

Explicit Quark-Hadron Duality in 1+1 Dimensions*†

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ABSTRACT

Explicit quark-hadron duality in the limit of heavy quark mass is studied using the 't Hooft model, where both partonic and hadronic amplitudes may be computed exactly. Results for weak decays of heavy mesons are presented for both standard spectator decays, where the duality limit is convincingly approached, and annihilation decays of the valence quark-antiquark pair, where the approach to asymptotic duality is much less precise.

1. Introduction

One of the central issues in the study of confinement is quark-hadron duality, the manner in which partial widths for exclusive hadronic processes sum to the corresponding inclusive total at the partonic level. In this talk I present a brief summary of recent advances^{1,2} in the study of this issue, obtained in collaboration with Benjamín Grinstein.

It is widely believed that the weak decay of a meson (“ \bar{B} ”) containing a very heavy quark (“ b ”) should be well approximated by the partonic decay of a free b . From the physical point of view, the heavy b quark is indifferent to the dynamics of the light spectator antiquark, and this reasoning is built into nonrelativistic quark models, as well as heavy quark effective theory and the “practical version”³ of the operator product expansion (OPE), which expands in powers of $1/m_b$. Exactly how this “duality limit” is achieved, or how closely it is satisfied, are open questions.

2. Method of Calculation

We address these issues in terms of a soluble toy theory chosen to resemble real QCD as closely as possible. The calculations are performed in the context of the 't Hooft model,⁴ which is the soluble theory of QCD in one space and one time dimension with an infinite number N_c of color charges. Here “soluble” means that each hadronic Green function (for example, meson wavefunctions and transition amplitudes) may be calculated exactly (albeit numerically) in terms of quark masses. In particular, the n th

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meson eigenfunction ϕ_n of mass eigenvalue μ_n for quarks of masses M and m is found to satisfy

$$\mu_n^2 \phi_n^{M\bar{m}}(x) = \left(\frac{M_R^2}{x} + \frac{m_R^2}{1-x} \right) \phi_n^{M\bar{m}}(x) - \int_0^1 dy \phi_n^{M\bar{m}}(y) \text{Pr} \frac{1}{(y-x)^2}, \quad (1)$$

where masses are written in units of the gauge coupling constant $g\sqrt{N_c/2\pi}$, since in 1+1 dimensions g has dimensions of mass, while in the large N_c limit⁵ it scales as $1/\sqrt{N_c}$. The renormalized quark mass is then $m_R^2 = m^2 - 1$, while x , the sole remaining kinematic invariant, represents the fraction of the meson momentum carried by the quark in light-cone coordinates.

Meson transition amplitudes, which are needed for decay widths, may be written in terms of integral double and triple overlaps of 't Hooft wavefunctions.⁶ The nonleptonic decays represented in these calculations are of the type $\bar{B} \rightarrow \pi_k \pi_m$, where, *e.g.*, π_k represents the k th radial excitation of the pion.

The partonic width, on the other hand, may be obtained through a straightforward calculation leading to a closed-form expression in terms of elliptic integrals with arguments that are functions of the quark masses. Results¹ of the hadronic and partonic calculations are compared in Fig. 1*a*.

Physics in 1+1 dimensions possesses a number of peculiar features, most notably the lack of spin (since spatial rotations are absent), as well as singularities in two-body phase space at threshold. These singularities appear as spikes in Fig. 1*a*, but remarkably do not affect the speed at which approximate duality is achieved.

That the exceptional agreement between hadronic and partonic calculations is not accidental may be verified by studying the partial width arising from each exclusive channel. In particular, in Fig. 1*b* we exhibit the partial width for the decay of \bar{B} into two ground-state π channels, which clearly does not saturate the result of Fig. 1*a*.

The small discrepancy between the two curves at large M has received attention in the literature. Numerically, it is well-fit by a relative correction of order $1/M$, which should not be present in expansions of the Ref.³ type. Recent work⁷ claims that such a correction is not present, partly on the basis of rigorous analytic studies of 't Hooft model solutions, but also partly on estimates of scaling behavior in hadronic threshold regions. The issue, for the moment, appears to be unresolved; however, even if Ref.⁷ proves to be absolutely correct, the presence of a numerical effect *simulating* a forbidden correction may provide valuable insight into processes in 3+1 dimensions that appear to violate duality at $O(1/M)$, such as the lifetime difference between Λ_b and \bar{B} .

3. Annihilation Decays

In the decays studied in the previous section, the light spectator quark of the \bar{B} is unaffected by the decay (except for binding effects). It is natural to consider also how well duality holds for decays in which it weakly annihilates with the b quark.²

In this case, one finds a new complication in that powers of $N_c \rightarrow \infty$ no longer appear homogeneously in the amplitude. The reason for this behavior is clear: There are special values of quark masses for which the \bar{B} can annihilate resonantly into a single, highly excited π meson. Since the presence of each additional meson in large N_c costs

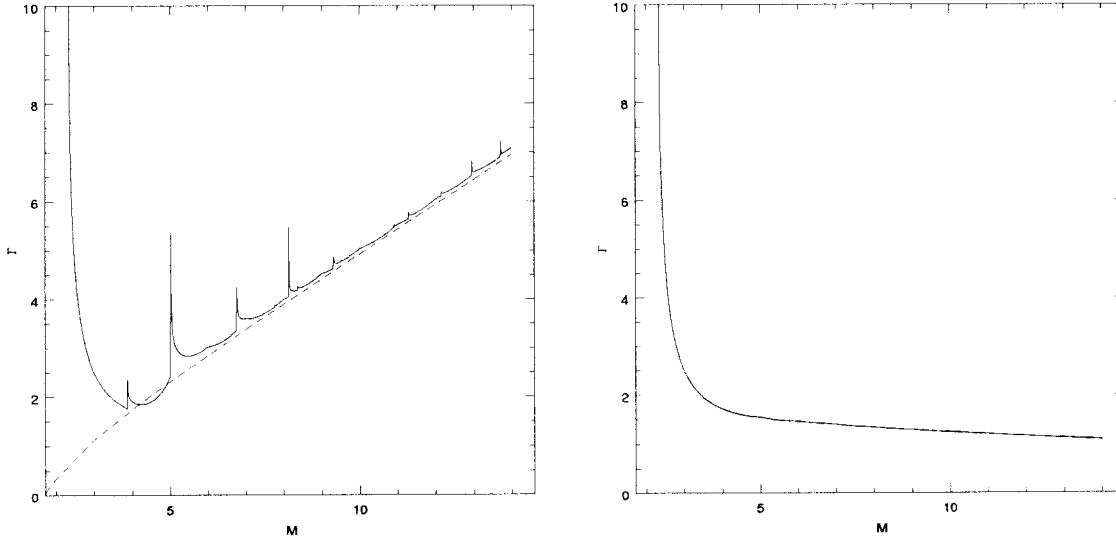


Fig. 1. (a) The full decay width for the sum of nonleptonic exclusive modes in the decay of \bar{B} as a function of heavy quark mass M , with light quark mass $m = 0.56$. The overall coefficient in the width is suppressed for convenience. The dashed line is the tree-level parton result. (b) The partial hadronic width from (a) in lowest exclusive channel.

an extra factor of $\sqrt{N_c}$ in the amplitude, one-meson decay widths should dominate the expected two-meson widths by a factor of N_c . However, this enhancement appears only very near the special values of b quark mass for which such resonances are allowed, since large N_c mesons have widths $\Gamma_{\text{res}} \propto 1/N_c$.

In a physically meaningful picture, these resonances must be “integrated out” into the two-meson channel, where they appear as intermediate mesons with Breit-Wigner widths in the form

$$\frac{i}{\mu_B^2 - \mu_{\text{res}}^2 + i\mu_{\text{res}}\Gamma_{\text{res}}} \quad (2)$$

The propagator, like meson masses, is $O(N_c^0)$ except in the immediate vicinity of $\mu_B = \mu_{\text{res}}$, where it is promoted to $O(N_c^1)$. It turns out that this is exactly the factor needed for large N_c counting consistency. On the other hand, one finds that different powers of N_c are necessarily mixed up in obtaining a physically reasonable picture of these decays, and a particular value of N_c must be chosen in order to exhibit numerical results. Since the 't Hooft model is exact for $N_c = \infty$, N_c is chosen as large as numerically practical.

The results for $N_c = 20$ are shown in Fig. 2, where the prominent cusps in Γ appear at values of M for which single-resonance decays can occur. In order to compare to the corresponding partonic rate, obviously some form of averaging, or “smearing,” process is required. This is achieved here by weighting the exact result by a series of Gaussian functions whose widths are chosen as small as possible, but that still produce a smooth result. Since larger N_c means sharper spikes, while the range in M over which to smear is finite, numerically allowed choices of N_c are bounded above. One then finds

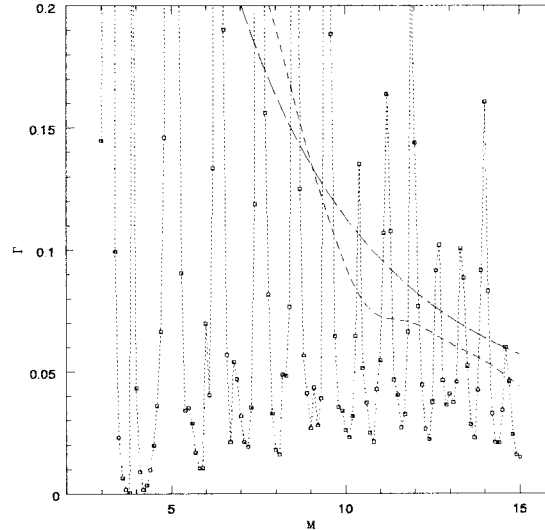


Fig. 2. Total width of \bar{B} meson through annihilation decays (dotted line) as a function of heavy quark mass M , with $N_c = 20$. The width smeared by Gaussian averaging is in short dashes, while the partonic result is in long dashes.

reasonable agreement between the two curves, to about 20%, but it is not nearly so startling as that presented in Fig. 1a. Also, the partonic and smeared hadronic curves do not yet even appear to have achieved the same asymptotic power law behavior.

4. Conclusions

The 't Hooft model provides a unique simplified environment in which to study many of the currently intractable issues of the strong interaction. Even the issue of quark-hadron duality raises a number of interesting subtleties, whose resolution may help in fathoming the complexities of the 3+1 dimensional problem.

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