

Coulomb sum rule for ${}^4\text{He}$

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Abstract. We determine the Coulomb sum for ${}^4\text{He}$ using the world data on ${}^4\text{He}(e, e')$ and compare the results to calculations based on realistic interactions and including two-body components in the nuclear charge operator. We find good agreement between theory and experiment when using *free-nucleon* form factors. The apparent reduction of the in-medium G_{ep} implied by IA-interpretation of the L/T-ratios measured in ${}^4\text{He}(e, e'p)$ and ${}^4\text{He}(\vec{e}, e'\vec{p})$ is not confirmed.

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Introduction. The conjecture of a modification of the form factors of a nucleon embedded in the nuclear medium has received considerable attention during the last few years. This conjecture first arose in connection with measurements of the Coulomb sum rule (CSR) in nuclei like C , Ca and Fe [1, 2]. These were found to be much too low relative to theoretical predictions based on the square of the free proton charge form factor G_{ep} , to which the CSR is approximately proportional. This discrepancy naturally led to speculations that the G_{ep} of a proton in medium may be reduced from its free value. Later, a more careful analysis of the longitudinal data on medium-weight nuclei [3] indicated the lack of any significant quenching of the CSR relative to expectations using the free proton G_{ep} . This conclusion was corroborated by the results of a scaling analysis of inclusive electron-nucleus scattering data [4]. It is important to stress at this point that no quenching of the CSR has been observed in the hydrogen and helium isotopes [5, 6].

However, it is fair to say that the question of in-medium modifications of the nucleon form factors is yet to be resolved satisfactorily, particularly in view of the results of more recent proton knock-out experiments with and without polarized electrons. For example, $(e, e'p)$ experiments [7, 8, 9] yielded separated longitudinal (L) and transverse (T) cross sections, the ratio of which differed from that expected for a free proton, even after corrections for the effect of final state interactions (FSI) were incorporated in the analysis of the data. This, in turn, led to speculations about a possible medium-modified ratio of

the proton charge and magnetic form factors, G_{ep}/G_{mp} .

Recently, polarization transfer experiments of the type $(\vec{e}, e'\vec{p})$ [10, 11] have measured ratios of asymmetries P'_x/P'_z which, in plane-wave-impulse-approximation (PWIA), are proportional to G_{ep}/G_{mp} . After correcting the data for FSI effects — for caveats concerning meson exchange currents (MEC), see below — ratios smaller than those for free protons have been inferred. In particular, this reduction is found to be of the order of $\sim 20\%$ in $(G_{ep}/G_{mp})^2$ for the experiments using ${}^4\text{He}$ as a target.

Recent calculations [12, 13] based on quark-models of the nucleon have also addressed this topic of an in-medium modification. Substantial changes of 10–20%, often increasing with momentum transfer q , have been found for G_{ep} and G_{mp} . Depending on the specific model, the changes can be of opposite sign for the charge and magnetic form factors. Large changes were also suggested by earlier studies [14, 15].

The various observables that, in PWIA, are related to the bound nucleon form factors are influenced by several hard-to-calculate corrections, mainly from FSI and MEC. The CSR is, in this respect, still the cleanest observable, as some corrections are absent and others accessible to a quantitatively reliable evaluation. In this letter we therefore concentrate on the CSR of the nucleus ${}^4\text{He}$ which, due to its high central density, has quite a large average density. As already mentioned, for this target the $(e, e'p)$ and $(\vec{e}, e'\vec{p})$ experiments have found large deviations from the free G_{ep}/G_{mp} [7]–[11].

Separation of the world data. The longitudinal and transverse response functions $R_L(q, \omega)$ and $R_T(q, \omega)$ have been extracted from the available world data on inclusive quasi-elastic scattering on ${}^4\text{He}$. Extensive sets of data by Zghiche *et al.* [16] and Reden *et al.* [17], which typically cover the region of large angles, are complemented with the forward angle data by Rock *et al.* [18], Day *et al.* [19], Sealock *et al.* [20], and Meziani *et al.* [21]. The combined world data have been used for an L/T-separation in the three-momentum transfer range $q=300\div 1050$ MeV/c. Details of the L/T-separation for low q have been given previously [22].

The following expression, valid in the plane-wave-Born-approximation (which is accurate enough for $Z=2$), is used for the L/T-separation:

$$\Sigma(q, \omega, \epsilon) = \frac{d^2\sigma}{d\Omega d\omega} \cdot \frac{1}{\sigma_{Mott}} \cdot \epsilon \cdot \left(\frac{q}{Q}\right)^4 = \epsilon \cdot R_L(q, \omega) + \frac{1}{2} \left(\frac{q}{Q}\right)^2 \cdot R_T(q, \omega),$$

where the longitudinal virtual photon polarization ϵ is defined as

$$\epsilon = \left(1 + \frac{2q^2}{Q^2} \tan^2 \frac{\vartheta}{2}\right)^{-1}$$

and varies from 0 to 1 as the electron scattering angle ϑ is varied from 180 to 0 degrees. Here, $d^2\sigma/d\Omega d\omega$ is the experimental cross section, ω is the energy transfer of the virtual photon, Q is the 4-momentum transfer, and σ_{Mott} is the Mott cross section. The equations show that measurements of the cross section at fixed ω and q but different ϵ allow for a

separation of the two response functions $R_L(q, \omega)$ and $R_T(q, \omega)$. The combined world data cover almost the full ϵ -range, with typical values ranging from 0.05 to 0.95; this allows for a more precise determination of $R_L(q, \omega)$ and $R_T(q, \omega)$ than when using single-experiment data, which typically only cover a region of 0.5 in ϵ . The various data sets agree quite nicely; the overall χ^2 for the L/T-fits amounts to 1.04 per degree of freedom when including the quoted systematic uncertainties.

The following expression is used to determine the contribution to the Coulomb sum for $\omega < \omega_{max}$:

$$S_L^{\omega_{max}}(q) = \frac{1}{Z} \int_{\omega_{th}}^{\omega_{max}} d\omega \frac{R_L(q, \omega)}{\tilde{G}_e^2(Q^2)},$$

where $\tilde{G}_e^2(Q^2)$ denotes the combination of form factors given by

$$\tilde{G}_e^2(Q^2) = [G_{ep}^2(Q^2) + (A - Z) G_{en}^2(Q^2)/Z] / (1 + Q^2/(4m^2)),$$

m being the nucleon mass. This factor accounts for the finite size of the nucleon as well as for relativistic effects, see discussion below. Thus, after factoring out $\tilde{G}_e^2(Q^2)$, the longitudinal response $R_L(q, \omega)$ is integrated from ω_{th} to ω_{max} , the maximal energy loss covered by the data. For $G_{ep}(Q^2)$ and $G_{en}(Q^2)$ the parameterizations of Mergell *et al.* [23] and Galster *et al.* [24] have been used, respectively.

For the determination of $S_L^{\omega_{max}}$ in the present work particular care was devoted to the choice of the ω -range. At large ω the coverage of the data is typically less complete; this leads to an increase of the errors in $R_L(q, \omega)$ with a corresponding increase of the relative error of the CSR. The choice of q and the ω -range was performed such as to minimize the error in the integral.

The statistical errors in $S_L^{\omega_{max}}$ follow from the statistical errors of the cross section data, while the systematic errors are determined by changing each data set by the reported systematic error, repeating the analysis and adding the resulting changes in quadrature. At $q=1050$ MeV/c the addition of the data by Day *et al.* [19] and Sealock *et al.* [20] improved the accuracy by more than a factor of two relative to that obtained in [21]. For both of these data sets spectra have been measured with typical q -values around 1050 MeV/c, thus minimizing the interpolation in the L/T-separation.

Correction for finite ω_{max} . The Coulomb sum S_L^∞ is defined as an integral to $\omega = \infty$. This limit cannot be reached experimentally by (e, e') scattering ($\omega < q$), furthermore the uncertainty of $R_L(q, \omega)$ at large ω becomes large. The strength beyond ω_{max} thus has to be calculated and added.

For ${}^4\text{He}$ Morita and Suzuki [25] have calculated the spectral function $S(k, E)$, using the ATMS approach and the Reid soft core v_8 nucleon-nucleon interaction. This spectral function contains components of large momentum k and large removal energy E that lead to strength beyond ω_{max} . We have used this spectral function to calculate, in PWIA,

the inclusive response, and the fraction of strength outside the region the experimental $R_L(q, \omega)$ was integrated over.

In order to estimate the uncertainty introduced by this correction, we have repeated the calculation using a much more approximate $S(k, E)$, obtained in LDA from the correlated spectral functions of nuclear matter at different densities [26] and the single-particle momentum distribution known from $(e, e'p)$. The difference in the integral amounts to a maximum of 3%, which we include in the final systematic error.

In an alternative estimate of the strength at $\omega > \omega_{max}$, we have used the procedure employed in Ref. [6]. It consists in parameterizing the longitudinal response for $\omega > \omega_{max}$ as

$$R_L(q, \omega > \omega_{max}) = R_L^{exp}(q, \omega_{max}) \left(\frac{\omega_{max}}{\omega} \right)^{\alpha(q)},$$

and in determining the (q -dependent) constant $\alpha(q)$ by requiring that the energy-weighted sum rule $W_L(q)$, defined as

$$W_L(q) = \frac{1}{Z} \int_{\omega_{th}}^{\infty} d\omega \omega \frac{R_L(q, \omega)}{\tilde{G}_e^2(Q^2)},$$

reproduces the theoretical value, which can be accurately computed, as discussed below. The form of $R_L(\omega > \omega_{max})$ has been suggested by a study of the high ω -tail of the deuteron longitudinal response, which can be reliably calculated [27].

We note that the tail-corrections are quite important, reaching 24% to 28% at the highest q , depending upon the estimate of the tail correction. These contributions need to be calculated with approaches that provide a realistic treatment of the correlations that move the strength to large ω .

Calculation. The Coulomb and energy-weighted sum rules can be accurately calculated, since they can be expressed, respectively, as

$$S_L(q) = [\langle 0 | \rho^\dagger(\vec{q}) \rho(\vec{q}) | 0 \rangle - | \langle 0 | \rho(\vec{q}) | 0 \rangle |^2] / [Z \tilde{G}_e^2(\vec{Q}^2)]$$

and

$$W_L(q) = \left[\frac{1}{2} \langle 0 | [\rho^\dagger(\vec{q}), [H, \rho(\vec{q})]] | 0 \rangle - \omega_{el} | \langle 0 | \rho(\vec{q}) | 0 \rangle |^2 \right] / [Z \tilde{G}_e^2(\vec{Q}^2)],$$

namely as expectation values on the ground state of appropriate combinations of the nuclear charge operator $\rho(\vec{q})$ and Hamiltonian H . These expectation values are easily computed with Monte Carlo techniques [5]. In the present work, the Hamiltonian consists of the Argonne v_{18} [28] two-nucleon and Urbana-IX [29] three-nucleon interactions. The corresponding ${}^4\text{He}$ ground state wave function is that obtained by the Pisa group [30, 31] with the correlated-hyperspherical-harmonics method, and is therefore quite accurate. The charge operator is that reviewed, most recently, in Ref. [22]. It contains one- and

two-body terms. The one-body term retains the Darwin-Foldy and spin-orbit relativistic corrections, as given in Eqs. (18) and (19) of Ref. [22], while the two-body term includes, in addition to the leading pion-exchange charge operator, also the operators arising from vector-meson (ρ and ω) exchanges.

A few comments are now in order. Firstly, the normalization factor $\tilde{G}_e(\tilde{Q}^2)$ included in the expression for S_L is such that in the limit in which two-body contributions to the charge operator are neglected, one finds

$$S_L(q; 1\text{-body}) \rightarrow 1 \quad q \rightarrow \infty ,$$

ignoring a tiny correction proportional to $\langle T \rangle / (12m^3)$ originating from the spin-orbit term ($\langle T \rangle$ is the average kinetic energy per nucleon, amounting to ~ 25 MeV in ${}^4\text{He}$). Note that the four-momentum transfer occurring in G_e as well as in the charge operator $\rho(\vec{q})$, via its implicit dependence on the nucleon form factors, is evaluated at the top of the quasi-elastic peak, hence the notation \tilde{Q}^2 above.

Secondly, it should be pointed out that the normalization $\tilde{G}_e(\tilde{Q}^2)$ used here differs from that of earlier studies such as, for example, Ref. [6], in which it was taken to be simply $G_{ep}(\tilde{Q}^2)$. The inclusion of $\tilde{G}_e(\tilde{Q}^2)$ is more natural, since it removes trivial corrections from the Darwin-Foldy term and the (small) neutron contribution in the one-body limit calculation of the sum rules.

Finally, we note that the dominant contribution to the energy-weighted sum rule is that associated with the kinetic energy operator. In the one-body limit for $\rho(\vec{q})$, its contribution to $W_L(q)$ is simply given by $q^2/(2m)$, again ignoring spin-orbit corrections. However, when fitting the experimental $W_L(q)$ to the calculated values as described above, we have taken the kinetic-energy contribution to be $(q^2 + m^2)^{1/2} - m$ rather than $q^2/(2m)$, given the high momentum transfers considered in this work. Additional relativistic contributions arising from the frame- (total pair momentum) dependence of the interaction have been ignored, they are expected to be small compared to the difference above.

In Fig. 1 we display the effect of two-body charge operators on the CSR by plotting the quantity

$$\Delta(q) = \left[\langle 0 | \rho^\dagger(\vec{q}) \rho(\vec{q}) | 0 \rangle - \langle 0 | \rho^{(1)\dagger}(\vec{q}) \rho^{(1)}(\vec{q}) | 0 \rangle \right] / [Z \tilde{G}_e^2(\tilde{Q}^2)] ,$$

as function of the three-momentum transfer. This quantity contains interference terms of the type $\rho^{(1)\dagger}(\vec{q}) \rho^{(2)}(\vec{q}) + \text{h.c.}$ as well as terms involving only the two-body charge operators $\rho^{(2)\dagger}(\vec{q}) \rho^{(2)}(\vec{q})$. The interference term produces a negative contribution to $\Delta(q)$ in the momentum transfer range $q < 550$ MeV/c. For larger values of q , the term $\rho^{(2)\dagger} \rho^{(2)}$ dominates, its contribution to $\Delta(q)$ amounts to ~ 0.2 at $q = 1$ GeV/c. This term is responsible for the enhancement of the CSR at large q . The increase with q is easily understood by considering the form of the pion-exchange charge operator, the dominant two-body operator. It is given by

$$\rho_\pi^{(2)}(\vec{q}) = \frac{3i}{2m} \sum_{i \neq j} e^{i\vec{q} \cdot \vec{r}_i} I_\pi(r_{ij}) \left[F_{1S}(\tilde{Q}^2) \vec{\tau}_i \cdot \vec{\tau}_j + F_{1V}(\tilde{Q}^2) \tau_{j,z} \right] \vec{\sigma}_i \cdot \vec{q} \vec{\sigma}_j \cdot \hat{r}_{ij} ,$$

where F_{1S} and F_{1V} are the isoscalar and isovector Dirac form factors of the nucleon (evaluated at \tilde{Q}^2), and $I_\pi(r)$ is the pion-range function defined in Ref. [6]. Then, in the high q limit, one finds

$$\langle 0 | \rho_\pi^{(2)\dagger}(\vec{q}) \rho_\pi^{(2)}(\vec{q}) | 0 \rangle = \left(\frac{3}{2m} \right)^2 q^2 \langle 0 | \sum_{i \neq j} I_\pi^2(r_{ij}) [F_{1S}(\tilde{Q}^2) \vec{\tau}_i \cdot \vec{\tau}_j + F_{1V}(\tilde{Q}^2) \tau_{j,z}]^2 | 0 \rangle .$$

The resulting q -dependence of $\Delta(q)$ and the CSR arises from the interplay among the q -dependence of F_{1S}^2 and F_{1V}^2 , that implicit in the combination \tilde{G}_e^2 , and the factor q^2 in the equation above.

It is interesting to note that, had we chosen to define $S_L(q)$ by dividing out G_{ep}^2 rather than \tilde{G}_e^2 , as was done in earlier studies [5, 6], then the enhancement of $S_L(q)$ due to the two-body charge operators would have been (accidentally) canceled by the reduction arising from the Darwin-Foldy term in the one-body charge operator. Indeed, ignoring the small neutron contribution in \tilde{G}_e^2 , one obtains $S_L^{\text{old}}(q) \simeq S_L(q)/[1 + \tilde{Q}^2/(4m^2)]$; for example, at $q = 1 \text{ GeV}/c$ $S_L^{\text{old}}(q) \simeq 1$.

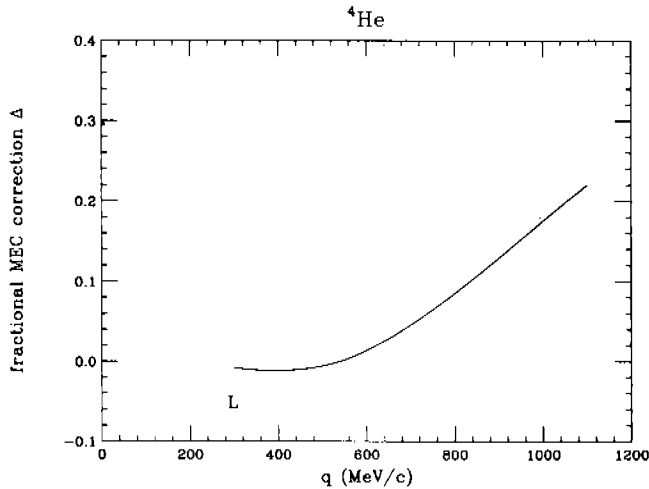


Figure 1: Contribution of MEC to the Coulomb sum.

Results. In Fig. 2 we show the experimental and theoretical results for the CSR. Both data and calculation increasingly differ from one towards low momentum transfer due to correlation effects [32]. Theory and experiment agree within the error bars.

Before addressing the question of the bound-nucleon form factor, we point out some important features of our results:

- Contrary to the usual approximation made when discussing the Coulomb sum — the neglect of two-body charge operators (MEC) — we find that the contribution of MEC to the integral is non-negligible, particularly beyond 600 MeV/c. In particular, at large

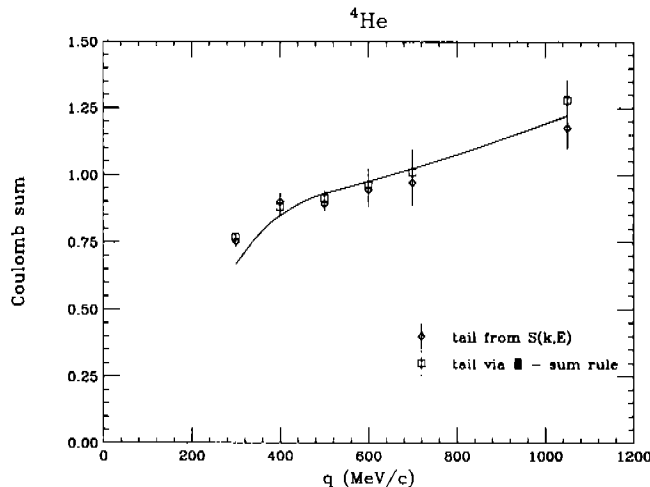


Figure 2: Experimental Coulomb sum compared to theory. The error bars include both the statistical and systematic uncertainties, the calculation includes the contribution of two-body charge operators.

q MEC lead to a significant increase. One should note that the MEC operators are the same ones that are crucial for reproducing the observed charge form factors of ${}^3\text{He}$ and ${}^4\text{He}$, for a review see Ref. [33].

- In the calculation of the MEC contribution to $R_L(q, \omega)$ and $R_T(q, \omega)$ it is very important to treat the $n - p$ short range and tensor correlations in both the initial and the final state wave functions [22]. When omitting these correlations — as done in most previous calculations of MEC contributions — we find, as these calculations did, a much smaller MEC effect (see also [34, 35]).
- For the transverse response we find large enhancements due to MEC, 65÷30% for $q=300\div 1050\text{MeV}/c$, falling only very slowly with increasing q [22].

As evidenced by Fig. 2, data and calculation are in agreement when using the *free nucleon* charge form factor — no in-medium reduction of G_{ep} is needed to reproduce the measured CSR. We should note that the total integrated strength $S_T(q)$ in the *transverse* response $R_T(q, \omega)$ is increased by two-body current contributions, indeed to a much more significant extent than for $S_L(q)$. The ratio $S_L(q)/S_T(q)$ is less than one, see Fig. 8 of Ref. [22]. One should note that the (e, e') cross section considered here to $\sim 95\%$ [69%] for $R_L(q, \omega)[R_T(q, \omega)]$ is due to the $(e, e'p)$ process that led to the G_{ep}^2/G_{mp}^2 ratios that differ from the free ones. This would suggest that the $\simeq 20\%$ reduction in the ratio G_{ep}^2/G_{mp}^2 compared to the value for a free proton, which was observed in $(e, e'p)$ and $(\bar{e}, e'\bar{p})$ experiments [7]–[11], may reflect inadequacies in the treatment of MEC in the analysis of those experiments.

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References

- [1] R. Altemus *et al.* *Phys. Rev. Lett.*, 44:965, 1980.
- [2] Z. Meziani *et al.* *Phys. Rev. Lett.*, 52:2130, 1984.
- [3] J. Jourdan. *Nucl. Phys. A*, 604:117, 1996.
- [4] I. Sick. *Phys. Lett.*, 157B:13, 1985.
- [5] R. Schiavilla *et al.* *Phys. Rev. C*, 40:2294, 1989.
- [6] R. Schiavilla *et al.* *Phys. Rev. Lett.*, 70:3856, 1993.
- [7] G. van der Steenhoven *et al.* *Phys. Rev. Lett.*, 57:182, 1986.
- [8] G. van der Steenhoven *et al.* *Phys. Rev. Lett.*, 58:1727, 1987.
- [9] D. Reffay Pikeroen *et al.* *Phys. Rev. Lett.*, 60:776, 1988.
- [10] S. Dieterich *et al.* *Phys. Lett. B*, 500:47, 2001.
- [11] S. Strauch. *World Scientific*, in print, 2002.
- [12] D.H. Lu *et al.* *Phys. Rev. C*, 60:068201, 1999.
- [13] M.R. Frank *et al.* *Phys. Rev. C*, 54:920, 1996.
- [14] J. V. Noble. *Phys. Rev. Lett.*, 46:412, 1981.
- [15] L.S. Celenza *et al.* *Phys. Rev. Lett.*, 53:891, 1984.
- [16] A. Zghiche *et al.* *Nucl. Phys. A*, 572:513, 1994.
- [17] K.F. vanReden *et al.* *Phys. Rev. C*, 41:1084, 1990.
- [18] S. Rock *et al.* *Phys. Rev. C*, 26:1592, 1982.
- [19] D. Day *et al.* *Phys. Rev. C*, 48:1849, 1993.
- [20] R.M.Sealock *et al.* *Phys. Rev. Lett.*, 62:1350, 1989.
- [21] Z.-E. Meziani *et al.* *Phys. Rev. Lett.*, 69:41, 1992.
- [22] J. Carlson *et al.* *Phys. Rev. C*, 65:024002-1, 2002.
- [23] P. Mergell *et al.* *Nucl. Phys. A*, 596:367, 1996.
- [24] S. Galster *et al.* *Nucl. Phys. B*, 32:221, 1971.
- [25] H. Morita and T. Suzuki. *Prog.Theo.Phys.*, 86:671, 1991, and priv. com.
- [26] I. Sick *et al.* *Phys. Lett. B*, 323:267, 1994.

- [27] J. Carlson and R. Schiavilla. *Phys. Rev. Lett.*, 68:3682, 1992.
- [28] R.B. Wiringa *et al.* *Phys. Rev. C*, 51:38, 1995.
- [29] B.S. Pudliner *et al.* *Phys. Rev. Lett.*, 74:4396, 1995.
- [30] M. Viviani *et al.* *Few-Body Syst.*, 18:25, 1995.
- [31] L.E. Marcucci *et al.* *Phys. Rev. C*, 63:015801, 2000.
- [32] R. Schiavilla *et al.* *Nucl. Phys. A*, 473:290, 1987.
- [33] J. Carlson and R. Schiavilla. *Rev. Mod. Phys.*, 70:743, 1998.
- [34] W. Leidemann and G. Orlandini. *Nucl. Phys. A*, 506:447, 1990.
- [35] A. Fabrocini. *Phys. Rev. C*, 55:338, 1997.