Excitations of the nucleon with dynamical fermions

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We measure the spectrum of low-lying nucleon resonances using Bayesian fitting methods. We compare the masses obtained in the quenched approximation to those obtained with two flavours of dynamical fermions at a matched lattice spacing. At the pion masses employed in our simulations, we find that the mass of the first positive-parity nucleon excitation is always greater than that of the parity partner of the nucleon.

The observed spectrum of excited nucleon states contains some puzzles, such as the anomalously light masses of the Roper $N^*(1440)$ and $\Lambda(1405)$. Neither of these states can be incorporated in the quark model, and this had led to speculation as to whether they are really three quark states. This spectrum, at least in principle, is accessible to lattice QCD calculations. In this exploratory study we extract the masses of the nucleon (N^+), its parity partner (N^-) and the lightest positive-parity excitation (N'^+) and examine their ordering.

Lattice determinations of the nucleon generally use the following three interpolation operators:

$$N_1^{1/2+} = \epsilon_{ijk}(u_i^T C\gamma_5 d_j)u_k, \quad (1)$$

$$N_2^{1/2+} = \epsilon_{ijk}(u_i^T C d_j)\gamma_5 u_k, \quad (2)$$

$$N_3^{1/2+} = \epsilon_{ijk}(u_i^T C\gamma_4\gamma_5 d_j)u_k. \quad (3)$$

These operators have an overlap with both positive- and negative-parity states, but on a lattice with (anti-)periodic boundary conditions we can use the parity projection operator $\frac{1}{2}(1 \pm \gamma_4)$ to delineate forward and backward propagating states of opposite parities. The time dependence of the parity projected correlators is then,

$$C_\pm(t) = \sum_n A_n^\pm e^{-M_n^\pm t} + A_n^\mp e^{-M_n^\mp (N_t - t)} \quad (4)$$

The "diquark" part of $N_{1,3}$ couples the upper spinor components, whilst the N_2 operator couples upper and lower components, and so vanishes in the non-relativistic limit. The expectation, confirmed by the data, is that the $N_{1,3}$ operators give a much better overlap with the nucleon ground state while the N_2 operator has an overlap with the lowest positive-parity nucleon excitation.

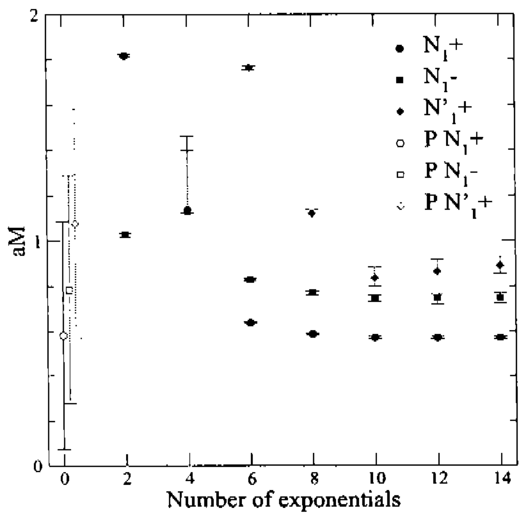

Figure 1. Mass versus number of exponentials. $\beta = 6.2$, $\kappa = 0.1346$.

For the negative-parity states the signal is short lived [1], so the choice of fit range is a substantial systematic error. The aim of Bayesian fitting is to eliminate this systematic uncertainty [2]. Furthermore, by fitting all the data, it may be pos-

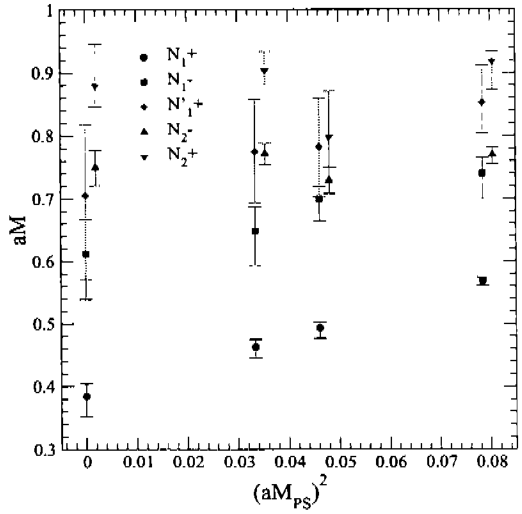

Figure 2. Quark mass dependence of baryon masses at $\beta = 6.2$

sible to measure the contribution from excited states. The number of exponentials $N_{\rm exp}$ is chosen such that the results for the ground and first excited states are stable if $N_{\rm exp}$ is increased. The ground-state priors were guessed from the correlator data at large temporal separations using the effective mass,

$$M^{\rm eff}(t) = \ln\left(\frac{C(t+1)}{C(t)}\right), \tag{5}$$

and priors for the mass splitting and excited state amplitudes obtained using $\Delta M = M_1^{\rm eff}(\tau) - M_0^{\rm eff}(t)$, where $\tau < t$, and $M_1^{\rm eff}$ is determined from the correlator C' with the ground state prior contribution subtracted. This ensures that the minimisation algorithm starts from the neighbourhood of the global minimum of χ^2. The width of the priors is chosen to be larger than the splitting. Furthermore, the fits are more stable if both M_0 and ΔM are constrained to be positive, so instead of fitting to eqn (4) we fit the following expression to the data

$$f(t) = \sum_n A_n \exp\left[-\sum_{m=0}^{n} e^{\lambda_m} t\right] \tag{6}$$

where $\lambda_k = \ln \mu_k$, $\mu_0 = m_0$ and $\mu_k = M_k - M_{k-1}$.

Table 1
Details of the data sets. a^{-1} in GeV, the m_π/m_ρ values for the DF configurations are at $\kappa^{\rm sea} = \kappa^{\rm val}$.

$\beta,\kappa^{\rm sea}$	cfgs	a^{-1}	V	m_π/m_ρ
6.2,0	216	2.91	$24^3 \times 48$	N/A
6.0,0	496	2.12	$16^3 \times 48$	N/A
5.93,0	623	1.90	$16^3 \times 32$	N/A
5.2,0.1350	202	1.91	$16^3 \times 32$	0.70(1)
5.26,0.1345	102	1.90	$16^3 \times 32$	0.78(1)
5.29,0.1340	101	1.94	$16^3 \times 32$	0.84(1)

The calculation was performed using the standard Wilson gluon action, and the non-perturbatively $\mathcal{O}(a)$-improved Sheikholeslami-Wohlert action. Lattice artefacts thus appear in the spectrum at $\mathcal{O}(a^2)$. The values of β and $\kappa^{\rm sea}$ for the coarsest quenched (Q) and the dynamical-fermion (DF) ensembles were chosen so that the lattice spacing is the same. The light hadron spectrum has already been determined by the UKQCD collaboration for $\beta = 6.2$ and $\beta = 6.0$ [3] and for DF matched ensembles [4]. The N^- masses have also been determined for $\beta = 6.2$ and $\beta = 6.0$ [1].

We perform simultaneous fits to both the local-local and smeared-local correlators of both parities. However, for the lightest two DF ensembles the limited number of configurations only allows a fit to the local-local correlator alone. The dependence of the extracted masses on the number of exponentials $N_{\rm exp}$ is shown in figure 1, together with the priors, shown as the open symbols. The results are stable when $N_{\rm exp} > 10$, corresponding to five states of each parity. In general we use $N_{\rm exp} = 12$, except for DF and matched Q ensembles where we use $N_{\rm exp} = 10$.

The spectrum as determined from the operators $N_{1,3}$ (N_1 hereafter) and N_2 is shown in figure 2; we find consistency between the masses of N^- using the two operators, and between the excited-state mass $N_1'^+$ and ground-state mass N_2^+. However, as the quark mass is decreased the data quickly become very noisy, and at lighter quark masses, the masses from the N_2 operator are larger than those from the N_1. Only a simple linear chiral extrapolation is attempted as the data is too noisy for a more sophisticated approach.

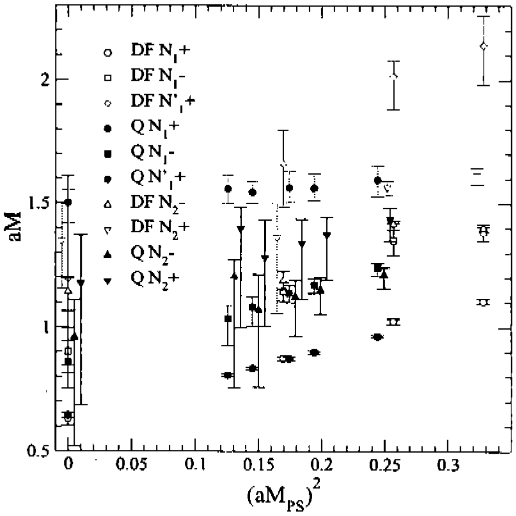

Figure 3. $N_f = 2$ (open symbols) versus $N_f = 0$ (closed symbols). Some of the symbols have been offset for clarity.

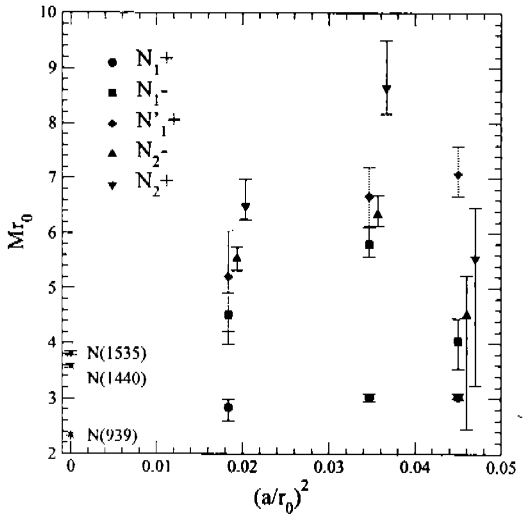

Figure 4. Lattice spacing dependence.

The comparison between the Q and DF ensembles at matched lattice spacing is shown in figure 3, revealing no systematic difference between the Q and DF results. In all cases the mass of N'^+ is above that of N^-. Furthermore the positive-parity excited state mass obtained from N_1 is consistent with the ground-state mass obtained from N_2, except at light quark masses where the latter becomes very noisy. Finally, the lattice spacing dependence of the measured masses on the quenched ensemble is shown in figure 4.

We have measured the nucleon spectrum and found that the masses are ordered according to $N'^+ > N^- > N^+$ for both the Q and DF data at all measured quark masses and lattice spacing, thus providing little evidence for the Roper resonance as a simple three-quark nucleon excitation. However, there are several caveats. The statistical quality of the data is poor. The spatial extent is only around 1.7 fm, though a comparison with a calculation of both the N^+ and N^- masses on a lattice of around twice that volume at $\beta = 6.0$ suggests that at these quark masses the finite-volume corrections are small [1]; such a comparison has not been performed for N'^+. Finally, the quark masses are still large, with the smallest DF $m_{PS}/m_V = 0.7$. Other determinations of nucleon resonances were presented at this conference [5,6], including one in the quenched approximation that exhibited a dramatic cross-over between N^- and N'^+ at light quark mass [7].

The nature of the Roper resonance remains a crucial question, but these studies have shown that lattice QCD can resolve the issue.

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