# Spin-3/2 baryons in lattice QCD

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We present first results for masses of spin-3/2 baryons in lattice QCD, using a novel fat-link clover fermion action in which only the irrelevant operators are constructed using fat links. In the isospin-1/2 sector, we observe, after appropriate spin and parity projection, a strong signal for the $J^P = \frac{3}{2}^-$ state, and find good agreement between the $\frac{1}{2}^+$ mass and earlier nucleon mass simulations with a spin-1/2 interpolating field. For the isospin-3/2 $\Delta$ states, clear mass splittings are observed between the various $\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ channels, with the calculated level orderings in good agreement with those observed empirically.

## 1. INTRODUCTION

Recent advances in computing power and the development of improved actions have enabled simulations of the spectrum of excited states of the nucleon in lattice QCD [1–3]. The lattice studies complement the high precision measurements of the $N^*$ spectrum under way at Jefferson Lab [4]. In a recent paper [5], results were presented for the excited nucleon and spin-1/2 hyperon spectra using the Fat-Link Irrelevant Clover (FLIC) fermion action [6] with an $\mathcal{O}(a^2)$ improved gluon action. Here we extend the analysis of Ref. [5] to the spin-3/2 sector, and present results in both the isospin-1/2 and 3/2 channels.

In the isospin-3/2 sector, our results for the $\Delta(\frac{3}{2}^+)$ state agree well with earlier simulations [7] using Wilson fermions. We find a clear signal for the $P$-wave $\Delta(\frac{3}{2}^-)$ parity partner of the $\Delta$ ground state, and a discernible signal for the $\Delta(\frac{1}{2}^\pm)$ states. In particular, the $\Delta(\frac{1}{2}^-)$ state is found to have a mass $\sim$ 350–400 MeV above the $\Delta(\frac{3}{2}^+)$, with the $\Delta(\frac{3}{2}^-)$ slightly heavier. The $\Delta(\frac{1}{2}^+)$ state is found to lie $\sim$ 200–300 MeV above these. This level ordering is consistent with experiment.

In the spin-3/2 nucleon sector, there is good agreement for the projected $\frac{1}{2}^+$ state with earlier nucleon mass calculations [5] using standard spin-1/2 nucleon interpolating fields. Furthermore, we find a good signal for the $N(\frac{3}{2}^-)$ state, with a mass splitting of $\sim$ 700–1000 MeV with the nucleon ground state.

## 2. SPIN-$\frac{3}{2}$ BARYONS ON THE LATTICE

The standard isospin-1/2, spin-3/2, charge +1 interpolating field is given by $\chi_\mu^{N^+}(x) = \epsilon^{abc}\left(u^{Ta}(x)\, C\gamma_5\gamma_\mu\, d^b(x)\right)\gamma_5 u^c(x)$, which transforms as a pseudovector under parity, in accord with a positive parity Rarita-Schwinger spinor. The quark field operators $u$ and $d$ act at Euclidean space-time point $x$, $C$ is the charge conjugation matrix, $a, b$ and $c$ are color labels, and the superscript $T$ denotes the transpose. The charge neutral interpolating field is obtained by interchanging $u \leftrightarrow d$.

The commonly used interpolating field for the $\Delta^{++}$ resonance is given by [8] $\chi_\mu^{\Delta^{++}}(x) = \epsilon^{abc}\left(u^{Ta}(x)\, C\gamma_\mu\, u^b(x)\right) u^c(x)$. Since the spin-3/2 Rarita-Schwinger spinor-vector is a tensor product of a spin-1 vector and a spinor, the spin-3/2 interpolating field contains spin-1/2 contributions. To project a spin-3/2 state one needs

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to use a spin-3/2 projection operator [9]. Following spin projection, the correlation function for a given spin $s$, $G^s_{\mu\nu}$, still contains positive and negative parity states. In an analogous procedure to that used in Ref. [5], where a fixed boundary condition is used in the time direction, positive and negative parity states are obtained by taking the trace of $G^s_{\mu\nu}$ with the operators $\Gamma_\pm = \frac{1}{2}(1 \pm \gamma_4)$, respectively.

## 3. RESULTS

Lattice simulations are performed on a $16^3 \times 32$ lattice at $\beta = 4.60$, corresponding to a lattice spacing of $a = 0.122(2)$ fm set by the string tension [10] with $\sqrt{\sigma} = 440$ MeV. We consider 392 configurations generated on the Orion cluster at the CSSM, U. Adelaide. A mean-field improved plaquette plus rectangle gauge action is used. For the quark fields, a Fat-Link Irrelevant Clover (FLIC) [6] action is implemented. We employ a highly improved definition of $F_{\mu\nu}$ [6,11] leaving errors of $\mathcal{O}(a^6)$. Mean-field improvement of the tree-level clover coefficient with fat links represents a small correction and proves to be quite adequate [6]. A fixed boundary condition in the time direction is used for the fermions by setting $U_t(\vec{x}, N_t) = 0 \; \forall \; \vec{x}$ in the hopping terms of the fermion action, with periodic boundary conditions imposed in the spatial directions. Gauge-invariant gaussian smearing in the spatial dimensions is applied at the fermion source (placed at time slice $t = 3$) to increase the overlap of the interpolating operators with the ground states.

In Fig. 1 the results for the spin-projected $\Delta(\frac{3}{2}^+)$ (triangles) and $\Delta(\frac{3}{2}^-)$ (diamonds) masses are shown as a function of the pseudoscalar meson mass, $m_\pi^2$. The trend of the $\Delta(\frac{3}{2}^+)$ data points with decreasing $m_q$ is consistent with the physical mass of the $\Delta(1232)$, with some nonlinearity in $m_\pi^2$ expected near the chiral limit. The mass of the $\Delta(\frac{3}{2}^-)$ lies some 500 MeV above that of its parity partner.

When performing a spin projection to extract the $\Delta(\frac{1}{2}^\pm)$ states, a discernible, although noisy, signal is detected. The $\chi_\mu^{\Delta^{++}}$ interpolating field has small overlap with spin-1/2 states, however,

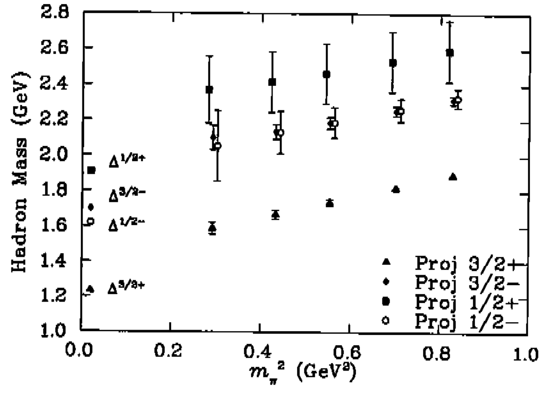


Figure 1. Masses of the spin-projected $\Delta(\frac{3}{2}^\pm)$ and $\Delta(\frac{1}{2}^\pm)$ states. The empirical masses are indicated along the ordinate.

with $\sim 400$ configurations we are able to extract masses for the spin-1/2 states at sufficiently early times. The $\Delta(\frac{1}{2}^+)$ (squares) and $\Delta(\frac{1}{2}^-)$ (open circles) are also displayed in Fig. 1. The lowest excitation of the ground state, namely the $\Delta(\frac{1}{2}^-)$, has a mass $\sim$ 350–400 MeV above the $\Delta(\frac{3}{2}^+)$, with the $\Delta(\frac{3}{2}^-)$ slightly heavier. The $\Delta(\frac{1}{2}^+)$ state is found to lie $\sim$ 200–300 MeV above these, although the signal becomes weak at smaller quark masses. This level ordering is consistent with that observed in the empirical mass spectrum.

In the isospin-1/2 sector, large statistical fluctuations make it difficult to obtain a clear signal, even with 392 configurations. A reasonable value of $\chi^2/N_{\rm DF}$ is obtained for time slices $t = 7$–8. These results are shown as a function of $m_\pi^2$ in Fig. 2. Parity projecting to extract the $N(\frac{3}{2}^+)$ state, we find that the correlation function changes sign and has a large negative contribution in the range $t = 7$–11. This behavior is an artifact associated with the quenched decay of the excited state into $N + \eta'$, and is further explored in Ref. [12].

The $N(\frac{1}{2}^+)$ channel displays the interplay of a quenched decay channel and the ground state contribution. A strong $P$-wave coupling of the $N(\frac{1}{2}^+)$ to $N\eta'$ forces the correlation function to be negative at small times, which then turns positive at larger times when the ground state contri-

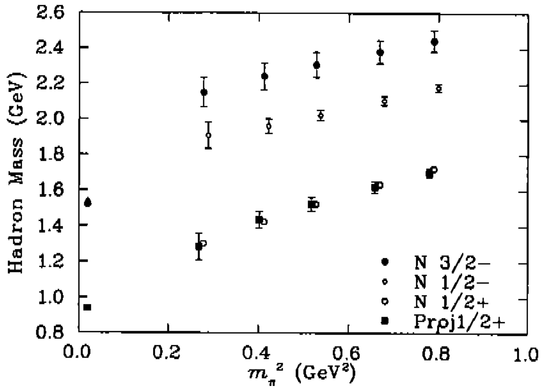


Figure 2. Masses of the spin-projected $N(\frac{3}{2}^-)$ and $N(\frac{1}{2}^+)$ states, compared with the nucleon and $N(\frac{1}{2}^-)$ masses from Ref. [5].

bution begins to dominate the correlation function. A good $\chi^2/N_{\rm DF}$ value is obtained for a fit to time slices $t = 10$–$14$. This suggests that the first excited state has strong coupling to the $\eta'$, which implies a gluon-rich structure for the Roper resonance [13].

The extracted masses of the $N(\frac{3}{2}^-)$ and $N(\frac{1}{2}^+)$ states are displayed in Fig. 2 as a function of $m_\pi^2$. Earlier results using the standard spin-1/2 interpolating field [5,6] are also shown in Fig. 2 for reference. There is excellent agreement between the spin-projected $\frac{1}{2}^+$ state obtained from the spin-3/2 interpolating field and the earlier $\frac{1}{2}^+$ results. A clear mass splitting is also seen between the $N(\frac{3}{2}^-)$ and $N(\frac{1}{2}^+)$ states obtained from the spin-3/2 interpolating field, with a mass difference of 700–1000 MeV.

## 4. CONCLUSION

First results for the spectrum of spin-3/2 baryons in the isospin-1/2 and 3/2 channels are reported, using a FLIC fermion action and an $\mathcal{O}(a^2)$ mean-field improved gauge action. Good agreement is found with earlier calculations for the $\Delta$ ground state, and clear mass splittings between the ground state and its parity partner are observed. A signal is also obtained for the $\Delta(\frac{1}{2}^\pm)$ states, with the level ordering consistent with the observed empirical mass spectrum.

For isospin-1/2 baryons, clear signals are obtained for both the $N(\frac{3}{2}^-)$ and spin-projected $N(\frac{1}{2}^+)$ states from a spin-3/2 interpolating field. The $\frac{1}{2}^+$ state in particular is in good agreement with earlier simulations of the nucleon mass using standard spin-1/2 interpolating fields.

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