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Moments of Isovector Quark Distributions in Lattice QCD

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Abstract. We investigate the connection of lattice calculations of moments of isovector parton distributions to the physical regime through extrapolations in the quark mass. We consider the one pion loop renormalisation of the nucleon matrix elements of the corresponding operators and thereby develop formulae with which to extrapolate the moments of the unpolarised, helicity and transversity distributions. These formulae are consistent with chiral perturbation theory in the chiral limit and incorporate the correct heavy quark limits. In the polarised cases, the inclusion of intermediate states involving the Δ isobar is found to be very important. The results of our extrapolations are in general agreement with the phenomenological values of these moments where they are known, and for the first time we perform an extrapolation of the low moments of the isovector transversity distribution which is consistent with chiral symmetry.

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1 Introduction

Until recently, state of the art studies [1] of the moments of parton distributions within lattice QCD have led to large discrepancies with experimental data [2]. Lattice calculations are performed at quark masses much greater than those of the physical light quarks. Consequently, an extrapolation to the physical mass regime must be made in order to compare with experiment. A naive linear extrapolation results in values for the first three non-trivial moments of the unpolarised $u-d$ distribution that are 50% above the phenomenological values [2]. For the polarised distributions the results are no better; g_A (the 0^{th} moment of $\Delta u - \Delta d$) is significantly underestimated by a linear extrapolation, while the lack of accuracy in the data (both experimental and lattice) on higher moments precludes definitive statements. Such a large disagreement in such basic hadronic observables casts doubt on the current reliability of the lattice approach to hadronic physics.

We present an improved extrapolation scheme [3–6] that for the first time resolves much of this discrepancy. Using constraints from chiral symmetry and the heavy quark limit, we develop a formalism for the extrapolation of the moments of the isovector, unpolarised, helicity and transveristy distributions. These are related to the forward nucleon matrix elements of various twist-2 operators which are calculated on the lattice through the operator product expansion. In particular we find that contributions from intermediate states involving the Δ isobar are large, and their effects cannot be ignored in any quantitative analysis. With their inclusion, we are able to make reliable, almost model-independent[1] extrapolations of the moments of parton distributions and make predictions for the low spin transversity moments which can be tested by future measurements.

2 Pion-Loop Renormalisation of Moments of Parton Distribution Functions

General constraints from the approximate chiral symmetry of QCD lead to the appearance of non-analytic terms in the quark mass expansion of many hadronic quantities. In particular, the moments of quark distributions behave as [8,9]

$$\langle x^n \rangle_{q,\Delta q,\delta q} \sim m_\pi^2 \log m_\pi^2, \tag{1}$$

where the Gell-Mann–Oakes–Renner relation, $m_q \sim m_\pi^2$, has been used to express the quark mass in terms of the pion mass. This behaviour arises from the infrared properties of the one-pion loop renormalisation of the matrix elements of the corresponding twist-2 operators $\mathcal{O}_{q,\Delta q,\delta q}$ (see Ref. [5] for their definitions and further details). That is,

$$\langle N|\mathcal{O}_i^{\mu_1\cdots\mu_n}|N\rangle_{\text{dressed}} = \frac{Z_2}{Z_i}\langle N|\mathcal{O}_i^{\mu_1\cdots\mu_n}|N\rangle_{\text{bare}}\,, \tag{2}$$

where Z_2 is the nucleon wavefunction renormalisation and Z_i, $i = q, \Delta q, \delta q$, are the operator renormalisations arising from πN, $\pi\Delta$, and $\pi(N\Delta$ transition) intermediate states.

[1] Results are independent of the shape of the pion-nucleon form factor to 1-2% [7].

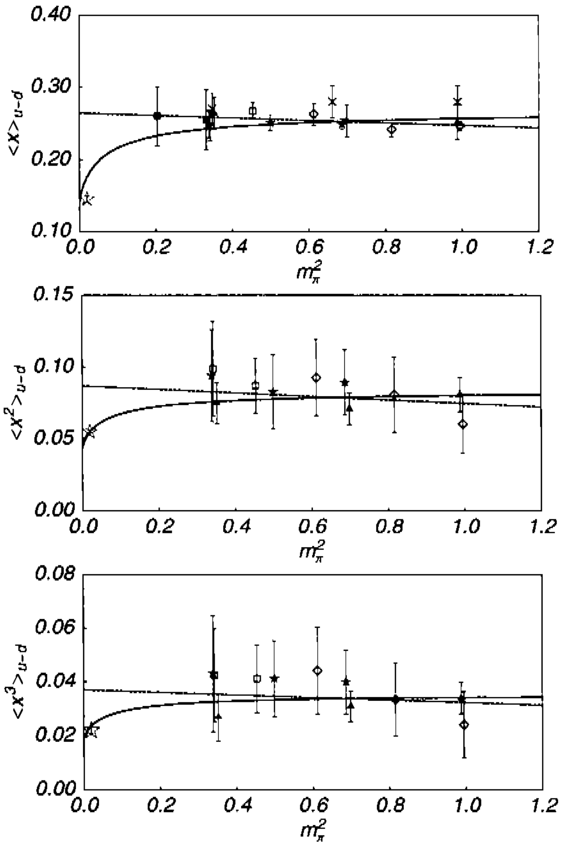

Fig. 1. Lowest three nontrivial moments of the unpolarised isovector parton distribution. Data are taken from various lattice simulations (see Ref. [5] for details). The linear extrapolation (dashed) significantly overestimates the experimental results (stars), whilst the extrapolation using Eq. (3) is in reasonable agreement.

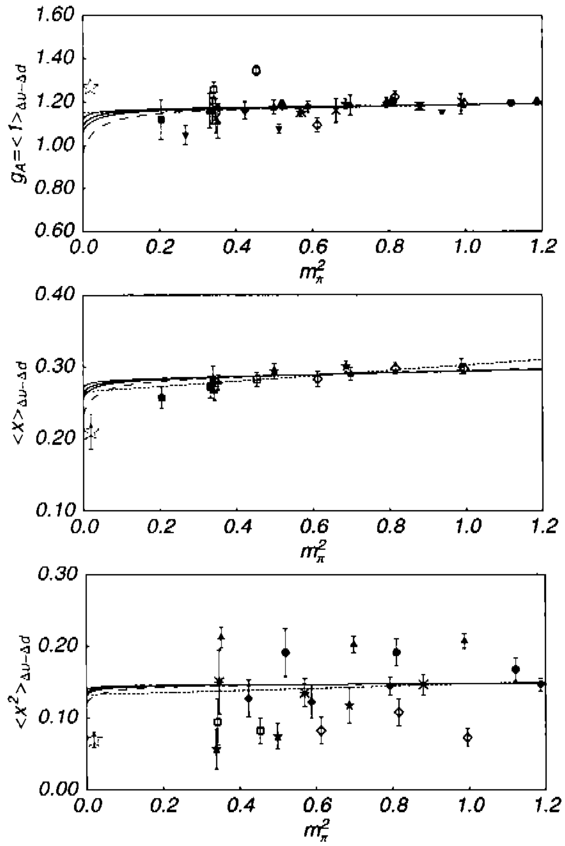

Fig. 2. Lowest three nontrivial moments of the helicity isovector parton distribution. Curves shown are (dotted line) linear extrapolation, (dashed line) extrapolation using the polarised analog of Eq. (3) but ignoring Δ contributions, and (solid lines) extrapolations for varying values of the $\pi N \Delta$ coupling. (Data as for Fig. 1.)

In the case of the unpolarised moments, simple one-loop calculations of the renormalisation using a variety of form factors (cutoff, monopole, dipole) for the pion–nucleon coupling all lead to a simple extrapolation formula

$$\langle x^n \rangle_{u-d} = a_n \left(1 + c_{\rm LNA} m_\pi^2 \log \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) + b_n m_\pi^2 \,, \quad (3)$$

with three free parameters, a_n, b_n, μ ($c_{\rm LNA}$ is fixed by chiral perturbation theory). Similar expressions are found for the helicity and transversity moments in Ref. [5]. The parameter μ describes the physical scale of the pion source [10] and ideally would be constrained by lattice data. However, until sufficiently accurate lattice data at low quark masses are available, alternative methods (such as the one used here) must be used to fix μ.

As shown by the solid curves in Fig. 1 this formula provides an excellent description of the lattice data, the experimental values of the moments and (with suitable modification – see Ref. [4]) the values known in the heavy quark limit. However, when a similar formula is applied to the moments of the helicity distributions, the extrapolation is considerably worse than a naive linear extrapolation, as shown by the dashed lines in Fig. 2.

When we turn to the moments of polarised parton distributions, there is considerable evidence from phenomenological models that suggests the Δ resonance will play an important role. Although the Δ contributions formally enter at higher order in m_π, the coefficients of these next-to-leading non-analytic terms are large, and they cannot be ignored in any quantitative analysis. This is clearly demonstrated by Fig. 3 where the pion mass dependence of the one-loop renormalisation of helicity matrix elements is shown calculated with a dipole form factor with varying values of the $\pi N \Delta$ coupling, $g_{\pi N \Delta}$. Full details are given in Ref. [5]. For these helicity (and transversity) operator matrix elements the difference between $g_{\pi N \Delta}/g_{\pi NN} = 0$ (no Δ) and $g_{\pi N \Delta}/g_{\pi NN} = 2$ (phenomenological value) is up to 15% — much greater than in the unpolarised case where the overall effect is less than 2% over the entire mass range studied here.

In order to incorporate these effects into the extrapolations of lattice data, we calculate the required renormalisations for the phenomenologically preferred dipole

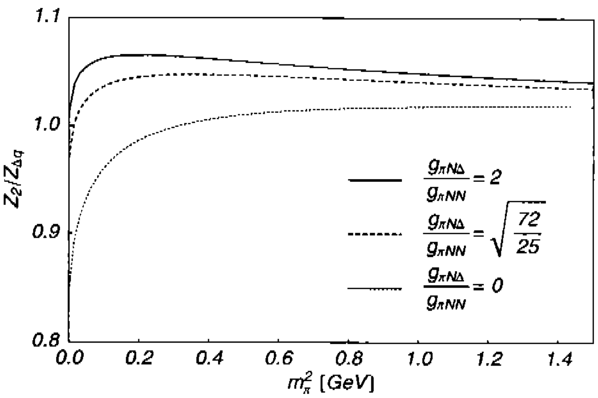

Fig. 3. One-loop renormalisation of the nucleon matrix elements of the polarised, isovector, twist-2 operators using a dipole form factor ($\Lambda = 0.8$ GeV) and varying values of the coupling ratio $g_{\pi N\Delta}/g_{\pi NN}$. The shaded region corresponds to variation of the Weinberg-Tomozawa coupling – see Ref. [5].

Moment	Experimental	Extrapolated
$\langle x\rangle_{u-d}$	0.145(4)	0.17(3)
$\langle x^2\rangle_{u-d}$	0.054(1)	0.05(2)
$\langle x^3\rangle_{u-d}$	0.022(1)	0.02(1)
$\langle 1\rangle_{\Delta u-\Delta d}$	1.267(4)	1.12(8)
$\langle x\rangle_{\Delta u-\Delta d}$	0.210(25)	0.27(3)
$\langle x^2\rangle_{\Delta u-\Delta d}$	0.070(11)	0.14(4)
$\langle 1\rangle_{\delta u-\delta d}$	?	1.22(8)
$\langle x\rangle_{\delta u-\delta d}$	?	0.5(1)

Table 1. Values of the unpolarized, helicity and transversity moments, extrapolated to the physical pion mass using Eq. (3). For comparison, experimental values of the moments, where known [2], are also listed.

parameter, $\Lambda = 0.8$ GeV, and for varying values of the coupling ratio $g_{\pi N\Delta}/g_{\pi NN}$. We then determine μ by fitting to the calculated renormalisations using Eq. (3) (with $c_{\rm LNA}$ calculated in the $\Delta M = M_\Delta - M_N \to 0$ limit). With μ thus fixed, we then use the lattice data to determine the fit parameters a_n and b_n. The resulting curves are then shown in each panel of Fig. 2 for $g_{\pi N\Delta}/g_{\pi NN} = 0$ (no Δ), $\sqrt{72/25}$, 1.85 and 2.[2]

Table 1 shows the resulting extrapolated values along with uncertainties resulting from the $\pi N\Delta$ and Weinberg-Tomozawa couplings, statistical and (estimated) systematic errors. Also listed are the experimental moments where known [2]. It is evident that the agreement between the experimental and extrapolated results is very good for the unpolarised moments, but significantly worse in the helicity sector. The extrapolated value of g_A is 10% below its experimental value, with 8% errors. However, there is some evidence that lattice simulations of this quantity are particularly susceptible to finite volume effects [11]. For the higher helicity moments, statistical errors on the lattice data are not yet sufficiently small to make definitive statements. Since the renormalisation of the transversity matrix elements is almost identical to that of the helicity matrix elements, we are also able to make an estimate of the values of these moments which can be compared with future experimental determinations.

3 Conclusion

We have investigated the extrapolation of lattice data on the low moments of the isovector, unpolarised, helicity and transversity parton distribution functions. In the case of the polarised moments, the Δ isobar, even though not leading non-analytic, was found to play a significant role and its inclusion in the operator renormalisation and extrapolation procedure is vital. With this accounted for, we have the surprising result that the effect of the non-analytic behavior is strongly suppressed for the polarised moments, and a naive linear extrapolation of the moments provides quite a good approximation to the more accurate form.

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