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Power-Law Wave Functions and Generalized Parton Distributions for Pion

A. MUKHERJEE[a,b,c], I.V. MUSATOV[d], H. C. PAULI[e], A. V. RADYUSHKIN[d,f]

[a] *Universität Mainz, J. J. Becher Weg 45, D-55099, Mainz, Germany*

[b] *Universität Regensburg, D-93040 Regensburg, Germany*

[c] *Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India*

[d] *Theory Group, Jefferson Lab, Newport News, VA 23606, USA*

[e] *Max Planck Institut für Kernphysik, 69117 Heidelberg, Germany*

[f] *Physics Department, Old Dominion University, Norfolk, VA 23529, USA.*

We propose a model for generalized parton distributions of the pion based on the power-law ansatz for the effective light-cone wave function.

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I. INTRODUCTION

Generalized parton distributions (GPDs) [1–3] are now the object of intensive theoretical studies, especially within the context of applications to deeply virtual [4,5] (for a detailed recent review see ref. [6]) and large momentum transfer processes [7,8]. The main advantage of GPDs is their universality allowing to connect different hard processes, both exclusive and inclusive. The price for this is the complexity of GPDs: they are functions of 3 variables, e.g., skewed parton distributions $\mathcal{F}_\zeta(X,t)$ or $H(x,\xi,t)$ depend on the fraction X (or x) of the momentum carried by the active quark, the skewedness parameter ζ (or ξ) and the invariant momentum transfer t. For this reason, the most promising approach to disentangling GPDs from experimental data is to construct realistic models for GPDs and fix their parameters by fitting the data. The crucial point for the model building is that, in specific limits, GPDs reduce to more familiar functions describing the hadronic structure, such as usual parton densities, form factors and distribution amplitudes. The "reduction" relations between GPDs and these functions have been used as a basis for building phenomenological models of GPDs [9]. Another fruitful idea used in the model-building is to construct GPDs from the light-cone (LC) wave functions [7,8]. The most popular ansatz [10] assumes a Gaussian dependence of the LC wave functions $\Psi(x,k_\perp)$ on the transverse momentum $k_\perp$. A pragmatic reason behind this ansatz is the simplicity of Gaussian integrals allowing to obtain many results in an analytic form. However, there are no a priori grounds to exclude wave functions with other types of transverse momentum dependence. In particular, the two-body (i.e., $\bar{q}q$) component of the pion wave function was calculated recently in a model [11] based on the one-gluon exchange approximation in the light-front framework. The wave function was found numerically, and it was observed that the fit is better if one uses a power-law form rather than a Gaussian [11,12]. Furthermore, the power-law wave functions were used some time ago in models for the nucleon form factors [13]. In the present paper, we show that a simple power-law ansatz for the pion LC wave function allows one to obtain explicit analytic expressions for the form factor and generalized parton distributions. In Section II, we specify the explicit model expression for the effective pion wave function, which is then used in Section III in deriving a parametric expression for the pion form factor. On phenomenological side, we show that the two parameters of the model, the constituent quark mass and the wave function width can be easily adjusted to provide a curve close to existing experimental data. On theoretical side, we note in Section IV that our parametric representation for the form factor has the form of the reduction relation connecting the pion form factor and the double distribution (DD) $F(x,y,t)$. As expected, the DD generated has correct spectral and symmetry properties. Moreover, it has the factorized structure proposed in Ref. [9]: it looks like a distribution amplitude with respect to the y variable and like a parton density with respect to the x variable. It also provides a nontrivial example of the interplay between x, y and t dependencies of DDs. With an explicit model for DDs at hand, one can calculate the relevant skewed distributions: the nonforward parton distribution $\mathcal{F}_\zeta(X,t)$ or Ji's off-forward parton distribution $H(x,\xi,t)$, see Section V. In Section VI, we analyze the asymptotic large-Q^2 behavior of the pion form factor. We consider both the massive $m \neq 0$ and massless $m = 0$ cases. We show that, in the latter case, the pion form factor in our model with a power-law wave function $\Psi(x,k_\perp) \sim 1/[\sqrt{x(1-x)}(1+k_\perp^2/(\lambda^2 x(1-x))^n]$ has the same asymptotic behavior $F_\pi(Q^2) \sim 1/Q^2$ for any power n. Though this behavior is generated by the soft (Feynman) mechanism, it formally coincides with the quark counting law dictated by the hard one-gluon exchange mechanism. In massive case, the true asymptotic behavior is $\ln(Q^2/\lambda^2)/Q^4$. However, for a wide range of accessible Q^2, the curve mimics the $1/Q^2$ behavior. The simple model that we use treats the pion as an effectively two-body system, which is not very realistic: one may expect that the parton densities at small x are affected by many-body components. Indeed, the valence parton density obtained in our model differs rather strongly from the phenomenologically established form. In Section VI, we fix this deficiency by adopting a model with a more realistic x-profile at $t=0$, but preserving the analytic structure of the interplay between x, y and t dependencies generated by the power-law ansatz. We note that by slightly changing the quark mass and the wave function width parameter it is still possible to get a good description of the pion form factor data. Our conclusions are formulated in Section VII. Summarizing, we note that in this paper, we demonstrate how to obtain a model for the C-odd pion double distribution $F_\pi(x,y,t)$ and the skewed parton distribution $\mathcal{F}_\pi(X,\zeta,t)$. By construction, the model GPDs satisfy all known constraints, such as reduction relations to usual parton densities and form factors, they have correct spectral and polynomiality properties, thus providing a realistic model that can be used in phenomenological applications.

II. WAVE FUNCTION

The $q\bar{q}$ wave function of the pion found numerically in [11] was parametrized analytically by a power law fit

$$\phi(\vec{k}) \sim \left(\frac{1}{1+k^2/\lambda^2}\right)^{\kappa}, \tag{2.1}$$

with $\kappa \sim 2$ rather than by a Gaussian $\phi(\vec{k}) \sim \exp(-k^2/p_G^2)$. Here $k^2 = k_z^2 + k_\perp^2$ is the square of the relative 3-momentum and λ is the parameter characterizing the width of the k^2-distribution, i.e., the size of the system. In the case of equal quark masses, there is a simple relation [12] between the variables $(k_z, k_\perp)$ and the infinite momentum frame (IMF) variables $(x, k_\perp)$

$$x = \frac{1}{2}\left[1 + \frac{k_z}{\sqrt{m^2 + k_z^2 + k_\perp^2}}\right], \tag{2.2}$$

where m is the effective quark mass.

The relation between $\phi(\vec{k})$ and the IMF wave function $\psi(x, k_\perp)$ is given formally by

$$\psi(x, k_\perp) = \phi(\vec{k})\,\frac{(1 + k^2/m^2)^{1/4}}{\sqrt{x(1-x)}}. \tag{2.3}$$

For light quarks, one may expect that the size parameter λ is close to the effective quark mass m. Then the factor $(1+k^2/m^2)^{1/4}$ can be essentially absorbed into a redefinition of the power κ, whose precise value, in fact, is not crucial for our purposes. Thus, in what follows, we will use a simplified power-law IMF wave function

$$\psi(x, k_\perp) = \frac{N}{\sqrt{x(1-x)}[a + bk_\perp^2]^2}, \tag{2.4}$$

where $a + bk_\perp^2$ is the IMF version of $(1 + k^2/\lambda^2)$ with

$$a = 1 + \frac{s^2(x-\frac{1}{2})^2}{x(1-x)}, \qquad b = \frac{s^2}{4m^2 x(1-x)}, \qquad s = \frac{m}{\lambda}, \tag{2.5}$$

and N is the normalization constant. To fix it, we write the pion form factor $F(Q^2)$ in the form suggested by the Drell-Yan formula [14]

$$F(Q^2) = \int \psi(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)\,\psi(x, \vec{k}_\perp)\,dx\,d^2k_\perp \tag{2.6}$$

for the two-body contribution in the $\bar{q} \leftrightarrow q$ symmetric case. Strictly speaking, the expression for the form factor contains contributions from all Fock components, with the two-body component responsible for a rather small portion of "1" in the normalization condition $F(0) = 1$ [10,12]. Hence, the wave function $\psi(x, k_\perp)$ normalized by

$$\int |\,\psi(x, k_\perp)\,|^2\,dx\,d^2k_\perp = 1 \tag{2.7}$$

should be treated as an effective wave function which includes also the contribution from the higher Fock components. The condition (2.7) gives an explicit expression for the normalization constant N of the effective wave function:

$$N^2 = \frac{3}{4\pi}\left(\frac{s}{m}\right)^2 \frac{1}{A(s)} \tag{2.8}$$

where

$$A(s) = \int_0^1 \frac{dx}{a^3} = \int_0^1 \frac{(1-z^2)^3}{[1-(1-s^2)z^2]^3}\,dz. \tag{2.9}$$

III. FORM FACTOR

The $k_\perp$ integral in the expression for the form factor

$$F(Q^2) = N^2 \int \frac{dx\, d^2k_\perp}{x(1-x)[a+b(k_\perp+(1-x)q_\perp)^2]^2[a+bk_\perp^2]^2} \tag{3.1}$$

can be done using either the Feynman parameters or the α-representation method briefly described below. To this end, we use

$$\frac{1}{A^\kappa} = \frac{1}{\Gamma(\kappa)} \int_0^\infty \alpha^{\kappa-1}\, e^{-\alpha A}\, d\alpha \ , \tag{3.2}$$

where $\kappa = 2$ in our case. After calculating the Gaussian integral over $k_\perp$, we arrive at the representation for the form factor in terms of two parameters α_1 and α_2

$$F(Q^2) = \pi N^2 \int_0^\infty \int_0^\infty \frac{\alpha_1 d\alpha_1\, \alpha_2 d\alpha_2}{(\alpha_1+\alpha_2)} \int_0^1 e^{-a(\alpha_1+\alpha_2)} e^{-b(1-x)^2 Q^2 \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}} \frac{dx}{bx(1-x)}, \tag{3.3}$$

where $Q^2 = q_\perp^2$. Changing the variables

$$\alpha_1+\alpha_2 = \lambda, \qquad \alpha_1 = \beta\lambda, \qquad \alpha_2 = (1-\beta)\lambda, \qquad d\alpha_1 d\alpha_2 = \lambda d\lambda d\beta \tag{3.4}$$

we obtain the parametric representation

$$F(Q^2) = \pi N^2 \int_0^1 \frac{dx}{bx(1-x)} \int_0^1 d\beta\, \beta(1-\beta) \int_0^\infty \lambda^2\, d\lambda e^{-[a\lambda + b\lambda(1-x)^2\beta(1-\beta)Q^2]}. \tag{3.5}$$

Integration over λ is easily performed to give

$$F(Q^2) = 2\pi N^2 \int_0^1 \frac{dx}{bx(1-x)} \int_0^1 d\beta\, \frac{\beta(1-\beta)}{[a+b(1-x)^2\beta(1-\beta)Q^2]^3}. \tag{3.6}$$

Incorporating the normalization condition, we get the final result

$$F_\pi(Q^2) = \frac{1}{A(s)} \int_0^1 dx \int_0^1 d\beta\, \frac{6\beta(1-\beta)}{[a+b(1-x)^2\beta(1-\beta)Q^2]^3}. \tag{3.7}$$

By construction, the form factor has the correct value at $Q^2 = 0$. However, its slope at this point depends on the values of the model parameters m and s. To obtain the analytic expression for the slope we note that, in the small Q^2 limit, one can expand the denominator of the β integration

$$\begin{aligned} F_\pi(Q^2)\mid_{Q^2\to 0} \ &= \frac{6}{A(s)} \int_0^1 \frac{dx}{a^3} \int_0^1 d\beta\, \beta(1-\beta) \Big[1 - \frac{3b}{a}\beta(1-\beta)Q^2(1-x)^2 \\ &\quad + \frac{6b^2}{a^2}(\beta(1-\beta))^2 Q^4 (1-x)^4 + \ldots\Big]. \end{aligned} \tag{3.8}$$

Using the normalization condition (2.7) for the wave function, taking the integrals over β and introducing the variable $z = 2x-1$ (like in Eq. (2.9)) we obtain

$$F_\pi(Q^2)|_{Q^2\to 0} = 1 - \frac{3}{20}\frac{Q^2}{m^2} s^2 \frac{B(s)}{A(s)} + \frac{9}{280}\frac{Q^4}{m^4} s^4 \frac{C(s)}{A(s)} + \dots \ , \tag{3.9}$$

where

$$B(s) = \int_0^1 \frac{(1+z^2)(1-z^2)^3}{(1-(1-s^2)z^2)^4}\, dz \tag{3.10}$$

and

$$C(s) = \int_0^1 \frac{(1+z^4+6z^2)(1-z^2)^3}{(1-(1-s^2)z^2)^5}\, dz. \tag{3.11}$$

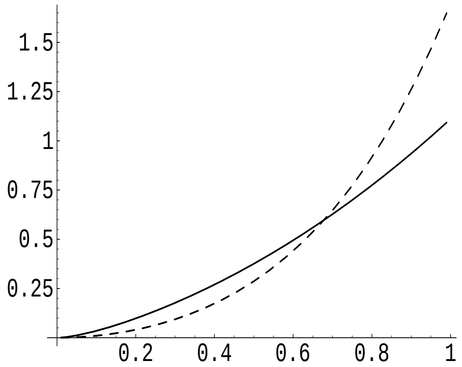

FIG. 1. Combinations $s^2B(s)/A(s)$ (solid) and $s^4C(s)/A(s)$ (dashed) as functions of the parameter s.

The integrals $A(s)$, $B(s)$ and $C(s)$ can be calculated in elementary functions, though the results are rather lengthy. Fig.1 shows the plot of the combinations $s^2B(s)/A(s)$ and $s^4C(s)/A(s)$ as functions of the parameter s.

The combination $s^2B(s)/A(s)$ is monotonically increasing from zero to infinity. Hence, after choosing the effective mass m we can always find such a parameter s that the slope $dF_\pi(Q^2)/dQ^2$ of the pion form factor at $Q^2 = 0$ has the experimental value $dF_\pi^{exp}(Q^2)/dQ^2 \approx 1/m_\rho^2$ [15]. For masses $m = 0.2$, 0.3 and 0.4 GeV, the parameters s fitting the pion charge radius are 0.56, 0.95 and 1.33, respectively.

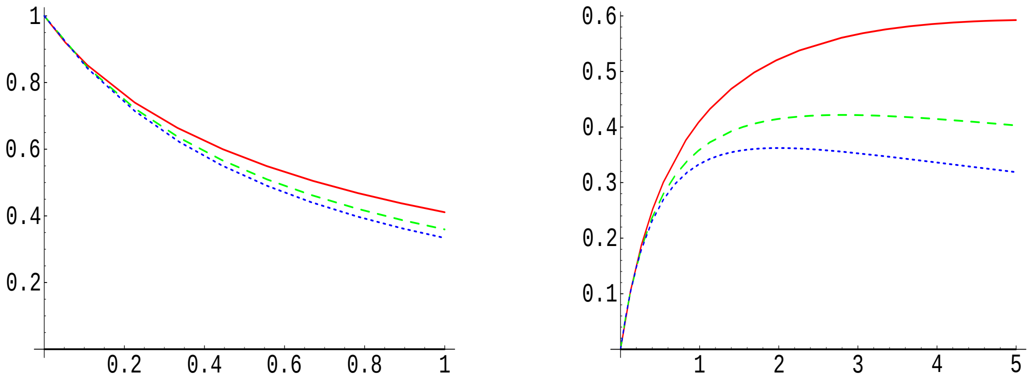

FIG. 2. Left: Form factor $F_\pi(Q^2)$ for three different parameterizations of the wave function: with $m = 0.2\,\text{GeV}$ (solid), 0.3 GeV (dashed) and 0.4 GeV (dotted). Right: $Q^2F_\pi(Q^2)$, with the same wave functions as in the left.

In Fig. 2a), we have plotted the form factor as a function of Q^2 in the low Q^2 region $Q^2 < 1\,\text{GeV}^2$ for these three different parametrizations of the wave function. Since they have the same slope at $Q^2 = 0$, the curves are rather close to each other. However, the difference between the curves becomes more pronounced as Q^2 increases. In Fig. 2b), the form factor calculated with these three effective wave functions is shown in the Q^2 region up to 5 GeV^2 relevant to future experiments at Jefferson Lab. Among these three choices, the most close to existing experimental data [16] is the curve corresponding to $m = 0.3\,\text{GeV}$ and $s = 0.95$.

IV. DOUBLE DISTRIBUTIONS

In a more explicit form, the expression for the pion form factor is

$$F_\pi(Q^2) = \frac{1}{A(s)} \int_0^1 dx \int_0^1 d\beta \, \frac{6\,\beta(1-\beta)}{\left[1 + \frac{s^2(x-1/2)^2}{x(1-x)} + \beta(1-\beta)\frac{s^2(1-x)Q^2}{4m^2 x}\right]^3} . \tag{4.1}$$

Introducing the variable $y = (1-x)\,\beta$, we can rewrite this formula as

$$F_\pi(Q^2) = \frac{1}{A(s)} \int_0^1 dx \int_0^1 dy \; \theta(x+y \le 1) \, \frac{6\,y(1-x-y)/(1-x)^3}{\left[1 + \frac{s^2(x-1/2)^2}{x(1-x)} + y(1-x-y)\frac{s^2 Q^2}{4m^2 x(1-x)}\right]^3} . \tag{4.2}$$

It may be treated as the standard representation [3]

$$F_\pi(Q^2) = \int_0^1 dx \int_0^{1-x} F(x,y;-Q^2)\, dy \tag{4.3}$$

of the pion form factor in terms of the double distribution

$$F(x,y;t) = \theta(x+y \le 1) \, \frac{6\,y(1-x-y)\,/(1-x)^3}{A(s)\,\left[1 + \frac{s^2(x-1/2)^2}{x(1-x)} - ty(1-x-y)\frac{s^2}{4m^2 x(1-x)}\right]^3} \tag{4.4}$$

(we switched to $t \equiv -Q^2$ to conform with the standard notations for generalized parton distributions). This double distribution has the correct spectral properties [3,5]: it vanishes outside the triangle $0 \le x, y, x+y \le 1$. It also satisfies the Munich symmetry condition [17]

$$F(x,y;t) = F(x, 1-x-y;t) \; . \tag{4.5}$$

Furthermore, for $t = 0$, it has the factorized form suggested in Ref. [9]

$$F(x,y;t=0) = \; \theta(x+y \le 1)\, h(x,y)\, f(x)\, dy \, , \tag{4.6}$$

in which the y-dependence appears only in the normalized profile function

$$h(x,y) = 6\,y(1-x-y)/(1-x)^3 \tag{4.7}$$

satisfying

$$\int_0^{1-x} h(x,y)dy = 1 \; . \tag{4.8}$$

The remaining factor

$$f(x) = \frac{1}{A(s)\,\left[1 + \frac{s^2(x-1/2)^2}{x(1-x)}\right]^3} \tag{4.9}$$

depends on x only, and may be interpreted as the parton distribution for the valence quarks inside the pion. For nonzero t, the profile function $h(x,y)$ also factorizes out in the expression for the double distribution, Eq.(4.4). However, it is multiplied by a function that has a nontrivial dependence on all three variables x, y and t. In Fig. 3, we plot $F(x,y;t)$ as a function of x and y for a few values of t.

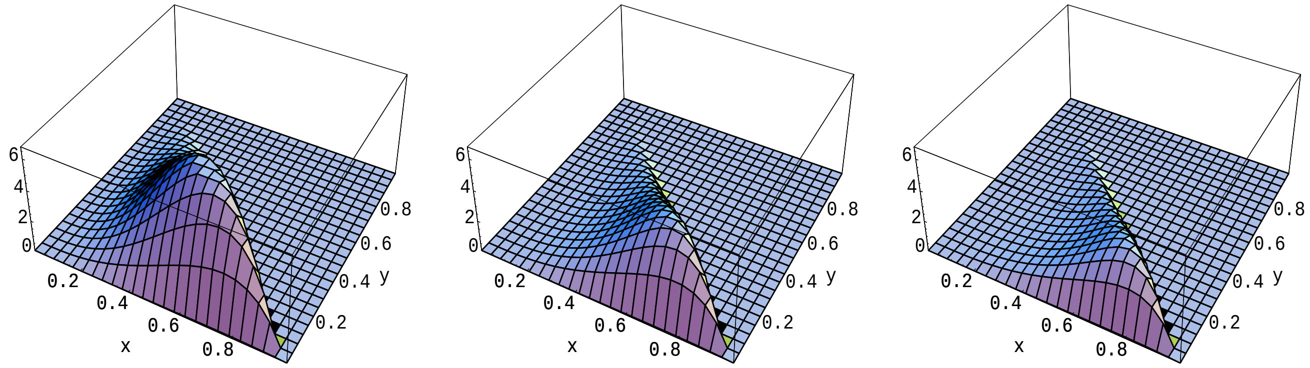

FIG. 3. $F(x, y; t)$ as a function of x and y for three values of $t = 0$, -0.5 and $-1\,\mathrm{GeV}^2$.

The form of the double distribution presented above corresponds to parametrization $k = xp_1 + yq$ of the active quark momentum k in terms of the initial pion momentum p_1 and the momentum transfer q. The final state pion has then the momentum $p_2 = p_1 - q$: the initial and final pions are not treated symmetrically in this formalism. To reinforce the symmetry, one should introduce the average momentum $P = (p_1 + p_2)/2$ (the initial and final pion momenta are then $P \pm q/2$) and write the active quark momentum as $k = xP + (1+\alpha)q/2$. To get the relevant double distribution $f(x, \alpha; t)$ we write the β variable as $\beta = (1+\eta)/2$ and then introduce α by $\alpha = (1-x)\eta$. This gives

$$f(x, \alpha; t) = \theta(|\alpha| \leq 1 - x)\, \frac{\frac{3}{4}[(1-x)^2 - \alpha^2]\,/(1-x)^3}{A(s)\, \left[1 + \frac{s^2(x-1/2)^2}{x(1-x)} - [(1-x)^2 - \alpha^2]\frac{s^2 t}{16m^2 x(1-x)}\right]^3} \,. \tag{4.10}$$

The normalized profile function in this case is $\frac{3}{4}[(1-x)^2 - \alpha^2]\,/(1-x)^3$.

If one uses a more general power-law LC wave function $\psi(x, k_\perp) \sim (a + bk_\perp^2)^{-\kappa}/\sqrt{x(1-x)}$, one obtains a DD with a κ-dependent profile $\sim [(1-x)^2 - \alpha^2]^{\kappa-1}/(1-x)^{2\kappa-1}$. The power of the denominator factor in Eq. (4.10) also changes from 3 to $2\kappa - 1$. The faster the decrease of $\psi(x, k_\perp)$ with $k_\perp$, the narrower the α-profile of the resulting DD. The purely exponential wave function gives an infinitely narrow profile function $\delta(\alpha)$.

V. SKEWED DISTRIBUTIONS

Having the expression for the double distribution, we can construct the nonforward distributions [5] in the standard way

$$\mathcal{F}_\zeta(X, t) = \int_0^1 dx \int_0^{1-x} F(x, y; t)\, \delta(X - x - \zeta y)\, dy \ . \tag{5.1}$$

Note, that for $F(x, y; t)$ given by Eq.(4.4), the integrations again can be performed in elementary functions. Hence, in this particular model, one can analytically study the interplay between X, ζ and t dependences of the nonforward parton distributions (though the expressions are now really lengthy).

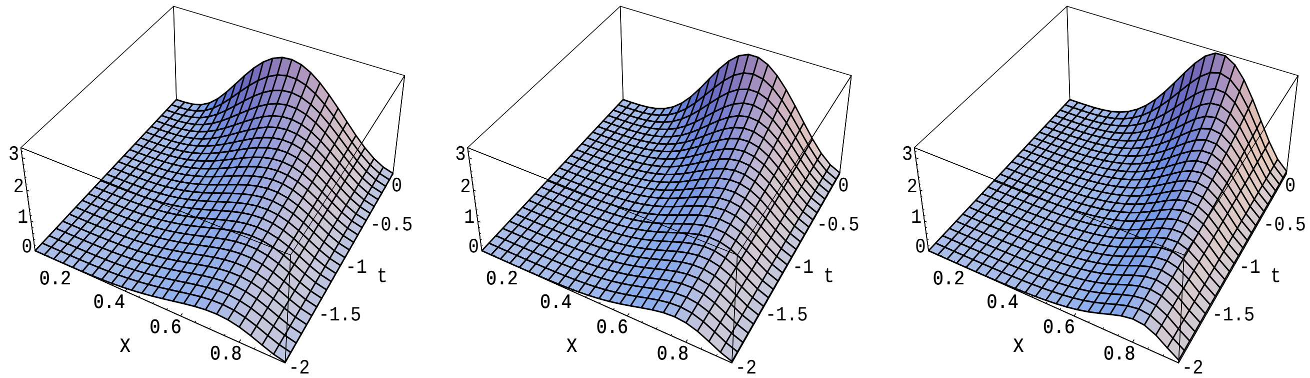

FIG. 4. $\mathcal{F}_\zeta(X,t)$ as a function of X and t for three values of $\zeta = 0.2$, 0.4 and 0.6.

In Fig. 4, we plot $\mathcal{F}_\zeta(X,t)$ as a function of X and t for some values of ζ. Note that for each value of ζ, the nonforward distributions satisfy the reduction formula

$$\int_0^1 \mathcal{F}_\zeta(X,t)\,dX = F_\pi(-t)\ . \tag{5.2}$$

An important point is that the right-hand side here has no dependence on the skewedness parameter ζ.

One can also use the symmetric double distribution $f(x,\alpha;t)$ and the relation

$$H(\tilde{x},\xi;t) = \int_0^1 dx \int_{-1+x}^{1-x} f(x,\alpha;t)\,\delta(\tilde{x}-x-\xi\alpha)\,d\alpha\ . \tag{5.3}$$

to obtain Ji's off-forward parton distributions $H(\tilde{x},\xi;t)$. Note that for the infinitely narrow profile function $f(x,\alpha;t) = F(x,t)\delta(\alpha)$ corresponding to the purely Gaussian wave function, the $t \to 0$ limit of GPD $H(\tilde{x},\xi,t)$ is given by the usual parton density, $H(\tilde{x},\xi,t=0) = f(\tilde{x})$: there are no skewedness effects. Thus, the GPDs obtained from the power-law ansatz have a much richer structure.

VI. ASYMPTOTIC BEHAVIOR OF THE PION FORM FACTOR

According to Fig. 2b), in the accessible energy range $Q^2 \lesssim 5\mathrm{GeV}^2$, the model curves show the behavior close to $1/Q^2$. Since the mass scales involved are rather small: $m^2, \lambda^2 \sim 0.1\mathrm{GeV}^2$, one may think that the form factor is already in the asymptotic region.

To analyze the asymptotic behavior of the form factor one can follow the approach described in Ref. [18]. The basic idea is that in the Drell-Yan formula (2.6) we deal with an overlap of two functions $\psi(x,k_\perp)$ and $\psi(x,k_\perp-(1-x)q_\perp)$ whose $k_\perp$ arguments are separated by a gap whose magnitude is $(1-x)Q$. Furthermore, $\psi(x,k_\perp)$ rapidly decreases with $k_\perp$. Hence, when Q^2 is large, the integral over $k_\perp$ in the form factor expression is dominated by two regions of phase space [18]:

1) $\mid k_\perp \mid \ll (1-x)Q$, where $\psi(x,k_\perp)$ is large,

2) $\mid k_\perp + (1-x)q_\perp \mid \ll (1-x)Q$, where $\psi(x,k_\perp+(1-x)q_\perp)$ is large.

In the first case, $k_\perp$ can be neglected compared to $(1-x)q_\perp$ in the wave function. The contribution from this region is then approximated by

$$F_\pi^{(1)}(Q^2) \sim \int_0^1 \psi(x,(1-x)Q)\,dx \int \theta\big(\mid k_\perp \mid < (1-x)Q\big)\psi(x,k_\perp)\,d^2k_\perp. \tag{6.1}$$

Since the wave function falls off rapidly at large transverse momenta, the major contribution to the integral comes from the region where $\mid k_\perp \mid$ is much smaller than $(1-x)Q$, and one may hope that the $k_\perp$ integral of $\psi(x,k_\perp)$ can be approximated by the pion distribution amplitude $\phi(x)$. The next statement usually made is that the large-Q behavior of the function $\psi(x,(1-x)Q)$ is determined by the large-$k_\perp$ behavior of $\psi(x,k_\perp)$ and, hence, the large-Q behavior of the form factor just repeats the large-$k_\perp$ behavior of the wave function; i.e., if $\psi(x,k_\perp) \sim 1/k_\perp^n$, then $F \sim 1/Q^n$. Note, that the last statement is only true if, after these substitutions, the integral over x converges. However, it is easy to derive that, after the $k_\perp$ integration in Eq.(6.1), the remaining integrand for the x-integration is proportional to

$$\frac{x^3(1-x)^5Q^2}{\left[x(1-x)+s^2\left(x-\frac{1}{2}\right)^2+(1-x)^2\frac{Q^2}{4\lambda^2}\right]^3}\,. \tag{6.2}$$

Neglecting the $x(1-x)$ and $s^2(x-1/2)^2$ terms compared to $(1-x)^2Q^2/4\lambda^2$, one would get the integral $dx/(1-x)$ logarithmically diverging in the $x \to 1$ region. Of course, this approximation is only true when $(1-x)^2Q^2/4\lambda^2 \gg 1$ or $x \ll 1-2\lambda/Q$. This cut-off converts the logarithmic divergence into $\ln(Q^2/\lambda^2)$. Hence, the asymptotic behavior is $\sim \ln(Q^2/\lambda^2)/Q^4$. This result can also be obtained from our representation (4.1) for the form factor, which we write now as

$$F_\pi(Q^2) = \frac{1}{A(s)}\int_0^1 dx \int_0^1 d\beta\, \frac{6\,\beta(1-\beta)x^3(1-x)^3}{\left[x(1-x)+s^2(x-1/2)^2+\beta(1-\beta)\frac{(1-x)^2Q^2}{4p^2}\right]^3}. \tag{6.3}$$

Again, there are two regions: $\beta \ll 4\lambda^2/[(1-x)^2Q^2]$ and $1-\beta \ll 4\lambda^2/[(1-x)^2Q^2]$ producing the leading large-Q^2 contribution $\sim \lambda^4/[(1-x)^2Q^2]^2$. Combined with other x-dependent factors, this gives the $dx/(1-x)$ divergence or, after a more accurate calculation, the $\ln(Q^2/\lambda^2)/Q^4$ asymptotic behavior. This behavior is not yet visible in the curves shown in Fig. 2a). The slow approach to asymptopia can be traced to rather small numerical factor $\beta(1-\beta)/4 \sim 1/16$ accompanying the Q^2 term. As a result, the effective scale governing the Q^2-behavior is something like $16\lambda^2 \sim 1.5\mathrm{GeV}^2$ rather than simply λ^2.

Let us also consider the case when quarks are massless, i.e., when $s=0$. Then

$$F_\pi(Q^2)\big|_{s=0} = \int_0^1 dx \int_0^1 d\beta\, \frac{6\,\beta(1-\beta)x^3}{\left[x+\beta(1-\beta)\frac{(1-x)Q^2}{4\lambda^2}\right]^3}, \tag{6.4}$$

and the situation drastically changes: the large-Q^2 behavior is dominated by integration over $1-x \ll 4\lambda^2/[\beta(1-\beta)Q^2]$ region. The remaining integral over β has no singularities, and we get $F_\pi(Q^2) \sim \lambda^2/Q^2$ for the asymptotic behavior. Clearly, the asymptotic behavior for massless quarks is governed by the soft (or Feynman) mechanism. One can easily check that the same power-law $F_\pi(Q^2) \sim 1/Q^2$ holds for the $\psi(x,k_\perp) \sim 1/[\sqrt{x(1-x)}(1+bk_\perp^2)^\kappa]$ wave functions with any power κ, and also for the exponential wave function $\psi(x,k_\perp) \sim e^{-bk_\perp^2}/\sqrt{x(1-x)}$.

An efficient way to obtain the asymptotic expansion in powers and logarithms of λ^2/Q^2 is based on the Mellin representation for the denominator factor:

$$\left[x+\beta(1-\beta)(1-x)\frac{Q^2}{4\lambda^2}\right]^{-3} = \frac{1}{2\pi i}\int_{-\delta-i\infty}^{-\delta+i\infty} \Gamma(-J)\Gamma(J+3) \\ \times \beta^J(1-\beta)^J(1-x)^J x^{-J-3}\left(\frac{Q^2}{4\lambda^2}\right)^J dJ\,. \tag{6.5}$$

Now, the β and x integrals can be calculated in Γ functions to give

$$F_\pi(Q^2) = \frac{1}{2\pi i}\int_{-\delta-i\infty}^{-\delta+i\infty} 6\,\frac{\Gamma(-J)\Gamma(1-J)}{(J+1)\Gamma(2J+5)}\Gamma^2(J+2)\Gamma^2(J+3)\left(\frac{Q^2}{4\lambda^2}\right)^J dJ\,. \tag{6.6}$$

The integrand has poles at integer J in the left half-plane. We explicitly displayed the rightmost of these poles $1/(J+1)$. It corresponds to $x \sim 1$ integration and gives the leading asymptotic contribution equal to $12\lambda^2/Q^2$. Expanding the integrand in the vicinity of $J=-2,-3$, etc., we can get subleading contributions. Note that the singularity at $J=-2$ is a double pole $1/(J+2)^2$. Hence, this contribution will have the $(\lambda^2/Q^2)^2\ln(Q^2/\lambda^2)$ term. One can also close the integration contour in the right half-plane. This procedure gives the small-Q^2 expansion of $F_\pi(Q^2)$ the first terms of which are explicitly written in (3.8).

VII. REALISTIC MODEL

The function $f(x)$, Eq.(4.9) was interpreted above as the valence quark distribution in the pion. However, this form strongly differs from the usual phenomenological parametrizations. At a normalization point $\mu \sim 1$ GeV, the latter have the form close to

$$f^R(x) = \frac{3}{4}(1-x)/\sqrt{x} \ ,$$

with the $1/\sqrt{x}$ reflecting the t-channel Regge exchanges which are not taken into account in the simplest two-body model. One may also ask at what normalization point the model quark distribution is taken. In particular, one can try to interpret Eq. (4.9) as a distribution taken at a very low normalization point, hoping that evolution to $\mu \sim 1$ GeV would make its shape more realistic. Note, that the form factor, being the matrix element of a conserved current (whose anomalous dimension vanishes) is not affected by the evolution: despite the fact that the shape of the double distribution changes, the form factor remains the same. This observation suggests another possibility: to "correct" the shape of the model DD at $t = 0$, but still preserve the analytic structure of the interplay between its t vs. x and y dependencies dictated by the simplest ansatz (4.4), i.e., to take the model

$$F^R(x, y; t) = F(x, y; t)\, \frac{f^R(x)}{f(x)} \tag{7.1}$$

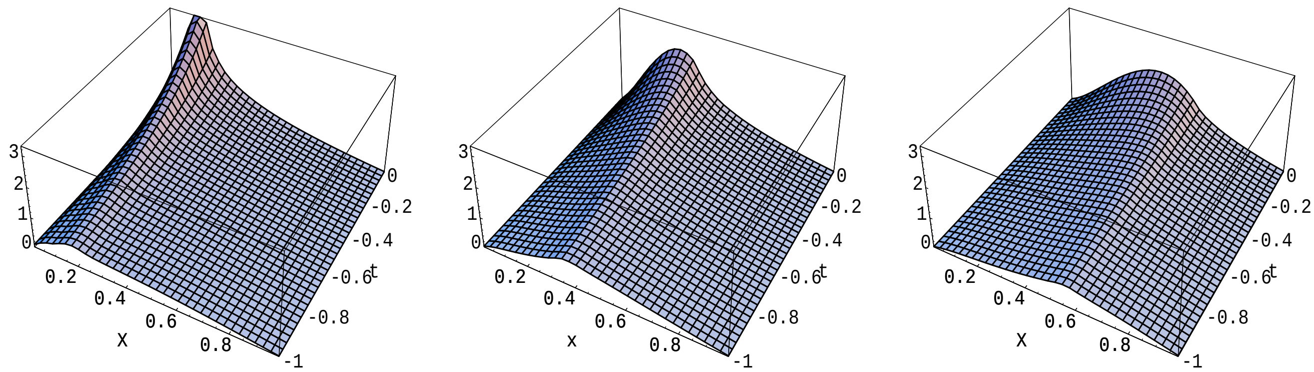

FIG. 5. $\mathcal{F}^R_\zeta(X, t)$ obtained with the "realistic" DD as a function of X and t for three values of $\zeta = 0.2$, 0.4 and 0.6.

With these new DDs, one can obtain a "realistic" model for SPDs $\mathcal{F}^R_\zeta(X, t)$ via (5.1). The SPD is presented in Fig. 5 as a function of X and t for some values of ζ.

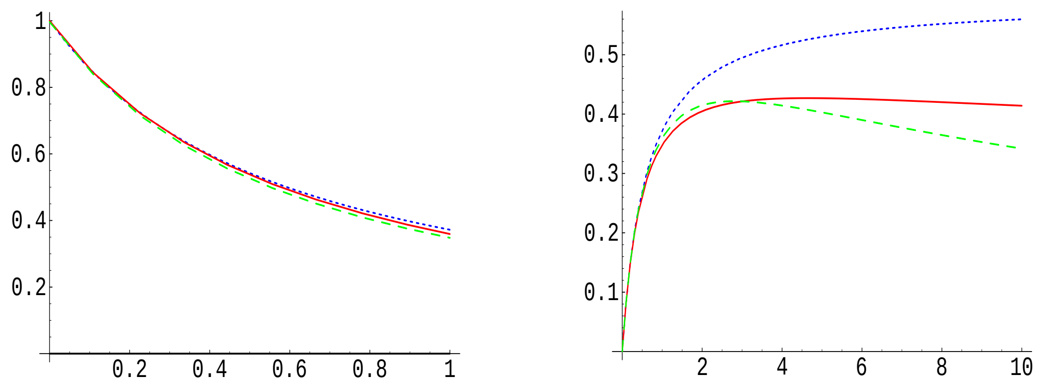

FIG. 6. Left: Form factor $F_\pi(Q^2)$ (left) and $Q^2 F_\pi(Q^2)\,\mathrm{GeV}^2$ (right) obtained with the "realistic" model for double distributions (solid line). For comparison, we present the results for the original model (dashed) and "ρ – meson fit" $Q^2/(1 + Q^2/(0.77\,\mathrm{GeV})^2)$ (dotted).

The parameters m and s of the new model should again be fixed by fitting the slope of the pion form factor at $t=0$ and its behavior in the $-t \sim 1\,\text{GeV}^2$ region. In Fig. 6, we show the curve for the pion form factor obtained with the ansatz (7.1) and the values $m=0.46$ and $s=0.81$. It practically coincides with the curve obtained within the original model for $m=0.3$ GeV and $s=0.95$ in the region of "not-so-high" transfer.

As a more explicit illustration of the t - dependence of SPDs, in Fig. 7 we show SPDs $F_\zeta(X;t)$ with different ζ's for two different values of t, both for the original and the "realistic" model.

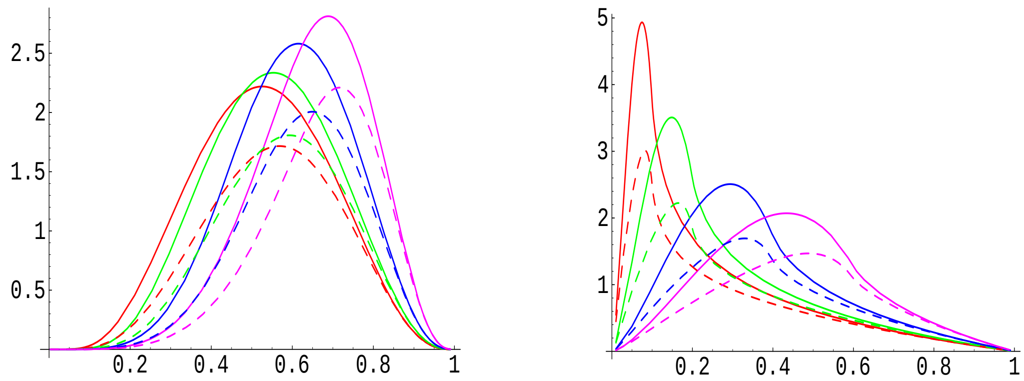

FIG. 7. SPDs $F_\zeta(X;t)$ with $\zeta = 0.1,\ 0.2,\ 0.4,\ 0.6$ are shown for two different values $t=0$ (solid lines) and $t=-0.2\,\text{GeV}^2$, both for the original (left) and "realistic" (right) model. The curves corresponding to larger ζ have maxima at higher x.

VIII. CONCLUSIONS

In this paper, we demonstrated how to obtain a model for the C-odd pion double distribution $F_\pi(x,y,t)$ and the skewed parton distribution $\mathcal{F}_\pi(X,\zeta,t)$ satisfying, by construction, all the known constraints imposed on GPDs, such as reduction relations to usual parton densities and form factors, spectral and polynomiality conditions. Our model GPDs have a nontrivial interplay between X,ζ and t dependence. Furthermore, they were adjusted to describe pion form factor for all available t, so we expect that our model correctly describes the t-dependence of the pion GPDs both for small and large t. The ability to have a unified model for GPDs from $t=0$ to $|t| \sim 10\,\text{GeV}^2$ is especially important in (future) applications to nucleons, for which GPDs are already being studied experimentally both for small (DVCS) and large t (wide-angle Compton scattering).

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