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Variational Calculation of Three-Nucleon Electroweak Capture Reactions

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Abstract. Recent advances in the study of the $p - d$ radiative and $\mu - {}^3$He weak capture processes are here presented and discussed.

1 Introduction

A number of electromagnetic (EM) and weak transitions in light nuclei have interesting astrophysical implications as well as important implications for an understanding of nuclear structure and dynamics. The theoretical description of these processes requires the knowledge of the initial and final nuclear states and the use of a realistic model for the EM and weak current operators. In particular, the trinucleon systems provide a unique "laboratory" due to the capability, achieved in the last few years, of obtaining very accurate bound and continuum nuclear wave functions [1, 2, 3]. Therefore, the study of EM and weak transitions in the three nucleon system does not suffer of uncertainties related to the computation of the wave functions and it is a direct test of the nuclear Hamiltonian H, from which the nuclear wave functions are obtained, and of the model used to describe the nuclear currents. Since the nuclear EM current is related to H through current conservation, it is clear that the two topics are inter-related. Furthermore, it is interesting to understand whether relativistic corrections as well as Δ-isobar and additional sub-nucleonic degrees of freedom play a role in these processes.

The model for the nuclear EM and weak current considered here has been recently reviewed in Refs. [4, 5, 6] and it includes one– and two–body operators. It has been tested in numerous few–nucleon processes and it is thought to be rather realistic. The one-body operators are obtained directly from the non–relativistic limit of the covariant single–nucleon vector and axial currents. In the study of the muon capture, the contribution coming from the induced pseudo–scalar term of the nucleon axial current has to be included (it gives a negligible contribution to β–decay processes). However, the experimental value of the corresponding form factor $G_{PS}(q_\sigma^2)$ is rather uncertain. Assuming pion-pole dominance, the partially conserved axial current (PCAC) hypothesis, and the Goldberger-Treiman relation, G_{PS} is predicted to be [7, 8, 9]

$$G_{PS}^{PCAC}(q_\sigma^2) = -\frac{2\, m_\mu\, m_N}{m_\pi^2 + q_\sigma^2}\, G_A(q_\sigma^2) \ , \tag{1}$$

where q_σ is the four-momentum transferred to the nuclear system, m_N, m_μ and m_π indicates the nucleon, muon and pion mass, respectively, and G_A is the axial form factor. In our calculation, we have assumed

$$G_{PS}(q_\sigma^2) = R_{PS}\, G_{PS}^{PCAC}(q_\sigma^2) \ , \tag{2}$$

where R_{PS} is a parameter which can be varied to study the sensitivity of our results to this form factor and to investigate to which extent $G_{PS}^{\mathrm{expt}} = G_{PS}^{\mathrm{PCAC}}$. As a starting point, we have assumed $R_{PS} = 1$.

The two-body EM current is separated in two terms. There is a "model-independent" part which is constructed consistently with the nucleon–nucleon interaction, in order to satisfy the current conservation relation [10]. The second part includes "model-dependent" contributions which come from the $\pi\rho\gamma$ and $\pi\omega\gamma$ processes and the Δ degrees of freedom. The latter contribution is included in the current and in the nuclear medium in an approximate way, by following the procedure described in Ref. [11]. The two-body weak vector current is then obtained from the isovector part of the EM current, in accordance with the conserved-vector-current (CVC) hypothesis.

Two-body terms have been taken into account in both the axial charge and current operators. The two-body axial charge operator has been obtained consistently with the two-nucleon interaction model, following the methods of Ref. [12]. The two-body axial current operators are derived from a meson-exchange model, including π- and ρ-exchanges and the $\rho\pi$-transition processes, as well as Δ-isobar excitation [5, 13]. The latter process gives the dominant contribution, and its strength is adjusted to reproduce the Gamow-Teller matrix element in tritium β-decay [5, 6, 13]. It has been shown that this procedure allows to significantly reduce the model dependence of the weak axial current contributions [5, 13].

The ^{3}He, ^{3}H bound and the $p-d$ continuum wave functions have been calculated by expanding on a basis of pair-correlated hyperspherical harmonic (PHH) functions [14]. Such a technique has been shown to be rather accurate [1]. Note that with the PHH technique the inclusion of the Coulomb potential, clearly very important in the energy range considered here, does not present any difficulty.

We discuss in the following sections the $p-d$ radiative capture above deuteron breakup threshold (DBT) and the muon capture on ^{3}He in the tritium channel.

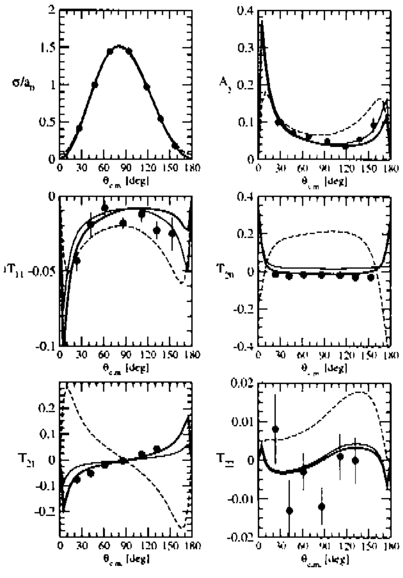

Figure 1. Differential cross section, proton vector analyzing power, and the four deuteron tensor analyzing powers for $p-d$ capture at $E_{\rm c.m.}$ = 3.33 MeV, obtained with the AV18/UIX Hamiltonian model and one-body only (dashed lines) or both one- and two-body currents (thin solid lines). The experimental data are from Ref. [18]. The results obtained in the LWA approximation for the spin-flip E_1 RME's are also shown (thick solid lines). In the first panel, $a_0 = \int d\Omega\, \sigma(\Omega)/(4\pi)$.

2 $p-d$ Radiative Capture

A recent extension of the PHH technique has allowed to compute $p-d$ scattering wave functions above the DBT [15] with comparable accuracy to the results below DBT. We can therefore compute $p-d$ capture observables at higher energies than previously published [16, 17]. We present here a preliminary study of the $p-d$ radiative capture at center-of-mass (c.m.) energy ($E_{\rm c.m.}$) of 3.33 MeV, for which high-quality experimental data, including differential cross sections, vector and tensor analyzing powers [18], exist. The theoretical calculation has been performed using the Argonne v_{18} (AV18) [19] two-nucleon and Urbana IX (UIX) [20] three-nucleon interactions. The results are shown in Fig. 1. By inspection of the figure, we note that the theoretical predictions for the differential cross section and the observables A_y and iT_{11} when one- and two-body contributions are included in the model for the current operator (thin solid line) are in good agreement with the experimental data. On the contrary, for the observables T_{20} and T_{21}, large discrepancies can be observed. A similar situation was found for the $E_{\rm c.m.}$ = 2 MeV observables [17]. The problem can be traced back to an overprediction of the spin-flip electric dipole E_1 reduced matrix elements (RME's). When the same RME's are computed in the long wavelength approximation (LWA) at leading order (thick solid lines in Fig. 1) also the observables T_{20} and T_{21} are rather well reproduced. Interestingly, an analysis of the next-to-leading order terms in the LWA performed in Ref. [17] has shown that they give a sizeable contribution to the spin-flip E_1 RME's. Therefore, the use of the leading order only for the calculation of this small spin-flip transition matrix elements should be considered inadequate. The origin of the discrepancies observed in the T_{20} and T_{21} observables is currently under investigation.

3 Muon Capture

The muon weak capture on ^{3}He can occur through three different hadronic channels [21], but we have considered only the reaction $\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$. When the triton polarization is not detected, the differential capture rate is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}(\cos\theta)} = \frac{1}{2}\Gamma_0 \left[1 + A_v P_v \cos\theta + A_t P_t \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) + A_\Delta P_\Delta\right] , \tag{3}$$

where Γ_0 is the total capture rate, A_v, A_t and A_Δ angular correlation parameters and θ is the angle between the muon polarization and the ^{3}H recoil direction. The coefficients P_v, P_t and P_Δ are linear combinations of the probabilities $P(f,\, f_z)$ of finding the $\mu^-\,{}^3$He system in the total-spin state $|f\, f_z\rangle$, and are defined in Refs. [22, 23].

The results for the total capture rate Γ_0 and angular correlation parameters A_v, A_t, and A_Δ are presented in Table 1. They are obtained using either the AV18/UIX interactions or the older Argonne v_{14} (AV14) two-nucleon interaction [25] in conjunction with the the Tucson-Melbourne (TM) three-nucleon interaction [26]. Note that both three-nucleon interactions have been adjusted to reproduce the triton binding energy.

Table 1. Total capture rate Γ_0 (sec^{-1}) and angular correlation parameters A_v, A_t, and A_Δ calculated using PHH wave functions corresponding to the AV18/UIX and AV14/TM Hamiltonian models are compared with the experimental results. The theoretical uncertainties, shown in parenthesis, reflect the uncertainty in the determination of the $N\Delta$ transition axial coupling constant. The experimental values of Γ_0 and A_v have been taken from Ref. [27] and [24], respectively. We assume $R_{PS} = 1$ in Eq. (2).

Observable	AV18/UIX	AV14/TM	Expt.
Γ_0	1484(8)	1486(8)	1496(4)
A_v	0.5350(14)	0.5336(14)	0.63(15)
A_t	-0.3650(9)	-0.3659(9)	
A_Δ	-0.1000(16)	-0.1005(17)	

The uncertainty (in parethesis) in the predicted values is due to the uncertainty in the determination of the $N\Delta$ transition coupling constant, as discussed in the Introduction. The latter reflects the experimental error in the Gamow-Teller matrix element of tritium β-decay.

Inspection of Table 1 shows that the theoretical determination of the total capture rate Γ_0 is within 1 % of the recent experimental result [27]. Furthermore, the model dependence in the calculated observables is very weak: the AV18/UIX and AV14/TM results differ by less than 0.5 %. The agreement between theory and experiment and the weak model dependence mentioned above reflect, to a large extent, the fact that both the AV18/UIX and AV14/TM Hamiltonian models reproduce: *i*) the experimental binding energies as well as the charge and magnetic radii [28] of the trinucleons; *ii*) the Gamow-Teller matrix element in tritium β-decay. The value for the angular correlation parameter A_v listed in Table 1 is also in reasonable agreement with the corresponding experimental result, which however has a rather large error [24].

An important motivation of the present work has been to test the sensitivity of the muon capture observables to the induced pseudo-scalar form factor G_{PS} and, eventually, infer its value from the Γ_0 measurement. By repeating the calculation using different values of the parameter R_{PS}, defined in Eq. (2), we have found that the angular correlation parameters, in particular A_t and A_Δ, are more sensitive to changes in R_{PS} than the total capture rate, as first pointed out in Ref. [22]. A precise measurement of these polarization observables could therefore be useful to ascertain the extent to which the induced pseudo-scalar form factors deviate from their PCAC values.

Finally, by enforcing perfect agreement between the experimental and theoretical values, taken with their uncertainties, for the total capture rate Γ_0, it is possible to obtain an estimate for the range of values allowed for R_{PS}, and we have found $R_{PS} = 0.94 \pm 0.06$. This 6 % uncertainty is smaller than that found in previous studies [23, 29, 30]. This substantial reduction in uncertainty can be traced back to the procedure used to constrain the (model-dependent) two-body axial currents discussed in the Introduction.

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