$\tau - \mu$ Flavor Violation as a Probe of the Scale of New Physics

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(April 24, 2002 and hep-ph/yymmnnn)

Abstract

Motivated by recent experimental indications of large $\nu_\mu - \nu_\tau$ mixing, we explore current bounds on the analogous mixing in the charged lepton sector. We present a general formalism for dimension-6 fermionic effective operators involving $\tau - \mu$ mixing with typical Lorentz structures and discuss their relationship to the standard model gauge symmetry. We find the low-energy constraints on the new physics scale associated with each operator, mostly from current experimental bounds on rare decay processes of τ, hadrons or heavy quarks. For operators involving at least one light quark (u, d, s), these constraints typically give a bound on the new physics scale of a few TeV or higher. Those operators with two heavy quarks turn out to be more weakly constrained at present, giving bounds of a few hundred GeV. A few scalar and pseudo-scalar operators are not subject to any current experimental constraints.

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1 Introduction

The flavor physics of quarks and leptons is of the most prominent mysteries in particle physics. The origin of fermion masses and their flavor mixing is totally unexplained in the Standard Model (SM) and many free parameters are introduced to parameterize this sector. Weak scale supersymmetry (SUSY) [1] as a leading candidate for physics beyond the Standard Model provides no further understanding of flavor physics. In fact, without additional assumptions for the flavor structure or for the SUSY breaking sector, supersymmetric theories often encounter phenomenolgical difficulties, known as the SUSY flavor problem [2]. In dynamical models of electroweak symmetry breaking, the phenomenological constraints on flavor-changing-neutral-currents (FCNC) preclude construction of successful models which generate observed heavy quark masses, unless new dynamics associated with the top quark is introduced [3]. It is quite challenging to satisfy the phenomenologically observed pattern of fermion masses and mixing in a predictive theoretical framework. Less ambitious approaches have been proposed to parameterize the new physics associated with flavor, and yet they do not shed light on its origin. For example, an ansatz [4] of fermion masses and mixing can be advocated. Phenomenologically viable FCNC couplings of fermions to the electroweak symmetry breaking sector were also introduced [5]. A non-linearly realized effective Lagrangian was also constructed [6] to allow FCNC interactions.

Recently, we have witnessed the exciting progress of flavor physics in the lepton sector, namely the observation of neutrino flavor oscillation [7]. The recent neutrino oscillation experiments seem to indicate maximal flavor mixing, and to prefer an interpretation of oscillation between the μ and τ neutrino flavors. This development has led to wide interest in model-building, with the hope of understanding the leptonic flavor sector[8].

In this paper, we wish to systematically explore the low energy constraints on the effective operators induced by $\tau - \mu$ mixing. This is strongly motivated by the neutrino oscillation results, in particular the large mixing between the two heavy generations. It is indeed tempting to search for lepton-flavor violating processes, other than neutrino oscillation experiments, especially in the charged lepton sector. In a SM framework with additional mass and mixing parameters in the neutrino sector, the $\tau - \mu$ mixing is typically of a size m_ν^2/M_W^2, which would be negligible. There are many possible sources for lepton-flavor mixing in theories beyond the SM. If the new physics scale is rather high, the effects on the charged lepton sector would be very small. On the other hand, both generic weak-scale SUSY models and composite fermion models have rich structures of flavor mixing, leading to potentially interesting new phenomena. Contemplating the lepton mass hierarchy, the conventional wisdom is that the new physics scale associated with the mass and mixing of the third generation leptons may be lower.

Assuming that the new physics scale is higher than that accessible by current experiments, we propose to adopt the effective operator approach, that can be obtained after integrating out the heavy degrees of freedom in a more complete theory. In Sec. 2, we present our general formalism for the dimension-6 fermionic effective operators involving $\tau - \mu$ flavor mixing with typical Lorentz structures

$$(\bar{\mu}\Gamma\tau)(\bar{q}_i\Gamma q_j), \tag{1.1}$$

where Γ is a generic Dirac operator. We discuss their relationship to the standard model gauge symmetry, and argue about the possible sizes of their coefficients, given a cutoff scale at which the effective theory breaks down and new physics sets in. We also comment on to what extent our formalism can be applicable to loop-induced processes. In Sec. 3, we systematically explore the constraints on the new physics scale associated with each operator, mostly from current experimental bounds on rare decays of τ, hadrons or heavy quarks. For the operators involving at least one light quark (u, d, s), the current low energy constraints on the new physics scale are typically a few TeV or higher. Those operators with two heavy quarks are at present subject to weaker constraints, only about a few hundred GeV. Some of the scalar and pseudo-scalar operators are not subject at present to any experimental constraint. We summarize our results in Table I and conclude in Sec. 4. An appendix is included to outline the relationship between our operators introduced in Sec. 2 and those with explicit SM gauge symmetry in both linear and non-linear realizations.

2 Formalism for Effective Operators

We consider an effective theory below the new physics scale Λ, which can be generally defined as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \Delta\mathcal{L} \tag{2.2}$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian and $\Delta\mathcal{L}$ denotes the new physics contribution via effective operators. For the current study, we will focus on the dimension-6 operators involving third and second family leptons (τ, μ),

$$\Delta\mathcal{L} = \Delta\mathcal{L}^{(6)}_{\tau\mu} = \sum_{j,\alpha,\beta} \frac{\mathcal{C}^{\alpha\beta}_j}{\Lambda^2} \left(\overline{\mu}\,\Gamma_j\,\tau\right)\left(\overline{q}_\alpha\,\Gamma_j\,q_\beta\right) \,+\, \text{H.c.}\,, \tag{2.3}$$

where $\Gamma_j \in (1,\, \gamma_5,\, \gamma_\sigma,\, \gamma_\sigma\gamma_5)$ denotes relevant Dirac matrices. We will not consider the possibility that Γ has tensor structure. Here, we consider Γ_j to be the same for both $\tau-\mu$ and $q_\alpha - q_\beta$ bilinears. Under these considerations, we can show that Eq. (2.3) is the most general form (containing one $\tau-\mu$ bilinear and one quark-bilinear) which is invariant under the unbroken $U(1)_{\rm em}$ gauge invariance.

The precise value of the dimensionless coefficient $\mathcal{C}^j_{\alpha\beta}$ in Eq. (2.3) should be derived from the underlying theory in principle. In the current effective theory analysis, we will invoke a power counting estimate. As shown in the Appendix, the operator (2.3) is formulated under the nonlinear realization of the SM gauge group $\mathcal{G}_{\rm SM}$, which provides a natural effective description of the strongly coupled electro-weak symmetry breaking (EWSB) sector and/or compositeness. In this scenario, the natural size of $\mathcal{C}^j_{\alpha\beta}$ for an effective dimension-6 four-Fermi operator such as (2.3) can be typically estimated as [3],

$$\mathcal{C}^j_{\alpha\beta} = 4\pi\,\mathcal{O}(1)\,, \qquad \text{(default)}, \tag{2.4}$$

which corresponds to an underlying theory with a strong gauge coupling $\alpha_S = g_S^2/4\pi = \mathcal{O}(1)$. Naive dimensional analysis (NDA) [10, 11] provides another way to estimate operators in the nonlinear realization. For the dimension-6 operator (2.3), the NDA gives

$$\mathcal{C}^j_{\alpha\beta}[\text{NDA}] =\lesssim (4\pi)^2\,\mathcal{O}(1)\,. \tag{2.5}$$

which corresponds to an underlying theory with a strong gauge coupling[1] $\alpha_S = g_S^2/4\pi \lesssim \mathcal{O}(4\pi)$. So, in general, for the nonlinearly realized effective theory, we expect $\mathcal{C}^j_{\alpha\beta} \gtrsim 4\pi$, and to be conservative

[1]In our current study, we will assume that α_S is always below its critical value so that there is no condensate formation for $(\overline{\tau}\mu)$ and $(\overline{q}_\alpha q_\beta)$ channels.

we will choose the estimate (2.4) as the "default" value of our analysis. With Eq. (2.4), all the phenomenological bounds derived in the next section can be translated into bounds on the new physics scale Λ. (The bounds with a different counting of $\mathcal{C}^j_{\alpha\beta}$ can be directly obtained from our default by simple rescaling.) The linear realization is more appropriate when the Higgs boson mass is relatively light, *i.e.*, well below the scale $4\pi v \simeq 3.1$ TeV. It may describe either a weakly coupled EWSB sector (such as typical models with supersymmetry [1]) or a composite Higgs sector (such as typical models with Top-color [3][2]) In the weakly coupled case, we have $\mathcal{C}^j_{\alpha\beta} \lesssim \mathcal{O}(1)$, while for the strongly coupled case, the $\mathcal{C}^j_{\alpha\beta}$ can be estimated by the above Eq. (2.4) [or Eq. (2.5)]. Since the new physics scale of a weakly coupled scenario is likely to be first determined by discovering the light new particles (such as a few low-lying states of superpartners), we will motivate the current study mainly by the strongly coupled case where the estimate in Eq. (2.4) [or (2.5)] can sensibly apply.

In our analysis, we will consider two types of bounds. Many will arise from the tree-level operators in Eq. (2.3) directly. Experimental constraints from various decays will bound these operators. In a few cases, the only significant bounds can be obtained by relating the operators involving one set of heavy quarks to those involving lighter quarks, through exchange of a W or Goldstone boson (see, for example, Figure 1 in Section 3). How does one handle loop effects in an effective theory? In such an effective theory calculation, some loop integrals are divergent and must be cut off at a scale Λ. In the case of W or Goldstone boson exchange, the divergence is logarithmic. Other diagrams, involving closed fermion loops, for example, may have quadratic divergences. In doing the analysis, we assume only one $(\bar{\mu}\Gamma_j\tau)(\bar{u}_1\Gamma_j u_2)$ operator to be non-zero[3]. We will cut off the loop integral by retaining the leading logarithmic terms in which the ultraviolet (UV) cutoff Λ would reliably represent the scale of new physics. What about other loop-induced processes which may have quadratic divergences? Unlike the logarithmic terms, such $\mathcal{O}(\Lambda^2)$ power corrections are not guaranteed to always represent the real contributions of the lowest heavy physical state of a mass $M_{\rm phys}$, so that to be conservative it has usually been suggested [12] that one only use the logarithmic terms ($\sim \ln\Lambda$) computed in the effective theory, for representing the real new physics contribution ($\sim \ln M_{\rm phys}$) from an underlying full theory. We will take this approach for the loop analysis in Section 3, though we keep in mind that retaining only leading logarithms may possibly underestimate the new physics loop-contributions if the $\mathcal{O}(M^2_{\rm phys})$ terms are not vanishing in a given underlying theory. This exception occurs only when the heavy mass effect in the underlying theory does not obey the usual decoupling theorem [14][4]. Thus, extracting the possible nonzero $\mathcal{O}(M^2_{\rm phys})$ terms is a highly model-dependent issue and is difficult to generally handle in a pure effective theory formalism. The traditional "leading logarithm" approach provides a conservative estimate for the effective theory analysis and is justified for those underlying theories in which the effects of the

[2]For the minimal Top-seesaw models [3], the composite Higgs mass is generally in the range around 0.4–1 TeV [13].

[3]If we were to keep other tree-level light-quark operators as well, then the loop contributions will essentially renormalize the tree-level light-quark operators. In that case, we could make a naturalness assumption–that there is no fine-tuned cancellation between the renormalized light-quark operators and the renormalized heavy-quark logarithmically cut-off operators. We can thus estimate the renormalized light-quark operator's effective coupling by only keeping the leading logarithmic terms, which correspond to cutting off the divergent integrals via simple replacement (in the $\overline{\rm MS}$ scheme) of $\log(\Lambda/M_{W,u,d}) \to \log(\mu/M_{W,u,d})$, where μ is the renormalization scale and will be set as the relevant energy scale invoked in the low energy process.

[4]One typical example is the models with heavy chiral fermions. For a heavy SM Higgs, the $\mathcal{O}(M_H^2)$ power corrections show up only at two-loop level due to the screening theorem [15], but an extended Higgs sector may possibly escape the screening theorem at one-loop. In general, for any heavy state of mass $M_{\rm phys}$, the nondecoupling occurs so long as $M_{\rm phys}$ is proportional to certain coupling of this state with light fields (which remain in the low energy theory).

heavy states (integrated out from the low enrgy theory) exhibit the decoupling behavior [14].

3 Phenomenological Constraints

We consider the general $\tau - \mu$ operators in Eq. (2.3) and take Γ_j to be the same for both the lepton and quark pieces. There are four types of operators to be considered, $\Gamma_j = (S, P, V, A)$, and for each type there are twelve combinations of $q_\alpha q_\beta$, $(uu, dd, ss, cc, bb, tt, ds, db, sb, uc, ut, ct)$. This gives a total of 48 operators for our analysis. We first consider operators involving two light quarks (uu, dd, ss, ds), then the non-diagonal operators involving one or more heavy quarks (db, sb, uc, ut, ct), and finally consider the diagonal operators involving two heavy quarks (cc, bb, tt). In our analysis, we will consider one operator to be nonzero at a time and derive the corresponding bound on the new physics scale Λ. This should provide a sensible estimate of the scale Λ under the *naturalness assumption* mentioned in Section 2, which states that there is no accidental cancellations among the contributions of different operators.

3.1 Operators with Two Light Quarks

For operators with two light quarks, the neutrinoless decay of the τ into a μ and one or more light mesons will provide the best bound. First, we establish our conventions: The PCAC condition for the pseudoscalar octet gives

$$\left\langle 0 \left| j_\mu^{(5)b}(0) \right| \phi^a(p) \right\rangle = i\delta^{ab} \frac{F_\pi}{\sqrt{2}} p_\mu, \tag{3.6}$$

where $F_\pi = 131$ MeV and the cartesian components of the axial vector current are

$$j^{\mu(5)b} = \bar{q}\gamma^\mu \gamma^5 \frac{\lambda^b}{2} q. \tag{3.7}$$

Here $\phi = \phi^a \lambda^a / \sqrt{2}$ with $a = 1 \cdots 8$ is the 3×3 matrix of pseudoscalar meson fields and λ^a are the Gell-Mann matrices normalized according to $\mathrm{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$. For current quark masses, we choose $m_u = m_d = 5$ MeV, and $m_s = 120$ MeV; the results are not particularly sensitive to these choices. For simplicity, the muon mass (and the pion mass, when applicable) will be neglected relative to the tau mass when calculating phase space.

Axial Operators

Bounds on the uu and dd axial operators can be obtained by looking at $\tau \to \mu\pi^o$. Using

$$\left\langle 0 \left| \bar{u}\gamma^\mu \gamma^5 u \right| \pi^0(p) \right\rangle = \left\langle 0 \left| \bar{q}\gamma^\mu \gamma^5 \left(\frac{1}{2} + \frac{\lambda^3}{2} \right) q \right| \phi^3(p) \right\rangle = i \frac{F_\pi}{\sqrt{2}} p_\mu \tag{3.8}$$

and noting that for $u \to d$ the right-hand side is the same except with an opposite sign, one finds

$$\Gamma\left(\tau \to \mu\pi^0\right) = \frac{1}{\Lambda^4} \frac{\pi}{2} F_\pi^2 m_\tau^2 < 0.908 \times 10^{-17} \quad \mathrm{GeV} \tag{3.9}$$

where the inequality comes from the experimental bound [16] on this decay mode. This then implies, for both the $\bar{u}\gamma^\mu\gamma^5 u$ and $\bar{d}\gamma^\mu\gamma^5 d$ operators, that $\Lambda > 11.3 \quad \mathrm{TeV}$.

For the ss axial operator, a bound is obtained from $\tau \to \mu\eta$. We have

$$\langle 0 \left| \bar{s}\gamma^\mu \gamma^5 s \right| \eta(p) \rangle = i \frac{-2}{\sqrt{3}} \frac{F_\eta}{\sqrt{2}} p^\mu \tag{3.10}$$

where $F_\eta = F_\eta^8 - \frac{1}{\sqrt{2}} F_\eta^0$ is defined using $\left\langle 0 \left| \bar{q}\gamma^\mu \gamma^5 \frac{\lambda^{8,0}}{2} q \right| \eta(p) \right\rangle = i \frac{F_\eta^{8,0}}{\sqrt{2}} p_\mu$. Here $\lambda^0 = \sqrt{\frac{2}{3}} 1$ and at NLO in chiral perturbation theory [17] $F_\eta^8 = 154$ MeV and $F_\eta^0 = 25$ MeV. We note that using these values we find that $F_\eta \approx F_\pi$, the value in the SU(3) limit. This gives

$$\Gamma(\tau \to \mu\eta) = \frac{2\pi}{3} \frac{\left(m_\tau^2 - m_\eta^2\right)^2}{m_\tau} \frac{F_\eta^2}{\Lambda^4} < 2.179 \times 10^{-17} \quad \text{GeV} \tag{3.11}$$

implying

$$\Lambda > 9.5 \text{ TeV}. \tag{3.12}$$

Note that this is also the bound when the effective operator is isospin invariant, i.e. $\frac{4\pi}{\Lambda^2} \left(\bar{\mu} A \tau\right) \left(\bar{u} A u + \bar{d} A d\right)$ with $A = \gamma^\mu \gamma^5$.

For the sd operator, the bound comes from $\tau \to \mu K^o$. We have

$$\langle 0 \left| \bar{s}\gamma^\mu \gamma^5 d \right| K^0 \rangle = i F_K p^\mu \tag{3.13}$$

where experimentally $F_K = 160$ MeV. This gives:

$$\Gamma(\tau \to \mu K^0) = \pi \frac{\left(m_\tau^2 - m_K^2\right)^2}{m_\tau} \frac{F_K^2}{\Lambda^4} < 2.27 \times 10^{-15} \quad \text{GeV} \tag{3.14}$$

and so

$$\Lambda > 3.60 \quad \text{TeV}. \tag{3.15}$$

Pseudoscalar Operators

Here, the Dirac equation is used to reduce the axial vector matrix elements to pseudoscalar matrix elements, and then we use the same processes as above.

One finds that

$$\langle 0 \left| \bar{u}\gamma^5 u \right| \pi^0(p) \rangle = - \langle 0 \left| \bar{d}\gamma^5 d \right| \pi^0(p) \rangle = \frac{i}{\sqrt{2}} \frac{m_\pi^2}{m_u + m_d} F_\pi \tag{3.16}$$

which then yields

$$\Gamma(\tau \to \mu\pi^0) = \frac{\pi}{8} \frac{F_\pi^2 m_\pi^4 m_\tau}{\Lambda^4 m_q^2} < 0.908 \times 10^{-17} \quad \text{GeV} \tag{3.17}$$

and hence

$$\Lambda > 11.7 \quad \text{TeV}. \tag{3.18}$$

For the strange quark operator, we find

$$\langle 0 \left| \bar{s}\gamma^5 s \right| \eta_8(p) \rangle = -i\sqrt{6} F_\eta^8 \frac{m_{\eta_8}^2}{m_u + m_d + 4m_s} \tag{3.19}$$

which gives (taking $\eta = \eta_8$)

$$\Gamma(\tau \to \mu\eta) = \frac{6\pi}{\Lambda^4} \frac{m_\eta^4 F_\eta^{8\,2}}{(m_u + m_d + 4m_s)^2} \frac{\left(m_\tau^2 - m_\eta^2\right)^2}{m_\tau^3} < 2.179 \times 10^{-17} \quad \text{GeV} \tag{3.20}$$

implying

$$\Lambda > 9.9 \quad \text{TeV}. \tag{3.21}$$

This is also the bound when the effective operator is isospin invariant, i.e. $\frac{4\pi}{\Lambda^2}\left(\bar{\mu}P\tau\right)\left(\bar{u}Pu + \bar{d}Pd\right)$ with $P = \gamma^5$.

Finally, we have

$$\left\langle 0\left|\bar{s}\gamma^5 d\right| K^0\right\rangle = i\frac{m_{K^0}^2}{m_d + m_s} F_K \tag{3.22}$$

which gives

$$\Gamma(\tau \to \mu K^0) = \frac{\pi}{\Lambda^4} \frac{m_K^4 {F_K}^2}{(m_d + m_s)^2} \frac{\left(m_\tau^2 - m_K^2\right)^2}{m_\tau^3} < 2.27 \times 10^{-15} \quad \text{GeV} \tag{3.23}$$

and hence

$$\Lambda > 3.7 \quad \text{TeV}. \tag{3.24}$$

Vector Operators

We take a simple SU(3) relation:

$$\left\langle 0\left|\bar{q}\gamma^\mu \frac{\lambda^a}{2} q\right| V^b\right\rangle \sim i\epsilon^\mu \delta^{ab} \frac{c}{\sqrt{2}}, \tag{3.25}$$

where $V_\mu = V_\mu^a \frac{\lambda^a}{\sqrt{2}}$ is the vector meson octet. Using vector meson dominance [18] we determine the dimensionless ratio $g \equiv m^2/c$ from $V \to e^+e^-$ for each vector $V = \rho, \omega, \phi$, which yields $g_\rho = 5.1$, $g_\omega = 17$ and $g_\phi = 12.9$. These phenomenological values indicate some SU(3) breaking, as expected. Assuming ideal mixing we get

$$\begin{aligned} \left\langle 0\left|\bar{u}\gamma^\mu u\right| \rho^0\right\rangle &= -\left\langle 0\left|\bar{d}\gamma^\mu d\right| \rho^0\right\rangle \sim i\epsilon^\mu K_\rho \\ \left\langle 0\left|\bar{s}\gamma^\mu s\right| \phi\right\rangle &\sim i\epsilon^\mu K_\phi \\ \left\langle 0\left|\bar{s}\gamma^\mu d\right| K^{0*}\right\rangle &\sim i\epsilon^\mu K_{K^*} \end{aligned} \tag{3.26}$$

where

$$K_\rho = \frac{m_\rho^2}{g_\rho}, \qquad K_\phi = \frac{3m_\phi^2}{g_\phi}, \tag{3.27}$$

and in the SU(3) limit one has $K_{K^*} = K_\rho$. Then we find

$$\Gamma(\tau \to \mu V) = \frac{\pi K_V^2}{\Lambda^4} \frac{m_\tau}{m_V^2} \left[m_\tau^2 \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_V^2}{m_\tau^2}\right) - 3\frac{m_\mu m_V^2}{m_\tau^3}\right] \tag{3.28}$$

which gives bounds as follows:

$$\begin{aligned}
\Gamma(\tau \to \mu\rho) &< 1.43 \times 10^{-17}\text{GeV} \Rightarrow \Lambda > 12.4 \quad \text{TeV} \qquad (uu, dd) \\
\Gamma(\tau \to \mu\phi) &< 1.59 \times 10^{-17}\text{GeV} \Rightarrow \Lambda > 14.3 \quad \text{TeV} \qquad (ss) \\
\Gamma(\tau \to \mu K^*) &< 1.7 \times 10^{-17}\text{GeV} \Rightarrow \Lambda > 12.8 \quad \text{TeV} \qquad (ds)
\end{aligned} \tag{3.29}$$

Scalar Operators

Scalar operators will lead to three-body decays of the τ into a μ and two mesons. Using the leading order chiral Lagrangian we get

$$\begin{aligned}
< 0|\bar{s}s(0)|K^+K^- >=< 0|\bar{u}u(0)|K^+K^- >= B_0 \\
< 0|\bar{u}u(0)|\pi\pi >=< 0|\bar{d}d(0)|\pi\pi >= B_0 \\
3 < 0|\bar{u}u(0)|\eta_8\eta_8 >= \frac{3}{4} < 0|\bar{s}s(0)|\eta_8\eta_8 >= B_0 \\
< 0|\bar{d}s(0)|\pi^+K^- >= B_0
\end{aligned} \tag{3.30}$$

where $m_\pi^2 = 2\hat{m}B_0$ and $m_{K^+}^2 = (m_u + m_s)B_0$ with $\hat{m} = m_u + m_d$. We take $m_u = m_d = 5$ MeV which gives $B_0 = 1.96$ GeV and $m_s = 120$ MeV. The differential decay widths are then

$$\begin{aligned}
d\Gamma(\tau \to \mu\pi^0\pi^0) &= \frac{1}{2}d\Gamma(\tau \to \mu\pi^+\pi^-) = \frac{B_0^2}{64\pi^3{\Lambda_{uu,dd}}^4}(E_1 + m_\mu)dE_1dE_2 \\
d\Gamma(\tau \to \mu\eta\eta) &= (\frac{B_0}{3})^2\frac{1}{64\pi^3{\Lambda_{uu,dd}}^4}(E_1 + m_\mu)dE_1dE_2 \\
&= (\frac{4B_0}{3})^2\frac{1}{64\pi^3{\Lambda_{ss}}^4}(E_1 + m_\mu)dE_1dE_2 \\
d\Gamma(\tau \to \mu K^+K^-) &= {B_0}^2\frac{1}{32\pi^3{\Lambda_{dd,ss}}^4}(E_1 + m_\mu)dE_1dE_2 \\
d\Gamma(\tau \to \mu K^+\pi^-) &= {B_0}^2\frac{1}{32\pi^3{\Lambda_{ds}}^4}(E_1 + m_\mu)dE_1dE_2
\end{aligned} \tag{3.31}$$

The best bounds on the $\bar{u}u$ and $\bar{d}d$ operators come from

$$\Gamma(\tau \to \mu\pi^+\pi^-) < 0.186 \times 10^{-16} \quad \text{GeV} \qquad \Rightarrow \Lambda_{uu,dd} > 2.6 \quad \text{TeV}, \tag{3.32}$$

the bound on the $\bar{s}s$ operator from

$$\Gamma(\tau \to \mu K^+K^-) < 0.341 \times 10^{-16} \quad \text{GeV} \qquad \Rightarrow \Lambda_{uu,ss} > 1.5 \quad \text{TeV}, \tag{3.33}$$

and the bound on the $\bar{s}d$ or $\bar{d}s$ operator from

$$\Gamma(\tau \to \mu K^+\pi^-) < 0.168 \times 10^{-16} \quad \text{GeV} \qquad \Rightarrow \Lambda_{ds} > 2.3 \quad \text{TeV}. \tag{3.34}$$

We see that the bounds on operators involving two light quarks range from 1.5 to 14.5 TeV. Improvement in the experimental limits on the branching ratios, of course, will increase these bounds as the fourth root of the branching ratio. Much greater improvement can be obtained from processes involving decays of heavy quarks, which we now consider.

3.2 Non-diagonal operators involving one or more heavy quark

Here we consider operators involving (uc, ut, ct, db, sb) quarks. The bounds involving an up quark and a charmed quark are problematic since the D^o can not (barely) kinematically decay into $\mu\tau$. The bounds on these operators will be discussed at the end of this subsection. We first turn to B-decays.

B-decays

Using the techniques described in the above subsection, one can bound the pseudoscalar and axial vector operators for $\bar{b}d$ quarks by looking at $B^o \to \mu\tau$. For the axial vector operator, we find that, using the experimental limit of 8.3×10^{-4} on the branching ratio

$$\Gamma(B \to \mu\tau) = \frac{\pi f_B^2 m_B m_\tau^2}{\Lambda^4}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \quad \Rightarrow \Lambda > 8.2 \text{ TeV} \tag{3.35}$$

and for the pseudoscalar operator,

$$\Gamma(B \to \mu\tau) = \frac{8 m_B^2 \beta_b^3}{\sqrt{\pi}\Lambda^4}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \quad \Rightarrow \Lambda > 9.3 \text{ TeV.} \tag{3.36}$$

In this latter case, we have used the result from Sher and Yuan [19]: $\langle 0|\bar{d}\gamma^5 b|B\rangle^2 = 4m_B\frac{\beta_B^3}{\pi^{3/2}}$, where $\beta_B \approx 300$ MeV is a variational parameter.

In a moment, we will consider the scalar and vector operators, but first let us look at the pseudoscalar and axial vector cases for $\bar{b}s$ quarks. Here, precisely the same analysis as for $B \to \mu\tau$ can be done for $B_s \to \mu\tau$, with the same result for the width, in the approximation where the masses of the constituent s and d quarks are equal. Alas, there are no published experimental bounds for $B_s \to \mu\tau$. Note that the lifetime of the B_s is given by 1.46 ± 0.06 picoseconds, compared with the B lifetime of 1.54 ± 0.02 picoseconds. There are consistent, as expected. But, if the rate for $B_s \to \mu\tau$ were too large, then the lifetime would be substantially shorter. A 10 percent branching ratio would shorten the lifetime by about 0.14 picoseconds, which would lead to a significant discrepancy. Without a detailed analysis, one can just conclude that there is a bound of five to ten percent on the branching ratio for $B_s \to \mu\tau$; we will give bounds assuming it is ten percent.

With a 10% bound, the above results scale as the fourth root of the branching ratio, giving a bound on the axial vector operator of $\Lambda > 2.5$ TeV and on the pseudoscalar operator of $\Lambda > 2.8$ TeV.

Note that here is a place where an experimental bound on $B_s \to \mu\tau$ would be very useful. This decay is particularly important because ALL of the quarks involved are second and third generation, and new physics effects might be substantial (especially if related to symmetry breaking); this decay also conserves "generation" number, and is thus particularly interesting.

We now turn to the scalar and vector operators. Here the matrix elements $\langle K|\bar{s}\gamma^\mu b|B\rangle$ and $\langle \pi|\bar{d}\gamma^\mu b|B\rangle$ are needed, along with their scalar counterparts. The vector matrix elements have been calculated in a quark model by Isgur, Scora, Grinstein and Wise [20]. They note that for a light pseudoscalar meson X,

$$\langle X(p_X)|\bar{q}\gamma^\mu b|\bar{B}(p_B)\rangle \equiv f_+(q^2)(p_B + p_X)^\mu + f_-(q^2)(p_B - p_X)^\mu \tag{3.37}$$

and present expressions (in their Appendix B) for f_+ and f_-. Here $q^2 = (p_B - p_X)^2$ and B and X are on-shell with $X \sim \bar{q}d$. The masses in these expressions are consituent quark masses, which we take to be $m_q = 300$ MeV, and we also take their variational parameters, β, to be 300 MeV. For example,

$$f_+(q^2) \approx -f_-(q^2) \approx \frac{3\sqrt{2}}{8}\sqrt{\frac{m_b}{m_q}}\exp[\frac{m_X - E_X}{2\kappa^2 m_X}] \tag{3.38}$$

where $\kappa \approx 0.7$ is a relativistic compensation and where $m_X \approx 2m_q$. As a result of these approximations, the matrix elements should be taken *cum grano salis*, with an error that could be a factor of 2 - 4 (which translates into a factor of 1.2 - 1.4 uncertainty in the final results for Λ). The uncertainty might be somewhat larger for the matrix elements involving the pion, since the relativistic compensation factors are suspect.

The result for the vector couplings is given (illustrating the $B \to K\mu\tau$ case) by

$$\frac{d\Gamma}{dE_\mu dE_\tau} = \frac{|\mathcal{M}^2|}{32\pi^3 m_B} \tag{3.39}$$

where

$$|\mathcal{M}^2| = \frac{18\pi^2 m_B}{\Lambda^4 m_d}\,(A_1A_2 - A_3)\; e^{-\frac{m_B - E_\tau - E_\mu - m_K}{m_d}} \tag{3.40}$$

with $A_1 = m_B^2 - m_K^2 - m_\tau^2 - 2m_BE_\mu$, $A_2 = m_B^2 - m_K^2 + m_\tau^2 - 2m_BE_\tau$ and $A_3 = m_K^2(-m_B^2 - m_\tau^2 + m_K^2 + 2(E_\mu + E_\tau))$.

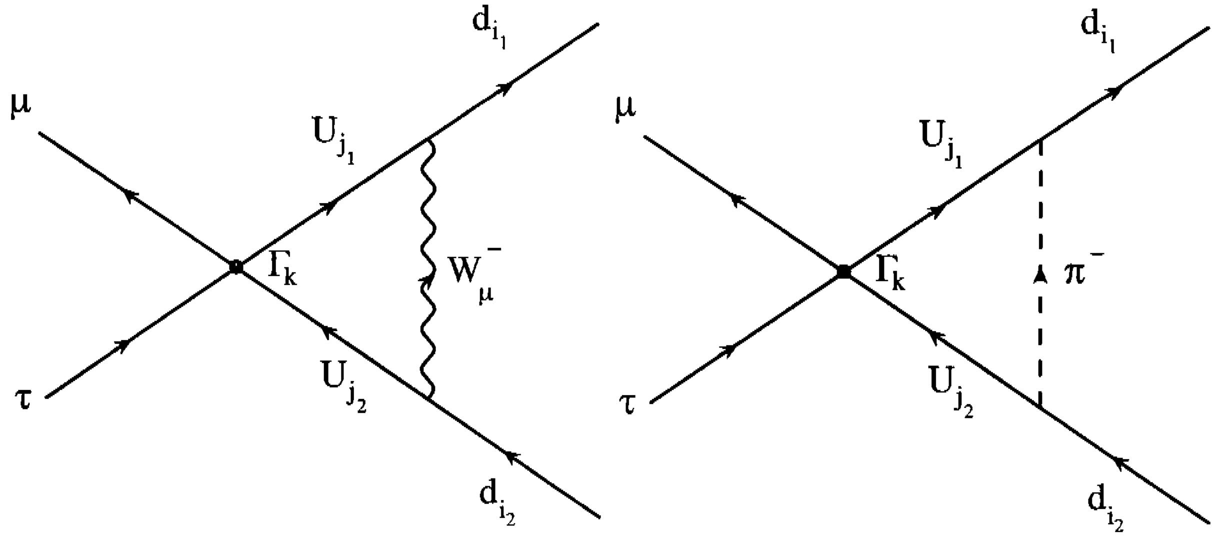
What about the experimental bounds for $B \to K\mu\tau$ and $B \to \pi\mu\tau$? None are listed. If the τ decays semi-hadronically (which occurs 65 percent of the time) then $B \to K\mu\tau$ will look like $B \to X_c\mu\nu$[21]. Then measurements of $B \to X_c\mu\nu$ would give a higher rate than for $B \to X_ce\nu$. These have been measured separately, with accuracies better than 0.5%, and thus an excess of μ-like events have not been seen with a sensitivity of 1.5% at 90% confidence level. If one assumes that the probability of classifying $B \to K\mu\tau$ decays as $B \to X_c\mu\nu$ is smaller by a factor of two, then, folding in the 65% branching fraction into hadrons, one would get a limit of 1.5% divided by 0.5 for acceptance and 0.65 for the branching fraction, which is about 5 percent. A very similar argument would apply to $B \to \pi\mu\tau$. Obviously, a more detailed analysis could yield a substantially better bound, however our result only scales as the fourth root of the branching ratio bound, and with the relatively large uncertainty in the matrix elements, one probably can't do much better. With a 5 percent branching ratio, we find that the bound on the (bs) vector operator is 2.6 TeV, and the bound on the (bd) vector operator is 2.2 TeV.

For the scalar operator, one must differentiate the vector matrix elements in Eq. 3.37, taking care to include the $\exp^{-i(p_B - p_K)\cdot x}$ factors properly. We find that, for example,

$$\langle K(p_K)|\bar{s}b|\bar{B}(p_B)\rangle \approx \frac{1}{m_b}\left[f_+(q^2)(m_B^2 - m_K^2) + f_-(q^2)(m_B^2 + m_K^2 - 2m_BE_K)\right] \tag{3.41}$$

Using this matrix element, we find the bound on the (bs) scalar operator is 2.6 TeV, and similarly that for the (bd) scalar operator is 2.2 TeV. One should keep in mind the relatively large uncertainties in these bounds due to the hadronic uncertainties discussed above.

Top Quark decays and Loop Contributions



(a) (b)

Figure 1: Diagrams which relate couplings involving u-type quark bilinears to d-type quark bilinears. The π^- is a Goldstone boson.

Due to the very short life-time of a top quark, bound state top mesons do not exist. Bounds on operators involving a top quark can be readily obtained by looking for decays $t \to c\mu\tau$ and $t \to u\mu\tau$. Neglecting the final state masses, one finds that the width is

$$\Gamma = \begin{cases} \frac{m_t^5}{96\pi\Lambda^4} & \text{for scalar and pseudo scalar couplings} \\ \frac{m_t^5}{24\pi\Lambda^4} & \text{for vector and axial vector couplings} \end{cases} \tag{3.42}$$

CDF [22] measures the ratio

$$\mathcal{R} = \frac{\mathrm{BR}(t \to Wb)}{\mathrm{BR}(t \to Wq)} \tag{3.43}$$

by counting b-tagged top events and all top events. The result is $\mathcal{R} = 1.23^{+0.37}_{-0.31}$, which translates into $\mathcal{R} > 0.92$ at one standard deviation. For the $t \to u\mu\tau$ channel, one considers this to be similar to Wq, and thus one has $\mathrm{BR}(t \to u\mu\tau) < 1 - 0.92 = 0.08$ at one standard deviation. This leads to a constraint

$$\Lambda > \begin{cases} 225 \text{ GeV} & \text{for scalar and pseudoscalar couplings} \\ 335 \text{ GeV} & \text{for vector and axial vector couplings} \end{cases} \tag{3.44}$$

One could improve on this substantially from non-observation of $t \to \text{jet} + \mu\tau$ decay, but this has yet to be done (and one should recall that our results only vary as the fourth root of the branching ratio).

For the vector and axial vector couplings, we will next see that a much stronger bound can be obtained by considering the loop contribution with $W^\pm$ exchange. This loop will also give good bounds on the (cu), (cc), (tt) operators, as will be discussed below.

Consider the loops in Figures (1a) and (1b), where π^- is the charged Goldstone boson. These loops will relate an operator of the form $\frac{C^k_{j_1j_2}}{\Lambda^2}(\bar{\mu}\Gamma_k\tau)(\bar{U}_{j_1}\Gamma_k U_{j_2})$ to an operator of the form $\frac{C^\ell_{i_1i_2}}{\Lambda^2}(\bar{\mu}\Gamma_\ell\tau)(\bar{d}_{i_1}\Gamma_\ell d_{i_2})$. Since we have very strong bounds on operators with light d_i quarks, this will give a method of obtaining bounds on heavy u_j quarks, and in many cases will provide the only bounds.

We consider the diagrams of Figure (1a) and (1b) and calculate the quark-bilinear

$$\delta C_k^{ij}\ (\bar{q}_i\Gamma_k q_j) \tag{3.45}$$

where δC_k^{ij} is the loop-induced form factor computed from the diagrams. It is easy to show that there will be no contribution to the scalar and pseudoscalar operators from the loop, and thus we will only consider vector and axial vector operators.

For $\Gamma_k = \gamma^\mu$, labelling the two external quarks with indices i_1, i_2 and the two internal quarks with indices j_1, j_2, we find that the vector and axial vector couplings are generated with corresponding induced form factors:

$$\delta C_V^{i_1,i_2} = -\delta C_A^{i_1,i_2} = \frac{g^2|V_{i_1,j_1}V^*_{i_2,j_2}|}{32\pi^2}\left(1 - \frac{M^2_{j_1}M^2_{j_2}}{2M_W^2}\right)\ln\frac{\Lambda}{M_W} \tag{3.46}$$

where V_{ab} is the relevant element of the CKM matrix. Note that since this is a leading log calculation, the finite terms are not considered, and thus the choice of M_W as the argument of the logarithm is somewhat arbitrary.

For $\Gamma_k = \gamma^\mu\gamma^5$, we find the analogues of the above equation:

$$\delta C_V^{i_1,i_2} = -\delta C_A^{i_1,i_2} = -\frac{g^2|V_{i_1,j_1}V^*_{i_2,j_2}|}{32\pi^2}\left(1 + \frac{M^2_{j_1}M^2_{j_2}}{2M_W^2}\right)\ln\frac{\Lambda}{M_W}. \tag{3.47}$$

We will ignore small masses of light quarks and the overall signs are also irrelevant. So, the above leading logarithmic contribution is universal except for the internal loop fields being both top quarks. It is useful that a single vector (or axial vector) coupling gives both vector and axial vector vertices, thus we can use either the vector- or axial-vector type of light-quark bounds to constrain both the vector- and axial-vector type of heavy-quark operators. (Note that in the limit where the muon mass is neglected, our VV and AA bounds from τ decays give identical bounds on VA and AV.)

In particular, the scales Λ associated with heavy and light operators, respectively, can then be related. Letting (X, Y) be the two internal quarks and (x, y) be the two external quarks, we derive, in an obvious notation,

$$\Lambda^\pm_{\mu\tau XY} = \Lambda^\pm_{\mu\tau xy}\sqrt{\frac{g^2}{32\pi^2}|V_{Xx}V^*_{Yy}|\ \ln\frac{\Lambda}{M_W}\left(1 \pm \frac{M_X M_Y}{2M_W^2}\right)} \tag{3.48}$$

where the $+(-)$ sign is for the axial vector (vector) coupling. Since, as discussed in Section 2, the loop cutoff Λ in the leading logarithmic terms can reliably represent the physical cutoff of the effective theory [12], we may set the above logarithmic cutoff Λ equal to the light-quark bound $\Lambda^\pm_{\mu\tau xy}$. The light-quark bound $\Lambda^\pm_{\mu\tau xy}$ varies in the range around $1.5 - 15\,\mathrm{TeV}$, and we can typically set it as $10\,\mathrm{TeV}$.

Now, we examine the vector- and axial-vector couplings for the $t-c$ and $t-u$ quark bilinears. With the internal quarks being the t and c (or u), the best bound comes from setting the external quarks to be b (which attaches to the internal t line) and d (which attaches to the internal c or u line). We then use the strong axial-vector bound 8.2 TeV for $b-d$ quark-bilinear, to obtain the bounds 310 GeV for the $t-c$ operator and 650 GeV for the $t-u$ operator which hold for both axial-vector and vector type of couplings. This bound is much stronger than that from t decays.

Charm Quark off-diagonal operators

The relevant charm quark operator is the $c-u$ operator. One can obtain bounds from the loop discussed above for the vector and axial vector operators. There are several possible choices for the external lines (and corresponding ss-vector, dd-vector or sd-axial bounds); the best comes from the vector ss-bound, i.e. from $\tau \to \mu\phi$. We find that the bound on both the vector and axial vector operators is 550 GeV.

The loop will not give a bound on the scalar or pseudoscalar operators. One does get a contribution from the finite parts of the loop, but they turn out to be negligible (the bounds are well below the W mass). For the pseudoscalar operator, one would get a strong bound from $D^o \to \mu\tau$, if it were kinematically accessible, but it falls 20 MeV short (the scalar operator would require an additional pion in the final state, which makes it even more inaccessible). We know of no bounds on the scalar and pseudoscalar operators.

3.3 Diagonal Operators

t-t Operators

The only possible way to bound the $t-t-\mu-\tau$ operators is through the loop discussed above. The best bound comes from the case in which the final state quarks are b and d. Due to the small CKM matrix elements, the bounds are not very strong. For the axial vector operator, we find that $\Lambda > 115$ GeV, and for the vector operator, $\Lambda > 75$ GeV. For the scalar and pseudoscalar operator, we know of no reasonable bounds at all.

b-b Operators

The loop contributions are negligible, in this case (primarily because the only final states, if the b's are internal, are u quarks, and the CKM matrix elements are very small). One can only look at $b-b$ bound states. No bound on $\Upsilon \to \mu\tau$ has been published. The ratio of the decay $\Upsilon(1s) \to \mu\tau$ through the vector operator to the decay $\Upsilon(1s) \to \mu^+\mu^-$ is independent of the matrix element (if the muon mass is negligible) and is given by $144\pi^2 M_\Upsilon^4/(e^4\Lambda^4)$. The $\Upsilon(1s) \to \mu^+\mu^-$ branching ratio is 2.5%. The upper bound on $\Upsilon(1s) \to \mu\tau$ can be estimated from looking at the paper [23] which measured $\Upsilon \to \tau\tau$, and comparing with the measurement of $\Upsilon(1s) \to \mu^+\mu^-$. If one assumes univerality, these will be equal. One can see that the excess of $\tau\tau$ events must (at 95% C.L.) be less than about 0.40%. One then asks what fraction of $\mu\tau$ events would pass the cuts of the $\tau\tau$ analysis. A conservative estimate [21] gives an upper bound of 4% on the branching ratio, which gives $\Lambda > 180$ GeV. We know of no bounds on the scalar, pseudoscalar, or axial vector couplings, since there is very little data on these bound states.

c-c Operators

The $c - c - \mu - \tau$ vector and axial vector operators can be bounded by the loop discussed in the last section, in which the final state quarks are both strange quarks. The bound, from $\tau \to \mu\phi$ decays, is 1.1 TeV, for both the vector and axial vector operators.

One can bound the scalar and pseudoscalar operators by looking at χ_c and η_c decays, respectively, however no bounds on these decays are published.

4 Summary

Motivated by the experimental evidence of large $\nu_\mu - \nu_\tau$ mixing, it is natural to consider the extent of allowed mixing between the second and third generations in the charged lepton sector. We have studied leptonic flavor-changing four-fermi interactions of the form $(\bar{\mu}\Gamma\tau)(\bar{q}_i\Gamma q_j)$. Such operators are interesting as they would arise in effective theories produced from various new physics scenarios. We treat each effective operator independently. We consider the Dirac matrices Γ to be the same for both the $\mu\tau$ and the $\bar{q}_i q_j$ terms. We performed systematic studies for those operators. For each possible choice of quark and antiquark, we look at existing experimental data for a variety of processes to establish the strongest bound on the energy scale Λ. Our results are summarized in Table 1.

For the light quarks it is fairly straightforward to find bounds using τ decays to μ and one or two light mesons since there are good experimental bounds for these processes. The bounds are of order 10 TeV for P, V and A, with the exception of the $\bar{s}d$ case where the P and A bounds are lower since the experimental bound is weaker. Since three-body decays are used for the S case the bounds are smaller and of order $1.5 - 2.5$ TeV.

For operators involving the charm quark we cannot use the obvious avenue of D decays since $m_D = 1865$ MeV and so just below the threshold for decay to $\mu\tau$. At tree level we also considered virtual τ decays $D \to \mu\tau^\star \to \nu_\tau$+hadron, but this width is proportional to Γ_τ and so no realistically obtainable experimental bound on $D \to \nu_\tau$+hadron will give anything more than a bound of a few GeV for Λ.

It turns out that we can get good bounds on both cu and cc for the V and A cases by considering loop contributions (shown in Fig. 1) to $\tau \to \mu\phi$. The bound is enhanced by the fact that $V_{cs} \approx 1$ - further details are given in Section 3. Unfortunately this does not help us for the S and P cases since the loop contribution is vanishing. Furthermore the pseudoscalar and scalar charmonium states are comparatively broad with several MeV uncertainties in their widths, so the prospect of obtaining a reasonable bound on a process as weak as decay to $\mu\tau$ is not promising.

The off-diagonal b-quark operators are easier to handle. Using approximate HQET matrix elements we use experimental bounds on $B \to \mu\tau$ and estimated bounds on $B \to \mu\tau P$ (P a light pseudoscalar meson) to obtain bounds in the 3-9 TeV range. Our P and A bounds for $\bar{b}s$ which are also good are actually based on conservative estimates of $B_s \to \mu\tau$. We also note that there is some error in our hadronic matrix elements, which could be improved, although since it is a fourth root which appears in our extraction of Λ the final error is not so great. The $\bar{b}b$ case is harder to pin down - in principle we could use the loop contribution which worked well for $\bar{c}c$, but this is suppressed by a CKM factor $|V_{ub}|^2$ and so does not lead to a useful bound. The only bound we can obtain on the V operator comes from Υ decay.

For the top quark, the best bounds for the S and P cases for the tu and for the S,P,V and A

Bound	1	γ_5	γ_σ	$\gamma_5\gamma_\sigma$
$\bar{u}u$	2.6 TeV $(\tau \to \mu\pi^+\pi^-)$	11.7 TeV $(\tau \to \mu\pi^0)$	12.4 TeV $(\tau \to \mu\rho)$	11.3 TeV $(\tau \to \mu\pi^0)$
$\bar{d}d$	2.6 TeV $(\tau \to \mu\pi^+\pi^-)$	11.7 TeV $(\tau \to \mu\pi^0)$	12.4 TeV $(\tau \to \mu\rho)$	11.3 TeV $(\tau \to \mu\pi^0)$
$\bar{s}s$	1.5 TeV $(\tau \to \mu K^+K-)$	9.9 TeV $(\tau \to \mu\eta)$	14.3 TeV $(\tau \to \mu\phi)$	9.5 TeV $(\tau \to \mu\eta)$
$\bar{s}d$	2.3 TeV $(\tau \to \mu K^+\pi^-)$	3.7 TeV $(\tau \to \mu K^0)$	12.8 TeV $(\tau \to \mu K^\star)$	3.6 TeV $(\tau \to \mu K^0)$
$\bar{b}d$	2.2 TeV $(B \to \pi\mu\tau)$	9.3 TeV $(B \to \mu\tau)$	2.2 TeV $(B \to \pi\mu\tau)$	8.2 TeV $(B \to \mu\tau)$
$\bar{b}s$	2.6 TeV $(B \to K\mu\tau)$	2.8 TeV $(B_s \to \mu\tau)$	2.6 TeV $(B \to K\mu\tau)$	2.5 TeV $(B_s \to \mu\tau)$
$\bar{t}c$	225 GeV $(t \to c\mu\tau)$	225 GeV $(t \to c\mu\tau)$	335 GeV $(t \to c\mu\tau)$	335 GeV $(t \to c\mu\tau)$
$\bar{t}u$	225 GeV $(t \to u\mu\tau)$	225 GeV $(t \to u\mu\tau)$	650 GeV $(B \to \mu\tau)$	650 GeV $(B \to \mu\tau)$
$\bar{c}u$	$\star$	$\star$	550 GeV $(\tau \to \mu\phi)$	550 GeV $(\tau \to \mu\phi)$
$\bar{t}t$	$\star$	$\star$	75 GeV $(B \to \mu\tau)$	115 GeV $(B \to \mu\tau)$
$\bar{b}b$	$\star$	$\star$	180 GeV $(\Upsilon \to \mu\tau)$	$\star$
$\bar{c}c$	$\star$	$\star$	1.1 TeV $(\tau \to \mu\phi)$	1.1 TeV $(\tau \to \mu\phi)$

Table 1: Bounds on four-fermi flavor-changing operators of the form $\bar{\mu}\Gamma\tau q_i\Gamma q_j$ where $\Gamma = \mathrm{S}, \mathrm{P}, \mathrm{V}, \mathrm{A}$. Combinations for which no bound has been found are marked with an asterisk. Otherwise we list the process which gives the strongest bound (see text for details).

cases for tc operators come directly from top quark decays $t \to c\mu\tau, u\mu\tau$. These widths scale as ${m_t}^5/\Lambda^4$ and so the bounds are actually non-trivial. For the V and A cases for tu (and also for $\bar{t}t$) the strongest bound comes from the appropriate contribution (see Fig. 1) to $B \to \mu\tau$. So far we have not found a way to treat the S and P operators for the $\bar{t}t$ case.

It is interesting to note that under the linear realization of the Standard Model gauge group $\mathcal{G}_{\rm SM}$, the allowed operators are restricted to V and A, as discussed in the Appendix. In this case, we have bounds for all but one of the allowed operators.

Of course tighter experimental bounds on τ decays would lead to stronger bounds on almost one half of the operators considered here. Also, as emphasized in Section 3, it would be particularly interesting to have an experimental bound on $B_s \to \mu\tau$ and also B decays to $\mu\tau P$. Since we have not found any way of bounding three of the four the $\bar{b}b$ operators, it would perhaps be helpful to have experimental bounds on scalar and pseudoscalar $\bar{b}b$ decay to $\mu\tau$. It would also be useful to have bounds on $t \to \mu\tau + {\rm jet}$ It would also be interesting to investigate how the present analysis constrains models giving rise to $\mu\tau$ flavor-changing operators of the kind studied here.

Acknowledgments

We are very grateful to Jon Urheim for numerous discussions and conversations about B-decays and Υ-decays, and to Jose Goity for useful discussions. D.B. wishes to acknowledge support from the Thomas Jefferson National Accelerator Facility operated by the Southeastern Universities Research Association (SURA) under DOE Contract No. DE-AC05-84ER40150.

A Appendix

We note that in principle the exact form of Eq. (2.3) depends on how the electroweak gauge symmetry is realized in $\mathcal{L}_{\rm SM}$. The general form (2.3) can be derived by using the nonlinear realization of the SM gauge symmetry. Under the nonlinear realization, the SM Higgs-Goldstone fields are parameterized as

$$\Phi = \frac{1}{\sqrt{2}}(v+H)U \quad , \qquad U = \exp[i\pi^a\tau^a/v] \tag{A.49}$$

which transforms, under $\mathcal{G}_{\rm SM} = SU(2)_L \otimes U(1)_Y$, as

$$\begin{aligned} &U \to U' = g_L U g_Y^\dagger \quad , \qquad H \to H' = H \quad , \\ &g_L = \exp[-i\theta_L^a\tau^a/2] \quad , \qquad g_Y = \exp[-i\theta_Y\tau^3/2] \quad . \end{aligned} \tag{A.50}$$

We introduce the following useful notations,

$$\begin{aligned} &D_\mu U = \partial_\mu U + igW_\mu^a\frac{\tau^a}{2}U - ig'UB_\mu\frac{\tau^3}{2} \\ &\mathcal{V}_\mu = (D_\mu U)U^\dagger\,, \quad \overline{\mathcal{V}}_\mu = U^\dagger(D_\mu U) = U^\dagger\mathcal{V}_\mu U\,, \quad \mathcal{T}^a = U\tau^aU^\dagger\,, \\ &\mathcal{W}_\mu^\pm = -\frac{i}{g}{\rm Tr}[\tau^\pm U^\dagger D_\mu U] = -\frac{i}{g}{\rm Tr}[\tau^\pm\overline{\mathcal{V}}_\mu] = -\frac{i}{g}{\rm Tr}[\mathcal{T}^\pm\mathcal{V}_\mu]\,, \\ &\mathcal{Z}_\mu^0 = -\frac{ic_{\rm w}}{g}{\rm Tr}[\tau^3U^\dagger D_\mu U] = -\frac{ic_{\rm w}}{g}{\rm Tr}[\tau^3\overline{\mathcal{V}}_\mu] = -\frac{ic_{\rm w}}{g}{\rm Tr}[\mathcal{T}^3\mathcal{V}_\mu]\,, \end{aligned} \tag{A.51}$$

where $c_w \equiv \cos\theta_W$ and $s_w \equiv \sin\theta_W$. It is easy to find that $\overline{\mathcal{V}}_\mu = i\left[\mathcal{W}^+_\mu\tau^- + \mathcal{W}^-_\mu\tau^+ + \mathcal{Z}^0_\mu\tau^3\right]$. Then, we can deduce the following transformation laws, under $\mathcal{G}_{\rm SM}$,

$$\begin{aligned}
&\mathcal{V}_\mu \to \mathcal{V}'_\mu = g_L\mathcal{V}_\mu g_L^\dagger\,, \quad \bar{\mathcal{V}}_\mu \to \overline{\mathcal{V}}'_\mu = g_Y\overline{\mathcal{V}}_\mu g_Y^\dagger\,,\\
&\mathcal{W}^\pm_\mu \to \mathcal{W}^{\pm\prime}_\mu = \exp[\mp i\theta_Y]\mathcal{W}^\pm_\mu\,, \quad \mathcal{Z}^0_\mu \to \mathcal{Z}^{0\prime}_\mu = \mathcal{Z}^0_\mu\,.
\end{aligned} \tag{A.52}$$

Since $(\mathcal{W}^\pm,\ \mathcal{Z}^0)$ feel only the unbroken $U(1)_{\rm em}$ gauge interaction, its covariant derivative is

$$D_\nu\mathcal{W}^\pm_\mu = (\partial_\nu \pm ieA_\nu)\mathcal{W}^\pm_\mu\,, \quad D_\nu\mathcal{Z}^0_\mu = \partial_\nu\mathcal{Z}^0_\mu\,, \tag{A.53}$$

In general, the non-linear composite field $(\mathcal{W}^\pm,\ \mathcal{Z}^0)$ can be expanded as

$$\begin{aligned}
\mathcal{W}^\pm_\mu &= W^\pm_\mu + \frac{1}{M_W}\partial_\mu\pi^\pm \pm \frac{i}{c_w v}\left[\left(c_w^2 - s_w^2\right)Z_\mu + 2c_w s_w A_\mu - c_w W^\pm_\mu\pi^0\right] + \cdots\,,\\
\mathcal{Z}^0_\mu &= Z^0_\mu + \frac{1}{M_Z}\partial_\mu\pi^0 + \frac{ic_w}{v}\left(\pi^- W^+_\mu - \pi^+ W^-_\mu\right) + \cdots\,,
\end{aligned} \tag{A.54}$$

so that in the unitary gauge, $(\mathcal{W}^\pm_\mu,\ \mathcal{Z}^0_\mu) = (W^\pm_\mu,\ Z^0_\mu)$, and

$$\mathcal{V}_\mu = \overline{\mathcal{V}}_\mu = \frac{ig}{2}\begin{pmatrix} c_w^{-1}Z_\mu & \sqrt{2}W^+_\mu \\ \sqrt{2}W^- & -c_w^{-1}Z_\mu\end{pmatrix}. \tag{A.55}$$

Now, we can rewrite the SM Lagrangian in terms of fields which feel only the unbroken $U(1)_{\rm em}$,

$$\begin{aligned}
\mathcal{L}_{\rm SM} &= \mathcal{L}^G_{\rm SM} + \mathcal{L}^H_{\rm SM} + \mathcal{L}^F_{\rm SM}\\
\mathcal{L}^G_{\rm SM} &= -\tfrac{1}{4}\left[W^a_{\mu\nu}W^{a\mu\nu} + B_{\mu\nu}B^{\mu\nu}\right]\big|_{W^\pm\to\mathcal{W}^\pm,\,Z^0\to\mathcal{Z}^0}\\
\mathcal{L}^H_{\rm SM} &= \tfrac{1}{2}\partial_\mu H\partial^\mu H - V_{\rm SM}(H) - \tfrac{1}{4}{\rm Tr}\left[\overline{\mathcal{V}}^\mu\overline{\mathcal{V}}_\mu\right]\left[v^2 + 2vH + H^2\right]\\
\mathcal{L}^F_{\rm SM} &= \overline{F_L}i\gamma^\mu\overline{\mathcal{V}}_\mu F_L + \overline{f_j}i\gamma^\mu(\partial_\mu + ig'Q_{fj}B_\mu)f_j - \overline{f_j}f_{j'}\left(M^f_{jj'} + \frac{y^f_{jj'}}{\sqrt{2}}H\right)\\
&= -\frac{g}{2}\overline{F_L}\gamma^\mu\left(\mathcal{W}^\pm_\mu\tau^\mp + c_w^{-1}\mathcal{Z}^0_\mu\tau^3\right)F_L + \overline{f_j}i\gamma^\mu(\partial_\mu + ig'Q_{f_j}B_\mu)f_j - \overline{f_j}M^f_{jj'}f_{j'}\left(1 + \frac{H}{v}\right)
\end{aligned} \tag{A.56}$$

where M^f is the general fermion mass-matrix and after diagonalization, $M_{fjj'} = m_{fj}\delta_{jj'}$. Here, the fermion fields only feel the electromagnetic $U(1)_{\rm em}$ gauge group and are connected to the SM linearly realized fermion via [6, 12]

$$\begin{aligned}
&F_L = U^\dagger F_L^{\rm linear}\,, \quad f_R = f_R^{\rm linear}\,,\\
&f_j \to f'_j = \exp[i\theta_Y Q_{f_j}]f_j\,, \quad (\text{under } \mathcal{G}_{\rm SM})\,,
\end{aligned} \tag{A.57}$$

with $Q_{f_j} = \left(I_{3j} + \frac{Y_{f_j}}{2}\right)$ denoting the electric charge of the fermion f_j and $F_L \equiv (f_{1L}, f_{2L})^T$. For the electroweak interactions, the non-linearly realized fermions have the $U(1)_{\rm em}$ covariant derivative

$$D_\mu f_j = \left(\partial_\mu + ieQ_{f_j}A_\mu\right)f_j \tag{A.58}$$

while for the linearly realized fermions we have

$$D_\mu F_L^{\text{linear}} = \left(\partial_\mu + ig\frac{\tau^a}{2}W_\mu^a + ig'\frac{Y_L}{2}B_\mu\right) F_L^{\text{linear}}, \quad D_\mu f_R^{\text{linear}} = \left(\partial_\mu + ig'\frac{Y_R}{2}B_\mu\right) f_R^{\text{linear}}. \tag{A.59}$$

Hence, the nonlinear SM Lagrangian (A.56) only poses an effective unbroken $U(1)_{\text{em}}$ gauge symmetry, and we see that under this formalism the dimension-6 $\tau - \mu$ operator (2.3) indeed has the most general form that involves one $\tau - \mu$ bilinear and one quark bilinear. This nonlinear formalism is particularly motivated when the Higgs sector of $\mathcal{L}_{\text{SM}}$ is strongly coupled or the Higgs does not exist (i.e., $M_H \to \infty$). In this case, the EWSB scale is bounded from the above, i.e., $\Lambda_{\text{EW}} \lesssim 4\pi v$ [24, 10, 11]. If the Higgs boson H^0 is relatively light, we may choose the linear realization for the $\mathcal{L}_{\text{SM}}$ [25], in which we consider $\Lambda \geq \Lambda_{\text{EW}}$ so that at the new physics scale Λ, the effective Lagrangian $\mathcal{L}_{\text{eff}}$ should be invariant under SM gauge group $\mathcal{G}_{\text{SM}} = SU(2)_L \otimes U(1)_Y$. This further restricts the form of the dimension-6 operator in Eq. (2.3) because of the requirement of both the isospin and hypercharge conservations. Thus, we have

$$\Delta\mathcal{L}^{\text{linear}} = \Delta\mathcal{L}_{\tau\mu(6)}^{\text{linear}} = \sum_{\alpha,\beta,\chi,\chi'}^{j} \frac{C_{\alpha\beta}^j}{\Lambda^2} \left(\overline{\mu_\chi}\, \Gamma_j\, \tau_\chi\right) \left(\overline{q}_{\chi'\alpha}\, \Gamma_j\, q_{\chi'\beta}\right). \tag{A.60}$$

where the chirality indices $\chi, \chi' = L, R$. This restricts $\Gamma \in (\gamma_\sigma,\, \gamma_\sigma\gamma_5)$, so that Eq. (A.60) only belongs to a sub-set of operators in Eq. (2.3). In the phenomenology analysis below we will first analyze bounds for the general form in Eq. (2.3) (under the nonlinear realization of $\mathcal{G}_{\text{SM}}$) and then comment on the restricted form in Eq. (2.3) (under the linear realization of $\mathcal{G}_{\text{SM}}$).

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