ELECTRON SCATTERING FROM NUCLEI

J. W. Van Orden
Department of Physics
Old Dominion University
Norfolk, Virginia 23529, USA
and
Thomas Jefferson National Accelerator Facility
12000 Jefferson Ave.
Newport News, Virginia 23606, USA

Abstract The description of nuclei at distances on the order of a fermi or less poses a difficult challenge for theoretical physicists. At larger distances the traditional description of the nucleus as a collection of interacting nucleons has been quite successful and substantial progress has been made in recent years in describing few-nucleon systems using this approach. However, it has been known for several decades that the nucleons themselves are composite objects which are believed to described by Quantum Chromodynamics (QCD). QCD is a complicated nonlinear strongly interacting field theory which can only be used for calculation in special circumstances. Due to the property of asymptotic freedom exhibited by QCD, perturbative calculations of QCD can be made at large momentum transfers and have achieved substantial success for a variety of processes. Understanding the transition from traditional pictures of nuclei to QCD is a substantial challenge. As an example of this problem, this paper describes recent calculations of elastic electron-deuteron scattering based on a relativistic extension of the traditional nuclear physics approach. The results of this work are compared to new data obtained at the Thomas Jefferson National Accelerator Laboratory and to the predictions of perturbative QCD.

1. Introduction

One of the outstanding challenges in nuclear physics is to understand the problem of the appropriate effective degrees of freedom at different distance scales.

The traditional view of the nucleus as a collection of nucleons interacting by means of potentials dates from 1932[1]. This approach has had considerable success in describing various nuclear properties and reactions at low energies. Indeed this approach when combined with modern computational techniques has been used to obtain wave functions and spectra for light nuclei which are exact within this context[2]. The two-nucleon potentials used in this approach are obtained by fitting the scattering matix to data using a general form for the potential and modern versions provide a fit to data with $\chi^2 \sim 1$ per degree of freedom [3, 4, 5]. These potentials contain terms proportional to $\boldsymbol{\tau} \cdot \boldsymbol{\tau}$, where $\boldsymbol{\tau}$ is a vector of the usual Pauli matrices operating in the isospin space of the nucleons. These terms allow for the exchange of charge between proton-neutron pairs. This is usually associated with the exchange of charged mesons between nucleons as part of the nucleon force. Reactions involving electromagnetic probes of the nucleus, such as electron scattering, can be sensitive to this flow of charge and models must include two-body meson-exchange currents to account for it. Effectively this extension of the traditional approach describes the nucleus as being composed of nucleons and mesons as the effective degrees of freedom.

We now understand that nucleons and mesons are themselves composite particles of quarks and gluons as described by the theory of quantum-chromodynamics (QCD). The difficulty with describing nuclei in terms of these underlying degrees of freedom is the result of the inherent complexity of QCD. QCD is a strong-coupling nonabelian gauge theory with nonlinear interaction terms. At present there are only two techniques for exploring the consequences of this theory. The most comprehensive of these is Lattice QCD where the field theory is discretized on a four-dimensional lattice and then the corresponding Feynman path integrals associated with observables are integrated using Monte Carlo techniques [6]. This is an extremely computationally intensive problem and current efforts in this direction are limited by the capabilities of computers. At present, these calculations are limited to describing the properties of single mesons and nucleons. With the rapid increase in the capabilities of computers for a given cost, it is likely that some limited attempts to describe light nuclei may be possible in the future.

The other technique for exploring QCD is perturbative QCD (PQCD) [7]. This approach takes advantage of the unusual properties of QCD. The interactions in field theories lead to the dressing of the "bare" particles of the lagrangian. For example in quantum electrodynamics (QED) a bare charge is dressed by polarizing the vacuum through the creation of virtual electrons and positrons from the vacuum. This dressing results of a screening of the charge of the electron. At low energies (long distances) a probe of the electron charge will see the usual physical charge e. As the energy is increased and the distance probed decreases, the probe sees less of the screening charge and the charge of the electron increases. For QED this changing of the magnitude of the charge with distance is very slow and is of no practical importance for most physical processes. The nonabelian character of QCD results in an opposite trend. The color charge decreases with decreasing distance as the result of anti-screening. This property is called asymptotic freedom. As a result, if processes can be found where all of the momenta are sufficiently large, then QCD can be calculated using perturbation theory just as for QED. PQCD has been used to study a large number of high energy processes in particle physics with considerable success. The main drawback to PQCD is that all physical process contain some contributions where some quarks and gluons are not of sufficiently high energy for this approximation to be applied. These "soft" contributions tend to determine the overall size of cross sections while the perturbative contributions tend to give the functional form of the change with momentum. Since the soft contributions are inherently nonperturbative, PQCD generally leads to information about trends in processes as a function of four-momentum transfers and models of the soft contributions are necessary to describe the size of processes.

We are then left with the following problem: While we know that the underlying degrees of freedom are quarks and gluons, we also know that at low energies we have had considerable success with a traditional nuclear physics of mesons and nucleons as effective degrees of freedom, whereas at high energies PQCD with explicit quark and gluon degrees of freedom is needed to describe data. In now becomes necessary to establish how we make the transition from the low energy model of mesons and nucleons to the high energy theory of quarks and gluons. That is, where does it become necessary to deal explicitly with the underlying degrees of freedom? Answering this question was one of the primary reasons for the building of the Continuous Electron Beam Facility (CEBAF) at the Thomas Jefferson National Accelerator Facility (Jefferson Lab.) As an example of how this search is progressing, this paper will concentrate on one process: elastic electron-deuteron scattering[8, 9, 10].

2. Elastic Electron-Deuteron Scattering

The Feynman diagram in Fig. 1 represents electron scattering from a nucleus in the one-photon-exchange approximation where k and k' are the initial and final electron four-momenta, $q = k - k'$ is the four-momentum of the virtual photon, and P and P' are the initial and final four-momenta of the nucleus. For electron scattering, the four-momentum of the virtual photon must be spacelike and it is traditional

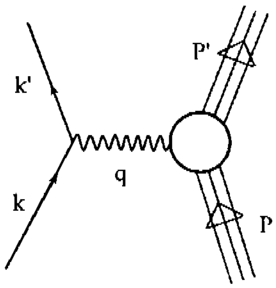

Figure 1. Feynman diagram representing electron-nucleus scattering.

to define a positive quantity

$$Q^2 = -q^2 = \mathbf{q}^2 - \nu^2 \tag{1}$$

where $\mathbf{q}$ is the photon three-momentum and ν is the photon energy.

For elastic electron-deuteron scattering Lorentz covariance, parity conservation and gauge invariance require that the current matrix element can depend on only three scalar form factor dependent upon Q^2. It is conventional to choose these three form factors to be the monopole charge form factor $G_C(Q^2)$, the dipole magnetic form factor $G_M(Q^2)$ and the quadrupole charge form factor $G_Q(Q^2)$ which are constrained such that

$$G_C(0) = 1 \tag{2}$$

$$G_M(0) = \frac{M_d}{m}\mu_d \tag{3}$$

$$G_Q(0) = M_d^2 Q_d \tag{4}$$

where m is the nucleon mass, M_d is the deuteron mass, $\mu_d = 0.857406(1)$ is the deuteron magnetic moment in nuclear magnetons and the deuteron quadrupole moment is $Q_d = 0.2859(3)$ fm^2.

The cross section for elastic electron-deuteron scattering in the one-photon-exchange approximation is the Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \sigma_{\mathrm{Mott}}\, f \left[A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}\right] \tag{5}$$

where θ is the electron scattering angle, the Mott cross section σ_{Mott} is the scattering cross section of an electron from an infinitely heavy unit charge,

$$f = \frac{1}{1 + \frac{2E}{M_d}\sin^2(\theta/2)} \tag{6}$$

is a recoil factor, and $A(Q^2)$ and $B(Q^2)$ are structure functions that contain all of the information about the deuteron electromagnetic currents. These structure functions can be represented in terms of the three form factors as

$$A(Q^2) \equiv G_C^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2) \tag{7}$$

and

$$B(Q^2) \equiv \frac{4}{3}\eta\,(1+\eta)\,G_M^2(Q^2) \tag{8}$$

where

$$\eta = \frac{Q^2}{4M_d^2}\,. \tag{9}$$

Although $B(Q^2)$ can be used to determine the magnetic factor, the charge monopole and quadrupole form factors can not be separated by using this in $A(Q^2)$. The total separation of the form factors requires that the cross section be measured for a polarized deuteron in either the initial or final state. Since the deuteron has a total angular momentum of 1, the deuteron is subject to both vector and tensor polarizations. The response which is typically chosen to effect the total separation of the form factors is tensor response $t_{20}(Q^2)$. This is defined as:

$$\begin{aligned} t_{20}(Q^2) &= -\left\{\frac{8}{3}\eta G_C(Q^2)G_Q(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2)\right. \\ &\quad \left. + \frac{1}{3}\eta\left[1+2(1+\eta)\tan^2\frac{\theta}{2}\right]G_M^2(Q^2)\right\} \\ &\quad \times\left[\sqrt{2}\left(A(Q^2)+B(Q^2)\tan^2\frac{\theta}{2}\right)\right]^{-1} \end{aligned} \tag{10}$$

3. Relativistic Meson-Nucleon Model of the Deuteron

Understanding this transition in effective degrees of freedom requires that the consequences of both pictures be examined in the transition region. Since this region is expected to be typified by Q^2 values of a few GeV^2, it is necessary that the conventional picture of nuclear physics be extended to accommodate these large four-momentum transfers. In particular, since the nucleon mass is on the order of 1 GeV, it is necessary to construct nuclear models that are consistent with the special theory of relativity.

One approach to construction of relativistic models of the deuteron is to start with some effective Lorentz invariant lagrangian constructed from nucleon and meson fields. The sum off all possible Feynman diagrams contribution to nucleon-nucleon scattering and the formation of the deuteron bound state can be formally expressed in terms of the Bethe-Salpeter equation[11]. This equation is constructed by separating the Feynman diagrams into two classes: all diagrams that cannot be separated into two pieces by simply cutting two nucleon lines are said to be two-particle irreducible while the remainder are said to be two-particle reducible. The sum off all possible diagrams can be produced by summing the infinite set of irreducible diagrams into the kernel of a four-dimensional integral equation (the Bethe-Salpeter equation) which then generates the sum of all reducible and irreducible diagrams.

In practice, however, it is impossible to evaluate all of the infinite number of irreducible diagrams so the kernel is approximated by only a few low-order diagrams. Most applications of this approach to two-nucleon systems approximate the kernel with single meson exchanges for some collection of mesons.

An additional problem with this approach is that it is an integral equation in the four-dimensional Minkowski space. This causes considerable complications in the solution of the integral equation due to the analytic structure of the scattering matrix. This problem can be addressed by analytic continuation to complex time although this imposes the additional complication of analytic continuation of the solution to the Bethe-Salpeter equation to real time.

A variant of the Bethe-Salpeter approach is the quasipotential approach. Quasipotential equations can be obtained by contraining the relative energy or time in the Bethe-Salpeter equation is such a manner that the equation is reduced to a three-dimensional integral equation that is more amenable to solution by traditional techniques common in nuclear physics. There are a large number of possible ways to accomplish this reduction resulting in a corresponding number of different quasipotential equations. Although this may appear to be a severe approximation to the Bethe-Salpeter equation it has been shown that quasipotential reductions may actually improve the convergence scattering matrix to the exact result with respect to approximations to the kernel[12].

In order to calculate the elastic scattering amplitude, it is necessary to know the current operator. Since the exchanged mesons can be charged, there will be currents associate with this flow of charge in addition to the currents of the individual constituent nucleons. These exchange or interaction currents can be determined in the Bethe-Salpeter and quasipotential approaches by identifying the two-particle irreducible diagrams associated with two nucleons absorbing a photon. Care must be taken to ensure that the approximations of kernels and interaction currents are consistent in order to guarantee current conservation.

An additional complication with these models is that the nucleons and mesons are themselves composite objects of approximately 1 fm in size. It is therefore necessary to include this phenomenologically by including form factors at all interaction vertices. This can also interfere with current conservation if not imposed with care.

Figure 2 shows several relativistic calculations of $A(Q^2)$, $B(Q^2)$ and $t_{20}(Q^2)$ in comparison to data. The curves labelled "Van Orden and Gross" are produced in the context of a quasipotential equation called the Spectator or Gross equation[13, 14, 16, 17, 18]. This approach is manifestly covariant and current conserving. A one-boson-exchange interaction with six mesons is fit to NN scattering data up to 300 MeV. Electromagnetic form factors for the nucleon are taken as parameterizations of nucleon electromagnetic form factor data[19] and according to a prescription that guarantees current conservation[20, 21]. The curves labelled "Hummel and Tjon" are the result of another quasipotential approach[22, 23] and the curves labelled "Forrest and Schiavilla" are for a semirelativistic potential model[24].

Clearly, "traditional" nuclear physics can account for the available data remarkably well up to $Q^2 = 6$ GeV^2. Some caution is required, however, in making this statement since the results shown in the figure reflect choices concerning the ingredients of the calculation some of which are not constrained by external data. In particular, the single-nucleon electromagnetic current operator used in this calculation has a piece that only contributes off mass shell. This contribution has a form factor that is constrained only at $Q^2 = 0$. It can not be fixed by other data for other values of Q^2. The curve labelled "Van Orden and Gross" uses the arbitrary choice that this form factor have a form similar to the on-shell form factors. This freedom to choose the form factor can be exploited to improve the description of the data. This has been done in the curve labelled "Van Orden and Gross, Offshell Fit." Unfortunately, this freedom severely limits the predictive power of these models.

4. Perturbative Quantum Chromodynamics

Perturbative QCD can be used to calculate the rate of fall of the the deuteron form factors for large values of Q^2. This follows from examination of diagrams such as that in Fig. 3 which shows six quarks interacting by exchanging gluons. If the photon transfers a large amount of momentum to the deuteron, this momentum must be distributed evenly among the quarks in order that the quarks can then be reconstituted

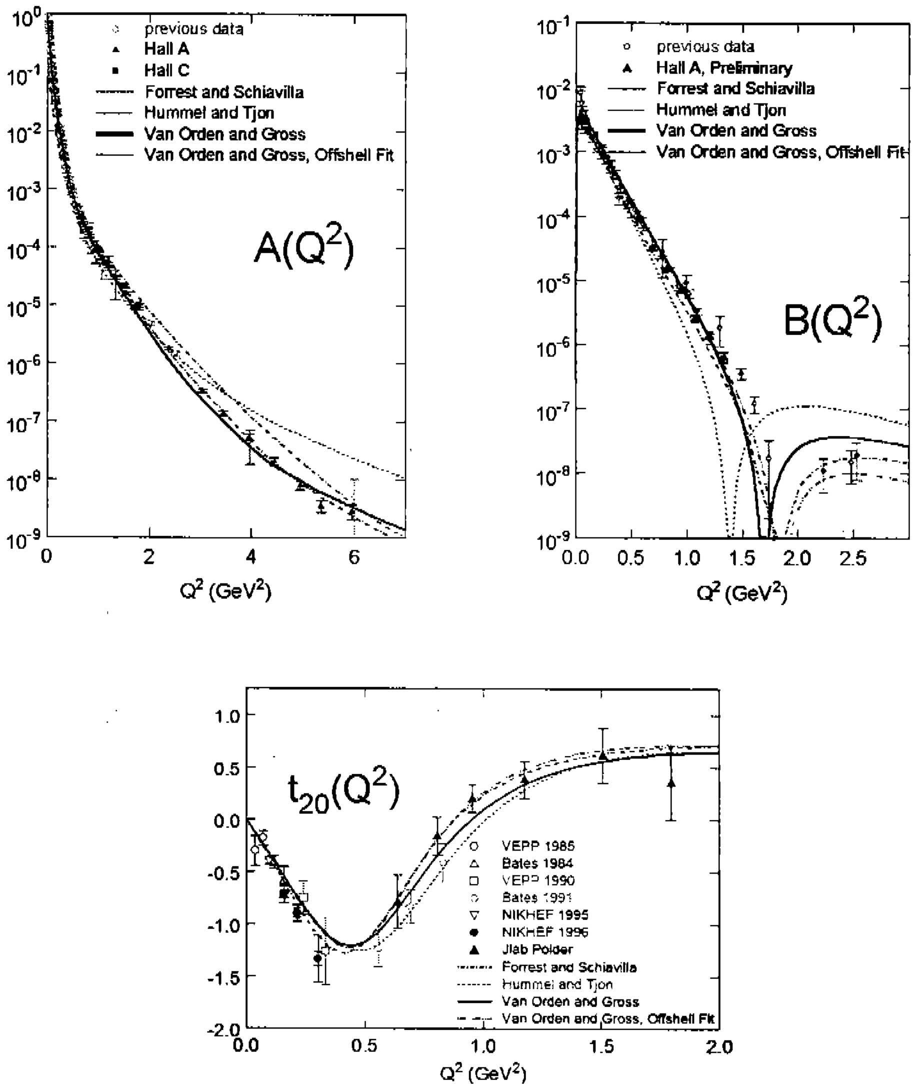

Figure 2. The structure functions $A(Q^2)$, $B(Q^2)$ and $t_{20}(Q^2)$ for several relativistic models.

as a deuteron in the final state. This requires that all of the quark and gluon propagators from the absorption of the photon and the final gluon exchange must also carry large four-momenta. Examination of the forms of the quark and gluon propagators then lead to a set of quark counting rules that determine the rate of fall of the form factors at large Q^2. For

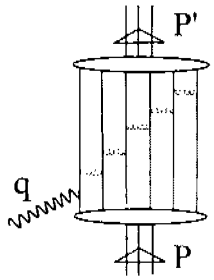

Figure 3. A typical diagram used in obtaining the PQCD quark counting rules for elastic electron deuteron scattering.

$A(Q^2)$ these rules predict that this structure function must fall as

$$A(Q^2) \sim \frac{c_0}{Q^{20}} \tag{11}$$

Figure 4 shows the data for $A(Q^2)$ for 3.0 GeV^2 $< Q^2 <$ 6.0 GeV^2 compared to Q^{2n} for three values of n with the curves normalized such

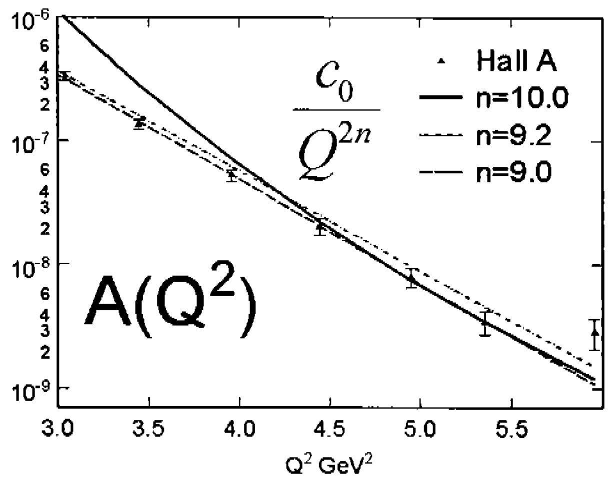

Figure 4. The structure function $A(Q^2)$ for several relativistic models.

they pass through the center of the penultimate data point. This semilog plot shows that all of the data with the exception of the last point appear to be along a straight line consistent with power law fall of the structure function. However, while the PQCD prediction of $n = 10$ is consistent with the the higher Q^2 points, the best fit to all of the data (with the exception of the last point) is for $n = 9.0$. The evidence for the onset of PQCD in this data would appear to be ambiguous at best.

5. Conclusions

The current status of the search for the quark degrees of freedom in elastic electron-deuteron scattering is unclear. Traditional nuclear physics model provide a reasonable description of elastic electron-deuteron scattering up to $Q^2 = 6$ GeV^2 but are subject to ambiguities that at present can not be eliminated phenomenologically. This limits their predictive power.

From the QCD side, nonperturbative calculations using lattice QCD for this process are not currently practical due to present capabilities of computers. While PQCD can predict the rate of fall of the form factors, it can not predict the normalization. The current data for $A(Q^2)$ are not unambiguously consistent with the predicted rate of fall.

The current situation suggests that it may be difficult to find evidence of an abrupt transition from the regime of traditional nuclear physics to the regime of QCD.

Acknowledgments

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