Magnetic polarizability of hadrons from lattice QCD * †

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We extract the magnetic polarizability from the quadratic response of a hadron's mass shift in progressively small static magnetic fields. The calculation is done on a $24 \times 12 \times 12 \times 24$ lattice at $a = 0.17$ fm with an improved gauge action and the clover quark action. The results are compared to those from experiments and models where available.

Polarizabilities are important fundamental properties of particles. They determine dynamical response of a bound system to external perturbations, and provide valuable insight into internal strong interaction structure. We discuss a lattice calculation for static magnetic polarizability β in the quenched approximation. This work is an extension of a lattice calculation for the electric polarizability [1]. For an updated calculation of the electric polarizability, see [2].

For small external magnetic fields, β can be defined via the energy (mass) shift of a particle in the fields

$$\Delta m = -\frac{1}{2}\beta \mathbf{B}^2 \tag{1}$$

We determine the magnetic polarizability by calculating the mass shift of hadrons

$$\Delta m = m(B) - m(0) \tag{2}$$

where $m(B)$ is the hadron mass in the presence of an magnetic field B. We fit the data with the polynomial

$$\Delta m(B) = c_1 B + \frac{1}{2} c_2 B^2 \tag{3}$$

We calculate mass shifts both in the field B and its inverse $-B$, then average them. The magnetic polarizabilities are the negative quadratic coefficients

$$\beta = -c_2 \tag{4}$$

The gauge action is the zero-loop, tadpole-improved Lüscher-Weiz action with a=0.17 fm (or 1/a=1159 MeV) set from the string tension. The tadpole factor is determined by the average plaquette as $u_0 = 0.877$. The clover quark action was used to put fermions on the lattice,

$$S_f = S_W - \frac{\kappa}{u_0^4} \sum_x \sum_{\mu<\nu} \left[\overline{\psi}(x)\ i\sigma_{\mu\nu} F_{\mu\nu} \psi(x) \right], \tag{5}$$

where S_W is the tadpole-improved Wilson fermion action.

In order to place an magnetic field on the lattice, we construct an analogy to the continuum case. There, the fermion action is modified by the minimal coupling prescription $\partial_\mu \rightarrow \partial_\mu + iqA_\mu$, where q is the charge of the fermion field and A_μ is the vector potential describing the external field. On the lattice, the prescription amounts to a phase factor e^{iaqA_μ} to the link variables [1]. Choosing $A_2 = Bx_1$ and $A_0 = A_1 = A_3 = 0$ a constant magnetic field $F_{12} = \partial_1 A_2 - \partial_2 A_1 = B_3 = B$ can be introduced in the z-direction. We

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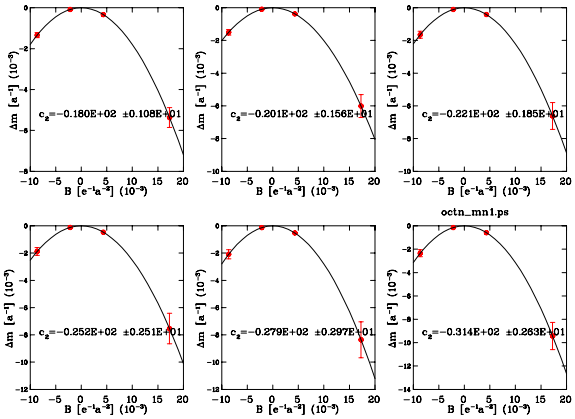

Figure 1. Mass shifts of the neutron as a function of the magnetic field B at the six quark masses.

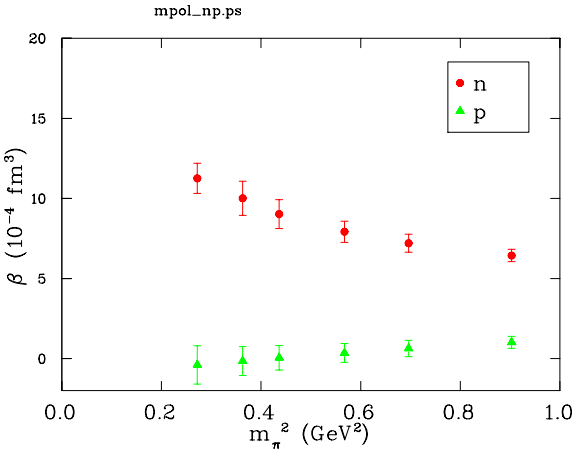

Figure 2. Magnetic polarizability of the neutron and proton as a function of m_π^2 in physical units.

introduce two dimensionless parameters: one is

$$\eta = qBa^2, \tag{6}$$

where $q = Qe$ with $Q = \pm 1/3, \pm 2/3$. The other is the integer lattice length,

$$\rho = x_1/a. \tag{7}$$

In terms of these two parameters the phase factor becomes:

$$e^{iaqA_2} = e^{i\eta\rho} \to (1 + i\eta\rho), \tag{8}$$

where we have linearized the phase factor to mimic the continuum prescription. Therefore, the field strength we use will be chosen to satisfy the linearization requirement. To summarize, our method to place the external magnetic field on the lattice is to multiply each gauge field link variable $U_\mu(x)$ in the z-direction with a x-dependent factor:

$$U_3(x) \to (1 + i\eta\rho)U_3(x) \tag{9}$$

This factor is applied only to the S_W part of the action, the clover term is untouched.

Periodic boundary conditions were imposed on the fermion fields in y and z directions. In the x and t directions, we have employed fixed (or Dirichlet) boundary conditions, i.e. the fermion fields are not allowed to propagate across the edges of x and t. To minimize the non-periodic boundary effects, we use a lattice size of $24 \times 12 \times 12 \times 24$ with the fermion source location $(x, y, z, t) = (12, 1, 1, 3)$. We analyzed 100 configurations.

Fermion propagators M^{-1} were constructed at six κ values, κ=0.1219, 0.1214, 0.1209, 0.1201, 0.1194, 0.1182 corresponding to m_π=450 to 820 MeV. The critical value $\kappa_c = 0.1232(1)$. We use six different values of the parameter $\eta = 0.0, +0.00036, -0.00072, +0.00144, -0.00288, +0.00576$. The η values in this sequence are related by a factor of -2. Thus we are able to study the response of a hadron composed of both up and down (strange) quarks to four different nonzero magnetic fields.

Typical mass shifts are displayed in Fig. 1 in the case of neutron. There is good parabolic behavior going through the origin, an indication that contamination from the linear term has been effectively eliminated by averaging results from B and $-B$. The factor

$$e^2a^3 = 0.358 \times 10^{-43}\text{cm}^3 \tag{10}$$

was used to convert the lattice numbers into physical polarizability in units of 10^{-4}fm^3. The curves in Fig. 2 and Fig. 3 represent our preliminary results for neutron, proton, π^0, and $\text{K}^\pm$. They are smooth functions of the quark mass. We did not attempt an extrapolation in the quark mass.

In Table 1, we show some experimental and

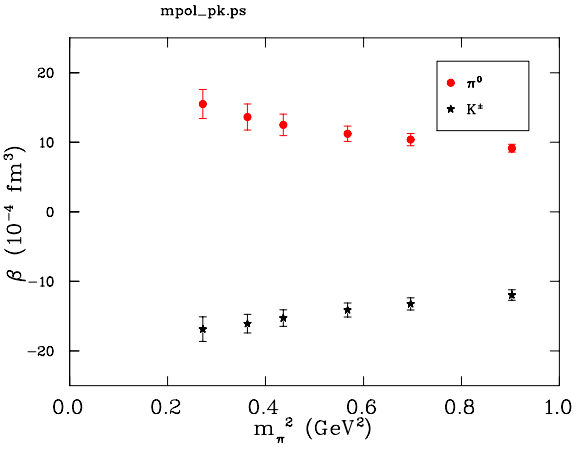

Figure 3. Magnetic polarizability for pion and kaon.

theoretical values for the magnetic polarizabilities β of neutron, proton, pion and kaon. The β for neutron is not well-determined. A recent experimental value [3] is 2.7 with a big error bar, while an earlier experimental data analysis [7] has it at 11. The range from models and experiments is from 1.22 to 14. Our lattice simulation result is at the higher end of this range. For the proton, the range is 1.6 to 4.4, while our lattice result lies in the range. For π^0, there is scant data. The number from Chiral Perturbation Theory calculation to $\mathcal{O}(p^6)$ is 0.5 to 1.7. Our result is quite high, about 20. For K^+, there is one theoretical value -6.2 from the NJL model. Our result is around -20.

In summary, we have computed the magnetic polarizability of hadrons for the first time on the lattice using the external field method. In additon to the particles described here, we have computed β for other mesons, the other members of the octet baryon, and the entire decuplet baryon family. We plan to present the final results elsewhere. Also under way is the extraction of magnetic moments from the same data set.

Table 1
A compilation of magnetic polarizabilitiy for selected hadrons from experiments or models. The numbers are in units of 10^{-4} fm^3.

Ref.	Particle	β	Approach
[3]	n	$2.7 \mp 1.8^{+0.6}_{-1.1} \mp 1.1$	Theo.
[4]	n	$6.7^{+1.3}_{-0.7}$	Theo.
[5]	n	7.8	Theo.
[6]	n	10.3 ∓ 2.0	Exp.
[7]	n	11 ± 3	Theo.
[8]	p	$3.3 \mp 2.2 \mp 1.3$	Exp.
[9]	p	$2.1 \mp 0.8 \mp 0.5$	Exp.
[5]	p	3.5	Theo.
[10]	p	$4.4 \pm 0.4 \pm 1.1$	Exp.
[11]	p	$3.58^{+1.19}_{-1.25}{}^{+1.03}_{-1.07}$	Exp.
[12]	π^0	0.50	Theo.
[12]	π^0	1.50 ± 0.20	Theo.
[13]	K^+	-6.2 (-2.9)	Theo.

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