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Note on Nuclear Shadowing at Low Q^2 and the Extraction of $\sin^2\theta_W$

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Abstract

In the light of a recent claim that models of nuclear shadowing which incorporate a vector meson dominance component at low Q^2 are incompatible with nuclear deep inelastic scattering data, we compare the predictions of such a model with high precision data on the Q^2 dependence of nuclear shadowing. Contrary to this claim, we find that models which incorporate both vector meson and partonic mechanisms are indeed consistent with both the magnitude and the Q^2 slope of the shadowing data.

In a recent note [1] Zeller, McFarland *et al.* have suggested that the nuclear shadowing model of Ref. [2] "is not supported by charged-lepton deep inelastic scattering (DIS) data", and predicts the wrong Q^2 dependence for nuclear shadowing in the NuTeV kinematic region ($1 < Q^2 < 140$ GeV^2). This assertion was used to cast doubt on the suggestion by Miller and Thomas [3] — see also Kovalenko *et al.* [4] — that a difference between the nuclear shadowing in neutral and charged current interactions may account for the apparent discrepancy between the value of $\sin^2\theta_W$ extracted from neutrino–nucleus scattering [5] and its value within the Standard Model. In view of the importance of this issue we carefully examine the predictions of Ref. [2] in comparison with the Q^2 dependence of the best available data on nuclear shadowing [6–8].

An extensive review of both data and models of nuclear shadowing was given recently by Piller and Weise [9]. We shall concentrate on the model which we published just before the release of the final NMC data [8]. Whereas that model was based on a two-phase picture of nuclear shadowing [10,11], similar to that pioneered by Kwiecinski and Badelek [12–14], Zeller *et al.* *misidentify* it as "a specific vector meson dominance shadowing model" [1]. To clarify the confusion apparent in Ref. [1], we briefly review this model.

At high virtuality the interaction of a photon with a nucleus is most efficiently parameterized through a partonic mechanism, involving diffractive scattering through the double and triple Pomeron [15]. For $Q^2 \gtrsim 2$ GeV^2, the contribution to the nuclear structure function F_2^A (per nucleon) from this mechanism can be written as

$$\delta^{(\mathbb{P})} F_2^A(x, Q^2) = \frac{1}{A} \int_{y_{min}}^{A} dy \ f_{\mathbb{P}/A}(y) \ F_2^{\mathbb{P}}(x_{\mathbb{P}}, Q^2) \ , \tag{1}$$

where $f_{\mathbb{P}/A}(y)$ is the Pomeron ($\mathbb{P}$) flux, and $F_2^{\mathbb{P}}$ is the effective Pomeron structure function [16]. The variable $y = x(1 + M_X^2/Q^2)$ is the light-cone momentum fraction carried by the Pomeron (M_X is the mass of the diffractive hadronic debris), and $x_{\mathbb{P}} = x/y$ is the momentum fraction of the Pomeron carried by the struck quark. The dependence of $F_2^{\mathbb{P}}$ on Q^2 at large Q^2, in the region where perturbative QCD can be applied, arises from radiative corrections to the parton distributions in the Pomeron [14,17], which leads to a weak, logarithmic, Q^2 dependence for the shadowing correction $\delta^{(\mathbb{P})} F_2^A$. Alone, the $\mathbb{P}$ contribution to shadowing would give a structure function ratio F_2^A/F_2^D that would be almost flat for $Q^2 \gtrsim 2$ GeV^2 [18].

On the other hand, the description of shadowing at low Q^2 requires a higher-twist mechanism, such as vector meson dominance (VMD), which can map smoothly onto the photoproduction limit at $Q^2 = 0$. The VMD model is empirically based on the observation that some aspects of the interaction of photons with hadronic systems resemble purely hadronic interactions [19,20]. In QCD language this is understood in terms of the coupling of the photon to a correlated $q\bar{q}$ pair with low invariant mass, which may be approximated as a virtual vector meson. One can then estimate the amount of shadowing in terms of the multiple scattering of the vector meson using Glauber theory. The corresponding VMD correction to F_2^A is

$$\delta^{(V)} F_2^A(x, Q^2) = \frac{1}{A} \frac{Q^2}{\pi} \sum_V \frac{M_V^4 \ \delta\sigma_{VA}}{f_V^2 (Q^2 + M_V^2)^2} \ , \tag{2}$$

where $\delta\sigma_{VA}$ is the shadowing correction to the vector meson–nucleus cross section, f_V is the photon–vector meson coupling strength [19], and M_V is the vector meson mass. In practice, only the lowest mass vector mesons ($V = \rho^0, \omega, \phi$) are important at low Q^2. (Inclusion of higher mass states, including continuum contributions, leads to so-called generalized vector meson dominance models [21].) The vector meson propagators in Eq. (2) lead to a strong Q^2 dependence of $\delta^{(V)} F_2^A$ at low Q^2, which peaks at $Q^2 \sim 1$ GeV^2, although one should note that the nucleon structure function itself also varies rapidly with Q^2 in this region. For $Q^2 \to 0$ and fixed x, $\delta^{(V)} F_2^A$ disappears because of the vanishing of the total F_2^A. Furthermore, since this is a higher twist effect, shadowing in the VMD model dies off quite rapidly between $Q^2 \sim 1$ and 10 GeV^2, so that for $Q^2 \gtrsim 10$ GeV^2 it is almost negligible — leaving only the diffractive partonic term, $\delta^{(\mathbb{P})} F_2^A$.

Zeller *et al.* suggest that the model can be tested by looking for "deviations from logarithmic Q^2 dependence of shadowing at low Q^2" [1], and point out that "the most precise data which overlaps NuTeV's kinematic region $\cdots$ comes from the NMC experiment" [7,8]. We follow this suggestion and examine the NMC data in detail. Actually, a detailed analysis of the Q^2 dependence of the NMC data, as well as the lower-Q^2 Fermilab-E665 data [22],

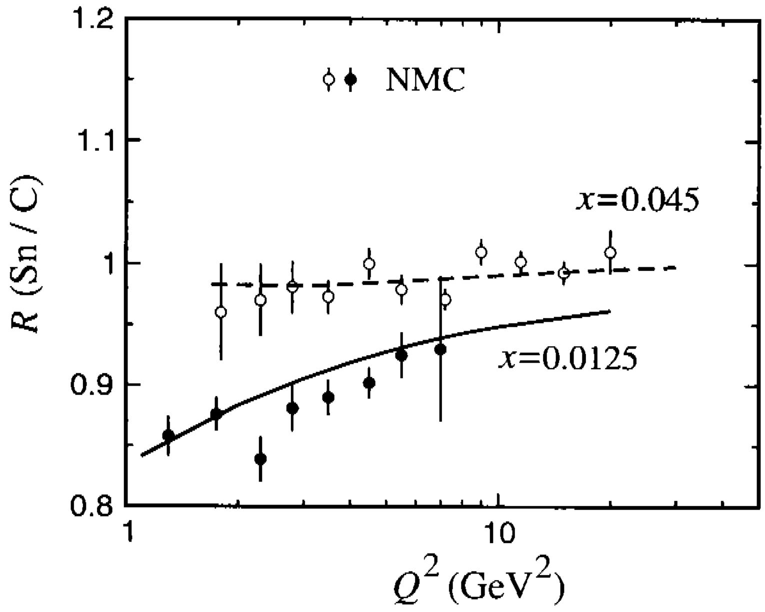

FIG. 1. Q^2 variation of the Sn/C structure function ratio in the model of Ref. [2] for $x = 0.0125$ (solid) and $x = 0.045$ (dashed). The data are from NMC [8], with statistical errors only.

was performed in Refs. [2,10] for a range of nuclei from $A = 2$ to $A = 208$ (*viz.*, for D, Li, Be, C, Al, Ca, Fe, Sn, Xe and Pb). Ratios of F_2^A/F_2^D were calculated [2,10] for a range of x ($10^{-5} \lesssim x \lesssim 0.1$) and Q^2 ($0.03 \lesssim Q^2 \lesssim 100$ GeV^2). Subsequent to these analyses, high precision data on the Q^2 dependence of Sn/C structure function ratios were published [8], which provided the first detailed evidence concerning the Q^2-dependence of nuclear shadowing.

In Fig. 1 we show the calculated ratio $R(\mathrm{Sn/C}) \equiv F_2^{\mathrm{Sn}}/F_2^{\mathrm{C}}$ as a function of Q^2 for $x = 0.0125$ (solid curve) and $x = 0.045$ (dashed), compared with the NMC data [8]. The overall agreement between the model and the data is clearly excellent. In particular, the observed Q^2 dependence of the ratios is certainly compatible with that indicated by the NMC data. At large Q^2 ($Q^2 \gtrsim 10$ GeV^2) the calculated curves become almost constant with Q^2, as expected from a partonic, leading-twist mechanism [2] – see also Refs. [23–27]. In the smallest x bins, however, the Q^2 values reach down to $Q^2 \approx 1$ GeV^2. The data on the C/D and Ca/D ratios analyzed in Ref. [2] at even smaller x ($x \gtrsim 0.0003$) extend down to $Q^2 \approx 0.05$ GeV^2. This region is clearly inaccessible to any model involving only a partonic mechanism, and it is essential to invoke a non-scaling mechanism here, such as vector meson dominance. One should also note that, even though the shadowing corrections may depend strongly on Q^2, because the nucleon structure function itself is rapidly varying at low Q^2, the Q^2 dependence of the ratio will not be as strong as in the absolute structure functions. In any case, the fact that the two-phase model [2] describes the NMC data over such a wide range of Q^2 gives one added confidence in applying the extension to $\nu(\bar{\nu})$ scattering [28] to the smaller range of Q^2 included in the NuTeV data sets.

To illustrate the Q^2 dependence of R over the full range of x covered in the NMC experiment, Arneodo *et al.* [8] parameterized the Sn/C ratio as $R(\mathrm{Sn/C}) = a + b \ln Q^2$, and

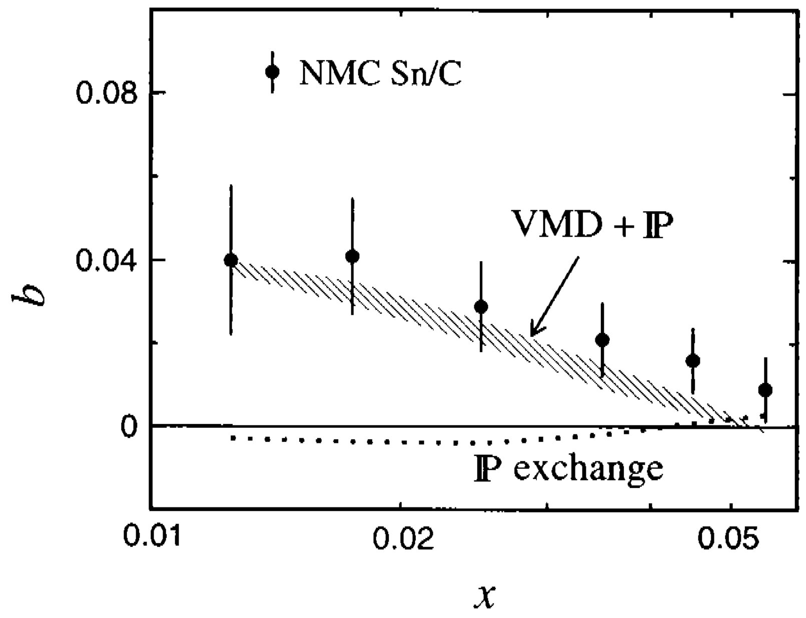

FIG. 2. Logarithmic slope, b, in Q^2 of the NMC Sn/C ratio as a function of x [8], compared with the nuclear shadowing model of Ref. [2]. The statistical and systematic errors are added in quadrature.

extracted the logarithmic slopes $b = dR/d\ln Q^2$ as a function of x. As illustrated in Fig. 2, the NMC find that "the slopes are positive and significantly different from zero in the region $0.01 < x < 0.05$, indicating that the amount of shadowing decreases with increasing Q^2" [8]. Based on the NMC data, "in the range $0.01 < x < 0.06 \cdots$ one expects a logarithmic slope b decreasing from about 0.03 at the smallest x value to zero at $x \approx 0.06$" [29]. The result of the model calculation [2] is perfectly consistent with the NMC data over the full range of x covered, as Fig. 2 demonstrates (see also Fig. 3(b) of Ref. [2]). In particular, the $\mathbb{P}$-exchange mechanism alone, modified by applying a factor $Q^2/(Q^2+Q_0^2)$ [13,30] to ensure that $\delta^{(\mathbb{P})}F_2^A \to 0$ as $Q^2 \to 0$, is clearly insufficient [18] to describe the logarithmic slope in Q^2 at low x, and a VMD component is necessary to describe the data (the shaded region indicates an estimate of the uncertainty in the model calculation).

As well as suggesting that the model of Ref. [2] is not consistent with the observed Q^2 dependence of the NMC data [6,7] — which we see from Figs. 1 and 2, as well as from Refs. [2,10], is clearly not the case — Zeller *et al.* assert that the "lack of evidence for strong Q^2 dependence of shadowing in the NuTeV kinematic region suggests that the conventional modeling of shadowing as a change in parton distribution functions is appropriate", citing Refs. [12] and [13] as examples of such "conventional modeling". However, as stated clearly for instance in the abstract of Ref. [13], "both the vector meson and parton contributions are considered for low and high Q^2 values" in their model. In essence, the models of Refs. [12,13] and [2] are the same so far as their implementation of VMD at low Q^2 to describe the transition to the photoproduction region, and parton recombination, parameterized via $\mathbb{P}$-exchange, at high Q^2. The assertion by Zeller *et al.* [1] that the VMD + $\mathbb{P}$-exchange model [2] is "highly disfavored from charged lepton DIS data" is evidently without factual basis.

In conclusion, whatever the final explanation for the NuTeV anomaly in $\sin^2\theta_W$, the alleged conflict between experiment and the nuclear shadowing model of Ref. [2], involving contributions from both VMD and partonic (diffractive $\mathbb{P}$-exchange) mechanisms, is not relevant to the argument.

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