i \mathbf{r}_i (\dot{\mathbf{r}}_i \beta_i + \dot{\mathbf{r}}_Y) \right] \end{aligned} \quad (40)$$

where we have defined  $\nu_i = 1/\tilde{\nu}_i$ , and  $\mathbf{r}_i = \mathbf{z}^{(i)} - \mathbf{r}_Y$ ,  $z_\mu^Y(t) = (t, \mathbf{r}_Y)$ .

As the next step we introduce the c.m. coordinate  $\mathbf{R}$  and Jacobi coordinates  $\boldsymbol{\xi}, \boldsymbol{\eta}$  as follows [36]

$$\begin{aligned} \dot{z}_k^{(1)} &= \dot{R}_k + \left( \frac{\mu\mu_3}{\mu_+(\mu_1 + \mu_2)} \right)^{1/2} \dot{\xi}_k - \left( \frac{\mu\mu_2}{\mu_1(\mu_1 + \mu_2)} \right)^{1/2} \dot{\eta}_k \\ \dot{z}_k^{(2)} &= \dot{R}_k + \left( \frac{\mu\mu_3}{\mu_+(\mu_1 + \mu_2)} \right)^{1/2} \dot{\xi}_k + \left( \frac{\mu\mu_2}{\mu_2(\mu_1 + \mu_2)} \right)^{1/2} \dot{\eta}_k \\ \dot{z}_k^{(3)} &= \dot{R}_k - \left( \frac{\mu(\mu_1 + \mu_2)}{\mu_+\mu_3} \right)^{1/2} \dot{\xi}_k, \end{aligned} \quad (41)$$

with the inverse expressions

$$\begin{aligned}\dot{R}_k &= \frac{1}{\mu_+} \sum_{i=1}^3 \mu_i \dot{z}_k^{(i)}, \quad \dot{\eta}_k = (\dot{z}_k^{(2)} - \dot{z}_k^{(1)}) \left( \frac{\mu_1 \mu_2}{\mu(\mu_1 + \mu_2)} \right)^{1/2} \\ \dot{\xi}_k &= \left( \frac{\mu_3}{\mu_+(\mu_1 + \mu_2)\mu} \right)^{1/2} (\mu_1 \dot{z}_k^{(1)} + \mu_2 \dot{z}_k^{(2)} - (\mu_1 + \mu_2) \dot{z}_k^{(3)})\end{aligned}\quad (42)$$

In (41), (42) the mass  $\mu$  is arbitrary and drops out in final expressions. Using (41) one can rewrite the kinetic part of the action as follows

$$\sum_{i=1}^3 K_i = \int_0^T dt \left[ \sum_{i=1}^3 \left( \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \frac{1}{2} \mu_+ \dot{\mathbf{R}}^2 + \frac{1}{2} \mu (\dot{\boldsymbol{\eta}}^2 + \dot{\boldsymbol{\xi}}^2) \right]. \quad (43)$$

The string part of the action can be transformed using (38) and integrating over  $\eta_i$  to the form

$$\begin{aligned}\exp\left(-\sum_{i=1}^3 \sigma S_i\right) &= \exp\left\{-\int_0^T dt \frac{1}{2} \sum_{i=1}^3 \int d\beta_i [\nu_i ((\dot{\mathbf{r}}^{(i)} \beta_i + \dot{\mathbf{z}}^{(Y)})^2 - \right. \\ &\quad \left. - [(\dot{\mathbf{r}}^{(i)} \beta_i + \dot{\mathbf{z}}^{(Y)}) \mathbf{r}^{(i)}]^2 \frac{1}{(\mathbf{r}^{(i)})^2} + \frac{\sigma^2 \mathbf{r}_i^2}{\nu_i}]\right\}.\end{aligned}\quad (44)$$

At this point it is important to note that  $\mathbf{z}^{(Y)}$  is not a dynamical variable, since it is defined by the minimum of the action.

Taking this minimum at a given moment, one arrives at the definition of  $\mathbf{z}^{(Y)}(t)$  as a Torricelli point, which is to be expressed through the positions  $\mathbf{z}^{(i)}(t)$ ;

$$\mathbf{z}^{(Y)}(t) = f(\mathbf{z}^{(1)}(t), \mathbf{z}^{(2)}(t), \mathbf{z}^{(3)}(t)) \quad (45)$$

where the function  $f$  is defined explicitly in [36]. Therefore  $\dot{\mathbf{z}}^{(Y)}(t)$  is also expressed through  $\dot{\mathbf{z}}^{(i)}(t)$ , or through  $\dot{\mathbf{R}}(t)$  and  $\dot{\mathbf{r}}^{(i)}(t)$ .

Below the simplified procedure will be used, where one identifies  $\mathbf{z}^{(Y)}$  with the c.m. coordinate  $\mathbf{R}$ , which is true on average for equal mass quarks. Explicit formulas for the general case  $\mathbf{z}^{(Y)} \neq \mathbf{R}$  are given in Appendix. We are now in position to get the final coordinates  $\boldsymbol{\eta}$ ,  $\boldsymbol{\xi}$  or their linear combinations  $\mathbf{r}^{(i)} \equiv \mathbf{z}^{(i)} - \mathbf{R}$  read from (41). To this end we replace in (44)  $\dot{\mathbf{z}}^{(Y)}$  by  $\dot{\mathbf{R}}$  and integrate over  $D\dot{\mathbf{R}}$  in both expressions (43), (44) in the same way, as was done in ([34], Eqs. (37)-(49)), with the result

$$\tilde{A} = \int_0^T dt \frac{1}{2} \left\{ \sum_{i=1}^3 \left( \frac{m_i^2}{\mu_i} \mu_i \right) + \mu (\dot{\boldsymbol{\eta}}^2 + \dot{\boldsymbol{\xi}}^2) + \sum_{i=1}^3 \int_0^1 d\beta_i [\nu_i (\beta_i) + \right.$$

$$+\frac{\sigma^2(\mathbf{r}^{(i)})^2}{\nu_i} + \nu_i \beta_i^2 \left[ (\dot{\mathbf{r}}^{(i)})^2 - \frac{(\dot{\mathbf{r}}^{(i)} \mathbf{r}^{(i)})^2}{(\mathbf{r}^{(i)})^2} \right]. \quad (46)$$

The last term on the r.h.s. of (46) can be rewritten as  $\nu_i \beta_i^2 \frac{(\dot{\mathbf{r}}^{(i)} \times \mathbf{r}^{(i)})^2}{\mathbf{r}^2}$  and disappears when the partial angular momentum  $l_i$  vanishes.

In this case (46) simplifies and using in (46)  $\dot{\mathbf{r}}^{(i)}$  instead of  $\dot{\boldsymbol{\eta}}, \dot{\boldsymbol{\xi}}$ , one gets in a standard way the Hamiltonian

$$H_0 = \frac{1}{2} \sum_{i=1}^3 \left[ \frac{m_i^2}{\mu_i} + \mu_i + \frac{\mathbf{p}_i^2}{\mu_i} + \int_0^1 d\beta_i \left( \nu_i(\beta_i) + \frac{\sigma^2(\mathbf{r}^{(i)})^2}{\nu_i} \right) \right]. \quad (47)$$

One can now apply to (47) the minimization procedure to define  $\mu_i, \nu_i$  from the conditions

$$0 = \frac{\partial H_0}{\partial \mu_i} = \frac{\partial H_0}{\partial \nu_i} \quad (48)$$

which yields

$$\mu_i = \sqrt{\mathbf{p}_i^2 + m_i^2}, \quad \nu_i = \sigma |\mathbf{r}_i|. \quad (49)$$

Note that in this case ( $l_i = 0, i = 1, 2, 3$ )  $\nu_i$  does not depend on  $\beta_i$  and play the role of potential. Inserting (49) in (47) one obtains the form well known from the standard relativistic quark model (RQM)[1]-[9]

$$H_{RQM} = \sum_{i=1}^3 \left( \sqrt{\mathbf{p}_i^2 + m_i^2} + \sigma |\mathbf{r}^{(i)}| \right). \quad (50)$$

Note that (50) is valid under assumptions that (1) string-junction  $\mathbf{z}^{(Y)}$  coincides with the c.m. (2)  $\sum_{i=1}^3 \mathbf{p}_i = 0$  (3) all angular momenta of quarks  $l_i, i = 1, 2, 3$  are zero, so that only radial part of momentum  $\mathbf{p}_i$  enters in (50). However, in RQM the form (50) is used without the condition (3). As one will see in what follows, at nonzero  $l_i$  the Hamiltonian  $H_0$  will be modified, and for not large  $l_i, l \leq 4$  this modification can be taken into account as a string correction  $\Delta H_{string}$  similarly to the meson case in [34, 27].

We consider now the general case of  $l_i > 0$ . To this end we separate for each  $\mathbf{r}^{(i)}$  transverse and longitudinal components as follows (omitting index  $i$  for a moment)

$$\dot{\mathbf{r}}^2 = \frac{1}{r^2} \{ (\mathbf{r}\dot{\mathbf{r}})^2 + (\dot{\mathbf{r}} \times \mathbf{r})^2 \} \quad (51)$$

and correspondingly define transverse and longitudinal momenta

$$p_r^2 = \frac{(\mathbf{p}\mathbf{r})^2}{r^2} = \frac{(\mu \mathbf{r}\dot{\mathbf{r}})^2}{r^2}, \quad (52)$$

$$p_T^2 = \frac{(\mathbf{P} \times \mathbf{r})^2}{r^2} = (\mu + \int_0^1 d\beta \beta^2 \nu(\beta))^2 \frac{(\dot{\mathbf{r}} \times \mathbf{r})^2}{r^2}. \quad (53)$$

One can now derive the Hamiltonian from (46) in the usual way

$$H = \sum_{i=1}^3 \left[ \frac{m_i^2 + p_{ri}^2}{2\mu_i} + \frac{\mu_i}{2} + \frac{\hat{l}_i^2 / r_i^2}{2(\mu_i + \int_0^1 d\beta_i \beta_i^2 \nu_i(\beta))} + \frac{\sigma^2}{2} \int_0^1 \frac{d\beta_i}{\nu_i(\beta_i)} r_i^2 + \frac{1}{2} \int_0^1 \nu_i(\beta_i) d\beta_i \right] \quad (54)$$

This is a general form for any values of  $l_i$ , the limit of  $l_i \rightarrow 0$  is obtained in (50). Now we shall derive the opposite limit  $l_i \rightarrow \infty$ . As in the meson case one can argue that in this case  $\mu_i \ll \nu_i$  and one can use the quasiclassical method and retain in (54) only the last three terms, expanding them around the stationary point at  $r^{(i)} = r_0^{(i)}$ , where

$$(r_0^{(i)})^2 = \left[ \frac{\hat{l}_i^2}{2(\mu_i + \int_0^1 d\beta_i \beta_i^2 \nu_i(\beta)) \sigma^2 \int_0^1 \frac{d\beta_i}{\nu_i(\beta_i)}} \right]^{1/2}. \quad (55)$$

Inserting (55) back into (54) one obtains the following quasiclassical energy of the  $i$ -th string,  $E = \sum_{i=1}^3 E_i$

$$E_i = \sigma \sqrt{\hat{l}_i^2} \left( \frac{\int_0^1 d\beta_i \nu_i^{-1}}{\int_0^1 d\beta_i \beta_i^2 \nu_i} \right)^{1/2} + \frac{1}{2} \int_0^1 \nu_i d\beta_i \quad (56)$$

and from the stationary point of  $E_i$ ,  $\frac{\delta E_i}{\delta \nu_i(\beta_i)} = 0$ , one has

$$\nu_i(\beta_i) = \sqrt{\frac{2}{\pi}} (1 - \beta_i^2)^{-1/2}. \quad (57)$$

Inserting (57) into (56) one obtains finally the energy of rotating string

$$E_i^2 = 2\pi\sigma \sqrt{\hat{l}_i^2}, \quad \hat{l}_i^2 \equiv l_i(l_i + 1). \quad (58)$$

This result shows that our general baryonic Hamiltonian indeed admits simple rotating string limit at large  $l_i$ , as it was in the case of mesonic Hamiltonian.

The difference from the mesonic case is however, that for the  $3q$  system one should be careful in proper exclusion of the c.m. motion and in quantizing

angular momenta  $l_i$ , which should add up to a total angular momentum  $\mathbf{L} = \sum_{i=1}^3 \mathbf{l}_i$ . To do this one should go over from  $\mathbf{r}^{(i)}, l_i$  to the independent Jacobi coordinates and momenta  $\boldsymbol{\xi}, \boldsymbol{\eta}, \mathbf{l}_\xi, \mathbf{l}_\eta$  which add up to  $\mathbf{L}$  as  $\mathbf{L} = \mathbf{l}_\xi + \mathbf{l}_\eta$ .

To accomplish this task, one should express  $r_k^{(i)} = z_k^{(i)} - R_k$  using (41) through  $\boldsymbol{\xi}, \boldsymbol{\eta}$  and  $l_i$  through  $l_\xi, l_\eta$ , and insert it into (54) which makes the whole expression rather complicated and not very tractable. Instead we adopt here another strategy and consider the contribution of string rotation as a correction, similarly to the case of mesons [34], where this approach has proved to be successful up to  $L = 4$  [27]. Therefore we shall represent the Hamiltonian (54) as a sum of unperturbed term  $H_0$  plus a string correction  $\Delta H_{string}$ , which should work with accuracy better than 5% up to  $l \approx 3 \div 4$ ,

$$H = H_0 + \Delta H_{string} \quad (59)$$

where we have defined

$$H_0 = \sum_{i=1}^3 \left( \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \frac{\mathbf{p}_\xi^2 + \mathbf{p}_\eta^2}{2\mu} + V_{conf.}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \quad (60)$$

and

$$V_{conf.} = \sigma \sum_{i=1}^3 |\mathbf{z}^{(i)} - \mathbf{z}^{(Y)}| = \sigma \sum_{i=1}^3 |\mathbf{z}_i + \mathbf{R} - \mathbf{z}^{(Y)}|, \quad (61)$$

where we have accounted for the possibility that  $\mathbf{R} \neq \mathbf{z}^{(Y)}$ .

The string correction  $\Delta H_{string}$  is computed to be

$$\Delta H_{string} = - \sum_{i=1}^3 \frac{\hat{l}_i^2 \sigma \langle r_i^{-1} \rangle}{6\mu_i(\mu_i + \frac{1}{3} \langle \sigma r_i \rangle)} \approx - \sum_{i=1}^3 \frac{\hat{l}_i^2 \sigma \langle r_i^{-1} \rangle}{10\mu_i^2}. \quad (62)$$

The total Hamiltonian for the bound 3q system in the c.m. coordinates, taking into account only valence quarks, can now be written as follows.

$$H_{tot} = H_0 + \Delta H_{string} + \Delta H_{coul.} + \Delta H_{self.} + \Delta H_{spin} \quad (63)$$

where  $H_0$  is given in (60),  $\Delta H_{string}$  in (62),  $\Delta H_{spin}$  is given in [40],  $\Delta H_{coul.}$  is easily computed allowing for perturbative gluon exchanges in  $W_3$ , resulting in a standard expression

$$\Delta H_{coul.} = - \frac{2\alpha_s}{3} \sum_{i < j} \frac{1}{|\mathbf{z}^{(i)} - \mathbf{z}^{(j)}|}. \quad (64)$$



As to  $\Delta H_{self}$ , it was found in [41] to originate from the  $\langle \sigma F \sigma F \rangle$  correlator referring to the same quark line. It has the form ([41], Eq.(36))

$$\Delta H_{self} = -\frac{2\sigma}{\pi} \sum_{i=1}^3 \frac{\eta_i}{\mu_i}. \quad (65)$$

where  $\eta_i = 1$  for light quarks.

## 5 The light-cone quantization of the 3q system. Derivation of the light-cone Hamiltonian

The general expression of the 3q Green's function allows to calculate Hamiltonian, corresponding to any prescribed hypersurface, with the evolution parameter  $T$  orthogonal to it, according to the equation (in Euclidean space-time)

$$\frac{\partial G}{\partial T} = -HG. \quad (66)$$

In the previous section the hypersurface was chosen to be  $z_4^{(i)} = const.$ , and the corresponding c.m. Hamiltonian was written in (60), (63). The obtained Hamiltonian is a 3q equivalent of the  $q\bar{q}$  c.m. Hamiltonian found earlier in [34, 47, 48].

The light-cone version of the  $q\bar{q}$  Hamiltonian was derived in [47] and solved numerically in [48].

In this section we shall follow the same technic as in [47] to obtain the 3q Hamiltonian on the light cone. To this end one should choose the hypersurface to be the plane with fixed values of  $z_+^{(i)}$ , where we use the following convention

$$ab = a_\mu b_\mu = a_i b_i - a_0 b_0 = a_\perp b_\perp + a_+ b_- + a_- b_+ \quad (67)$$

and  $a_\pm = \frac{a_3 \pm a_0}{\sqrt{2}}$ .

The same decomposition of quark coordinate  $z_\mu^{(i)}$  will be used as in (41), but for the light cone (l.c.) coordinates (67). Again for simplicity we shall identify  $R_\mu$  and  $z_\mu^{(Y)}$ , so that

$$r_\mu^{(i)} = z_\mu^{(i)} - R_\mu = z_\mu^{(i)} - z_\mu^{(Y)} \quad (68)$$

Some kinematical properties of l.c. coordinate to be used below are

$$\begin{aligned}
w_\mu^{(i)}(z_+, \beta_i) &= z_\mu^{(i)} \beta_i + z_\mu^{(Y)} (1 - \beta_i) = \\
&= r_\mu^{(i)} \beta_i + z_\mu^{(Y)} \cong r_\mu^{(i)} \beta_i + R_\mu, \\
r_\mu^2 &= r_\perp^2, \quad r_+ \equiv 0; \quad \frac{\partial w_\mu^{(i)}}{\partial \beta_i} = r_\mu^{(i)}; \\
\dot{w}_\mu^2 &= \dot{w}_\perp^2 + 2\dot{w}_-; \quad \dot{w}_\perp = \dot{r}_\perp \beta + \dot{R}.
\end{aligned} \tag{69}$$

Having this in mind one can directly obtain the l.c. action from (40) (cf [47] for the equivalent derivation of  $q\bar{q}$  l.c. action)

$$\begin{aligned}
A_{lc} &= \int_0^T dz_+ \sum_{i=1}^3 \left\{ \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} ((\dot{R}_\perp + \dot{r}_\perp^{(i)})^2 + \right. \\
&\quad \left. + 2(\dot{R}_- + \dot{r}_-^{(i)})) + \frac{1}{2} \int_0^1 d\beta_i \left[ \frac{\sigma^2 (r_\perp^{(i)})^2}{\nu_i} + \right. \right. \\
&\quad \left. \left. + \nu_i ((\dot{r}_\perp^{(i)} \beta_i + \dot{R}_\perp)^2 + 2(\dot{r}_-^{(i)} \beta_i + \dot{R}_-)) + \nu_i \eta_i^2 r_\perp^{2(i)} - \right. \right. \\
&\quad \left. \left. - 2\nu_i \eta_i ((\dot{r}_\perp^{(i)} \beta_i + \dot{R}_\perp) r_\perp^{(i)} + r_-^{(i)}) \right] \right\},
\end{aligned} \tag{70}$$

where  $\dot{r}_\mu^{(i)}$ , ( $\mu = \perp, -$ ) can be expressed in terms of  $\dot{\xi}$ ,  $\dot{\eta}$  using parametrization

$$\dot{R}_\mu = \sum_{i=1}^3 x_i z_\mu^{(i)}, \quad \sum_{i=1}^3 x_i = 1, \quad x_i \geq 0 \tag{71}$$

$$\begin{aligned}
\dot{z}_\mu^{(1)} - \dot{R}_\mu &= \left( \frac{x_3}{x_1 + x_2} \right)^{1/2} \dot{\xi}_\mu - \left( \frac{x_2}{x_1(x_1 + x_2)} \right)^{1/2} \dot{\eta}_\mu \\
\dot{z}_\mu^{(2)} - \dot{R}_\mu &= \left( \frac{x_3}{x_1 + x_2} \right)^{1/2} \dot{\xi}_\mu + \left( \frac{x_1}{x_2(x_1 + x_2)} \right)^{1/2} \dot{\eta}_\mu \\
\dot{z}_\mu^{(3)} - \dot{R}_\mu &= - \left( \frac{x_1 + x_2}{x_3} \right)^{1/2} \dot{\xi}_\mu.
\end{aligned} \tag{72}$$

One can rewrite (70) in the same form as in [47]

$$A_{lc} = \frac{1}{2} \int_0^T dz_+ \{ a_1 \dot{R}_\perp^2 + 2a_1 \dot{R}_- + 2a_{2\perp} \dot{R}_\perp + 2a_{2-} -$$

$$\begin{aligned}
& -2c_{1\perp}\dot{R}_\perp + \sum_{i=1}^3 [-2c_{2i}\dot{r}_\perp^{(i)}r_\perp^{(i)} + a_{3i}(\dot{r}_\perp^{(i)})^2 - \\
& \quad -2c_{1i}r_-^{(i)} + a_{4i}(r_\perp^{(i)})^2 + \frac{m_i^2}{\mu_i}] \} \tag{73}
\end{aligned}$$

where we have defined

$$\begin{aligned}
a_1 &= \sum_{i=1}^3 (\mu_i + \int_0^1 \nu_i(\beta) d\beta) = \sum_{i=1}^3 a_{1i} \\
a_{2\perp} &= \sum_{i=1}^3 (\mu_i + \int_0^1 \nu_i(\beta) \beta d\beta) \dot{r}_\perp^{(i)} \\
a_{2-} &= \sum_{i=1}^3 (\mu_i + \int_0^1 \nu_i(\beta) \beta d\beta) \dot{r}_-^{(i)} = \sum_{i=1}^3 a_{2i} \dot{r}_-^{(i)} \\
c_{1\perp} &= \sum_{i=1}^3 \int_0^1 d\beta \nu_i(\beta) \eta_i r_\perp^{(i)}, \quad c_{1i} = \int_0^1 \nu_i(\beta) \eta_i d\beta \tag{74}
\end{aligned}$$

$$\begin{aligned}
c_{2i} &= \int_0^1 d\beta \nu_i(\beta) \beta \eta_i, \quad a_{3i} = \mu_i + \int_0^1 \nu_i(\beta) \beta^2 d\beta; \\
a_{4i} &= \int_0^1 d\beta (\nu_i \eta_i^2 + \frac{\sigma^2}{\nu_i}). \tag{75}
\end{aligned}$$

We now require as in [47] that transverse velocity should be diagonalized, i.e. the mixed term  $a_{2\perp}$  to vanish. This gives conditions on coefficients  $x_i$ , when  $a_{2\perp}$  is expressed in terms of two independent velocities:  $\dot{\xi}_\perp$  and  $\dot{\eta}_\perp$ .

This immediately yields expressions for  $x_i$ :

$$x_i = \frac{\mu_i + \int_0^1 \nu_i(\beta) \beta d\beta}{\sum_{i=1}^3 (\mu_i + \int_0^1 \nu_i(\beta) \beta d\beta)}. \tag{76}$$

Now one can integrate over  $\prod_{i=1}^3 D\eta_i$  in the same way, as if was done for the  $q\bar{q}$  system in [47] with the result

$$\begin{aligned}
A'_{lc} &= \frac{1}{2} \int_0^T dz_+ \{ a_1 (\dot{R}_\perp^2 + 2R_-) + \sum_{i=1}^3 [\int_0^1 d\beta \frac{\sigma^2}{\nu_i} + \frac{m_i^2}{\mu_i} + a_{3i} (\dot{r}_\perp^{(i)})^2 - \\
& \quad - \frac{(r_-^{(i)} + r_\perp^{(i)} \dot{R}_\perp + \langle \beta \rangle_i \dot{r}_\perp^{(i)} r_\perp^{(i)})^2 \int \nu_i d\beta}{(r_\perp^{(i)})^2} - \frac{(\dot{r}_\perp^{(i)} r_\perp^{(i)})^2}{(r_\perp^{(i)})^2} \int_0^1 \nu_i(\beta) (\beta - \langle \beta \rangle_i)^2] \}. \tag{77}
\end{aligned}$$

Here we have defined

$$\langle \beta \rangle_i = \int_0^1 \nu_i(\beta) \beta d\beta / \int_0^1 \nu_i(\beta) d\beta. \quad (78)$$

Our next step is the integration over  $D\dot{R}$ , which is done in the same way as in [47], and choosing the system where transverse total momentum vanish,  $\mathbf{P}_\perp = 0$ , one obtains

$$A_{lc} = \frac{1}{2} \int_0^T dz_+ \sum_{i=1}^3 \left\{ \frac{m_i^2}{\mu_i} + \int_0^1 d\beta \frac{\sigma^2(r^{(i)})^2}{\nu_i} + a_{3i}(\dot{r}_\perp^{(i)})^2 - \right. \\ \left. - \langle \nu_i^{(2)} \rangle \frac{(\dot{r}_\perp^{(i)} r_\perp^{(i)})^2}{(r_\perp^{(i)})^2} - \frac{\langle \nu_i^{(0)} \rangle a_{1i} (r_\perp^{(i)} + \langle \beta \rangle_i \dot{r}_\perp^{(i)} r_\perp^{(i)})^2}{(r_\perp^{(i)})^2 (a_{1i} - \langle \nu_i^{(0)} \rangle)} \right\}. \quad (79)$$

where we have defined

$$\langle \nu_i^{(k)} \rangle = \int_0^1 \nu_i(\beta) (\beta - \langle \beta \rangle_i)^k d\beta, \quad a_{1i} = \mu_i + \int_0^1 \nu_i(\beta) d\beta. \quad (80)$$

Integration over  $D\dot{R}_-$  with  $\exp(iP_\perp \int_0^T \dot{R}_- dz_+)$  yields important constraint  $\delta(a_1 - P_+)$ , i.e.

$$a_1 = P_+ \quad (81)$$

and integration over  $DR_+$  trivial, since  $A_{lc}$  does not depend on  $R_+$ .

Before doing calculations for the l.c. Hamiltonian, one should go over to the Minkowskian action, which is achieved by replacements

$$\mu_i \rightarrow -i\mu_i^M, \quad \nu_i \rightarrow -i\nu_i^M, \\ a_i \rightarrow -ia_i^M, \quad A \rightarrow -iA^M. \quad (82)$$

Omitting the superscript  $M$  in what follows one obtains for the Minkowskian action

$$A_{lc}^{(M)} = \frac{1}{2} \int_0^T dt_+ \sum_{i=1}^3 \left\{ -\frac{m_i^2}{\mu_i} + a_{3i}(\dot{r}_\perp^{(i)})^2 - \int_0^1 \frac{\sigma^2 d\beta}{\nu_i} (r_\perp^{(i)})^2 \right. \\ \left. - \langle \nu_i^{(2)} \rangle \frac{(\dot{r}_\perp^{(i)} r_\perp^{(i)})^2}{(r_\perp^{(i)})^2} - \frac{\langle \nu_i^{(0)} \rangle a_{1i} (r_\perp^{(i)} + \langle \beta \rangle_i \dot{r}_\perp^{(i)} r_\perp^{(i)})^2}{r_\perp^{(i)2} (a_{1i} - \langle \nu_i^{(0)} \rangle)} \right\}. \quad (83)$$

From (83) one can define in a direct way the l.c. Hamiltonian, writing

$$A_{lc}^{(M)} = \int dt_+ L^{(M)}, \quad H = \sum \mathbf{p}_\perp \dot{\mathbf{q}}_\perp - L^{(M)}. \quad (84)$$

One cannot choose  $\mathbf{q}_\perp^{(i)}$  strictly speaking as a set of canonical coordinates  $\mathbf{r}_\perp^{(i)}$ , since they are not independent variables, subject according to (72) to a condition

$$\sum_{i=1}^3 x_i \dot{\mathbf{r}}_\perp^{(i)} = 0. \quad (85)$$

Instead the pair of coordinates  $\dot{\xi}_\perp, \dot{\eta}_\perp$  is independent, and one can define canonical momenta  $\mathbf{p}_\perp^{(\xi)}, \mathbf{p}_\perp^{(\eta)}$ , as

$$\mathbf{p}_\perp^{(\xi)} = \frac{1}{i} \frac{\partial}{\partial \xi_\perp}, \quad \mathbf{p}_\perp^{(\eta)} = \frac{1}{i} \frac{\partial}{\partial \eta_\perp}. \quad (86)$$

We can use nevertheless  $\mathbf{p}_\perp^{(i)} = \frac{1}{i} \frac{\partial}{\partial \mathbf{r}_\perp^{(i)}}$ . Then

$$\mathbf{p}_\perp^{(\xi)} = \sum_{i=1}^3 \mathbf{p}_\perp^{(i)} c_{i\xi}, \quad \mathbf{p}_\perp^{(\eta)} = \sum_{i=1}^3 \mathbf{p}_\perp^{(i)} c_{i\eta}, \quad (87)$$

where  $c_{i\xi}, c_{i\eta}$  are listed from (72)

$$\begin{aligned} c_{1\xi} &= \left( \frac{x_3}{x_1 + x_2} \right)^{1/2} = c_{2\xi}, \quad c_{3\xi} = - \left( \frac{x_1 + x_2}{x_3} \right)^{1/2} \\ c_{1\eta} &= - \left( \frac{x_2}{x_1(x_1 + x_2)} \right)^{1/2}, \quad c_{2\eta} = \left( \frac{x_1}{x_2(x_1 + x_2)} \right)^{1/2}, \quad c_{3\eta} = 0 \end{aligned} \quad (88)$$

We are now in the position to use (84) and calculate the l.c. Hamiltonian,

$$\begin{aligned} H &= \sum_{i=1}^3 \left\{ \frac{m_i^2}{2\mu_i} + \frac{1}{2} \int_0^1 \frac{\sigma^2 d\beta}{\nu_i} + \frac{(\mathbf{p}_\perp^{(i)})^2 - (\mathbf{p}_\perp^{(i)} \mathbf{r}_\perp^{(i)})^2 / (\mathbf{r}_\perp^{(i)})^2}{2a_{3i}} + \right. \\ &\quad \left. + \frac{\langle \nu_i^{(0)} \rangle a_{1i} (r_\perp^{(i)})^2}{2(r_\perp^{(i)})^2 \mu_i} + \frac{(p_\perp^{(i)} r_\perp^{(i)} + \frac{1}{\mu_i} \langle \nu_i^{(0)} \rangle a_{1i} r_\perp^{(i)})^2 \mu_i^2}{2(r_\perp^{(i)})^2 a_{3i} a_{2i}^2 (2\mu_i - a_{2i})^2} \times \right. \\ &\quad \left. \left[ \mu_i a_{3i} + \frac{(a_{2i} - \mu_i)^2}{\mu_i} (a_{3i} - 2\mu_i) \right] \right\}, \end{aligned} \quad (89)$$

where  $a_{mi}$  are defined in (74), (75).

Momenta  $\mathbf{p}_\perp^{(i)}$  are not linearly independent, and from (85) expressing  $\dot{\mathbf{r}}_\perp^{(i)}$  through  $\mathbf{p}_\perp^{(i)}$  one obtains the connection

$$\sum_{i=1}^3 \frac{x_i}{a_{3i}} (\mathbf{p}^{(i)} - C\mathbf{r}^{(i)} + \mathbf{r}^{(i)} D \frac{\mathbf{p}^{(i)} \mathbf{r}^{(i)} - C(\mathbf{r}^{(i)})^2}{a_{3i} - (\mathbf{r}_\perp^{(i)})^2 D}) \quad (90)$$

where we have defined

$$C = -\frac{\langle \nu_i^{(0)} \rangle a_{1i} r_-^{(i)} \langle \beta \rangle_i}{(\mathbf{r}_\perp^{(i)})^2 (a_{1i} - \langle \nu_i^{(0)} \rangle)}, \quad (91)$$

$$D = \frac{\langle \nu_i^{(2)} \rangle (a_{1i} - \langle \nu_i^{(0)} \rangle) + \langle \nu_i^{(0)} \rangle a_{1i} \langle \beta \rangle_i^2}{(\mathbf{r}_\perp^{(i)})^2 (a_{1i} - \langle \nu_i^{(0)} \rangle)} \quad (92)$$

To understand better the structure of the Hamiltonian (89), consider first the limit of heavy quarks  $m_i \gg \sqrt{\sigma}$ , in which case as was shown in [47], the inequality holds  $\mu_i \gg \nu_i, i = 1, 2, 3$ . One has from (74),(75)  $a_{1i} = a_{2i} = a_{3i} = \mu_i$  and the Hamiltonian assumes the form

$$H_{HQ} = \sum_{i=1}^3 \left\{ \frac{m_i^2}{2\mu_i} + \frac{1}{2} \int_0^1 \frac{\sigma^2 d\beta}{\nu_i} (\mathbf{r}_\perp^{(i)})^2 + \frac{(\mathbf{p}_\perp^{(i)})^2 - (\mathbf{p}_\perp^{(i)} \mathbf{r}_\perp^{(i)})^2 / (\mathbf{r}_\perp^{(i)})^2}{2\mu_i} + \right. \\ \left. + \frac{(r_-^{(i)})^2}{2(r_\perp^{(i)})^2} \int_0^1 \nu_i d\beta + \frac{p_\perp^{(i)} r_\perp^{(i)} + \langle \nu_i^{(0)} \rangle (r_-^{(i)})^2}{2(r_\perp^{(i)})^2 \mu_i} \right\}. \quad (93)$$

Introducing the dimensionless quantity  $y_i \equiv \frac{\nu_i}{P_+}$  (which will be shown to independent of  $\beta$  and small,  $y_i \ll 1$  one has from (76) and (81)

$$x_i = \frac{1}{1-Y} \left( \frac{\mu_i}{p_+} + \frac{1}{2} y_i \right), \quad \sum_{i=1}^3 \left( \frac{\mu_i}{P_+} + y_i \right) = 1 \quad (94)$$

where  $Y = \frac{1}{2} \sum y_i$ .

This enables one to expand the mass term in (93) around stationary points in  $x_i$ , and keeping the first order term in  $y_i$  one obtains

$$\sum_{i=1}^3 \frac{m_i^2}{2\mu_i} = \frac{1}{2P_+} \{ M^2 (1 + 2Y) + 2M \sum (x_i - \frac{m_i}{M})^2 \frac{1}{2m_i} \} \quad (95)$$

where  $M = \sum_{i=1}^3 m_i$ .

We now define as in [47] the  $z$ - component of momenta

$$M(x_i - \frac{m_i}{M}) \equiv p_z^{(i)} \quad (96)$$

Keeping now in expansion of (93) only leading terms one obtains  $H_{HQ} = \frac{\mathcal{M}^2}{2P_+}$  with total mass operator

$$\begin{aligned} \mathcal{M}^2 = M^2 + 2M \sum_{i=1}^3 \frac{(\mathbf{p}^{(i)})^2}{2m_i} + M^2 \sum_{i=1}^3 \frac{y_i}{(r_{\perp}^{(i)})^2} \left[ (r_{\perp}^{(i)})^2 + \right. \\ \left. + \frac{(P_+ r_z^{(i)})^2}{M^2} + \frac{\sigma^2}{M^2} \left( \frac{(r_{\perp}^{(i)})^2}{y_i} \right)^2 \right]. \end{aligned} \quad (97)$$

One can now define the stationary point of (97) with respect to  $y_i$ ,

$$y_i^{(0)} = \frac{\sigma(r_{\perp}^{(i)})^2}{M r_z^{(i)}}, \quad (r_z^{(i)})^2 = (r_{\perp}^{(i)})^2 + r_z^{(i)}, \quad r_z^{(i)} \equiv \frac{P_+ r_z^{(i)}}{M}. \quad (98)$$

Inserting  $y_i = y_i^{(0)}$  back into (97) one arrives at the familiar nonrelativistic expansion

$$\mathcal{M}^2 = M^2 + 2M \sum_{i=1}^3 \left[ \frac{(\mathbf{p}^{(i)})^2}{2m_i} + \sigma r_z^{(i)} \right]. \quad (99)$$

Connection between  $\mathbf{p}^{(i)}$  also simplifies for  $y_i \ll 1$ , so that (90) goes over into a simple relation  $\sum_{i=1}^3 \mathbf{p}^{(i)} = 0$ , as expected.

We now turn to the case of light quarks, where relations (76) and (81) hold, and observe that three strings contribute to the total momentum  $P_+$  an amount,  $P_+^{str} \equiv \sum_i \int_0^1 \nu_i(\beta) d\beta$  which can be significant and comparable to that of valence quarks,  $\sum_{i=1}^3 \mu_i$ . The numerical value of  $\langle y \rangle \approx 0.2$  obtained in [48] for a light meson, suggests that a larger value can be obtained for  $P_+^{str}/P_+$ , which can be comparable to the 55% of the energy-momentum sum rule, observed in DIS experiment on nucleons. We suggest at this point following [48] that this number usually associated with gluon contribution, is mostly due to the string contribution  $P_+^{str}$  from all Fock components of nucleon, most importantly from the ground state strings, and from hybrid baryon excitation, where the ratio  $\frac{P_+^{str}}{P_+}$  is even larger.

This point will be elaborated elsewhere [52].

Another topic connected to the l.c. Hamiltonian (89) is the whole range of dynamical calculations similar to those done for mesons in [48]. One can solve for the eigenfunctions of (89) and calculate form-factor and structure function (valence part of) of the baryon-ground and excited states. To compute the full structure function however one needs higher Fock components and first of all lowest hybrid excitations. Here comes in the important problem of small  $x$  - Regge-type behaviour and connection of  $t$ -channel Regge poles (including Pomeron) with  $s$ -channel summation over baryon resonances (primarily hybrid excitation) which is planned to discuss in [52].

## 6 Discussion of the c.m. wave-function properties

In this chapter we consider possible strategies and first estimates in the solution of the c.m. Hamiltonian (63). The later is the sum of spin-independent part (the first four terms on the r.h.s. of (63)) and  $\Delta H_{spin}$ , which is calculated in [40] and has a full relativistic Dirac  $4 \otimes 4 \otimes 4$  structure.

At this point one can apply two different approaches in treating  $\Delta H_{spin}$ . In the most part of this section we shall consider  $\Delta H_{spin}$  as a correction which should be taken into account in the first order of perturbation theory. This is especially consistent for the perturbative part of  $\Delta H_{spin}$ , which is known for light quarks only to the order  $O(\alpha_s)$ . At the end of this section we also consider another strategy, when  $\Delta H_{spin}$ , and especially its hyperfine part, is treated in a full matrix form.

We start with the first approach and concentrate on the first term  $H_0$  in (63), which is given in (60). This term was considered before in [36] and numerical solution of the  $\Delta$ -type states is presented there including the study of Regge trajectories.

Since  $H_0$  does not depend on spin and isospin and color degrees of freedom are already integrated out, one should look for fully symmetric wave functions depending on spin variables  $\sigma$ , isospin variables  $\tau$ , and coordinate  $\xi, \eta$ . Relativistic effects are taken into account in kinematics, where einbein fields  $\mu_i$  are introduced in (60), which upon stationary point optimization in  $\mu_i$ , yield relativistic energies as in (50). However it is more advantageous to solve (60), which has nonrelativistic form without square roots, and do optimization in  $\mu_i$  for the resulting total mass  $M_0(\mu_i)$ . The accuracy of this



procedure for mesons was checked in [53]) to be around or better than 5%. This type of procedure also simplifies calculation of all 4 corrections in (63), which contain  $\mu_i$  explicitly.

Hence one can follow the construction of the fully symmetric wave function as was done in [9, 36], which we slightly simplify and adopt for notations used before. Namely Jacobi coordinates  $\xi, \eta$  (41) are chosen to be symmetric (s) and antisymmetric (a) with respect to interchange indices 1 and 2, and belong to the two-dimensional mixed representation of the permutation group  $S_3$ , denoted  $\psi''$  and  $\psi'$  respectively while one-dimensional ones are  $\psi^s$  and  $\psi^a$ . The same holds true for isospin wave functions  $\eta'', \eta', \eta^s, \eta^a$  and spin-isospin wave functions  $\varphi'', \varphi', \varphi^s, \varphi^a$  and finally the full coordinate-spin-isospin wave function which should be symmetric in interchange of all 3 indices is

$$\Psi(\mathbf{z}^{(i)}, \sigma, \tau) = \psi^s \varphi^s + \psi^a \varphi^a + \psi'' \varphi'' + \psi' \varphi'. \quad (100)$$

An additional requirement is that  $\varphi^{(i)}$  and  $\psi^{(i)}$ ,  $i = u, t, s, a$ , must belong to given total angular momentum  $L, m_L$  and total spin  $S, m_S$  and isospin  $I, I_3$ .

Inclusion of  $\Delta H_{spin}$  serves to diagonalize the wave function into the eigenfunctions of total momentum  $J, m_J$ .

Since the construction of spin-isospin functions for 3 quarks is well-known [1]-[9], we consider here only the coordinate part  $\psi^{(i)}(\xi, \eta)$ . As in [2]-[4], [9, 36] we shall use the hyperspherical formalism (54) which has proved to be very accurate for the  $3q$  case, namely the lowest hyperspherical function [2, 10, 54].

One can introduce hyperradius  $\rho$  in the following way (note the difference from definition in [36], where the case of equal masses  $\mu_i$  was considered).

$$\rho^2 = \sum_{i=1}^3 \frac{\mu_i (\mathbf{z}_i - \mathbf{R})^2}{\mu} = \xi^2 + \eta^2. \quad (101)$$

The coordinate wave function  $\psi(\xi, \eta)$  can be expanded in an infinite series of hyperspherical functions  $u_K^\nu(\Omega)$  depending on angular variables  $\Omega$ , with  $K$ - grand angular momentum,  $K = L, L+2, L+4, \dots$ , and  $\nu$  - the set of all other quantum numbers, see [54] for a review,

$$\psi(\xi, \eta) = \frac{1}{\rho^2} \sum_{K, \nu} u_K^\nu(\Omega) \psi_K^\nu(\rho). \quad (102)$$

writing (60) as

$$H_0 = \sum_{i=1}^3 \left( \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + h_0. \quad (103)$$

One can reduce the equation  $h_0\psi = E\psi$  to a system of equations

$$\begin{aligned} \frac{d^2\psi_K^\nu}{d\rho^2} + \frac{1}{\rho} \frac{d\psi_K^\nu}{d\rho} + \left[ 2\mu E - \frac{(K+2)^2}{\rho^2} \right] \psi_K^\nu &= \\ &= 2\mu \sum_{K',\nu'} U_{KK'}^{\nu\nu'}(\rho) \psi_{K'}^{\nu'}(\rho), \end{aligned} \quad (104)$$

where it was defined

$$U_{KK'}^{\nu\nu'}(\rho) = \int u_{K'}^{\nu'+}(\Omega) V_{conf}(\boldsymbol{\xi}, \boldsymbol{\eta}) u_K^{\nu}(\Omega) d\Omega. \quad (105)$$

The confining potential  $V_{conf}$  was considered in [36] assuming linear  $Y$ -type form. The analysis of matrix elements (105) is also given in [9, 54], and here we shall use only the simplest form, namely the so-called hypercentral component, which for the equal mass case ( $\mu_i = \mu$ ) is

$$U_{00}^{00}(\rho) = 1.118\sqrt{2}\sigma\rho = 1.58\sigma\rho. \quad (106)$$

The lowest order equation (105) for  $K = K' = 0$  was solved numerically in [9, 2]. Below we shall demonstrate a simpler approach which allows to obtain eigenvalues of this equation analytically with accuracy of 1% for lowest states. To this end we reduce (105) for  $K = K'$  (neglecting nondiagonal coupling) to the form  $\psi_K^\nu(\rho) = \frac{\bar{\psi}_K^\nu(\rho)}{\sqrt{\rho}}$ ,

$$-\frac{1}{2\mu} \frac{d^2\bar{\psi}_K^\nu(\rho)}{d\rho^2} + W_{KK}(\rho)\bar{\psi}_K^\nu(\rho) = E_{Kn}\bar{\psi}_K^\nu(\rho) \quad (107)$$

with

$$W_{KK}(\rho) = b\rho + \frac{d}{2\mu\rho^2}, \quad b = \sigma\sqrt{\frac{2}{3}}\frac{32}{5\pi}, \quad d = \left(K + \frac{3}{2}\right) \left(K + \frac{5}{2}\right). \quad (108)$$

The eigenvalue  $E_{Kn}$  can be found using oscillator-well approximation near the minimum of  $W_{KK}(\rho)$

$$\left. \frac{dW_{KK}(\rho)}{d\rho} \right|_{\rho=\rho_0} = 0, \quad \rho_0 = \left(\frac{d}{\mu b}\right)^{1/3} \quad (109)$$

which yields

$$E_{Kn} \cong W_{KK}(\rho_0) + \omega \left(n + \frac{1}{2}\right) \equiv \frac{\sigma^{2/3}}{\mu^{1/3}} c_n \quad (110)$$

where for  $K = 0$

$$W_{00}(\rho_0) = \frac{3}{2} \left( \frac{b^2 d}{\mu} \right)^{1/3}, \quad \omega = \frac{\sqrt{3d}}{\mu \rho_0^2}. \quad (111)$$

The spectrum  $\omega n$  corresponds to the so-called "breathing modes", when baryon is excited in its  $\rho$ -dependent mode only.

Finally adding other terms in (60) one has for  $M_{Kn}$  - the eigenvalue of  $H_0$ ,

$$M_{Kn} = \frac{3}{2} \mu + E_{Kn}(\mu) \quad (112)$$

At this stage one defines  $\mu$  from the stationary point of (112),  $\frac{\partial M_{Kn}}{\partial \mu} |_{\mu=\mu_0} = 0$ , which yields

$$\mu_0 = \left( \frac{2}{9} c_n \right)^{3/4} \sqrt{\sigma}, \quad M_{Kn}(\mu_0) = \sqrt{\sigma} 6 \left( \frac{2}{9} c_n \right)^{3/4}. \quad (113)$$

The total spin-averaged mass of baryon corresponding to the Hamiltonian (63) is

$$M_{Kn}^{tot} = M_{Kn}(\mu_0) + \langle \Delta H_{string} \rangle + \langle \Delta H_{coul} \rangle + \langle \Delta H_{self} \rangle. \quad (114)$$

For the lowest states with  $L = 0, 1$  one can neglect  $\langle \Delta H_{string} \rangle$ , while the other two terms are

$$\langle \Delta H_{self} \rangle = -\frac{6\sigma}{\pi\mu_0}, \quad \langle \Delta H_{coul} \rangle = -\frac{\lambda b^{1/3}}{(2\mu_0)^{2/3} \rho_0} \quad (115)$$

where  $\langle \Delta H_{self} \rangle$  is given in [41], while  $\langle \Delta H_{coul} \rangle$  is in [9, 37]. Here notation is used

$$\lambda = \alpha_s \frac{8}{3} \left( \frac{10\sqrt{3}\mu_0^2}{\pi^2\sigma} \right)^{1/3}. \quad (116)$$

Now  $M_{Kn}(\mu_0)$  is defined in (113) and one should choose the only input parameters (for light quarks we put all  $m_i = 0$ )  $\sigma$  and  $\alpha_s$ . The string tension  $\sigma$  is renormalized due to the presence of nondiagonal terms (23) and therefore is smaller than in the mesonic case (see [46] for comparison with lattice data and more discussion). For simple estimate below we choose  $\sigma = 0.15 \text{ GeV}^2$  (the same value as in [8]) and take  $\alpha_s = 0.4$ , which near its saturated value [55].

Results of calculations made according to Eqs. (??)-(116) are given in Table 1.

**Table 1**  
 Baryon masses (in GeV) averaged over hyperfine spin splitting for  
 $\sigma = 0.15 \text{ GeV}^2, \alpha_s = 0.4, m_i = 0.$

State	$M_{Kn} + \langle \Delta H \rangle_{self}$	$\langle \Delta H \rangle_{coul}$	$M_{Kn}^{tot}$	$M^{tot}(\text{exp})$
$K = 0, n = 0$	1.36	-0.274	1.08	1.08
$K = 0, n = 1$	2.19	-0.274	1.91	?
$K = 0, n = 2$	2.9	-0.274	2.62	?
$K = L = 1, n = 0$	1.85	-0.217	1.63	1.6
$K = 2, n = 0$	2.23	-0.186	2.04	?

As it seen from the Table the calculated spin-averaged mass  $\frac{1}{2}(M_N + M_\Delta)$  agrees well with the experimental average, the same is also true for lowest negative parity states with  $K = L + 1$ , which should be compared with  $\frac{1^-}{2}, \frac{3^-}{2}$  states of  $N$  and  $\Delta$  respectively.

We also notice that breathing modes ( $n > 0$ ) have excitation energy around 0.8 GeV while orbital excitations  $K = L = 1$  have energy interval around 0.5 GeV.

## 7 Problem of Spin-dependent forces

We are now turning to the spin-dependent interaction. For the  $3q$  case the corresponding nonperturbative and perturbative terms are given in [40]. They have been derived under the only assumption of Gaussian dominance, i.e. only contribution of the bilocal correlator (represented by scalar functions  $D$  and  $D_1$ ) was retained in (9), Gaussian dominance being supported by recent lattice data [25, 26]. The resulting spin-dependent forces have in general the form of a product of two  $4 \times 4$  matrices, one for each interacting quark, and this is the most general relativistic spin interaction.

The expansion in powers of inverse quark mass was not used in [40], and for light quarks the spin-dependent interaction is proportional to the terms  $\frac{1}{\mu_i \mu_j}$  and higher inverse mass terms, where  $\mu_i$  are the same as in (103) and (113) and have the meaning of constituent quark masses, which grow with excitation. For the lowest states  $\mu_0 \cong 0.37 \text{ GeV}$  and grow fast with increasing  $K, L$  and  $n$ .

Now one could use two types of strategy to implement spin-dependent forces.

1). Since all terms in (63) except the last one  $\Delta H_{spin}$  are diagonal in Dirac indices, then one can calculate spin-independent wave-functions and account for spin effects calculating matrix elements  $\langle \Delta H_{spin} \rangle_{KLn}$ . This procedure is actually used by most authors, and one can mention two positive moments associated with it. First of all in this procedure one treats spin-dependent forces as a perturbation, and it should work at least for high enough excitation, when  $\mu_i \mu_j$  in the denominator become large. Secondly, the perturbative spin-dependent forces are known for light quarks only to the lowest order in  $\alpha_s$  and therefore it is illegitimate to account those terms in higher than first order approximation.

However doing so one immediately comes to a serious contradiction. Namely the theoretical estimates of perturbative hyperfine splitting for a reasonable value of  $\alpha_s \approx 0.4$  yield values around a hundred of MeV instead of 300 MeV for the  $\Delta - N$  case [9]. The phenomenological remedy used is to take  $\alpha_s \sim 1$  and smearing the hyperfine  $\delta$ -function take the resulting potential to higher orders, which was criticized above.

To resolve this contradiction it is suggested first of all to take into account the nonperturbative part of hyperfine interaction, which was derived in [40]. It is known to yield a large part of hyperfine splitting in light mesons [27, 28] and may be large also for baryons. Secondly, it is suggested to use another strategy discussed below.

2). In case when spin forces are important, as was discussed in the hyperfine case, one should take into account that the same type matrix element which creates hyperfine splitting, also connects lower and higher components of Dirac bispinor. Physically this means excitation of negative energy components of quark wave function in baryon, which is also associated with the backward in time propagation of quarks.

Therefore the strong hyperfine splitting implies also strong negative energy component excitation, and the solution of the total Hamiltonian (63) should be sought for in the form

$$\Psi_B = \sum C_{ikl}^{\alpha\beta\gamma} \psi_\alpha^{(i)} \psi_\beta^{(k)} \psi_\gamma^{(l)},$$

where  $\alpha, \beta, \gamma$  are Dirac bispinor indices and  $i, k, l$  refer to the excitation state of a given quark.

This strategy is equivalent to the full relativist 3-body Bethe-Salpeter equation, which was studied in the quasipotential form in [56].

Another possible treatment of the same problem was recently initiated in

[38, 39], where 3-fold Dirac equations were derived from the QCD Lagrangian for the baryon Green's function.

## 8 Conclusions.

We have derived the  $3q$  Hamiltonian both in the c.m. and in the l.q. coordinate systems. It was demonstrated that the c.m. Hamiltonian can be written conveniently as a sum of a main term  $H_0$  and four corrections in (63), representing rotating string energy, Coulomb energy, nonperturbative selfenergy correction and spin interaction respectively. The explicit form of all terms is given above, except for the last one, published recently in [40].

The spin-averaged energy levels have been calculated analytically, using hyperspherical formalism yielding accuracy around 1% for energy levels in linear confining potential. Results for  $\Delta - N$  system are in good agreement with experiment. The present paper is meant to be a starting point of a new treatment of baryons, where all types of forces are derived explicitly from the first principles under the only assumption of the Gaussian dominance. The spin-dependent forces derived for the first time in its totality in [40] constitute an essential part of this new approach.

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